Heat Transfer in a channel bounded by a stretching sheet and Partially Filled with Porous Medium

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Abstract- The study in this paper involves the steady two dimensional flows and heat transfer through a channel bounded by a stretching sheet and a highly porous layer with impermeable bottom. The expressions for thestream function, velocity, temperature distribution, coefficient of skin friction and the rates of heat transfer at the lower permeable surface and the upper sheet, have been obtained and discussed.
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Index Terms- Incompressible fluid, Impermeable, Porous, Stretching sheet

I. INTRODUCTION

The flow due to a stretching sheet has many scientific and engineering applications, and therefore, it was investigated by researchers. Flow in the boundary layer on a continuous solid surface with constant speed was studied by sakiadis (1961).McCormack and crane (1973), explained its importance in detail, in many situations arising in industries where it is essential to account the stretching of the sheet. Crane (1970),Wang (1984) studied flow past a stretching of Newtonian fluid, whereas Ali (1994,1996), Magyari and Keller (199b) studied the temperature distribution in boundary layer over a stretching sheet. Mc Leod and Rajgopal (1987) worked on the uniqueness of flow of a Navier Stokes fluid due to stretching boundary. Ariel (1995) gave a second solution of flow of viscoelastic fluid over a stretching sheet. Magyari and Kellar (2000) investigated the exact solution for the self -similar two dimensional steady boundary layer flows induced by permeable stretching walls. Many researchers such as Kumaran,Tiruchirappalli and Ramanaiah(1996),Raptics, C. Perdikis(2006)Ali and Al-Yousef (2002), Ishak, Amin and Pop(2004), Liu (2006), Ishak, Nazar and Pop (2008,2009),Makinde and Aziz(2011),etc. investigated flows in similar boundaries and matching conditions, whose one wall is a stretching sheet. In viscous flows through and across porous media,the heat transfer was much investigated, however, in a stretching channel the literature is scanty when the one boundary wall is highly porous layer.
The study in this paper involves the steady two dimensional flows and heat transfer through a channel bounded by a stretching sheet and a highly porous layer with impermeable bottom. At the fluid porous interface a modified set of boundary conditions, is applied taking effective medium considerations in the permeable layer). The expressions for the velocity, temperature distribution, stream function, Pressure coefficient and the rates of heat transfer at the lower permeable surface and the upper sheet, have been obtained and discussed.

II. FORMULATION OF PROBLEM

A viscous incompressible fluid is confined between a stretching sheet and a highly porous layer of thickness ‘a’, with impermeable bottom. A Cartesian coordinate system is used with origin at the lower porous interface and axis of y normal to it. The upper stretching sheet is at y = h, and it is stretched by introducing two equal and opposite forces so that the position of the point (0, h) on the sheet remains unchanged. The lower porous layer is saturated with fluid. It is impermeable bottom and the upper sheet, are maintained at constant temperatures, T_1 and T_2, respectively. All the variables are assumed to be independent of z.
The momentum and energy equations in free fluid region $(0 \le y \le h)$ are :

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

$$\rho Cp\left(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}\right)u=\kappa\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}\right)+\mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right\}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right] \tag{3}$$

And the equation of continuity

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (4)$$

The momentum and energy equations in the porous region $(-a \le y \le 0)$ are :

$$\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{\upsilon U}{K_0}+\frac{\bar{\mu}}{\rho}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)=0 \quad (5)$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y}-\frac{\upsilon V}{K_0}+\frac{\bar{\mu}}{\rho}\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)=0 \quad (6)$$

$$\rho Cp\left(U\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)u=\bar{\kappa}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+ \bar{\mu}\left[2\left\{\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial y}\right)^2\right\}+\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)^2\right]+\frac{\mu U^2}{K_0} \quad (7)$$

And the equation of continuity is

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \quad (8)$$

Where, u, v and U ,V are the fluid velocity components along the x and y directions in the free fluid and porous regions respectively, p and P are the pressures in the free fluid and porous regions, respectively; t and T are the temperature in the free fluid and porous region respectively; ρ, is the density of the fluid; μ ,the viscosity; υ ,kinematic viscosity; $\bar{\mu}$, the effective viscosity in the porous medium; κ ,the thermal conductivity; $\bar{\kappa}$,the effective thermal conductivity; and Cp, is the specific heat on constant pressure.

