

Study of Nuclear Structure of $^{24,26}\text{Na}$ isotopes by using USDB interaction

Ali .K. Hasan¹ and Azhar.N.Rahim²

Department of Physics, College of Education for Girls, University of Kufa, Iraq
alikh.alsinayyid@uokufa.edu.iq¹ and AzharNathem1992@gmail.com²

Abstract:

In this article, we assess the accuracy of theoretical shell model in calculating the excited states of Sodium isotopes $^{24,26}\text{Na}$ on the basis of recently reported experimental results. The assessments rely on the calculations of the energy levels, reduced electric quadrupole transition probabilities $B(E2)$ and reduced magnetic dipole transition probabilities $B(M1)$ are based on OXBASH shell model code by applying USDB interaction. Applying the program for above isotopes using the defined codes introduces several files which each file contains a set of data. Mean while the ground state of excitation energy evaluated by OXBASH code together with energy levels and reduced electric quadrupole transition probability $B(E2)$ and Magnetic dipole transition probability $B(M1)$ and also probable places for nucleons' placements in each energy level. A compilation of SD-shell energy levels calculated with the USD Hamiltonian and has been published around 1988. A comparison had been made between our results and the available experimental data to test theoretical shell model description of nuclear structure in Sodium isotopes. The calculated energy spectrum is in good agreement with the available experimental data.

1.Introduction:

Obtaining the nuclear structure and energy levels of nuclei is one of the criteria to improve investigations of nuclei properties. Nuclear models have the property to help us to better understanding of nuclear structure which contains main physical properties of nuclei, and shell-model is one of the most prominent and successful nuclear models. This model can be compared with the electron shell model for atoms. As atomic behavior and properties can be described with valance electrons which exist out of a closed shell, similarly, valance nucleons (protons or neutrons) in a nucleus which are placed out of close shells (with magic numbers 2,8,20,28,50,82 and 126) play important roles in determining nuclear properties. Nuclei with magic numbers are very stable and have completely different properties comparing with their neighbors[1].The nuclear shell model has been very successful in our understanding of nuclear structure: once a suitable effective interaction is found, the shell model can predict various observables accurately and systematically. For light nuclei, there are several "standard" effective interactions such as the Cohen- Kurath and the USD interactions for the p and SD shells, respectively. Analysis of neutron-rich SD nuclei has been of intense curiosity in recent years as they present new aspects of nuclear structure [2].Traditional shell-model studies have recently received a renewed interest through large scale shell-model computing in no-core calculations for light nuclei .Because of the quite importance of the $(0d_{5/2}, 1s_{1/2}, 0d_{3/2})$ space for variety of problems in nuclear structure, this space is a region where the shell model can play an indispensable role and is at the frontier of our computational abilities[3].The shell model calculations of the neutron-rich Sodium isotopes have been developed using the OXBASH code[4].

2.Theory:

The nuclear shell model, introduced almost 50 years ago by Mayer Haxel, Jensen, and Suess, has been very successful in describing the properties of nuclei with few valance nucleons[5]. These properties include the energy levels, magnetic and quadrupole moments, electromagnetic transition probabilities, beta decay, and cross section for various reactions. The basic assumption of the nuclear shell model is that, to a first approximation each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus. The complete Schrodinger equation for A nucleons reads as [6].

In the realistic shell model, we have to take into account (H) this part of the nuclear Hamiltonian that was omitted in the mean-field description. Nucleon configurations are mixed by this residual interaction. Interactions between nucleons make them jump from one orbital to another with conserve (T, J^π) , so that the wave function contains several configurations. So, we should solve the eigenvalues problem[7].

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle$$

Configuration mixing leads to the wave functions to consist of more than just one Slater determinant. So, we are looking for the wave function of the system in the form.

$$|\Psi_i\rangle = \sum_{k=1}^n a_{ki}|\Phi_k\rangle \quad , i = 1, 2, \dots, n$$

where g the number of pure configurations considered and it is related to the valance space used, and a_k is amplitude(weight) the wave function $|\Phi_k\rangle$. Usually, the valance space incorporates all possible configurations of valance protons and valance neutrons in the partially filled orbitals, while the rest is considered as an inert core (usually, we take a double magic numbers). So we treat only the valance nucleons. This theory efficient for few numbers of valance nucleons (smaller than five valance nucleons). It is clear that the valance space becomes quickly huge for numerical treatment as the number of valance nucleons increases [8].

There are many programs that implement the calculations of the shell model, which differ in their methods of calculation as well as the language in which they were built, and differ in the systems that work on them and their speed in the completion of calculations, including OXBASH. And to run the calculations of the shell model using OXBASH. It was necessary to know how to install the program and how to operate it to ensure the accuracy of calculations and results. In order to calculate the nuclear structure properties of both ground and excited states based on the nuclear shell model one needs to have wave functions of those states. These wave functions are obtained by using the shell-model code OXBASH [9]. OXBASH (Oxford- Buenos Aires Shell Model Code) is a powerful computer code to calculate the energy levels, reduced electric quadrupole transition probabilities $B(E2)$ and Magnetic quadrilateral transition probability $B(M1)$ of light and medium nuclei. By using it, we can measure the energy levels of the nucleus and compare it with experimental data as well [10].

