A comment on "FUZZY GRAHPS ON COMPOSITION, TENSOR AND NORMAL PRODUCTS"

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Abstract

In a paper in this journal by Nirmala and Vijaya [Fuzzy graphs on Composition, Tensor and Normal products, 2(2012)1-7], some operations on two fuzzy graphs were defined and the degree of a vertex in fuzzy graphs were obtained. They also proved some properties on fuzzy graphs and illustrated these properties through examples.

The purpose of this paper is to show by examples that some definitions and theorems in mentioned paper contain some flaws and in general they are not true.

Key word: Graph, Fuzzy graph, Degree of a vertex.

1. Introduction

In 1965, fuzzy set was introduced by Lotfi A. Zade [6] for representing uncertainty.

A fuzzy subset of a non-empty set S is a mapping $\mu: S \to [0,1]$. Fuzzy set theory was applied in different fields such as management, optimization, engineering, neural network, graph theory and etc.

Graph theory has since been used to study modern science such as operations research, transportation and cluster analysis.

Rosenfeld introduced fuzzy graphs [5] based on fuzzy set in 1975

A fuzzy graph G is a pair of functions (μ, ρ) where μ is a fuzzy subset of a set S and $\rho: S \times S \to [0,1]$ be symmetric fuzzy relation on μ such that $(x, y) \leq min\{\mu(x), \mu(y)\}$.

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. Many interesting graphs are composed of simpler graphs via several operations (also known as graph products) for more information on graph products see [4]. Especially fuzzy operations was used to form a new fuzzy graph [1].

Nirmala and Vijaya [4] studied normal product, tensor product and composition on two graphs and also they obtained the degree of a vertex in new fuzzy graph. They proved theorems to express some properties in fuzzy graphs that formed by these operations.

The purpose of this paper is to show by contra- examples that some definitions and theorems in [4] contain some flaws and in general they are not true.

2. Degree of a vertex in fuzzy graph

Nirmala and Vijaya [4] studied normal product, tensor product and composition on two fuzzy graphs and also they obtained the degree of a vertex in new fuzzy graphs. They proved theorems to express some properties in fuzzy graphs that formed by these operations. In this section we proceed to illustrate two fuzzy graphs and their new fuzzy graphs obtained from graph operations normal product, tensor product and composition and also calculate the degree of vertex in fuzzy graphs formed by these operations based on both figures and formulas proved in [4] and we will show that the amount of degree of vertex in two cases are not equal.

Example 1: Consider two following fuzzy graphs G_1 and G_2:

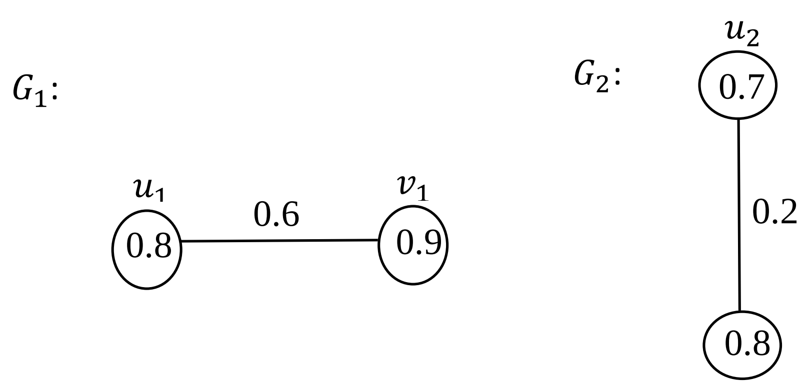

Figure1: Two fuzzy graphs $G_1$and G_2

By using definition 3.1[4], we present fuzzy normal product $G_1 \circ G_2$ of G_1 and G_2, as follows:

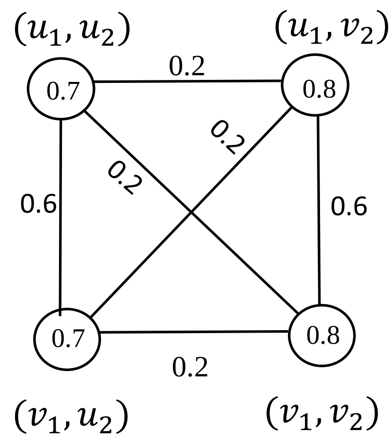

Figure2: The fuzzy normal product of G_1 and G_2

Based on theorem 3.4[4]:

$$d_{(G_1\circ G_2)}(u_1,u_2) = P_2 d_{G_1}(u_1) + d_{G_2}(u_2) = 2 \times 0.6 + 0.2 = 1.4$$

and by definition of degree for a vertex (u_1,u_2) in a fuzzy graph, we have:

$$d_{(G_1\circ G_2)}(u_1,u_2) = \sum_{(u_1,u_2)(x,y)\in E(G_1\circ G_2)} \rho(u_1,u_2)(x,y) = 0.6 + 0.2 + 0.2 = 1$$

it is clear that they are not equal, so it is a contradiction.

