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An Empirical Study for Competition Case between Iraqi Telecommunication Companies Using Game Theory

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Abstract - Telecommunication companies in Iraq are experiencing a huge competition in the market nowadays. Each company is striving to attract more customers through providing satisfactory and competitive services. In this study, Game Theory, which is one of the operational research modeling techniques is selected then used to evaluate performance of a number of telecommunication companies in Iraq after reconstruction. This research is set out to analyze the competition models of two leading telecommunication companies using Game Theory. Two local telecommunication companies based in Iraq; Asia Cell and Korek is used as a case study. Game theory methods are applied to provide some advices on how to maximize profits of the selected companies. This is enabled by studying the strategies (services) offered by each company, and formulating a payoff matrix of competition between these companies. The payoff matrix size is (9 × 5) represented by (9) strategies adopted by Asia Cell Company, and (5) strategies adopted by Korek Company. The manual method shows that there is no saddle point for the payoff matrix. As there is no saddle point found for the payoff matrix, size of this matrix is reduced to (2 × 3) to enable solving it using the graphical method. As a conclusion, Asia Cell should always use strategies of 8th (selling the International Line) and the 9th (selling the New Khalat Line), to get the expected optimal value (higher profit expected), is equal to v = 68053291 ID. While, Korek should always use the 2nd (selling the Data Offers) and the 3rd (selling Diary Line) strategies to get the expected optimal value (which will bring less losses), is equal to v = 68053291 ID. TORA package is used to provide results which are close to the manual solution method.

Index Terms - Operations Research, Game Theory, Telecommunication Industry, Payoff Matrix, Profit.

I. Introduction

The general description of the business environment is the ability to compete. The reason behind this description is that the aim of a business is to increase profits by any means, taking into consideration the business ethics. Game theory is a collection of mathematical techniques that is used to study the interactive decision problems between more than rational player (here they are companies). Furthermore, it also assists in predicting the possible outcome of the interactive decision problem. As the Game theory emerged from the analysis of competitive scenarios, these interactive problems are called games and the participants are called players. These techniques are not limited to competitive situations and hence it can be applied to solve different real life problems not just those faced by the sport industry. However, game theory deals with any problem in which each player's strategy depends on what other players do.

In this research game theory techniques will be practically applied to analyze the competition models of two leading telecommunication companies. This research aims at applying a number of Game Theory techniques to solve problems that are currently being faced by two Iraqi telecommunication companies. In order to deliver this aim, a number of objectives are set as follow:

1) Reviewing a number of related literatures in the area of Game theory and its applications in the telecommunication industry.
2) Applying game theory methods in practical way by formulating a payoff matrix of competition between the two companies (Asia Cell and Korek).
3) Investigating the strategies (services) available to use by each company, in an attempt to provide some advices to maximize the profits for both of them.
4) Understanding how players may be chosen to organize themselves in a game in order to get joint-maximizing profits.

II. Review of Game Theory

The beginning of the Game Theory (GT) as a separate subject of study is dated back to the first half of the 20th century and it is connected with the development of mathematical aggregates theory and mathematical economy [1].

"At the end of the 1990s, a high-profile application of game theory has been the design of auctions. Prominent game theorists have been involved in the design of auctions for allocating rights to the use of bands of the electromagnetic spectrum to the mobile telecommunications industry. Most of these auctions were designed with the goal of allocating these resources more efficiently than traditional governmental practices, and additionally raised billions of dollars in the United States and Europe". [8, page 5]

Game theory is a bag of analytical techniques designed to assist in understanding the phenomena observed when decision-makers interact. The basic assumptions that underlie the theory are that decision-makers pursue well-defined exogenous objectives and take into account their knowledge or expectations of other decision-makers' behavior. The models of game theory are highly abstract representations of classes of real-life situations. A game is a description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests, but does not specify the actions that the players do take. A solution is a systematic description of the outcomes that may emerge in a family of games [6].