The boundary conditions are:

At y = h , u = cx, v =0 , t = T_2

At y = 0 , p = P, u =U, v = V, $\mu\frac{\partial u}{\partial x}=\bar{\mu}\frac{\partial U}{\partial y}$, t =T , $\kappa\frac{\partial t}{\partial y}=\bar{\kappa}\frac{\partial T}{\partial y}$,

At y = -a , U = V=0, t = T_1 (9).

III. SOLUTION OF PROBLEM

We Assume

$$u=Cxf'(\eta),\quad v=Chf(\eta),\quad \eta=\frac{y}{h}$$

$$U=CxF'(\eta),\quad V=-ChF(\eta),$$

$$t=T_1+\frac{C^2h^2}{RC_p}\left[s(\eta)+\frac{x^2g(\eta)}{h^2}\right],$$

$$T=T_1+\frac{C^2h^2}{RC_p}\left[S(\eta)+\frac{x^2G(\eta)}{h^2}\right],\quad R=\frac{Ch^2}{\upsilon} \quad (10)$$

Substituting (9) into (1) to (8) and solving, we get

$$f'''-R\left(f'^2-ff''\right)=A(\text{Constant}), \quad (11)$$

$$\phi_1^{-1}F'''-\beta^{-1}F'=D(\text{Constant}), \quad (12)$$

And

$$s''=-\left[2g+R\Pr\left(4f'^2+fs'\right)\right], \quad (13)$$

$$g''=R\Pr\left[\left(2f'g-fg'\right)-f''^2\right], \quad (14)$$

$$S''=-\left[2G+R\Pr\phi_2\left(4\phi_1^{-1}F'^2+FS'\right)\right], \quad (15)$$

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$$G'' = R\Pr\phi_2\left[\left(2F'G - FG'\right) - \phi_1^{-1}F''^2 - \beta^{-1}F'^2\right], \tag{16}$$

Where, a prime denotes a differentiation with respect to η

$$\Pr = \frac{c_p \rho \upsilon}{k}, \quad \phi_1 = \frac{\mu}{\bar{\mu}}, \quad \phi_2 = \frac{k}{\bar{k}} \text{ and } \beta = \frac{k_0}{h^2} \tag{17}$$

The corresponding boundary conditions are:

$$\begin{aligned}
&\text{At } \eta = 1, \; f' = 1, \; f = 0, \; s = \phi, \; g = 0,\\
&\text{At } \eta = 0, \; f' = F', \; f = F, \; \phi_1 f'' = F'', \; s = S, g = G, \; \phi_2 s' = S', \; \phi_2 g' = G',\\
&\text{At } \eta = -a^*, \; F' = 0, F = 0, \quad s = 0, g = 0,
\end{aligned} \tag{18}$$

Where $a^* = \frac{a}{h}$, and $\phi = \frac{1}{R\,Ec.}$

For small values of the stretching parameter R, a regular perturbation scheme can be developed by expanding of f, F, A, D, s, g, S,G in ascending power of R as.

$$f(\eta) = \sum_{n=0} R^n f_n(n), \qquad A = \sum_{n=0} R^n A_n,$$

$$\begin{aligned}
&F(\eta) = \sum_{n=0} R^n F_n(\eta), \qquad D = \sum_{n=0} R^n D_n,\\
&s(\eta) = \sum_{n=0} R^n s_n(\eta), \qquad g(\eta) = \sum_{n=0} R^n g_n(\eta),\\
&S(\eta) = \sum_{n=0} R^n S_n(\eta), \qquad G(\eta) = \sum_{n=0} R^n G_n(\eta),
\end{aligned} \tag{19}$$