3. Shell model calculations:

The calculations have been conducted using the code OXBASH for Windows. The code uses an m-scheme Slater determinant basis. Using a projection technique, wave functions with good angular momentum J and isospin T are constructed. The effective interactions of the USDB Hamiltonians with SD model space. The SD model spaces consists of $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ above the $Z = 8$ and $N = 8$ closed shells for protons and neutrons, respectively. single-particle energies (SPEs) for every Hamiltonian used in this work (MeV). Use in this work USDB interaction and single-particle energies (SPEs) are $\{0d_{5/2}=-3.926, 1s_{1/2}=-3.208$ and $0d_{3/2}=2.112\}$ respectively [11].

4. Results and Discussion:

Shell model calculations for low lying energy states of $^{24,26}\text{Na}$ isotopes have been performed for the space model ($0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$) with neutrons ($N=13$ and 15) above the ^{16}O close core for above isotopes. The calculations are based on the Universal (sd-shell) Hamiltonian (USDB). we have used the OXBASH code in both m-scheme and jj-coupling. The object of this present study is to calculate energy levels and reduced electric quadrupole transition probabilities $B(E2)$ and Magnetic quadrilateral transition probability $B(M1)$ by employing harmonic oscillator potential (HO, b), $b < 0$ all isotopes. The effects of core polarization have been taken into account in the calculations by effective charges of both protons and neutrons.

4.1 Energy Levels:

4.1.1. ^{24}Na nucleus:

According to the shell model, the ground state of ^{24}Na nucleus is a closed ^{16}O core with eight nucleons distributed as three protons and five neutron in sd space, which is similar for other Na isotopes ($13 \leq N \leq 11$) for the closed core and proton distribution. Excited states are formed by the configuration of these nucleons in the sd-shell model space. Table (1) show the comparison between theoretical and available experimental data of ^{24}Na nucleus [12] by using the USDB interaction.

From the down table both USDB Hamiltonians agree reasonable well with empirical data compared with the ^{24}Na energy levels with the experimental values. The ground state was confirmed 4_1^+ . The agreement is good for the states of with empirical values. $J^\pi = 1_1^+, 2_1^+, 2_2^+, 1_2^+, 5_1^+, 2_3^+, 3_2^+, 3_4^+, 4_2^+, 4_2^+, 1_3^+, 1_4^+, 3_5^+$ and 2_8^+ as compared with the experimental data, respectively. The $J^\pi = 1_6^+$,

$1_7^+, 1_8^+$ and 0_5^+ levels confirmed as positive parity. This study also confirmed the $J^\pi = (2_4^+), (4_3^+), (2_5^+)$ and 2_6^+ levels with the calculated energies (2.787, 3.001, 3.604 and 4.045) MeV, respectively. The levels of for which the angular momentum and parity are yet unknown $J^\pi = 3_3^+, 1_5^+, 5_4^+, 2_7^+, 2_9^+, 7_1^+, 6_2^+, 0_2^+, 4_8^+, 1_9^+, 5_7^+, 4_{10}^+, 6_6^+, 5_8^+, 7_2^+, 5_9^+, 5_{10}^+, 6_7^+, 7_3^+, 6_8^+, 7_4^+, 8_1^+, 0_6^+, 8_2^+, 7_6^+, 7_7^+, 8_3^+, 8_4^+, 9_1^+, 8_6^+, 9_2^+$ and 8_7^+ , respectively. New energy levels were predicted at the states ($J^\pi = 8_8^+, 9_4^+, 8_9^+, 10_2^+, 9_5^+, 8_{10}^+, 9_6^+, 9_7^+, 9_8^+, 10_3^+, 10_4^+, 11_1^+, 9_9^+, 9_{10}^+, 10_5^+, 10_6^+, 11_2^+, 10_7^+, 10_8^+, 10_9^+, 11_3^+, 10_{10}^+, 11_4^+, 11_5^+, 12_1^+, 11_6^+, 11_7^+, 11_8^+, 11_9^+, 12_2^+, 11_{10}^+$ and 12_3^+).

Table 1: Comparison of the experimental excitation energies[13] and excitation energies predictions for ^{24}Na nucleus by using USDB interactions