"Game theory is a collection of techniques for predicting outcomes of a group of interacting agents where an action of a single agent directly affects the payoff of other participating agents". [5]

"The theory of games is basically concerned with strategic behavioral interactions as opposed to individual maximization, typically found in decision theory. Thus each participant attempts to maximize a function of which he/she does not control all variables". [3, page 63]

Game theory provides mathematical tools for modeling and analyzing interactive decisions. It considers players that represent decision-making firms or persons, and their possible actions and models their incentives by payoffs that the players want to maximize. The players may have different information about that state and about the actions taken by the other players [2].

Game theory and its applications in the telecommunication industry investigated by a number of researchers, for example, [7] analyses three oligarchs' co-competition in China mobile market by modified Court and Strackelbery Models in game theory, applying with business development data of 2G and 3G for verification. According to the verification and more oligarchs' co-competition game analyzing in the future, the conclusion is: on the basis of competition, the best choice for each oligarch is cooperation. Co-competing of the telecom operators can effectively configure all aspects of resources to produce more intelligent terminals, making the promotion of telecommunication industry progress, to some extent, can promote the development of artificial intelligence in mobile internet era.

Game theory provides a convenient framework to model and helps us to interpret the behavior of participants in such strategic interactions. Hence it can be applied to solve a wide variety of problems involving diverse areas such as industrial relations, markets, auctions, online retail, cold war, paying taxes, bargaining, elections, portfolio management, social interactions etc. Game theory could thus be viewed as a study of strategic decision making [10].

III. Game Theory Assumptions

In game theory (generally non-cooperative game theory) players usually make the following assumptions [4, 9]:

1) Each player has two or more well-specified strategies.
2) Every player has possible combinations of moves / strategy that lead to an optimum response (End-state like win, lose or draw) in a given game.
3) Each player has a specified payoff for each optimum response.
4) All players are rational; that is, each player, given the two moves/strategies, will choose that one that gives him/her the better payoff.

There are many fundamental definitions (terminologies) [4, 8]:

1) Player: each participant is called player.
2) Strategy: the decision rule by which a player determines his course of action is called a strategy.
3) Pure strategy: if a player decides to use only one particular course of action during every play, he is said to use a pure strategy.
4) Mixed strategy: if a player decides, in advance, to use all or some of his available courses of action in some fixed proportion he is said to use mixed strategy.
5) Payoff: it is the outcome of playing the game.
6) Payoff matrix: it is a table showing the amounts received by the player named at the left side after all possible plays of the game.

IV. Characteristics of Two-Person Zero-Sum

The characteristics of two-person zero-sum games are [9, 11]:

1) There are two players (called the row player and the column player).
2) The row player must choose 1 of m strategies. Simultaneously, the column player must choose 1 of n strategies.
3) If the row player chooses his i^{th} strategy and the column player chooses his j^{th} strategy, then the row player receives a reward of and the column player loses an amount. Thus, it is thought that the row player's reward is as if coming from the column player.

Such game is called a two-person zero-sum game, which is represented by the matrix. A two-person zero-sum game has the property that for any choice of strategies, the sum of the rewards to the players is zero. In a zero-sum game, every dollar that one player wins comes out of the other player's pocket, so the two players have totally conflicting interests [11].

In zero-sum games, the total benefit to all players in the game, for every combination of strategies, always adds to zero (more informally, a player benefits only at the expense of others). Games, as studied by economists and real-world game players, are generally finished in a finite number of moves. Pure mathematicians are not so constrained, and set theorists in particular study games that last for infinitely many moves, with the winner (or other payoff) not known until after all those moves are completed. The focus of attention is usually not so much on what is the best way to play such a game, but simply on whether one or the other player has a winning strategy [10].

V. Reduce Game by Dominance

If no pure strategies exist, the next step is to eliminate certain strategies (rows and / or columns) by dominance. The principle of dominance stated that if one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored [4, 11].