Example 2: Consider two fuzzy graphs G_1 and G_2, in figure 3:

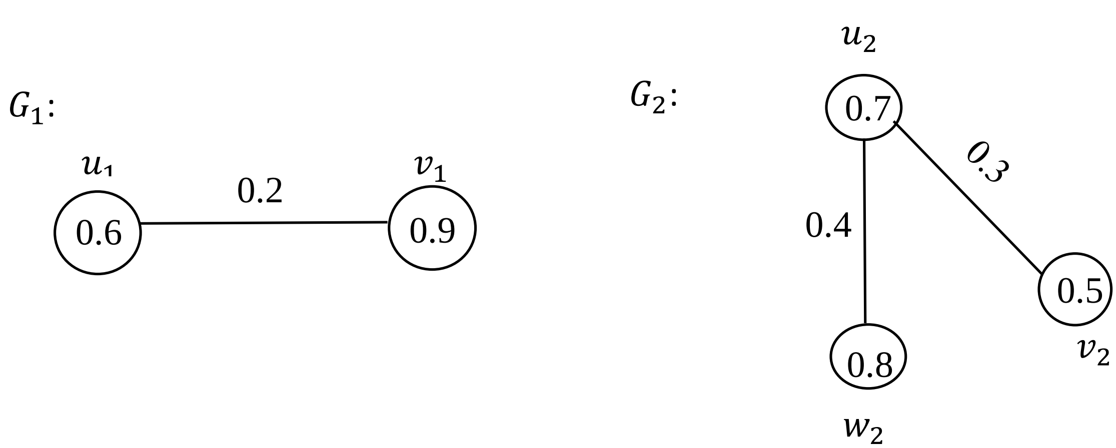

Figure3: Two fuzzy graphs G_1andG_2

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By definition 3.2[4], the tensor product $G_1 \otimes G_2$ of G_1 and G_2 is presented in figure 4:

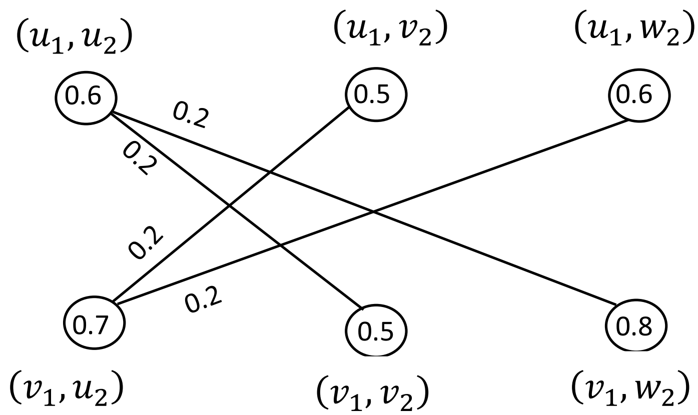

Figure 4: The fuzzy tensor product $G_1 \otimes G_2$ of G_1 andG_2

And it is evident that the degree of vertex (u_1, u_2) in $G_1 \otimes G_2$ obtained as follows:

$$d_{(G_1 \otimes G_2)}(u_1, u_2) = \sum_{(u_1,u_2)(x,y) \in E(G_1 \otimes G_2)} \rho(u_1, u_2)(x, y) = 0.2 + 0.2 = 0.4$$

and according to theorem 3.3[4]:

$$d_{(G_1 \otimes G_2)}(u_1, u_2) = d_{G_1}(u_1) = 0.2$$

Consequently, it is a contradiction.

According to [2], the Composition or lexicographic product $G_1[G_2]$of two graphs $G_1$and $G_2$is defined as a graph with the vertex set $V(G_1) \times V(G_2)$ and vertex (u_1, u_2) is adjacent with vertex (v_1, v_2) if u_1 is adjacent with v_1 in G_1 or $u_1 = v_1$ and u_2 is adjacent with v_2 in G_2, which is different form definition 2.5 in [4].

In [4], the authors applied the definition of deleted lexicographic product instead of composition of two graphs. We refer the reader to [1] for more information on deleted lexicographic product.

