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Extreme Value Control Charts in Some Life Testing Models

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Abstract- Variable control charts are based on sub group statistics and variation in the values of the sub group statistics between sub groups. In this paper extreme order statistic of the sub group are considered to develop to the control limits to decide upon the in control status of the process. The quality characteristic is assumed to follow Exponential/Weibull Distributions. Relevant comparisons are also presented.

***Index Terms*-** Extreme value control Chart, Extreme order statistic, Variable control charts

I. INTRODUCTION AND REVIEW OF REVIEW OF LITRATURE

Variable control charts are popularly used to monitor the variability in the quality control data by developing graphical procedures for subgroup statistics such as mean, range, standard deviation etc. Depending on the subgroup size and the sampling distribution of the subgroup statistics separate control chart constants and hence control limits would be constructed in practice. However, some research works appeared in literature that deals with control charts for non-normally distributed process variates (Edgemen (1989), Kantam and Sriram (2001) and the references therein). At the same time, control charts for individual observations in the case of normal process variates are developed and the principle was made use of to propose a procedure for comparison of multiple means known as Analysis of Means (OTT (1967)). On similar lines the well known Gamma and Exponential distributions are assumed as models of process variate and the corresponding ANOM procedure along with control charts for individual observations are developed by Sriram (2004). Motivated from these aspects we extend the same principle to two probability models of exponential and Weibull with shape parameter(2) **(**Rayliegh distribution) developed alternative pairs of control limits for variable control charts and compare their appropriateness with that of existing pairs of limits in literature. The preliminary narration of the two models, the percentiles of extreme order statistics in samples from the models, their use in developing limits for variable control charts are presented in the section 2. In section 3, we compare our control limits with those existing in literature with an example.

II. RESEARCH METHODOLOGY

Extreme Order Statistics Based Control Limits:

With as the conjecture, we made an attempt to get the pair of control limits using extreme order statistics in samples from Exponential and Weibull with shape parameter (2).

The cumulative distribution functions of these models in standard form are

$$F(X) = 1 - e^{-x}, X \geq 0 \quad \text{.......}(1)$$

$$F(X) = 1 - e^{-x^C}, X \geq 0, c > 0 \quad \text{.......}(2)$$

The percentiles of extreme order statistics for standard exponential distribution can be obtained from the following steps.

$$P[X_n \leq U] = 0.99865$$

$$i.e, \left[1 - e^{-U}\right]^n = 0.99865$$

$$\Rightarrow \quad U = -\ln(1 - A)$$

$$where A = (0.99865)^{\frac{1}{n}}$$

$$P[X_1 < L] = 0.00135$$

$$i.e, \left(1 - e^{-nl}\right) = 0.00135$$

$$\Rightarrow \quad L = -\ln A$$

$$Where A = (0.99865)^{\frac{1}{n}}$$

Similarly the results for Weibull with shape parameter (2) are as follows:

$$P[X_n \leq U] = 0.99865$$

$$i.e, (1 - e^{-U^2})^n = (0.99865)$$

$$(1 - e^{-U^2}) = (0.99865)^{\frac{1}{n}}$$

$$\Rightarrow \quad U = \left[-\ln[1 - A]\right]^{1/2} \quad where \quad A = (0.99865)^{\frac{1}{n}} \quad \text{.......}(3)$$

$P[X_1 < L] = 0.00135$

$i.e, 1 + e^{-nL^2} = 0.00135$

$L = [-\ln A]^{1/2} \quad where \quad A = (0.99865)^{\frac{1}{n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$

The values of U and L so obtained for a standard probability model can be used to construct control chart constants as follows.

P{L< min Z_i < max Z_i < U} = 0.9973

where Z_i is standard ordered i^{th} variate, $Z_i=X_i/\sigma$, σ being scale parameter.

i.e, P{L < min (X_i/σ) < max (X_i/σ) < U} = 0.9973

i.e, P{Lσ < min X_i < max X_i < Uσ} = 0.9973

If σ is unknown, using an appropriate estimate of σ say $\hat{\sigma}$ the above probability statement would be

P{L $\hat{\sigma}$ < all X_i < U $\hat{\sigma}$ } = 0.9973

Recalling that σ is scale parameter of the model under consideration it can be estimated unbiasedly with sample mean, sample range from the following formula.

In this case Exponential distribution, if σ is not known, it is estimated by $\frac{\bar{R}}{d_2}$ where d_2 =E[Z_n-Z_1] taken from Gupta (1960).