The concept of dominance is especially useful for the evaluation of two-person zero-sum, games where a saddle point does not exist. Generally, the dominance property is used to reduce the size of a large payoff matrix. The rule as follows:

1) If all the elements of a column (say i^{th} column) are greater than or equal to the corresponding elements of any other columns (say j^{th} column), then the i^{th} column is dominated by the j^{th} column and can be deleted from the matrix.
2) If all the elements of a row (say i^{th} row) are less than or equal to the corresponding elements of any other row (say j^{th} row), then the i^{th} row is dominated by the j^{th} row and can be deleted from the matrix.

VI. Mixed Strategies (2×2 Game)

In case where, there is no saddle point and dominance has been game matrix, Arithmetic Method is used for finding optimum gamers. It consists of the following steps [4]:

1) Subtract the two digits in column 1 and write them under column 2, ignoring sign.
2) Subtract the two digits in column 2 and write them under column 1, ignoring sign.
3) Similarly proceed for the two rows.

These values are called oddments. They are the frequency players who must use their courses of action in their optimum strategy.

VII. Mixed Strategies (2 × M) & (M × 2)

The game of one player has only two courses of action while the other has more than two which is called (2 × M) or (M × 2) games. If these games do not have a saddle point or are reducible by the dominance method, it can be still solved by Algebraic Method and Graphical Method. Thus, co-operation between the two players would not occur [11].

VIII. Research Population and Sample

The population of this research is all Iraqi's communication companies, where the researchers selected only two of them as a sample (Asia Cell and Korek) in Erbil. In order to obtain the required sampling data such as costs, the researchers have used different sources like; scratch cards sales, offered services, and also the profit related to the services. This has been implemented by: firstly, surfing the data available on the Internet through the websites of both companies. Secondly, by visiting many of authorized sales centers related to the two companies. In Erbil, there are 18 sales centers for Korek and 20 for Asia Cell. Only 15 from Korek and 18 from Asia Cell have accepted to provide the researchers with the required information.

IX. Asia Cell Telecom Packages[1]

Asia Cell is a telecom company was founded in 1999 in Sulaymanya. In 2006, Asia Cell started working in Erbil. The subscribers were more than 1050000 as in figure (1). Asia Cell services are explained in the following:

1) Youth line: it is one of Asia Cells services; the price is 3000 ID (including 1200 ID free balance). It provides a discount in prices 0.25 ID/second from 12:00am to 8:00am, and for the other times it costs 1 ID/second and for a message between (9:00am – 12:00am) it charges 25 ID/SMS.
2) Gold line: Customers can buy it by 3000 ID with 1200 ID free credits. It costs 1 ID/second for normal call and 25 ID for a normal message.
3) Hala Line It is visitor's line and costs 4000 ID (Including 1000 Free Airtime). It charges 1 ID/second and for a normal message 25 ID.
4) Almas line: It offers a good discount in regular calls 50% discount after the 3rd minute of each call. In Asia cell network, it costs only 1 ID/second.
5) Hali Service for Friends and Family: By using this offer, customers can select 5 Asia cell numbers and make calls for only 1 ID/second.
6) Student line: For university students 50% free balance through 4 years, this can be used for data and voice.
7) Smart Line: Is available at all Asia Cell sell points with only 5000 ID.
8) International Lines: International calls to Turkey, China and India with up to 50% discount.
9) New Khalat lines: It provides calls to all local networks for just 1 ID/second.