Using (19) in equation (11) to (16), comparing coefficients of like powers of R,and on solving under the corresponding boundary conditions, We obtain

$$f(\eta) = f_0(\eta) + Rf_1(\eta)$$

$$= \frac{A_0\eta^3}{6} + \frac{a_1\eta^2}{2} + a_2\eta + a_3 + R\left[\frac{A_0{}^2\eta^7}{2520} + \frac{A_0 a_1\eta^6}{360} + \frac{a_1^2\eta^5}{120} + \frac{(a_1a_2 - a_3A_0)\eta^4}{24} + \frac{\left(A_1 + a_2^2 - a_1a_3\right)\eta^3}{6} + \frac{b_1\eta^2}{2} + b_2\eta + b_3\right]$$

$$F(\eta) = F_0(\eta) + RF_1(\eta)$$

$$= \frac{c_1e^{\sigma\eta}}{\sigma} - \frac{c_1e^{-\sigma\eta}}{\sigma} - D_0\beta\eta + c_3 + R\left[\frac{m_1e^{\sigma\eta}}{\sigma} - \frac{m_2e^{-\sigma\eta}}{\sigma} - D_1\beta\eta + m_3\right]$$

$$s(\eta) = s_0(\eta) + Rs_1(\eta)$$

$$= \frac{\eta + \phi_2 a}{REc\left(\phi_2 a + 1\right)} + R\left[-\frac{A_0{}^2\eta^6}{36} - \frac{20\left(A_0a_1 + h_1A_0\right)\eta^5}{120} - \frac{\left(6a_1^2 + 8A_0a_2 + h_1a_1\right)\eta^4}{24} - \frac{\left(8a_1a_2 - h_1a_2\right)\eta^3}{6} + a_2^2\eta^2 - \frac{L_1\eta^3}{3} - L_2\eta^2 + M_1\eta + M_2\right]$$

$$S(\eta) = S_0(\eta) + RS_1(\eta)$$

$$S(\eta) = S_0(\eta) + RS_1(\eta)$$

$$= \frac{\phi_2(\eta + a)}{REc(\phi_2 a + 1)} + Rpr[\phi_2[(\frac{1}{\beta\sigma^2} - 7\phi_1^{-1})\frac{(c_1^{\,2}e^{2\sigma\eta} + c_2^{\,2}e^{-2\sigma\eta})}{8\sigma^2} - \{A_0\beta(\frac{4}{\beta\sigma^2} - 8\phi_1^{-1}) - \frac{\phi_2 h_1}{\sigma}\}\frac{c_2 e^{-\sigma\eta}}{\sigma^2}$$

$$-\{A_0\beta(\frac{4}{\beta\sigma^2} - 8\phi_1^{-1}) + \frac{\phi_2 h_1}{\sigma}\}\frac{c_1 e^{-\sigma\eta}}{\sigma^2} - 2c_1c_2(\phi_1^{-1}\sigma^2 - \frac{1}{\beta})\frac{\eta^4}{12} + A_0^{\,2}\beta\frac{\eta^4}{12} + \phi_2 h_1 A_0 \beta \frac{\eta^3}{6}$$

$$-4\phi_1^{-1}(A_0^{\,2}\beta^2 + 2c_1c_2)\frac{\eta^2}{2} - \phi_2 h_1 c_3 \frac{\eta^2}{2}] - \frac{L_3\eta^3}{3} - L_4\eta^2 + M_3\eta + M_4]$$

$$g\left(\eta\right) = g_0(\eta) + Rg_1(\eta)$$

$$= -R\,\mathrm{Pr}\left[A_0^{\,2}\frac{\eta^4}{12} + A_0 a_1\frac{\eta^3}{3} + a_1^{\,2}\frac{\eta^2}{2} - L_1\eta - L_2\right]$$

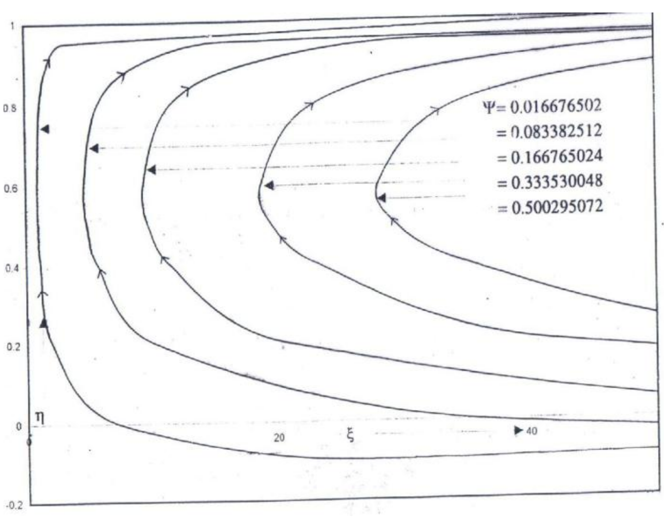
$$G(\eta) = G_0(\eta) + RG_1(\eta) - R\,\mathrm{Pr}[\phi_2\{\phi_1^{-1}\sigma^2(\frac{c_1^{\,2}e^{2\sigma\eta} + c_2^{\,2}e^{-2\sigma\eta}}{4\sigma^2} - c_1c_2\eta^2 + \frac{1}{\beta}(\frac{c_1^{\,2}e^{2\sigma\eta} + c_2^{\,2}e^{-2\sigma\eta}}{4\sigma^2}$$

