Minimum Cost Spanning Tree using Matrix Algorithm

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Abstract- A spanning tree of a connected graph is a sub graph that is a tree and connects all the verticestogether. A single graph can have many different spanning trees. We can also assign a weight to each edge, which is a number representing how unfavorable it is, and use this to assign a weight to a spanning tree by computing the sum of the weights of the edges in that spanning tree. A minimum spanning tree (MST) or minimum weight spanning tree is then a spanning tree with weight less than or equal to the weight of every other spanning tree. More generally, any undirected graph (not necessarily connected) has a minimum spanning forest, which is a union of minimum spanning trees for its components. Our objective is to find minimum cost (weight) spanning tree using the algorithm which is based on the weight matrix of weighted graph.

Index Terms- Simple Graph, Weight Graph, Minimum Cost Spanning Tree.

I. Introduction

A minimum cost of the spanning tree is spanning tree but it has weight or length associated with the edges and total weight of the tree is minimum. For example a telecommunications company laying cable to a new neighborhood. If it is constrained to bury the cable only along certain paths, then there would be a graph representing which points are connected by those paths. Some of those paths might be more expensive, because they are longer, or require the cable to be buried deeper; these paths would be represented by edges with larger weights. A spanning tree for that graph would be a subset of those paths that has no cycles but still connects to every house. There might be several spanning trees possible. A minimum spanning tree would be one with the lowest total cost.

In this paper our objective is to find the minimum cost spanning tree using the matrix algorithm based on the weight matrix of the weighted graph (cost).[1].

II. BASIC DEFINITIONS AND APPLICATIONS

Definition 2.1

Spanning tree

A tree T is said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all vertices of G.

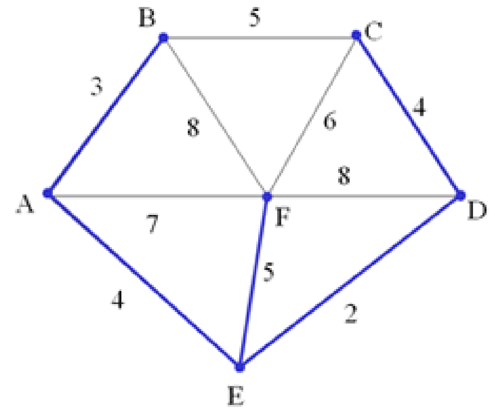

Figure 1: spanning tree

Definition 2.2:

Minimum Spanning Tree

A *minimum spanning tree (MST)* of an edge-weighted graph is a spanning tree whose weight (the sum of the weights of its edges) is no larger than the weight of any other spanning tree.

Application:

NetworkDesign.

The standard application is to a problem like phone network design. You have a business with several offices; you want to lease phone lines to connect them up with each other; and the phone company charges different amounts of money to connect different pairs of cities. You want a set of lines that connects all your offices with a minimum total cost. It should be a spanning tree, since if a network isn't a tree you can always remove some edges and save money.

The another one application would be, Suppose, we have a group of islands that we wish to link with bridges so that it is possible to travel from one island to any other in the group, the set of bridges which will enable one to travel from any island to any other at minimum capital cost to the government is the minimum cost spanning tree.

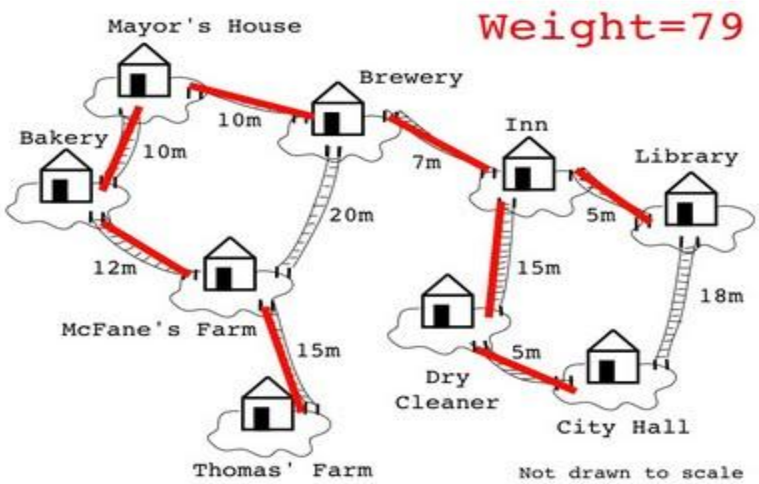

Figure 2: Network

III. CONSTRUCTION OF MINIMUM COST SPANNING TREE

Let, G = (V, E) be an undirected connected weighted graph with n vertices, where V is the set of vertices, E is the set of edges and W be the set of weights (cost) associated to respective edges of the graph. Where

$e_{ij}$the edge adjacent to vertices $v_i$and v_j.

w_{ij}= the weight associated to the edge e_{ij}.

The Weight Matrix M of the graph G is constructed as follows: If there is an edge between the vertices $v_i$to $v_j$in G then Set,

$M_{[i,j]} = w_{ij}$

Else

Set, $M_{[i,j]} = 0$

Algorithm:

Input: the weight matrix $M = [w_{ij}]_{n \times n}$ for the undirected weighted graph G

Output: Minimum Cost Spanning Tree T of G.

