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Every *u-v* path of NP-complete Steiner graphs contains exactly 2n-edges

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Abstract- Complexity theory has many facts. Its motivations and goals however, are similar; to determine the computational "difficulty" or complexity of a problem. In this work, we propose an NP- completeness proof for the Steiner tree problem in graphs

Index Terms- Steiner problem in graphs- NP-complete-3-satisfiability- Clause

I. Introduction

For many decision problems, Steiner Problem in Graphs and satisfiability, no polynomial time algorithm is known. Nevertheless some of these problems have a property which is not inherent to every decision problem, there exists algorithm which, if presented with an instance of the problem.(i.e., a graph G, a terminal set K, and a bound B, respectively a Boolean formula F) and in addition with a potential solution x (i.e., a sub graph T of G, respectively a truth assignment τ for the variables in F) these algorithms verify in polynomial time whether x is a valid solution (i.e. whether T is a Steiner tree for K-terminals contains at most B edges, respectively whether τ satisfies F). The decision problems with this property form the NP. This abbreviation comes from Non deterministic polynomial time. A very important concept in complexity theory is the concept of reducibility. It allows showing that one problem is at least as different as another one.

The Steiner tree problem in graph is called for brevity ST, defined in decisional form as follows

* an undirected graph G = (V, E)
* a subset of the vertices R⊆V, called terminal nodes.
* a number K € N.

There is a subtree of G that includes all the vertices of R (i.e. a spanning tree for R) and that contains at most k edges.

This problem has many applications especially when we have to plan a connecting structure among different terminal points. For example, when we want to find an optional way to build roads and railways to connect, a set of cities or decide routing policies over the internet for multicast traffic, usually from a source to many destinations. Unfortunately, this problem has shown to be intractable in the sense that there exists no polynomial algorithm to Solve it, unless P = NP. The goal of this exercise is to propose an NP – Completeness, proof for the Steiner tree problem.

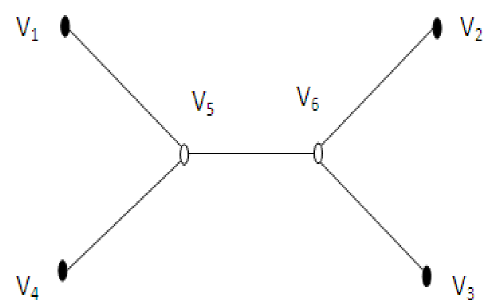
II. Preliminaries

Definition **2.1:** Let a connected graph G= (V, E) and a set K ⊆ V of terminals. Then the Steiner minimum tree for K in G that is a Steiner tree T for K such that
$|E(T)| = \min \{|E(T^I)| / T^I$ is a steiner tree for K in G$\}$

In the Steiner minimal tree problem, the vertices are divided into two parts, terminals and non terminal vertices.

The terminals are the given vertices which must be included in the solution.

Example 2.1:



V_1, V_2, V_3 and V_4 are terminals and V_5 and V_6 is non terminals.

Fig.1: Steiner minimal tree

Definition 2.2:

A tree is a connected graph which is acyclic. A forest is an acyclic graph. A tree is connected acyclic graph. A leaf is a vertex of degree.

Example 2.2:

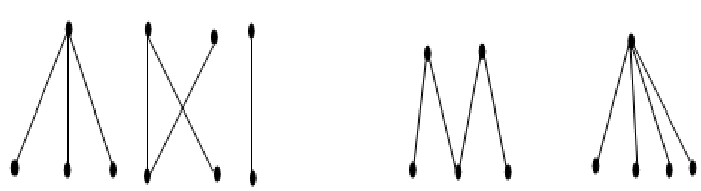


Fig.2: Tree

A tree is a connected forest and every component of a forest is a tree.

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Definition 2.3:

A tree is said to be a spanning tree of a connected graph G, if T is a sub graph of G and T contains all vertices of G.

Example 2.3:

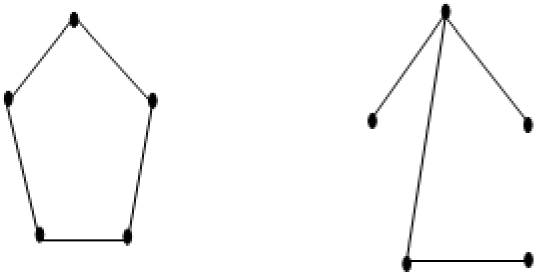


Fig.3: Spanning Tree

Definition 2.4:

Let G be a connected graph each of whose edges is assigned a number called the cost or weight. We denoted the weight of an edge e of G by w(e). Such a graph is called weighted graph. For each sub graph H of G, the weight W of H is defined as the sum of the weight of its edges that is W (H) = $\sum_{e \in E(H)} w(e)$

We seek a spanning tree of whose weight is minimum among all spanning tree of G. Such a spanning tree is called minimum spanning tree.