J^π (OXBASH)	Energy (OXBASH) USDA (MeV)	Energy (exp.) (MeV)	J^π (exp.)
4_1^+	0.000	0.000	4+
1_1^+	0.540	0.472	1+
2_1^+	0.629	0.563	2+
2_2^+	1.107	1.341	2+
3_1^+	1.338	1.344	(3)+
1_2^+	1.324	1.346	1+
5_1^+	1.546	1.512	5+
2_3^+	1.807	1.846	2+
3_2^+	1.803	1.885	3+
3_3^+	2.348	1.960	-----
3_4^+	2.627	2.513	3+
4_2^+	2.649	2.562	4+
2_4^+	2.787	2.977	(2+, 3+)
4_3^+	3.001	3.216	(4+, 2+)
1_3^+	3.346	3.413	1+
1_4^+	3.621	3.589	1+
3_5^+	3.459	3.628	3+
2_5^+	3.604	3.655	(2+, 1+)
0_1^+	3.527	-----	-----
4_4^+	3.707	-----	-----
5_2^+	3.860	-----	-----
6_1^+	4.003	-----	-----
3_6^+	3.844	3.936	-----
5_3^+	4.112	-----	-----
2_6^+	4.045	3.977	(1-, 2+)
1_5^+	4.366	4.220	-----
3_7^+	4.679	4.526	3-
4_6^+	4.721	4.690	-----
5_4^+	4.782	4.772	-----
5_5^+	4.897	-----	-----
3_8^+	4.781	-----	-----
2_7^+	4.908	4.891	-----
2_8^+	5.020	5.031	2+, 3, 4+
1_6^+	4.816	5.045	(1, 2, 3)-
2_9^+	5.323	5.180	-----
7_1^+	5.313	5.308	-----
1_7^+	5.184	5.397	(1, 3)-
6_2^+	5.399	5.408	-----
4_7^+	5.340	-----	-----
6_2^+	5.659	5.479	1-
0_2^+	5.567	5.571	-----
6_3^+	5.556	5.585	-----
0_3^+	5.820	5.720	-----
4_8^+	5.979	5.896	-----
3_{10}^+	5.904	5.918	1-, 3+
6_4^+	6.530	-----	-----
5_6^+	5.995	5.953	-----
4_9^+	6.234	-----	-----

1 ₉ ⁺	6.192	6.183	-----
5 ₇ ⁺	6.139	6.222	-----
1 ₁₀ ⁺	6.724	-----	-----
6 ₅ ⁺	6.794	-----	-----
4 ₁₀ ⁺	6.586	6.578	-----
6 ₆ ⁺	7.442	6.715	-----
5 ₈ ⁺	6.638	6.787	-----
0 ₄ ⁺	6.770	-----	-----
7 ₂ ⁺	6.695	6.846	-----
5 ₉ ⁺	7.236	7.090	-----
0 ₅ ⁺	7.103	7.085	0-
5 ₁₀ ⁺	7.483	7.324	-----
6 ₇ ⁺	7.442	7.433	-----
7 ₃ ⁺	7.497	7.511	-----
6 ₈ ⁺	7.826	7.708	-----
7 ₄ ⁺	7.843	7.832	-----
7 ₅ ⁺	8.565	-----	-----
8 ₁ ⁺	8.482	8.390	-----
0 ₆ ⁺	8.554	8.610	-----
6 ₉ ⁺	8.722	8.860	-----
8 ₂ ⁺	8.772	-----	-----
7 ₆ ⁺	9.008	-----	-----
7 ₇ ⁺	9.022	-----	-----
6 ₁₀ ⁺	9.065	-----	-----
0 ₇ ⁺	9.094	9.280	-----
8 ₃ ⁺	9.870	9.630	-----
7 ₈ ⁺	10.031	-----	-----
0 ₈ ⁺	10.360	-----	-----
7 ₉ ⁺	10.342	-----	-----
0 ₉ ⁺	10.519	-----	-----
8 ₄ ⁺	10.724	-----	-----
7 ₁₀ ⁺	10.583	-----	-----
8 ₅ ⁺	10.966	-----	-----
0 ₁₀ ⁺	10.845	-----	-----
8 ₆ ⁺	11.762	11.610	-----
9 ₁ ⁺	12.185	12.190	-----
9 ₂ ⁺	12.953	11.900	-----
9 ₃ ⁺	13.396	-----	-----
8 ₇ ⁺	12.592	12.540	-----
8 ₈ ⁺	12.611	-----	-----
9 ₄ ⁺	14.097	-----	-----
8 ₉ ⁺	12.997	-----	-----
10 ₂ ⁺	16.507	-----	-----
9 ₅ ⁺	14.606	-----	-----
8 ₁₀ ⁺	13.207	-----	-----
9 ₆ ⁺	14.097	-----	-----
9 ₇ ⁺	14.870	-----	-----
9 ₈ ⁺	15.161	-----	-----
10 ₃ ⁺	15.014	-----	-----
10 ₄ ⁺	16.092	-----	-----
11 ₁ ⁺	17.019	-----	-----
9 ₉ ⁺	15.161	-----	-----
9 ₁₀ ⁺	15.748	-----	-----
10 ₅ ⁺	17.375	-----	-----
10 ₆ ⁺	17.577	-----	-----
11 ₂ ⁺	18.461	-----	-----
10 ₇ ⁺	18.136	-----	-----
10 ₈ ⁺	18.485	-----	-----
10 ₉ ⁺	19.287	-----	-----
11 ₃ ⁺	19.372	-----	-----

10 ₁₀ ⁺	19.482	-----	-----
11 ₄ ⁺	20.540	-----	-----
11 ₅ ⁺	21.396	-----	-----
12 ₁ ⁺	22.031	-----	-----
11 ₆ ⁺	22.675	-----	-----
11 ₇ ⁺	23.884	-----	-----
11 ₈ ⁺	24.330	-----	-----
11 ₉ ⁺	24.577	-----	-----
12 ₂ ⁺	25.057	-----	-----
11 ₁₀ ⁺	25.211	-----	-----
12 ₃ ⁺	29.988	-----	-----

4.1.2. ²⁶Na nucleus:

The ground state of ²⁶Na nucleus is a close ¹⁶O core plus six nucleons distributed as three protons and three neutron in sd space at 0d_{5/2}, 1s_{1/2}, and 0d_{3/2} configurations. At table (2) show the comparison between theoretical and available experimental data of Na¹⁶ nucleus[12] by using the USDB interaction.