$$+(A_0^{\,2}\beta + 2c_1c_2)\frac{\eta^2}{2} - \frac{2A_0\beta}{\sigma^2}(c_2e^{-\sigma\eta} + c_1e^{\sigma\eta}))\} - L_3\eta - L_4]$$

The constant of integration $L_1, L_2, L_3, L_4, M_1, M_2, M_3, M_4, D_1, D_2, D_3, D_4, D_4, D_5, D_6, D_7, D_8, D_9$ and D_{10} are obtained by the corresponding boundary conditions and are not reported for the sake of brevity

IV. DISCUSSION

Figure 1. shows the streamline patterns for the flow due to a stretching sheet in the channel bounded by a highly porous layer. Due to the stretching of the upper deformable sheet of the channel, the fluid near it is thrown out axially, causing an adverse pressure gradient developed at a large distance and to keep continuity the fluid rushes from infinity towards the mouth of the channel through lower porous medium.



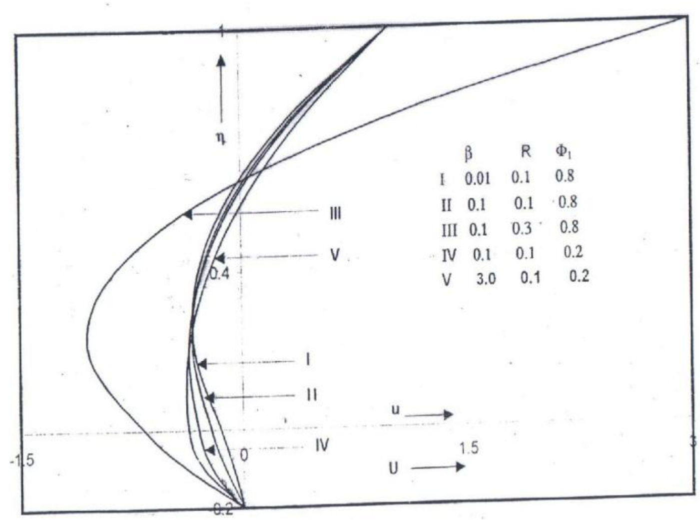


Fig. 1. Stream lines pattern for R=0.1, $\beta = 0.1, \phi_1 = 0.8$

Fig. 2. u vs η *for* $\xi = 10$

Figure 2. shows the longitudinal velocity profiles, u, for various parameters. The stretching of the upper sheet induces a backward flow in the channel it is found that the stretching parameter are increases u near the upper plate, however, there is a back flow near the lower porous surface and also inside the porous medium which increases by increasing R.This back flow in the channel also shift towards the upper plate by increasing R. The permeability parameter β also increases the flow near the upper plate and the back flow near the porous surface and inside porous medium, however, back flow decreases in the middle by increasing β by decreasing the viscosity ratio ϕ_1 ,back flow also increases.