Example 2.4:

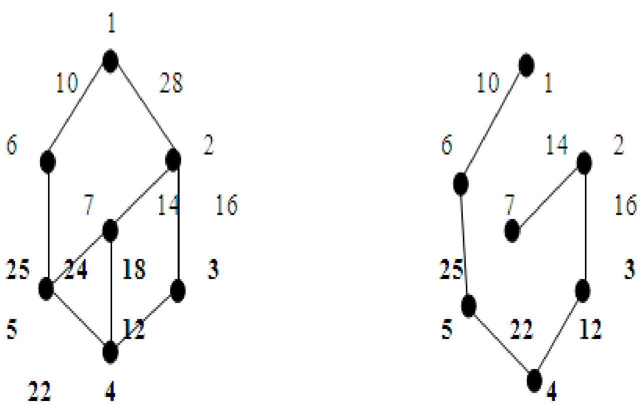
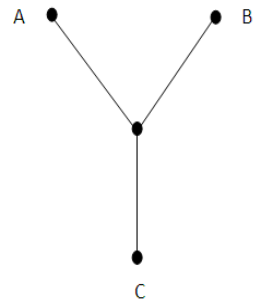


Fig.4: Weighted Graphs

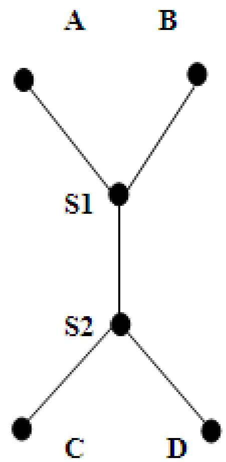
Definition 2.5:

A Steiner tree is a tree in a distance graph which spans a given subset of vertices (Steiner point with the minimum total distance on its edges)

Example 2.5:



Steiner tree for three points A, B and C [note there is no direct connection between A, B, and C.]



Solution for four points [note that there are two Steiner points s_1 and s_2]

Fig.5: Steiner Tree

Definition 2.6:

A formula is satisfiable if there is an assignment of truth values to the variables that makes every clause tree.

Definition 2.7:

A set of logical variables u = {uj} and a set c={ci} of clauses where each clauses consists of three literal being a variable ui or its negations ui.

Definition 2.8:

The class of problems solvable by non-deterministic polynomial time algorithm is called NP.

Definition 2.9:

A problem is NP-complete if

1. It is an element of the class NP
2. Another NP –complete problem is polynomial time reducible to it.

III. EVERY U-V PATH OF NP-COMPLETE STEINER GRAPH CONTAINS EXACTLY 2N EDGES.

Result: 1

Steiner problems in graph is NP complete

Result: 2

Every u-v path of NP-complete Steiner graph contains exactly 2n edges.

Theorem 3.1:

Every u-v path of NP-complete Steiner graph contains exactly 2n edges, if the Steiner problem in graph is NP-complete

Proof**;** Let the Steiner problem in graphs is € NP, it sufficient to show that Steiner problem in graphs is in fact NP-complete.

To see this, we reduce 3 SAT to Steiner problem in graphs.

Let x_1,x_2,x_3-------x_n be the variables c_1,c_2,----c_m the clauses in an arbitrary instance of 3SAT.Our aim is to construct a graph G=(V, E) a terminals set K, and a bound B such that G contains Steiner tree T for k at size at most B if and only if the given 3SAT instance is satisfiable

The graph G is constructed as follows. First we connect two vertices u and v by a variable path as shown in the figure 6. First we consider taking the variable x_1 to x_{10}, and then we connect two vertices u & v by a variable path as shown in figure 6.

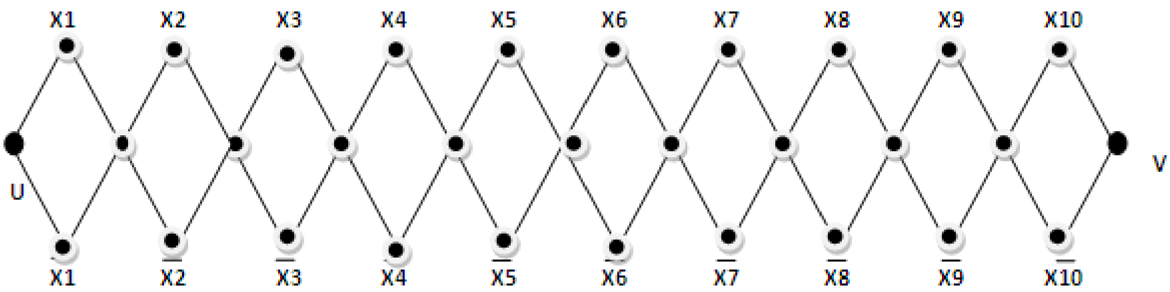


Fig.6: Transforming 3SAT to Steiner problem in graph: the variable path

Clause gadget consisting of a c_i vertex connected to the literals contained in the clause c_i by path of length t=2n+1

As terminal set we choose K= {u, v} U {c_1 ----c_6} and set B = 2n+t.m.

