

# A Novel Method to Tournament Scheduling

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**Abstract-** The scheduling of tournaments is an intriguing and complex problem that can be solved in many branches of mathematics. Some of these include graph theory, constraint and integer programming, and statistical techniques. This paper focuses on scheduling tournaments using graph theoretic techniques and introduces a novel method called the “Points Order Method”. The method aims to address the challenges associated with scheduling tournaments efficiently and fairly. The Points Order Method optimizes the schedule to ensure a balanced and competitive tournament by considering various factors, such as team strengths, competitiveness, and fairness. The author evaluates the effectiveness of this method through theoretical analysis and simulation studies by applying this method to two different types of tournaments: the Football World Cup (FIFA) 2026 and the National Basketball Association (NBA) in the United States of America.

**Index Terms-** Graph Theory, Tournament Scheduling, Graph Coloring, Round Robin, Elimination

## I. INTRODUCTION

In the modern world, sports are an industry worth billions of dollars that is twice as big as the automobile sector and seven times as big as the film industry. Additionally, the majority of income for the sports industry comes from TV networks.

Tournament scheduling is an important problem in the field of sports management, as it involves designing a schedule that determines the dates, times, and locations for a series of games or matches. As it requires balancing a number of constraints, including fairness, efficiency, and logistical considerations, this task can be challenging in large tournaments involving many teams or players. However, due to the advancement of transportation, logistical considerations are now considerably more straightforward.

Numerous studies have been done in mathematics, computer science, and operations research on the challenging problem of tournament scheduling. Despite the advancements made, numerous issues still need to be resolved in the tournament schedule. For instance, many scheduling issues include balancing numerous goals, such as reducing the number of days required to finish the tournament, reducing the time teams must travel, and ensuring that each team plays a reasonable number of games at various times and locations. It is a challenging endeavour that involves careful consideration of numerous different aspects to strike a balance between these goals that is both efficient and fair.

## II. METHODOLOGY

Although it's important to note that there are many alternative strategies and approaches for scheduling tournaments, in this research study, we primarily concentrate on using graph theory in the context of sports scheduling. These techniques include integer optimization, orthogonal Latin squares, and many other techniques. Our primary focus is on studying how graph theory might be used to overcome the difficulties and complexity involved in sports scheduling.

### 1. Graph Theory

A popular and successful method for handling the challenging problem of managing sporting event scheduling is graph theory. Graph theory provides a well-structured framework for modelling and improving match schedules. In this sense, teams or players are called nodes, and the matches are called edges on a graph. Various graph theory algorithms can be used to design schedules that guarantee equity, balance, and effectiveness. In round-robin events, when each team must play every other team, this technique is beneficial because it ensures that no team has an undue advantage or disadvantage due to the scheduling. Graph theory is crucial in this process since it allows for incorporating restrictions like venue availability and travel issues and avoiding conflicts. Graph colouring and perfect matching are two systematic methods for creating round-robin tournaments in graph theory.

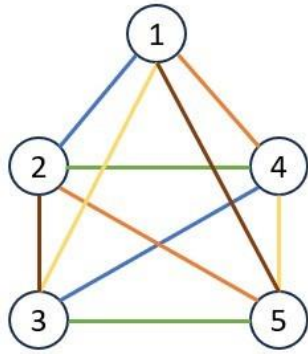


Figure 1: Graph colouring for the tournament with five teams.

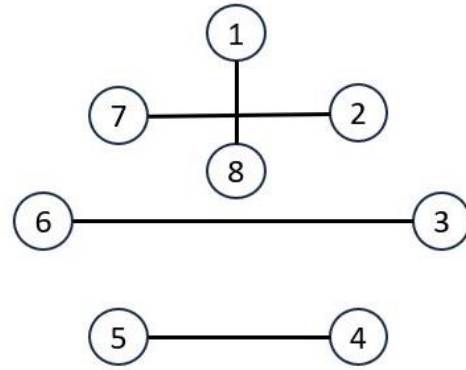


Figure 2: Perfect matching for the tournament with eight teams.