From the Down table both USDB Hamiltonians agree reasonable well with empirical data compared with the ²⁶Na energy levels with the experimental values. The ground state was confirmed 3₁⁺. The agreement is good for the states of with empirical values. J^π=1₁⁺, 2₁⁺ and 2₂⁺ as compared with the experimental data, respectively. The J^π = 3₉⁺ levels confirmed as positive parity. This study also confirmed the J^π=(1₂⁺), (3₂⁺), (4₁⁺), (5₁⁺), (2₄⁺), (2₅⁺), (4₇⁺) and 2₁₀⁺ levels with the calculated energies (1.281, 1.708, 1.628, 2.977, 2.236, 3.065, 4.683 and 5.303) MeV, respectively. When compared with the process values. The levels of for which the angular momentum and parity are yet unknown at the states J^π=0₁⁺, 4₂⁺, 1₃⁺, 1₄⁺, 3₃⁺, 0₂⁺, 4₄⁺, 3₄⁺, 3₅⁺, 1₆⁺, 4₅⁺, 2₈⁺, 1₇⁺, 1₉⁺, 6₅⁺ and 0₈⁺, respectively. New energy levels were predicted at the states J^π=7₄⁺, 7₅⁺, 0₉⁺, 0₁₀⁺, 7₆⁺, 7₇⁺, 7₈⁺, 8₂⁺, 9₁⁺, 7₉⁺, 8₃⁺, 7₁₀⁺, 8₄⁺, 8₅⁺, 8₆⁺, 8₇⁺, 8₈⁺, 9₂⁺, 8₉⁺, 8₁₀⁺, 9₃⁺, 10₁⁺, 9₄⁺, 10₂⁺, 9₅⁺, 9₆⁺, 9₇⁺, 9₈⁺, 10₃⁺, 9₉⁺, 9₁₀⁺, 10₄⁺, 10₅⁺, 11₁⁺, 10₆⁺, 10₇⁺, 10₈⁺, 10₉⁺, 11₂⁺, 10₁₀⁺, 11₃⁺, 11₄⁺, 11₅⁺, 11₆⁺, 12₁⁺, 11₇⁺, 11₈⁺, 11₉⁺, 12₂⁺, 11₁₀⁺, 12₃⁺, respectively.

Table 2: Comparison of the experimental excitation energies[14] and excitation energies predictions for ²⁶Na nucleus by using USDB interactions

J ^π (OXBASH)	Energy (OXBASH) USDA (MeV)	Energy (exp.) (MeV)	J ^π (exp.)
3 ₁ ⁺	0.000	0.000	3 ⁺
1 ₁ ⁺	0.004	0.082	1 ⁺
2 ₁ ⁺	0.108	0.232	2 ⁺
2 ₂ ⁺	0.325	0.406	2 ⁺
1 ₂ ⁺	1.281	1.509	(1 ⁺)
4 ₁ ⁺	1.628	1.660	-----
3 ₂ ⁺	1.708	1.808	(3 ⁺)
0 ₁ ⁺	1.740	-----	-----
4 ₂ ⁺	1.988	1.996	(4 ⁺)
2 ₃ ⁺	2.059	2.045	-----
5 ₁ ⁺	2.224	2.118	(5 ⁺)
2 ₄ ⁺	2.236	2.192	(2 ⁺)
1 ₃ ⁺	2.450	2.452	-----
1 ₄ ⁺	2.677	2.720	(1 ⁺)
3 ₃ ⁺	2.726	2.803	-----
5 ₂ ⁺	2.977	-----	-----
4 ₃ ⁺	2.998	-----	-----
2 ₅ ⁺	3.065	3.222	(2 ⁺)
0 ₂ ⁺	3.231	3.304	-----
4 ₄ ⁺	3.434	3.417	-----
1 ₅ ⁺	3.530	-----	-----