Whether or not, now we focus on definition 2.5 in [4]

$$E = \{(u_1, u_2)(v_1, v_2) \mid u_1 = v_1, u_2v_2 \in E_2 or\ u_2 \neq v_2\ , u_1v_1 \in E_1\}$$

and

$$\mu_1 \times \mu_2\big((u_1, u_2)(v_1, v_2)\big) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2) & if\ \ u_1 = v_1, u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1v_1) & if\ \ u_2 = v_2, u_1v_1 \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1u_1) & if\ \ u_2 \neq v_2, u_1v_1 \in E_1 \end{cases}$$

Based on E, the edges of $G_1[G_2]$ are divided two parts whereas $\mu_1 \times \mu_2$ is included three parts and it is clear that "$\sigma_2(u_2) \wedge \mu_1(u_1v_1)$ if $u_2 = v_2, u_1v_1 \in E_1$" must be deleted.Consequently, we modify the definition of $\mu_1 \times \mu_2\big((u_1, u_2)(v_1, v_2)\big)$ as follows:

$$\mu_1 \times \mu_2\big((u_1, u_2)(v_1, v_2)\big) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2) & if\ \ u_1 = v_1, u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1u_1) & if\ \ u_2 \neq v_2, u_1v_1 \in E_1 \end{cases}$$

Now consider two fuzzy graphs G_1 and G_2 in example 3.2, we illustrate $G_1[G_2]$ in figure 5 based on definition of set E which is different form figure 2 in [4].

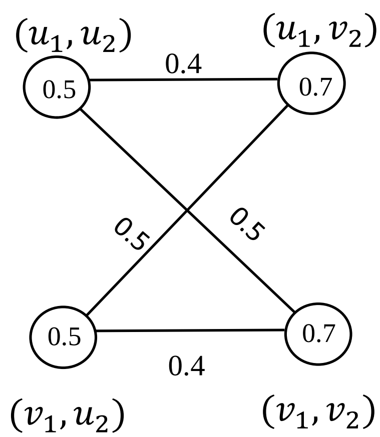

Figure5: The fuzzy composition $G_1[G_2]$ of two fuzzy graphs G_1 and G_2illustrated [4]

Example 3: Let G_1 and G_2 be two fuzzy graphs illustrated in figure 6:

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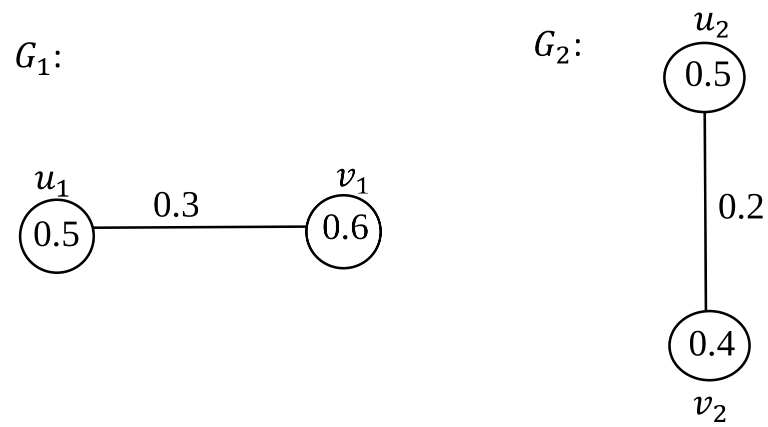

Figure 6: Two fuzzy graphs $G_1$and G_2.

and $G_1[G_2]$ is shown in figure 7, it is necessary to express that $G_1[G_2]$ is formed by E in definition 2.5 [4]

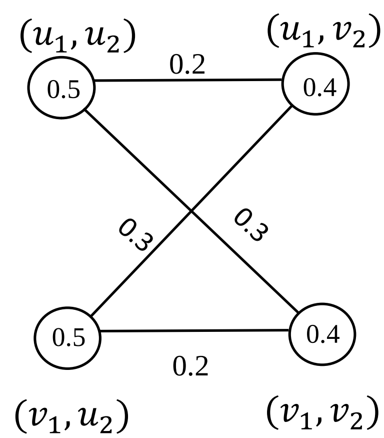

Figure7: The fuzzy composition $G_1[G_2]$ of G_1 and G_2

By applying theorem 3.2, we compute the degree of a vertex (u_1, u_2) in figure 7, as follows:

$$d_{(G_1[G_2])}(u_1, u_2) = P_2 d_{G_1}(u_1) + d_{G_2}(u_2) = 2 \times 0.3 + 0.2 = 0.8$$

and according to figure7:

$$d_{(G_1[G_2]G_2)}(u_1,u_2) = \sum_{(u_1,u_2)(x,y)\in E(G_1{}^{\circ}\, G_2)} \rho(u_1,u_2)(x,y) = 0.2 + 0.3 = 0.5$$

and they are not the same.

REFERENCES

[1] B. Frelih, S. Miklavič, Edge regular graph products, the electronic journal ofcombinatorics, 20 (2013) 62-66.

[2] W. Imrich, S. Klavžar, Product Graphs: Structure and Recognition, Wiley, New York (2000).

[3] J.N. Mordeson, C.-S. Peng, Operations on fuzzy graphs, Information Sciences 79 (1994) 159–170.

[4] G. Nirmala and M. Vijaya "Fuzzy graphs on composition, Tensor and Normal Products" International Journal of Scientific and Research Publications, 2(2012)1-7.

[5] A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, M. Shimura (Eds.), Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975, pp. 77–95.

[6] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.