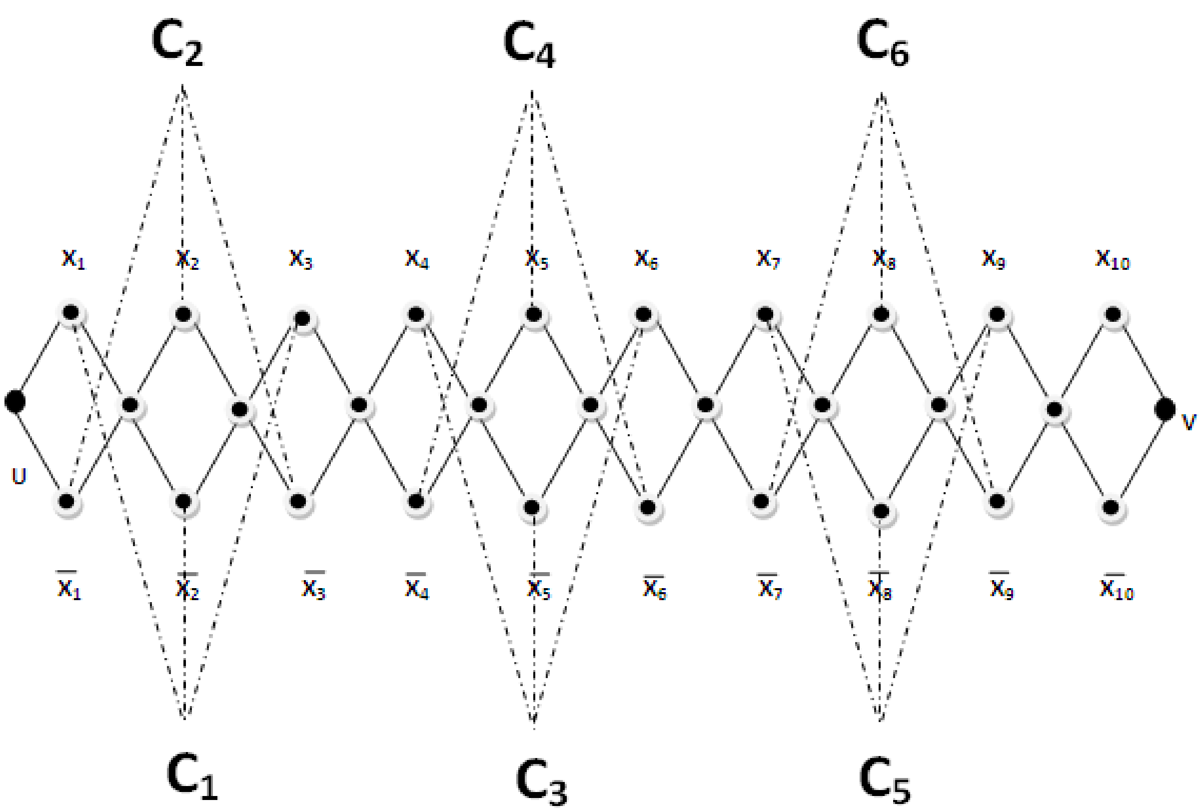


Fig. 7. The clause gadget for the clause C_i=-x_1 Vx_2, Vx_3. The dashed lines indicate paths of length t=2n+1 from c_i to the appropriate variable path.

Now we take n =10. To form the clauses {c_1, c_2, c_3, ----c_6} and the terminals set K = {u, v} Ụ {c_1 … c_6} and set B = 2n+t.m
B=2n+t.m
t =2n+1
t =2(10) +1
=21
B=2(10) +21(6)
=20+126
=146

Assume first that the 3SAT instance is satisfiable. To construct a Steiner tree for K. we start with a u-v path p reflecting a satisfying assignment.

We let x_i € p if x_i is set to true in this assignment, x_i € p otherwise. Next observe that for every clause the vertex c_i can be connected to P by a path of length t. In this way we obtain a Steiner tree for K of length 2n+t.m=B.

To see the other direction assume now that T is a Steiner tree for K of length at most B. Trivially, for each clause to the vertex c_i has to be connected to the variable path.

Then $|E(t)| \geq (m+1).t > B$
$|E(t)| \geq (6+1).21>B$
$\geq 7.21>146$
$\geq 147>146$

So this can't be. This shows that u and v can only be connected along the variable path, which requires at least 2n edges.

In this graph u-v path contain 20 edges and that each clause gadget is connected to this path using exactly t edges. Thus the u-v path reflects a satisfying assignment.

Result: 1. Steiner problems in graph is NP complete

Result: 2. every u-v path of NP-complete Steiner graph contains exactly 2n edges.

Result: 3. if u & v are two arbitrary verities of a NP-complete graph with n-variables then the u-v path contains exactly 2n edges.

Result: 4. structure of the Steiner minimum tree is simple.

IV. CONCLUSION

By using 3-satisfiability to a NP-complete Steiner graph, it is found that every u-v path of the graph contains exactly 2n edges and that each clause gadget connected to this path exactly contain t edges where t=2n+1.

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