2 ₆ ⁺	3.542	-----	-----
3 ₄ ⁺	3.644	3.603	-----
3 ₅ ⁺	3.791	3.814	-----
4 ₅ ⁺	3.985	3.966	-----
1 ₆ ⁺	4.086	-----	-----
3 ₆ ⁺	4.243	4.188	-----
2 ₇ ⁺	4.269	-----	-----
3 ₇ ⁺	4.429	-----	-----
2 ₈ ⁺	4.433	4.440	-----
4 ₆ ⁺	4.461	-----	-----
2 ₉ ⁺	4.525	-----	-----
6 ₁ ⁺	4.591	-----	-----
4 ₇ ⁺	4.683	-----	-----
5 ₃ ⁺	4.703	4.702	-----
1 ₇ ⁺	4.851	-----	-----
3 ₈ ⁺	4.872	-----	-----
0 ₃ ⁺	4.873	-----	-----
3 ₉ ⁺	4.938	-----	-----
5 ₄ ⁺	4.960	4.915	-----
6 ₂ ⁺	5.117	-----	-----
4 ₈ ⁺	5.202	4.970	(4 ⁺)
1 ₈ ⁺	5.213	-----	-----
5 ₅ ⁺	5.243	-----	-----
2 ₁₀ ⁺	5.303	5.080	(2 ⁺)
3 ₁₀ ⁺	5.349	5.480	-----
1 ₉ ⁺	5.634	-----	-----
6 ₃ ⁺	5.715	-----	-----
1 ₁₀ ⁺	5.724	-----	-----
4 ₉ ⁺	5.791	-----	-----
7 ₁ ⁺	5.914	-----	-----
5 ₆ ⁺	5.940	-----	-----
4 ₁₀ ⁺	6.162	-----	-----
0 ₄ ⁺	6.181	-----	-----
5 ₇ ⁺	6.257	-----	-----
5 ₈ ⁺	6.414	-----	-----
5 ₉ ⁺	6.578	-----	-----
6 ₄ ⁺	6.583	-----	-----
5 ₁₀ ⁺	6.793	-----	-----
7 ₂ ⁺	6.999	-----	-----
6 ₅ ⁺	7.009	-----	-----
0 ₅ ⁺	7.034	7.200	-----
6 ₆ ⁺	7.522	-----	-----
6 ₇ ⁺	7.802	-----	-----
0 ₆ ⁺	7.847	-----	-----
6 ₈ ⁺	8.134	-----	-----
0 ₇ ⁺	8.150	-----	-----
6 ₉ ⁺	8.202	-----	-----
7 ₃ ⁺	8.347	-----	-----
8 ₁ ⁺	8.513	-----	-----
6 ₁₀ ⁺	8.660	-----	-----
7 ₄ ⁺	8.846	-----	-----
0 ₈ ⁺	9.060	9.000	-----
7 ₅ ⁺	9.068	-----	-----
0 ₉ ⁺	9.207	-----	-----
7 ₆ ⁺	9.357	-----	-----
0 ₁₀ ⁺	9.693	-----	-----
7 ₇ ⁺	9.726	-----	-----
9 ₁ ⁺	9.859	-----	-----
7 ₈ ⁺	9.990	-----	-----
8 ₂ ⁺	10.014	-----	-----

7 ₉ ⁺	10.343	-----	-----
8 ₃ ⁺	10.446	-----	-----
7 ₁₀ ⁺	10.581	-----	-----
8 ₄ ⁺	11.104	-----	-----
8 ₅ ⁺	11.441	-----	-----
8 ₆ ⁺	11.549	-----	-----
8 ₇ ⁺	11.790	-----	-----
9 ₂ ⁺	12.415	-----	-----
8 ₈ ⁺	12.428	-----	-----
8 ₉ ⁺	12.738	-----	-----
9 ₃ ⁺	12.738	-----	-----
8 ₁₀ ⁺	12.839	-----	-----
9 ₄ ⁺	13.252	-----	-----
10 ₁ ⁺	13.456	-----	-----
9 ₅ ⁺	13.605	-----	-----
9 ₆ ⁺	14.287	-----	-----
9 ₇ ⁺	14.824	-----	-----
9 ₈ ⁺	15.222	-----	-----
10 ₂ ⁺	15.441	-----	-----
9 ₉ ⁺	15.844	-----	-----
9 ₁₀ ⁺	15.886	-----	-----
10 ₃ ⁺	16.217	-----	-----
10 ₄ ⁺	16.892	-----	-----
11 ₁ ⁺	17.297	-----	-----
10 ₅ ⁺	17.888	-----	-----
10 ₆ ⁺	18.340	-----	-----
10 ₇ ⁺	18.565	-----	-----
11 ₂ ⁺	19.050	-----	-----
10 ₈ ⁺	19.097	-----	-----
10 ₆ ⁺	19.293	-----	-----
10 ₇ ⁺	19.531	-----	-----
11 ₂ ⁺	19.582	-----	-----
11 ₃ ⁺	21.628	-----	-----
11 ₄ ⁺	22.932	-----	-----
11 ₅ ⁺	23.594	-----	-----
11 ₆ ⁺	23.669	-----	-----
12 ₁ ⁺	23.758	-----	-----
11 ₇ ⁺	24.865	-----	-----
11 ₆ ⁺	25.336	-----	-----
12 ₂ ⁺	25.736	-----	-----
11 ₈ ⁺	26.025	-----	-----
12 ₃ ⁺	31.664	-----	-----

4.2 : B(E2) and B(M1)

Transition rates are a sensitive indicator for most modern effective interactions developed to describe the sd-shell region. This sensitivity resulting from the adoption of transition rates on the single particle wave function (Hamiltonian eigenvectors). In this section, the theoretical and experimental reduced electric quadrupole transition probability B(E2) (in units of e²fm⁴) and reduced magnetic dipole transition probability B(M1) (in units of μ₂, μ Bohr magneto) values for ^{24,26}Na isotope [15]. The comparison between theoretical and experimental B(E2) shows an advantage for USDB calculations for many states. The reduced magnetic dipole transition probabilities B(M1) results gave a clear advantage to the USDB calculations compared to the other Hamiltonians results. The transition strengths calculated in this work performed using the harmonic oscillator potential HO for each in-band transition by assuming pure B(E2) transition. Core polarization effect were included by choosing the effective charges for proton and for neutron e_p = e_n = 0.350e. We also calculated magnetic quadrilateral transition probability B(M1), Values of effective charge are (e_p = e_n = 0.350e) and the free nucleon g factors are g_s(p) = 5.586, g_s(n) = -3.826, g_l(p) = 1 and g_l(n) = 0. New electric and magnetic { B(E2), B(M1), } transitions were expected in Our results by using (USDB) interaction are listed in Tables (3 and 4) for ²⁴Na nucleus and Tables (5 and 6) for ²⁶Na nucleus, transition probabilities gives agreement comparing with experimental data.