## 2. Points Order Method

One new approach that addresses the benefits and drawbacks of round-robin and elimination tournaments is the Points Order Method, which combines graph theory and counting techniques. The tournament scheduling technique offers a mix between being fairer than elimination events and shorter than round robin. Tournament organizers should consider this novel approach as it has the potential to enhance the competitive experience across multiple domains.

The structure of the Points Order Method is straightforward. In the case of tournaments with an even number of teams ( $N$ ), which is expected.

### I. Initial Pairings Based on Past Performance:

- Order teams by past performance in descending order.
- Pairings:
  - 1<sup>st</sup> team vs  $[\frac{N}{2}+1]$ <sup>th</sup> team
  - 2<sup>nd</sup> team vs  $[\frac{N}{2}+2]$ <sup>th</sup> team
  - Continuing in this pattern
  - Last pair:  $[\frac{N}{2}-1]$ <sup>th</sup> team vs  $N^{\text{th}}$  team

### II. Creation of Performance Tables:

- Create a points table with various performance measures.
- Derive a points order table for the next round's pairings.

### III. Handling Equal Points:

- If teams have equal points, use other performance measures to rank them.

### IV. Subsequent Pairings:

- Pair teams as follows:
  - 1<sup>st</sup> team with 2<sup>nd</sup> team
  - 3<sup>rd</sup> team with 4<sup>th</sup> team
  - Continue in this pattern
- Ensure teams do not compete against prior opponents in the same round:
  - If 1<sup>st</sup> and 2<sup>nd</sup> teams have met before, pair 1<sup>st</sup> with 3<sup>rd</sup>, 2<sup>nd</sup> with 4<sup>th</sup> and so on.

### V. Repeat Pairings:

- Continue this method up to the square root of the total number of teams.
- Ensures a variety of opponents and fewer rounds than a standard round-robin format.

### III. RESULTS AND DISCUSSION

In the rest of the study, we will explore the use of a new approach (point order method) in two different tournament schedules, namely the upcoming FIFA World Cup 2026 and the present NBA scheduling.

#### A. FIFA World Cup

Instead of the 32 teams that have previously competed in World Cups, the FIFA World Cup anticipates up to 48 teams in 2026. Considering these 48 teams, there are three stages in this novel schedule.

1. Preliminary Stage
2. Points Order Stage
3. Extended Elimination Stage

Let's examine each of these three stages separately from now on.

#### 1. Preliminary Stage

This stage includes all 48 teams and is then divided into 16 groups. It is fairer than previous versions in that it allows both teams with strong performances and those with worse ones equal opportunities. Figure 3 illustrates the construction.