Step 1: Start

Step 2: Repeat Step 3 to Step 4 until all (n-1) elements matrix of M are either marked or set to zero or in other words all the nonzero elements are marked

Step 3: Search the weight matrix M either column-wise or row-wise to find the unmarked nonzero minimum element $M_{[i,j]}$, which is the weight of the corresponding edge eij in M.

Step 4: If the corresponding edge eij of selected $M_{[i,j]}$ forms cycle with the already marked elements in the elements of the M then Set $M_{[i,j]} = 0$ Else Mark $M_{[i,j]}$

Step 5: Construct the graph T including only the marked elements from the weight matrix M which shall be the desired Minimum cost spanning tree of G.

IV. NUMERICAL EXAMPLE

Consider the following graph and its shows the various steps involved in the construction of the minimum cost spanning tree.

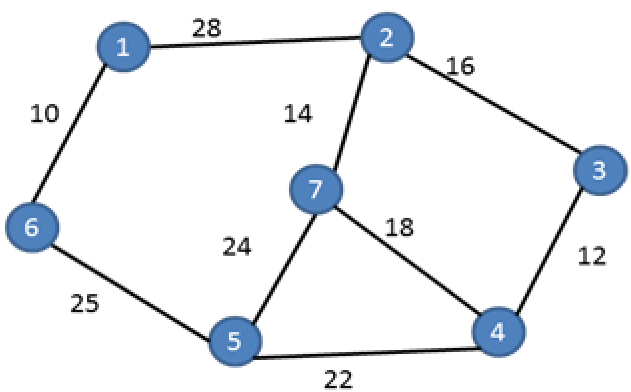

Figure 3: Graph G

$$\begin{bmatrix} 0 & 28 & 0 & 0 & 0 & 10 & 0 \\ 28 & 0 & 16 & 0 & 0 & 0 & 14 \\ 0 & 16 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 22 & 0 & 18 \\ 0 & 0 & 0 & 22 & 0 & 25 & 24 \\ 10 & 0 & 0 & 25 & 0 & 0 & 0 \\ 0 & 14 & 0 & 18 & 24 & 0 & 0 \end{bmatrix}$$

Figure 4 : Matrix of Graph G

From Figure. 3 Graph G = (V, E) where V is the set of vertices and E represents the edge with weigh are also given. Here V=7 and Edge = 9.

Fig 4 represents the adjacency matrix for the given graph.

0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	18
0	0	0	22	0	25	24
10	0	0	25	0	0	0
0	14	0	18	24	0	0

Figure 5: Graph G

From Fig.5. The minimum element 10 is selected and the corresponding edges (1, 6) are also marked by the color pink. Repeat the process till the iteration will exist

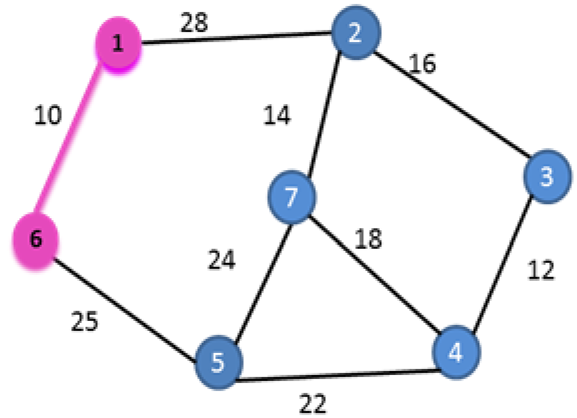

Figure 6: weight matrix

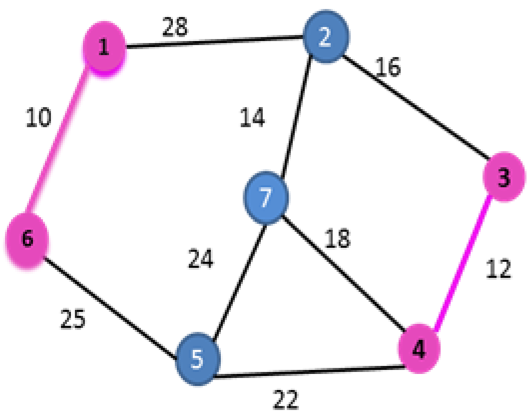

Figure 7:

0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	18
0	0	0	22	0	25	24
10	0	0	25	0	0	0
0	14	0	18	24	0	0

Figure. 8: weight matrix

From Fig 7 and 8 the next non zero minimum element 12 is marked and corresponding edges are also colored.

0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	18
0	0	0	22	0	25	24
10	0	0	25	0	0	0
0	14	0	18	24	0	0

Figure 9

The next non zero minimum element 14 is marked it is shown in the Fig 9. The corresponding marked edges are shown in below Fig 10.