Table 3. Comparison of the B(E2) results in unit e² fm⁴ for ²⁴Na nucleus with the experimental data[13].

$J_i \rightarrow J_f$	B (E2)our Results for USDB (e ² fm ⁴)(e _p =0.350, e _n =0.350)	B (E2; ↓) Exp. Results(e ² fm ⁴)
2 ₁ ⁺ →4 ₁ ⁺	17.71	-----
2 ₂ ⁺ →4 ₁ ⁺	9.963	-----
3 ₁ ⁺ →4 ₁ ⁺	1.263	1.685
5 ₁ ⁺ →4 ₁ ⁺	40.84	-----
3 ₂ ⁺ →4 ₁ ⁺	6.479	-----
4 ₂ ⁺ →4 ₁ ⁺	3.778	-----
5 ₂ ⁺ →4 ₁ ⁺	1.618	-----
6 ₁ ⁺ →4 ₁ ⁺	7.237	-----
6 ₂ ⁺ →4 ₁ ⁺	3.444	-----
2 ₁ ⁺ →1 ₁ ⁺	33.81	-----
2 ₂ ⁺ →1 ₁ ⁺	10.65	-----
2 ₃ ⁺ →1 ₁ ⁺	3.275	4.934
3 ₁ ⁺ →1 ₁ ⁺	13.38	-----
3 ₂ ⁺ →1 ₁ ⁺	5.527	-----
2 ₂ ⁺ →2 ₁ ⁺	17.67	-----
3 ₁ ⁺ →2 ₁ ⁺	30.73	1.233
3 ₂ ⁺ →2 ₁ ⁺	2.929	2.878
4 ₂ ⁺ →2 ₁ ⁺	19.93	-----
0 ₁ ⁺ →2 ₁ ⁺	12.59	-----
0 ₂ ⁺ →2 ₁ ⁺	0.9825	-----
1 ₂ ⁺ →3 ₁ ⁺	0.5084	-----
3 ₂ ⁺ →3 ₁ ⁺	2.590	-----
5 ₁ ⁺ →3 ₁ ⁺	14.27	-----
4 ₂ ⁺ →3 ₁ ⁺	19.24	-----
3 ₂ ⁺ →5 ₁ ⁺	0.4727	-----
4 ₂ ⁺ →5 ₁ ⁺	2.732	-----
5 ₂ ⁺ →5 ₁ ⁺	2.530	-----
6 ₁ ⁺ →5 ₁ ⁺	40.85	-----
7 ₁ ⁺ →5 ₁ ⁺	10.20	-----
6 ₂ ⁺ →5 ₁ ⁺	0.5711	-----
7 ₁ ⁺ →6 ₁ ⁺	26.25	-----
6 ₂ ⁺ →6 ₁ ⁺	0.9019	-----
7 ₂ ⁺ →6 ₁ ⁺	8.791	-----
8 ₁ ⁺ →6 ₁ ⁺	16.21	-----
8 ₂ ⁺ →6 ₁ ⁺	1.142	-----
6 ₂ ⁺ →7 ₁ ⁺	0.6661	-----
7 ₂ ⁺ →7 ₁ ⁺	6.243	-----
8 ₁ ⁺ →7 ₁ ⁺	15.35	-----
8 ₂ ⁺ →7 ₁ ⁺	18.65	-----

Table 4. Comparison of the B(M1) results in unit μ² for ²⁴Na nucleus with the experimental data[13].

$J_i \rightarrow J_f$	B(M1) Cal. Results USDB	Exp. results
3 ₁ ⁺ →4 ₁ ⁺	0.2697	0.3401
5 ₁ ⁺ →4 ₁ ⁺	0.3228	-----
4 ₂ ⁺ →4 ₁ ⁺	0.06733	-----
2 ₁ ⁺ →1 ₁ ⁺	1.135	-----
2 ₂ ⁺ →1 ₁ ⁺	0.6632	-----
2 ₃ ⁺ →1 ₁ ⁺	0.006045	0.020406
3 ₁ ⁺ →2 ₂ ⁺	0.1770	-----
3 ₂ ⁺ →3 ₁ ⁺	0.03513	-----
4 ₂ ⁺ →3 ₁ ⁺	0.8562	-----
0 ₁ ⁺ →1 ₁ ⁺	0.2025	-----