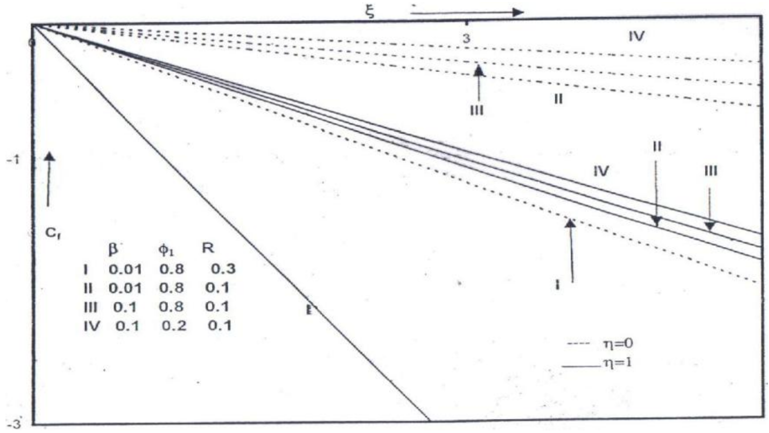
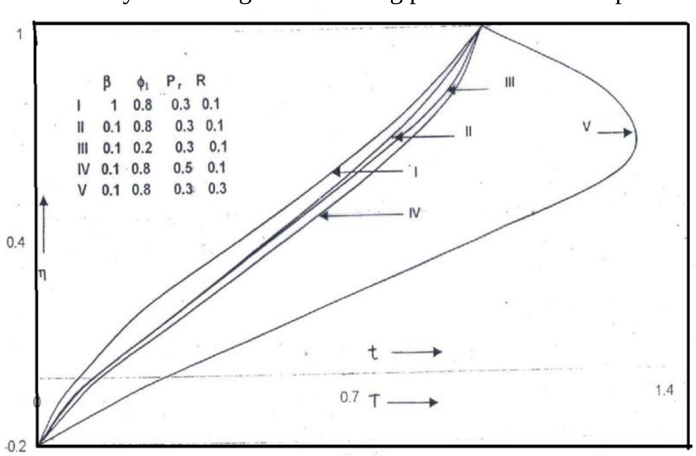


Fig. 3. C_f vs ξ

Fig.3 shows the variation of the coefficient of skin-friction C_{f0} and C_{f1}. It is observed that both C_{f0} and C_{f1} decrease numerically by increasing permeability of the porous medium, however, both increase substantially by increasing R. By increasing ϕ_1 both C_{f0} and C_{f1} also increase.

Fig.4 and Fig. 5 show the variation of temperature θ , and the rate of heat transfer at both walls of the channel for various parameters. It is found that the permeability of the porous matrix is to decrease the temperature in the channel, while temperature in the channel is increases by increasing the stretching parameter R or the prandtl number Pr.



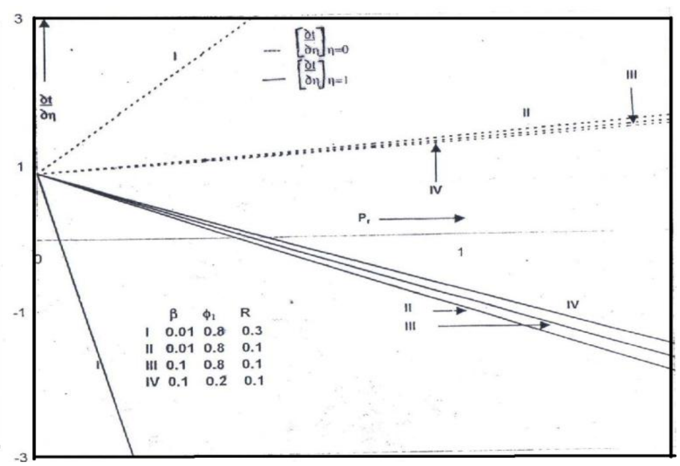


Fig.4. Temperature distribution for $\phi_2 = 0.6, E_c = 0.7 and \xi = 10.$ Fig.5.Rate of heat transfer for $\phi_2 = 0.6, E_c = 0.7 and \xi = 10.$

It decreases by increasing ϕ_1 . It is observed that the rate of heat transfer at the porous surface decreases by increasing β ,whereas it increases by increasing R and ϕ_1 . The rate of heat transfer at the upper wall increases by increasing β for small values of Pr, as Pr increases it changes sign at certain Pr and then decreases numerically by increasing β ,whereas at small Pr, it decreases by increasing R or ϕ_1 , then at certain Pr changes sign and increases numerically afterwards. The result may find applications in the polymer industry, bio-engineering, and many other industrial manufacturing processes, such as hot rolling, glass fiber and paper production and drawing the plastic films.

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