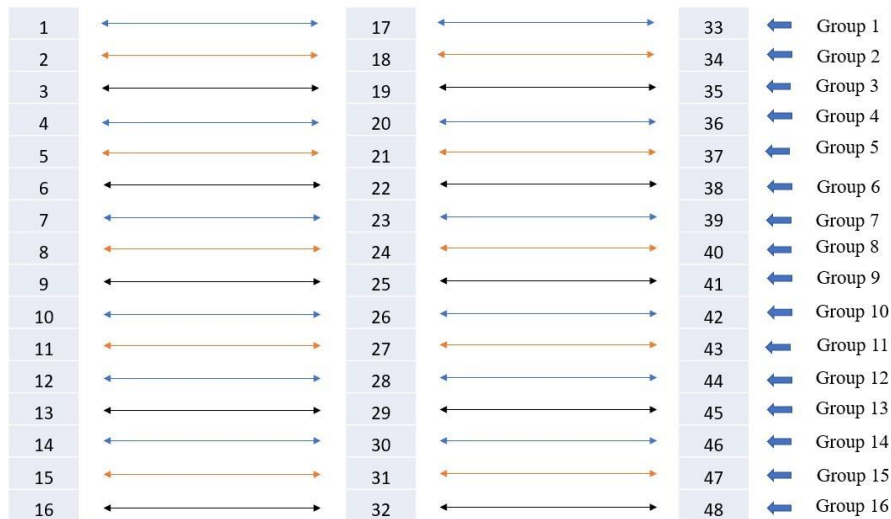


Figure 3: Preliminary stage groups

After this, the single round-robin method was used in these 16 groups. Graph coloring, Kirkman tournaments, and Steiner tournaments can be used to design single round-robin tournaments. Because graph coloring is more straightforward with fewer groups than other approaches, we utilized it in this situation. Figure 4 describes graph coloring for group 1, and a similar process can be performed with the other 15 groups.

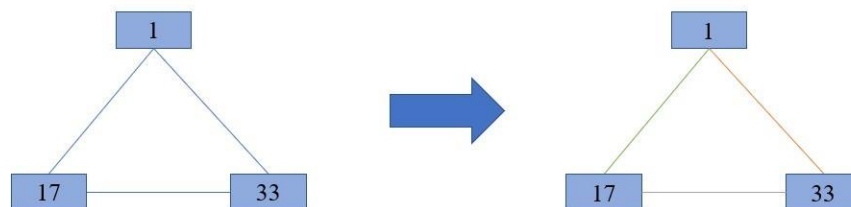


Figure 4: Graph coloring process

Each group has three games, and in the preliminary round, there are 48 games. After completing this stage, only two teams per group, i.e. only 32 teams, will advance to the points order stage.

#### 2. Points Order Stage

This method is novel to the tournament scheduling and also to the FIFA World Cup. Thirty-two teams should be divided into four groups as the initial step in this stage of construction. It can be done systematically, i.e. all the group winners and runners-up in the preliminary stage are equally distributed among these four groups: A, B, C, and D. This illustration is given in Table 1.

Table 1: Points order stage teams

Group A	Group B	Group C	Group D
Group 1 Winner	Group 2 Winner	Group 3 Winner	Group 4 Winner
Group 4 Runner-up	Group 3 Runner-up	Group 2 Runner-up	Group 1 Runner-up
Group 5 Winner	Group 6 Winner	Group 7 Winner	Group 8 Winner
Group 8 Runner-up	Group 7 Runner-up	Group 6 Runner-up	Group 5 Runner-up
Group 9 Winner	Group 10 Winner	Group 11 Winner	Group 12 Winner
Group 12 Runner-up	Group 11 Runner-up	Group 10 Runner-up	Group 9 Runner-up
Group 13 Winner	Group 14 Winner	Group 15 Winner	Group 16 Winner
Group 16 Runner-up	Group 15 Runner-up	Group 14 Runner-up	Group 13 Runner-up

Particularly in this stage, it is essential to consider each team's match points, goal differences, and average ball possession. Therefore, the 1st round of this stage can be done according to the points table of the preliminary stage. Then, the 2nd round considers the points table of the 1st round, the 3rd round considers the points table up to the 2nd round, and so on. Another specialty of this method is organizers can decide on the rounds at this stage. Therefore, assume that organizers decided to have four rounds and 1st round configuration as in Table 2.

Table 2: First round configuration of Group A: Points order stage

<b>First Round</b>
Group 1 winner-Group Five winner
Group 9 winner-Group 13 winner
Group 4 runners-up-Group 8 runners-up
Group 12 runners-up-Group 16 runners-up

Then, before the second round, we have to consider the points table. Assume Table 3 is the points table after the 1st round. GD means the sum of goal differences between each game and Avg. BP means average goal possession. We construct the points order table while examining this table. When constructing a points order table, we consider the goal difference and the average ball possession if there is any tie in match points between teams.