X. Korek Telecom Packages[2]

Korek telecom is a mobile company and was established in 16/8/2000. It has more than 2000 employees all over Iraq more than 500 of them in Erbil, as in figure (2). Korek services are described below:

1) Prepaid plan it started with Korek in 16/8/2001. This plan have 4500,000 subscribes but only 3700,000 of them use it because sim-cards are in market or unusable.
2) Data offers: It has 8 offers:
    1. Extra plus: Works for 30 days and costs 90000 ID. It provides internet service with 40 GB.
    2. Extra VIP: Works for 30 days and costs 60000 ID. It provides internet service with 20 GB.
    3. Extra: Works for 30 days and costs 40000 ID. It provides internet service with 10 GB.
    4. Alshabab: Works for 30 days and costs 30000 ID. It provides internet service with 7GB from 12:00 pm to 6:00 am for free.
    5. Monthly: Works for 30 days and costs 20000 ID. It provides internet service with 2 GB.
    6. Weekly 1000: Works for 7 days and costs 1000 ID. It provides internet service with 1000 MB.
    7. Weekly 500: Works for 7 days and costs 5000 ID. It provides internet service with 500 MB.
    8. Pay as you go: Works for 30 days and costs 0.60 ID/KB.
3) Diary line: was established in 2013, it has around 30000 subscribers and it charges 65 ID for a normal call and 25 ID for a normal message. This line work only for 15 days. For renewing, a customer should pay 1000 ID, 50 ID for a normal call and 20 ID for a normal message.
4) Bangladesh line was established in 2015 and it is only for international calls, the cost is 35 ID/second. This line is only for foreign workers in Iraq. It has around 3000 subscriber.
5) International caller to Korek: It started in 16/8/2001, and its cost depends on the contract between Korek and customers. It costs 3$ for one minute and 0.225$ for an international message.

XI. Appling Game Theory Method on Korek and Asia Cell

1. Solving the Problem Using Graphical Method

The two competing companies in Iraqi local market (Asia Cell and Korek) have a usage rate in Erbil city which is presented in figures 1 and 2. Player [A] represents the rows of Asia Cell Company; it includes the 9th most important services (Strategies). While, player [B] represents the columns of Korek Company; it includes the 5th most important services.

From the original data and the authorized sales centers of the two companies, the researchers can get the payoff matrix below. The cells (A1 , B1) are explained as below:

When player A (Asia Cell) play strategy A1 (Youth line service), it has got a profit equals to (63302180 ID). Whereas player B (Korek) has got profit equals to (58801400 ID) when it plays strategy B1 (Prepaid plan service). This is explained as follows:

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[1] https://www.asiacell.com/pages.php?lang=&pid=47

[2] http://www.korektel.com/top-links/about-us

- (A1 , B1) cell = A1 Profit – B1 Profit = 63302180 – 58801400 = 4500780 ID
- (A1 , B2) cell = A1 Profit – B2 Profit = 63302180 – 59317180 = 3985000 ID

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- (A9 , B5) cell = A9 Profit – B5 Profit = 154223000 – 80241585 = 73981415 ID

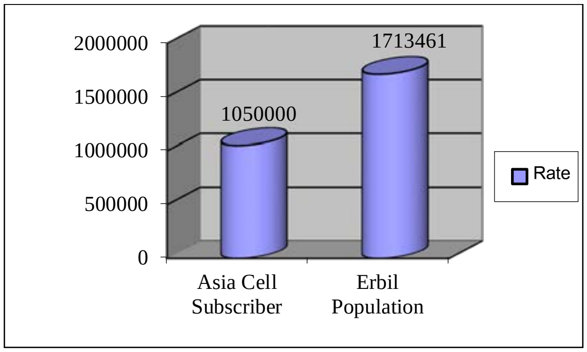

Figure 1: Asia Cell user rate in Erbil

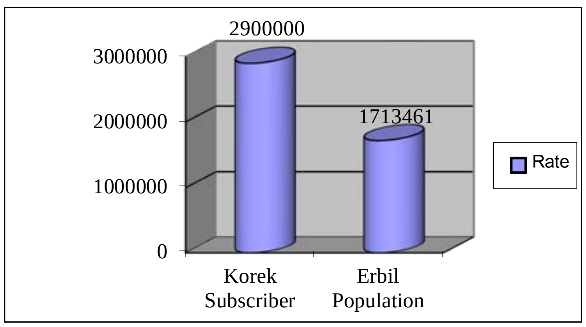

Figure 2: Korek user rate in Erbil

Now, the payoff matrix for the two companies can be set as follow in table (1).