$0_2^+ \rightarrow 1_1^+$	0.7856	-----
$0_1^+ \rightarrow 1_2^+$	0.2719	-----
$0_2^+ \rightarrow 1_2^+$	0.0002657	-----
$4_2^+ \rightarrow 5_1^+$	0.3917	-----
$6_1^+ \rightarrow 5_1^+$	0.2873	-----
$6_2^+ \rightarrow 6_1^+$	0.02435	-----
$7_1^+ \rightarrow 6_1^+$	0.3229	-----
$7_2^+ \rightarrow 6_1^+$	0.02950	-----
$6_2^+ \rightarrow 7_1^+$	0.0002075	-----
$7_2^+ \rightarrow 7_1^+$	0.01240	-----
$8_1^+ \rightarrow 7_1^+$	0.001071	-----
$8_2^+ \rightarrow 7_1^+$	0.006340	-----
$3_2^+ \rightarrow 2_1^+$	0.7838	0.4654
$3_3^+ \rightarrow 4_1^+$	0.002568×10^{-3}	0.003938

Table5. Comparison of the B (E2) results in unit $e^2 \text{ fm}^4$ for ^{26}Na nucleus with the experimental data

$J_i \rightarrow J_f$	B (E2)our . Results ($e^2 \text{ fm}^4$) $e_n=0.350, e_p=0.350$	B (E2)Exp. Results ($e^2 \text{ fm}^4$) [16]
$1_1^+ \rightarrow 3_1^+$	19.05	16.47
$2_1^+ \rightarrow 3_1^+$	20.02	50.325
$2_2^+ \rightarrow 3_1^+$	14.05	13.725
$1_2^+ \rightarrow 3_1^+$	18.11	-----
$3_2^+ \rightarrow 3_1^+$	3.897	-----
$4_2^+ \rightarrow 3_1^+$	4.745	-----
$4_1^+ \rightarrow 3_1^+$	2.317	-----
$5_1^+ \rightarrow 3_1^+$	23.66	-----
$5_2^+ \rightarrow 3_1^+$	2.499	-----
$2_1^+ \rightarrow 1_1^+$	5.662	-----
$3_2^+ \rightarrow 1_1^+$	48.58	86.925
$1_2^+ \rightarrow 1_1^+$	2.121	-----
$3_2^+ \rightarrow 1_1^+$	10.69	-----
$2_2^+ \rightarrow 1_1^+$	25.01	2.287
$2_2^+ \rightarrow 2_1^+$	2.342	-----
$1_2^+ \rightarrow 2_1^+$	0.1018	-----
$0_1^+ \rightarrow 2_1^+$	25.04	-----
$3_2^+ \rightarrow 2_1^+$	14.00	-----
$4_1^+ \rightarrow 2_1^+$	8.920	-----
$4_2^+ \rightarrow 2_1^+$	14.14	-----
$1_2^+ \rightarrow 2_2^+$	16.17	-----
$0_1^+ \rightarrow 2_2^+$	0.6580	-----
$3_2^+ \rightarrow 2_2^+$	0.7810	-----
$4_1^+ \rightarrow 3_2^+$	0.1757	-----
$4_2^+ \rightarrow 3_2^+$	22.25	-----
$0_2^+ \rightarrow 2_2^+$	1.683	-----
$5_1^+ \rightarrow 3_2^+$	1.778	-----
$5_2^+ \rightarrow 3_2^+$	0.8589	-----
$4_2^+ \rightarrow 4_1^+$	0.3302	-----
$5_1^+ \rightarrow 4_1^+$	0.9571	-----
$5_2^+ \rightarrow 4_1^+$	50.15	-----
$6_1^+ \rightarrow 4_1^+$	13.23	-----
$6_2^+ \rightarrow 4_1^+$	0.2773	-----
$5_2^+ \rightarrow 5_1^+$	2.499	-----
$6_1^+ \rightarrow 5_1^+$	0.3603	-----
$6_2^+ \rightarrow 5_1^+$	2.326	-----
$7_1^+ \rightarrow 5_1^+$	23.08	-----
$7_2^+ \rightarrow 5_1^+$	6.905	-----
$6_2^+ \rightarrow 6_1^+$	0.02135	-----

$7_1^+ \rightarrow 6_1^+$	27.73	-----
$7_2^+ \rightarrow 6_1^+$	5.222	-----
$8_1^+ \rightarrow 6_1^+$	30.70	-----
$8_2^+ \rightarrow 7_1^+$	0.1888	-----
$8_2^+ \rightarrow 6_1^+$	0.1721	-----
$8_1^+ \rightarrow 7_1^+$	16.38	-----
$7_2^+ \rightarrow 7_1^+$	2.497	-----
$9_1^+ \rightarrow 7_1^+$	25.83	-----
$9_2^+ \rightarrow 7_1^+$	0.01602	-----
$8_2^+ \rightarrow 8_1^+$	2.309	-----
$9_1^+ \rightarrow 8_1^+$	18.54	-----
$9_2^+ \rightarrow 8_1^+$	1.897	-----

Table6. Comparison of theB(M1) results in unit μ^2 for ^{26}Na nucleus with the experimental data