In case of Weibull Distribution σ is estimated by MLE i.e $\sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$ say s under repeated sub-grouping $\hat{\sigma} = \bar{s}$

For Exponential distribution the control limits are $\left[D_3^*\bar{R}, D_4^*\bar{R}\right]$, Where $D_3^* = \frac{L}{d_2}$ and $D_4^* = \frac{U}{d_2}$ D_3^* and D_4^* are given in the following table

For Weibull with shape parameter (2) the control limits are , $\sqrt{L^*}$ and $\sqrt{U^*}$ are given in the following table

III. CONTROL CHART CONSTANTS FOR R-CHART

Variable control chart

Constants of R-chart
Exponential Distribution (Sriram (2004))

n	m_3	m_4
2	0.00013	6.60765
3	0.02495	4.86697
4	0.06388	4.20318
5	0.10214	3.83686
6	0.13587	3.50848
7	0.16495	3.42810
8	0.19006	3.29866
9	0.21190	3.19609
10	0.23108	2.03453

Exponential Distribution

n	D_3^*	D_4^*
2	0.00067	7.30046
3	0.00030	5.13721
4	0.00018	4.36006
5	0.00013	3.67832
6	0.000099	3.94395
7	0.000079	3.49101
8	0.000065	3.35016
9	0.000055	3.23943
10	0.000048	3.14942

Weibull with shape parameter (2)

n	$\sqrt{L^*}$	$\sqrt{U^*}$
2	0.0259854	2.7019364
3	0.0212098	2.7759346
4	0.0183774	2.8272670
5	0.0164372	2.8664523
6	0.0150066	2.8980420
7	0.0138917	2.8917755
8	0.0129948	2.9472871
9	0.0122516	2.9671998
10	0.0116229	2.9848999

IV. STUDY AND FINDINGS

COMPARISION OF ABOVE CONTROL CHARTS EXPONENTIAL DISTRIBUTION (Sriram (2004)) WITH WEIBULL WITH SHAPE PARAMETER (2) (RAYLIEGH DISTRIBUTION) WITH EXAMPLES

EXAMPLE.1

Consider the following data

Sample No	Sample Observations	Total	Sample Range	Sample S.D (s)
1	42 65 75 78 87	347	45	71.09
2	42 45 68 72 90	317	48	65.87
3	19 24 80 81 81	285	62	63.97
4	36 54 69 77 84	320	48	66.27
5	42 51 57 59 78	287	36	58.62
6	51 74 75 78 132	410	81	86.27
7	60 60 72 95 138	425	78	89.95
8	18 20 27 42 60	167	42	39.92
9	15 30 39 62 84	230	69	52.05
10	69 109 113 118 153	562	84	115.54
11	64 90 93 109 112	468	48	95.15
12	61 78 94 109 136	478	75	99.01
	Total		716	903.71

The Control limits for exponential Distribution are $[m_3\bar{R}, m_4\bar{R}]$ according Sriram (2004)

For Sub-group size 5 the control chart constants from the above table are

$m_3 = 0.10214$, $m_4 = 3.83686$

$$\left[D_3^*\bar{R}, D_4^*\bar{R}\right]$$

$D_3^* = 0.00013$, $D_4^* = 3.67832$,

(From above Exponential Distribution table at n=5)

$\bar{R} = 52.67$

$[m_3\bar{R}, m_4\bar{R}] = [5.37971, 202.08741]$

$\left[D_3^*\bar{R}, D_4^*\bar{R}\right] = [0.00685, 207.7276]$

The Control limits for Weibull with shape parameter (2) are

$$\left[\sqrt{L^*}\,\bar{s}, \sqrt{U^*}\,\bar{s}\right]$$

$s = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2}{n}}$, $\bar{s} = \dfrac{\sum s}{k}$, k is the Sample number

$\sqrt{L^*} = 0.0164372$, $\sqrt{U^*} = 2.8664523$

(From above Weibull Distribution (2) Distribution table at n=5)

$\bar{s} = 75.31$

$\left[\sqrt{L^*}\,\bar{s}, \sqrt{U^*}\,\bar{s}\right] = [1.2379, 215.8725]$

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EXAMPLE.2

Consider the following data

Sample No	Sample Observations	Total	Sample Range	Sample S.D (s)
1	14 8 12 12	46	6	11.70
2	11 10 13 8	42	5	10.65
3	11 12 16 13	52	5	13.13
4	15 12 14 11	52	4	13.09
5	10 10 8 8	36	2	9.05
	Total		22	57.62

The Control limits for exponential Distribution are $[m_3 \overline{R}, m_4 \overline{R}]$ according Sriram (2004)

For Sub-group size 4 the control chart constants from the above table are

$m_3 = 0.06388$, $m_4 = 4.20318$

$$\left[D_3^* \overline{R}, D_4^* \overline{R}\right]$$

$D_3^* = 0.00018$, $D_4^* = 4.36006$,

(From above Exponential Distribution table at n=4)

$\overline{R} = 4.4$

$[m_3 \overline{R}, m_4 \overline{R}] = [0.281072, 18.493992]$

$\left[D_3^* \overline{R}, D_4^* \overline{R}\right] = [0.00792, 19.184264]$

The Control limits for Weibull with shape parameter (2) are $\left[\sqrt{L^*}\,\overline{s}, \sqrt{U^*}\,\overline{s}\right]$

$s = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2}{n}}$, $\overline{s} = \dfrac{\sum s}{k}$, k is the Sample number

$\sqrt{L^*} = 0.0183774$, $\sqrt{U^*} = 2.8272670$

(From above Weibull Distribution (2) Distribution table at n=4)

$\overline{s} = 11.524$

$\left[\sqrt{L^*}\,\overline{s}, \sqrt{U^*}\,\overline{s}\right] = [0.21175, 32.58142]$

Since the whole computation is carried out with an example. The conclusions from the above calculations about preferability of one over the other cannot generalize. However, it gives a hint that our control chart limits are preferable.

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