Step one: Testing the existence of a saddle point:

69060500 ≠ 67745960 No saddle point

MaxiMin ≠ MiniMax ➔ $67745960 \leq v \leq 69060500$

1) Selected strategies for each player: $\frac{A}{9}\ \frac{B}{2}$. This means that player [A] will choose the 9th strategy which will bring a profit of (67745960) ID, while player [B] will choose the 2nd strategy, which will bring less losses of (69060500) ID.

Since the matrix size is (9 × 5) the researchers try to reduce it as much as possible through applying the principle of control; to facilitate the process of obtaining the best solution.

2) Applying the principle of control for the payment matrix in table (1):
   a) Comparing rows [A] if all of rows values are less than or equal to the corresponding grade values in another row, the latest row can be deleted.
      - Comparing the values A1A2 ➔ values A1 less than A2 corresponding, deleting A1 strategy for being controlled by A2 strategy.
      - Deleting A2 strategy for being controlled by A3 strategy.
      - Comparing A3A4 ➔ value A2 less than A3 corresponding values; deleting A2 strategy for being controlled by A3 strategy.
      - Comparing A3A4➔ values A3 less than A4 corresponding values; deleting A3 strategy for being controlled by A4 strategy.
      - Comparing A4A5 ➔ does not work.

      And so on …
   b) Comparing the columns [B] but in this case the biggest column will be deleted.
      - B1B2 ➔ does not work.
      - B1B3 ➔ does not work.
      - B1B4 ➔ does not work.
      - B1B5 ➔ values B5 is greater than the corresponding values of B1; deleting B5 strategy for being controlled by B1 strategy.

      And so on …

The matrix has become (2×3) after downsizing i.e. only two rows (A8, A9) and three columns (B1, B2, B3). It cannot be reduced more using the principle of rate strategies.

Step two: Second trial to test the existence of a saddle point as in table (2). It can be noticed that there is no stability, and the game value is: $67745960 \leq v \leq 69060500$.

Note: The arithmetic method of payoff matrix cannot be applied because of the type the resulting matrix (2×3), so graphical method will be used to illustrate the work style of the method as such kind of arrays.

From table (2) it has been noted that the saddle point is not available and the resulting matrix is (2×3). Thus, graphical method will be applied as follows:

Table 1: Payoff matrix for the two companies

[A] Asia Cell Company	[B] Korek Company: B1	B2	B3	B4	B5	Min. Row
A1	4500780	3985000	5011000	4150000	5950000	3985000
A2	8039075	7037490	7371920	8001500	10011250	10011250
A3	18620245	18318660	17953090	19552780	18950340	17953090
A4	28535415	28233830	31887760	30950000	30112900	28233830
A5	32054915	32253330	27868260	35052950	34005000	27868260
A6	29658585	24357000	28991430	25451020	30567000	24357000
A7	28798795	35497210	51131640	36925411	29970600	28798795
A8	56262085	69060500	50594930	70091340	61120400	50594930
A9	72047545	67745960	73380390	71050450	73981415	67745960 ← MaxiMin
Max. Col.	72047545	69060500 (↑ MiniMax)	73380390	71050450	73981415	

	B1	B2	B3	B4	B5	Min. Row
A1	4500780	3985000	5011000	4150000	5950000	3985000
A2	8039075	7037490	7371920	8001500	10011250	10011250
A3	18620245	18318660	17953090	19552780	18950340	17953090
A4	28535415	28233830	31887760	30950000	30112900	28233830
A5	32054915	32253330	27868260	35052950	34005000	27868260
A6	29658585	24357000	28991430	25451020	30567000	24357000
A7	28798795	35497210	51131640	36925411	29970600	28798795
A8	56262085	69060500	50594930	70091340	61120400	50594930