$J_i \rightarrow J_f$	B(M1) our. Results	B(M1) Exp. Results[16]
$2_1^+ \rightarrow 3_1^+$	0.1531	0.00179
$2_2^+ \rightarrow 3_1^+$	0.6216	0.002685
$4_1^+ \rightarrow 3_1^+$	0.0008276	-----
$3_2^+ \rightarrow 3_1^+$	0.1328	-----
$4_2^+ \rightarrow 3_1^+$	0.2971	-----
$2_1^+ \rightarrow 1_1^+$	0.08654	0.00537
$2_2^+ \rightarrow 1_1^+$	0.4041	0.000716
$0_1^+ \rightarrow 1_1^+$	0.4252	-----
$0_2^+ \rightarrow 1_1^+$	0.3141	-----
$2_2^+ \rightarrow 2_1^+$	0.3470	-----
$3_2^+ \rightarrow 2_2^+$	0.02708	-----
$3_2^+ \rightarrow 2_1^+$	0.02708	-----
$4_2^+ \rightarrow 4_1^+$	0.004789	-----
$1_2^+ \rightarrow 1_1^+$	0.05102	-----
$5_1^+ \rightarrow 4_1^+$	0.08585	-----
$5_2^+ \rightarrow 5_1^+$	0.05243	-----
$6_1^+ \rightarrow 5_1^+$	0.001111	-----
$6_2^+ \rightarrow 6_1^+$	0.1046	-----
$7_1^+ \rightarrow 6_1^+$	0.04680	-----
$7_2^+ \rightarrow 7_1^+$	0.2052	-----
$8_1^+ \rightarrow 7_1^+$	0.06191	-----
$8_2^+ \rightarrow 7_1^+$	0.01022	-----

5. Conclusions

Full sd-space shell model calculations were performed using the code OXBASH for Windows. The SD model space are employed with the effective interactions(USDB) to reproduce the level spectra , reduced electric quadrupole transition probability B(E2) and magnetic quadrilateral transition probability B(M1)for the nuclei $^{24,26}\text{Na}$. Good agreement were obtained by comparing these calculations with the recently available experimental data for the level spectra using USDB effective interaction. Calculation of the transition strengths prove that USDB is more consistent in for the(sd-shell) region.

6. References

[1] Mohammadi, Saeed, B. N. Giv, and N. S. Shakib. "Energy Levels Calculations of ^{24}Al and ^{25}Al Isotopes." Nucl. Sci 2.1 (2017): 1-4.

- [2] Dean, D. J., et al. "Effective interactions and the nuclear shell-model." *Progress in Particle and Nuclear Physics* 53.2 (2004): 419-500.
- [3] Al-Sammarräie, A. A., Inche Ibrahim, M. L., Ahmed Saeed, M., Sharrad, F. I., & Abu Kassim, H. "Inelastic electric and magnetic electron scattering form factors of ^{24}Mg nucleus: Role of g factors." *International Journal of Modern Physics E* 26.05 (2017): 1750032.
- [4] Aissaoui, L., F. Benrachi, and D. Boumala. "Pairing gap energy correction in Shell model for the neutron-rich tin isotopes." *Brazilian Journal of Physics* 39.4 (2009): 663-666.
- [5] Steer, S. J., Podolyak, Z., Pietri, S., Górska, M., Regan, P. H., Rudolph, D., ... & Wollersheim, H. J. "Single-particle behavior at $N=126$: Isomeric decays in neutron-rich Pt 204." *Physical Review C* 78.6 (2008): 061302.
- [6] Hala .Wattar, Michael .Farah and Nawras .Alhoujami "Excited States and Electromagnetic Transition of $A=78$ Calculated Using Shell Model and BCS Theory" *C* 5.2 (2017): 78-88 .
- [7] Pan, Xing-Wang, Da Hsuan Feng, and Michel Vallières. "Contemporary Nuclear Shell Models." *Contemporary Nuclear Shell Models*. Vol. 482. 1997.
- [8] J. Suhonen, *From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory*, Springer Science & Business Media (2007).
- [9] Brown, B. Alex. "The nuclear shell model towards the drip lines." *Progress in Particle and Nuclear Physics* 47.2 (2001): 517-599.
- [10] Brown, B. Alex, et al. "Oxbash for Windows PC." MSU-NSCL Report 1289 (2004).
- [11] Brown, B. Alex, and W. A. Richter. "New "USD" Hamiltonians for the sd shell." *Physical Review C* 74.3 (2006): 034315.
- [12] D. R. Tilley, C. Cheves, J. Kelley, S. Raman and H. Weller, *Nucl. Physics*, Vol. A636, No. 249, (1998).
- [13] R.B. FIRESTONE, *Nuclear. Data Sheets* 108, Vol. 127, No. 2319, (2007).
- [14] M. Shamsuzzoha Basunia and A.M. Hurst, *Nucl. Data Sheets*, Vol. 134, No. 1, (2016).
- [15] Al-Sammarräie, A. A., Sharrad, F. I., Aziz, A. A., Yusof, N., & Kassim, H. A. "Application of USDA and SDBA Hamiltonians in calculating the excited states of odd-A magnesium isotopes." *The European Physical Journal Plus* 129.6 (2014): 125.
- [16] M.S. Basunia and A.M. Hurst, *Nuclear. Data Sheets* 134, Vol. 127, No. 1, (2016).