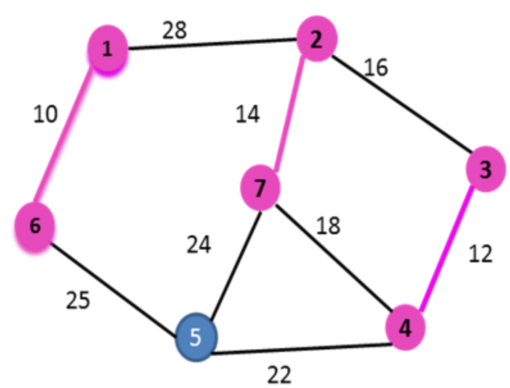

Figure: 10

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0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	18
0	0	0	22	0	25	24
10	0	0	25	0	0	0
0	14	0	18	24	0	0

Figure 11: adjacency matrix

From Fig 11. the next minimum element 16 is marked.

From Fig 12 the next minimum non zero element18 is marked. But while drawing the edges it forms the circuit so we remove and mark it as 0 instead of 18.

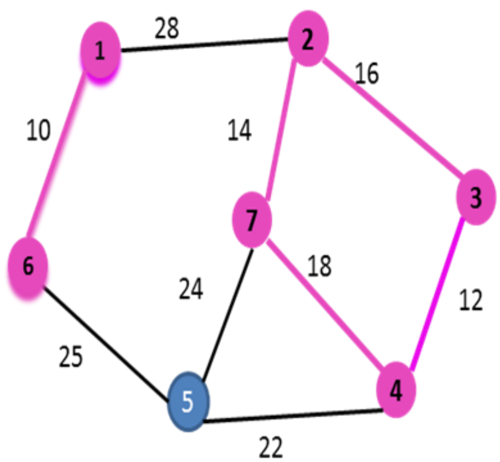

Figure 13: circuit {2, 3, 4, 7}

The next minimum element is 24 while marking that its forms a circuit so we mark 0 instead of 24 and we move to the final marking process that is the minimum element is 25 is shaded and edges are colored.

0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	~~18~~ 0
0	0	0	22	0	25	~~24~~ 0
10	0	0	25	0	0	0
0	14	0	18	24	0	0

Figure 15: adjacency matrix

0	28	0	0	0	10	0
28	0	16	0	0	0	14
0	16	0	12	0	0	0
0	0	12	0	22	0	~~18~~ 0
0	0	0	22	0	25	24
10	0	0	25	0	0	0
0	14	0	18	24	0	0

figure 12: adjacency matrix

Next we go to next minimum element that is 22 it is marked and shown in same Fig 13. and the both marking process of the edges 18 and 19 were shown below figs.

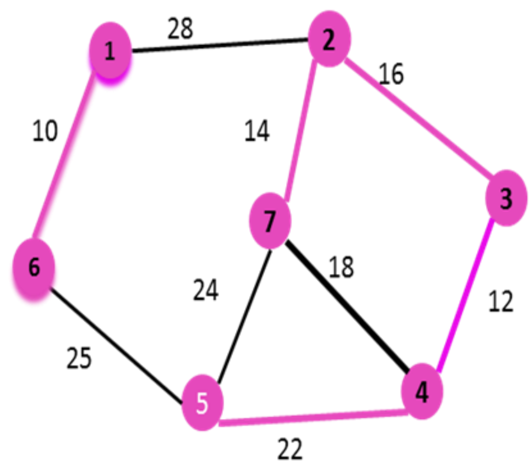

Figure. 14

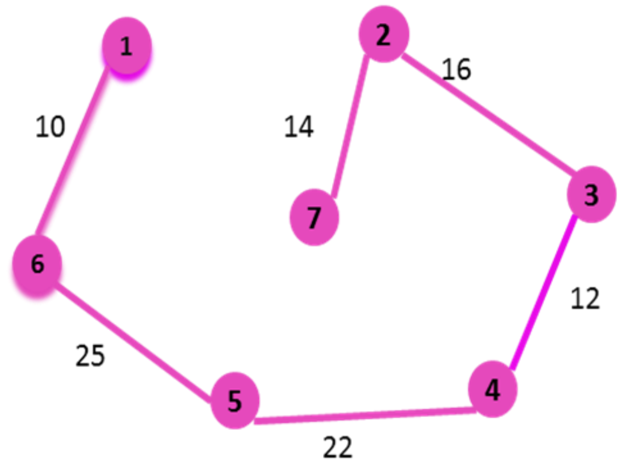

Figure 16 : final stage

We have to study the minimum cost spanning tree using the Matrix Algorithm and find the minimum cost is 99 [1]. So the final path of minimum cost of spanning is {1, 6}, {6, 5}, {5, 4}, {4, 3}, {3, 2}, {2, 7}.

V. CONCLUSION

The paper presented a new simple efficient technique to find the minimum cost spanning tree of an undirected connected weight graph. As there are several application s of minimum spanning tree. Algorithm presented here is based entirely on adjacency matrix. Using the matrix algorithm we Find the minimum cost is 99 so the final path of minimum cost of spanning tree is {1, 6}, {6, 5}, {5, 4}, {4, 3}, {3, 2}, {2, 7}. In future we shall concentrate to solve other constrained spanning tree problems using matrix algorithm

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