1) Player [A] have two strategies A8, A9 and two possibilities item for them p_1, p_2 respectively. Player [A] trying to increase profits by making the value of *v* is greatest.
2) The expected results for player [A] whether player [B] applied torrent free various strategies are as follows:
   a) The expected value of the player [A] in case of a player [B] play the 1st strategy B1:
   $(56262085)\ p_1 + (72047545)\ p_2 \geq v$
   b) The expected value of the player [A] in case of a player [B] play the 2nd strategy B2:
   $(69060500)\ p_1 + (67745960)\ p_2 \geq v$
   c) The expected value of the player [A] in the case of a player [B] play the 3rd strategy B3:
   $(50594930)\ p_1 + (73380390)\ p_2 \geq v$

Where $p_1 \geq 0$, $p_2 \geq 0$. Substitute $p_2 = 1 - p_1$ in the above we get table (3).

Table 3: Free strategies for Korek Company

[B] free strategies for Korek	[A] expected value [outcome] for Asia Cell
1	$72047545 - 15785460\ p_1$
2	$67745960 + 1314540\ p_1$
3	$73380390 - 22785460\ p_1$

Depending on drawing lines represented in table 1, the researchers can determine the optimal value of the game as shown in graphic formats figure (3). For the region (A, B, C, D, E) identified in bold, which was formed from the intersection of lines (2 & 3). It is also called Lower Envelope, where it represents the area of suitable solutions for Asia Cell Company [A]. It has been noted that point (B) represents MaxiMin match point, has been obtained from the intersection of the 2nd strategy with the 3rd. The way of getting the best value p_1^* is equal to (0.23379378), in addition to the value of the game (v^*= 68053291).

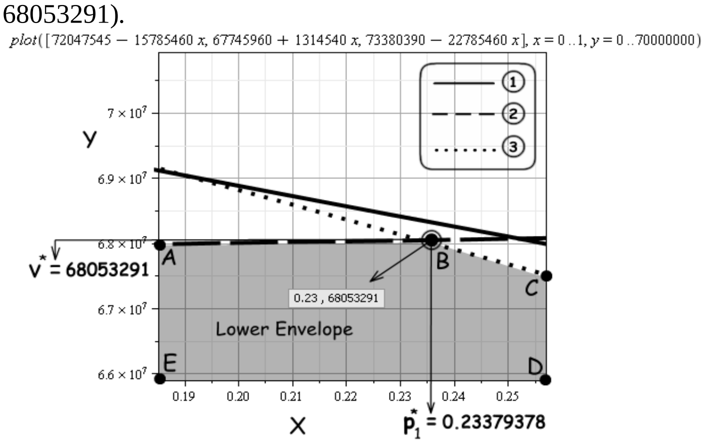

Figure 3: Area of optimal solutions to Asia Cell Company
Source: Created by the researchers using Maple 11 program

Also the researchers can get the game value to substitute the optimal value p_1^* in intersecting lines that formed MaxiMin point (strategies 2 & 3 for Korek company) as shown in table (3):

$$v^* = \begin{cases} 67745960 + 1314540\,p_1 = \\ 67745960 + 1314540 \times 0.23379378 = 68053291.28 \\ 73380390 - 22785460\,p_1 = \\ 73380390 - 22785460 \times 0.23379378 = 68053291.18 \end{cases}$$

$$\because p_1 + p_2 = 1 \Rightarrow p_2 = 1 - p_1 = 0.76620622$$

Thus, the optimal strategy for player [A] which should always be used is the 8th and 9th with probabilities 0.23379378 & 0.76620622 respectively, to get the expected optimal value (higher profit expected) is equal to *v* = 68053291 ID.

On the other hand, player [B] can compete with player [A] through the 2nd and the 3rd strategies that have formed MaxiMin point for player [A]. For this reason, the value of q_1 will be equal to zero because the first strategy of the first line has not participated in determining MaxiMin point. And thus the equations are as follows:

$$\because q_2 + q_3 = 1 \Rightarrow \therefore q_3 = 1 - q_2$$

$$56262085 q_1 + 69060500 q_2 + 50594930 q_3$$

$$56262085(0) + 69060500 q_2 + 50594930 q_3$$

$$18465570 q_2 + 50594930 ...(1)$$

$$72047545 q_1 + 67745960 q_2 + 73380390 q_3$$

$$72047545(0) + 67745960 q_2 + 73380390 q_3$$

$$-5634430 q_2 + 73380390 ...(2)$$

The expected values of player [B] as a result of the selection player [A] strategy for free, are shown in the table (4) below:

Table 4: The free strategies for Asia Cell Company

[A] free strategies for Asia cell	[B] expected value [outcome] for Korek
1	$50594930 + 18465570\ q_2$
2	$73380390 - 5634430\ q_2$

Where MiniMax point can be determined through the intersection above the upright as shown in figure (4). The solutions area (A, B, C, D, E) defined by the intersection of the upright, also called Upper Envelope, and represent possible losses area (area of possible solutions) for the player [B].

Since the player [B] always wants to reduce the large losses, they try to choose the strategies that make them move down direction pocket top area. Through drawing note that the lowest point in the top pocket area is the point (B) under which will get the optimal expected value as the MiniMax value of the game, including the determine the optimal value of q_2^* which is equal to (0.945454771), and the value of q_2^* (using the free strategies in table 4) can also be obtained as follows:

Table 2: Second trial for payoff matrix

[B] Korek Company

		B1	B2	B3	Min. Row	
[A] Asia Cell Company	A8	56262085	69060500	50594930	50594930	
	A9	72047545	67745960	73380390	67745960	←MaxiMin
	Max. Col.	72047545	69060500	73380390		
			↑ MiniMax			

	B1	B2	B3	Min. Row	
A8	56262085	69060500	50594930	50594930	
A9	72047545	67745960	73380390	67745960	←MaxiMin

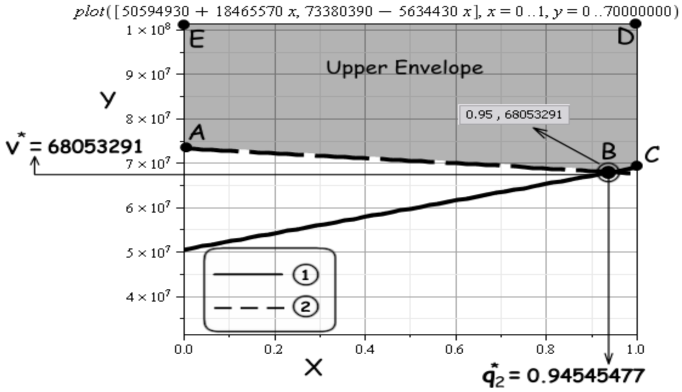

Figure 4: Area of optimal solutions to Korek Company
Source: Created by the researchers using Maple 11 program

$$18465570q_2^* + 50594930 = -5634430q_2^* + 73380390$$

$$\therefore 24100000q_2 = 22785460 \Rightarrow \therefore q_2 = 0.945454771$$

Solving the above equation will provide the value of the probability of the 1st strategy q_2* which is equal to (0.945454771), and the optimal probability for the 2nd strategy q_3* as follows:

$$q_2^* + q_3^* = 1 \Rightarrow 0.94545771 + q_3^* = 1 \Rightarrow q_3^* = 0.054545228$$

Through the MiniMax point for player [B], which has been identified from the intersection of the lines in table (4); the researchers can substitute q_2* in one of the equations straight cross, to obtain the optimum expected value as shown below:

$$v^* = \begin{cases} 18465570q_2^* + 50594930 = 68053291.26 \\ -5634430q_2^* + 73380390 = 68053291.27 \end{cases}$$

2. SOLVING THE PROBLEM USING TORA SYSTEM 7TH PACKAGE

This program has been provided by Hamdy A. Taha (2007) to solve many of the operations research models, including problems of (linear programming, transportation, game theory,...). The graphical method can be used in TORA to solve the research problem, where the steps have been introduced as follows:[3]

1) After entering the payoff matrix in table 5, we can select SOLVE Menu button, and then Graphical method:[4]

Table 5: Payoff matrix for player A

	B1	B2	B3
A1	0.56262085	0.69060500	0.50594930
A2	0.72047545	0.67745960	0.73380390

2) From (Select Output Format), determine the number of digits entered before and after the decimal point and then pressing (Go to Output Screen). Figure (5) shows the area of optimal solutions to Asia Cell Company and the final results for the game problem.

3) Where it is noted that the point has been obtained from the first intersection of lines with MaxiMin represent the second match point, and through it the optimal value p_1* is obtained which is equal to (0.2337938). In addition the value of the game (v* = 0.6805329) have been received.

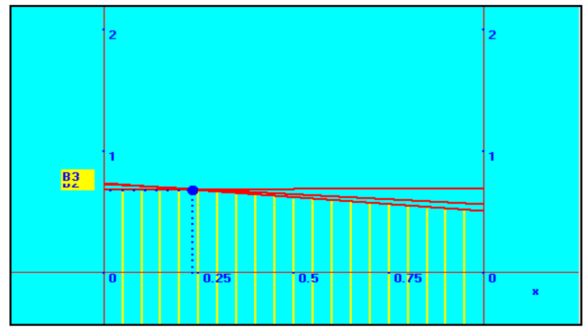

Figure 5: Area of optimal solutions to Asia Cell Company

Zero-Sum Game results:
Player A's expected payoffs:
Strategy B1: 0.72047545 – 0.15785460x
Strategy B2: 0.67745960 + 0.01314540x
Strategy B3: 0.73380390 – 0.22785460x
Optimal Solution: Value of the game = 0.68053291
Player A's mix:
Strategy A1: 0.23379378
Strategy A2: 0.76620622
Player B's mix:
Strategy B1: 0.00000000
Strategy B2: 0.94545477
Strategy B3: 0.05454523

XII. CONCLUTIONS

This research has formulated a matrix of competition between Korek and Asia Cell telecom companies by studying the strategies used by each company. This has been accomplished by providing suggestions for each company related to using best strategies to maximize the profit. The payoff matrix size was (9×5) represented by 9 rows related to the strategies adopted by Asia Cell Company, and 5 columns related to the strategies adopted by Korek Company. The manual method show that there is no saddle point for the payoff matrix where: MaxiMin ≠ MiniMax (69060500 ≠ 67745960). This means that the value of the game lies between: 67745960 ≤ v ≤ 69060500. To solve this matrix the researchers have reduced it to two rows and three columns, using the principle of control and the same game value have been received. Here the arithmetic method of payoff matrix cannot be used because the resulting payoff matrix of type (2×3). So the graphical method was used. As a conclusion, it has been found that the optimal strategy for player [A] which should always be used is the 8th (selling the International Line) and 9th (selling the New Khalat Line) with probabilities 0.23379378 & 0.76620622 respectively from competition time to get the expected optimal value (higher profit expected) is equal to v = 68053291 ID. While, the optimal strategy for player [B] which should always be used is the 2nd (selling the Data Offers) and 3rd (selling Diary Line) with probabilities 0.945454771 &

[3] A. T. Hamdy, "Operations Research: An Introduction", by Pearson Education, Inc., 8th edition, 2007.

[4] The maximum digit accepted by TORA program is 7 degrees, but all numbers in payoff matrix has 8 degree, se we divided each number in payoff matrix by 10^{-8} to be acceptable and that will not effect on the original value.

0.054545228 respectively from competition time to get the expected optimal value (which will bring less losses) is equal to v = 68053291 ID. TORA package gave results close to the manual method for the two companies; this confirms the validity of the results that have been obtained.

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