Table 3: Points table after the first round

Team	Round 1	Points	GD	Avg. BP
Group 1 Winner	2	2	2	65
Group 5 Winner	0	0	0	35
Group 9 Winner	2	3	3	72
Group 13 Winner	0	0	0	28
Group 4 Runner-up	0	0	0	13
Group 8 Runner-up	2	3	3	87

Group 12 Runner-up	1	0	0	55
Group 16 Runner-up	1	0	0	45

Table 3 shows the point order table, as shown in Table 4.

Table 4: Points order table after the first round

No	Points Order
1	Group 8 Runner-up
2	Group 9 Winner
3	Group 1 Winner
4	Group 12 Runner-up
5	Group 16 Runner-up
6	Group 5 Winner
7	Group 13 Winner
8	Group 4 Runner-up

After this process, pair the teams in the following manner for the 2nd round: 1-2, 3-4, 5-6, 7-8. But we don't pair teams with those who have already met. Assume 1-2 met in the previous round, then we don't pair them and pair 1-3 and 2-4.

Then, the remaining rounds should follow a similar procedure. In the other three groupings, similar architectures can be perceived. Only three teams from each group will advance to the final stage after the end of the points order stage.

### 3. Extended Elimination Stage

The extended elimination method is also a newly added construction to the FIFA World Cup. Here, the highest points taker from each group in the previous stage will directly move on to the quarterfinals. In contrast, the other two teams in each group will have to compete in the elimination round as the 1st round of the extended elimination stage. The FIFA 2026 champion will be determined after all the rounds in this stage.



Figure 5: Extended elimination stage construction

**B. NBA League**

The NBA (National Basketball Association) is one of the world's most popular and prestigious professional basketball leagues. Known for its high-flying dunks, skilled athletes, and intense competition, the NBA brings together the best basketball players from around the globe. The league follows a regular season schedule that typically begins in October and extends through April. During this time, teams play a series of games against each other, aiming to secure a spot in the playoffs. The NBA schedule includes exciting matchups, rivalries, and marquee games that captivate fans worldwide. The playoffs, which start in April, feature the top teams from each conference competing for the coveted NBA championship. The NBA's schedule provides basketball enthusiasts with months of thrilling action and serves as a platform to witness exceptional athleticism and sportsmanship.

Figure 6 shows the team construction of this novel schedule for the NBA. Here,  $A_i, B_i, C_i, D_i,$  and  $E_i$ , where  $i = 1, 2, 3, 4, 5, 6$ , denotes all the teams in the NBA league.

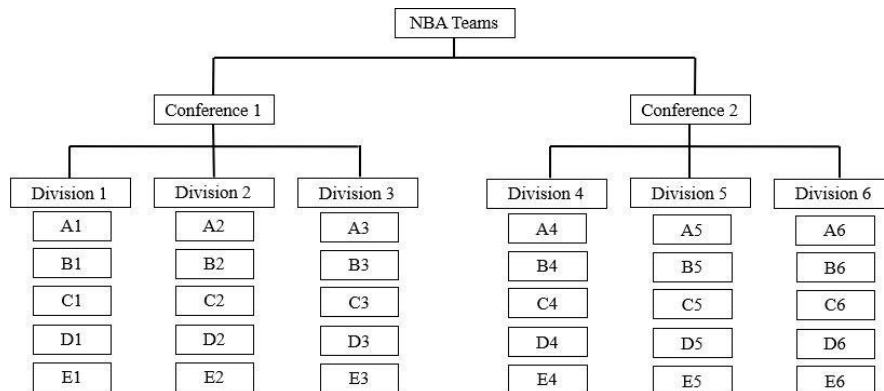


Figure 6: Team configuration of NBA league

According to Figure 6, there are three types of games.

1. Intra-divisional games
2. Inter-divisional games within the conference
3. Inter-conference games

Moreover, this schedule was created using the Points Order method.

1. Intra-divisional games

Each division contains five teams, and under this setup, games within a division are played using the points order method. Assume Division 1 teams A1, B1, C1, D1, and E1 are performing in descending order. Since there are an odd number of teams in this situation, we must give one team a bye in each round and award the relevant bye team one point. Therefore, this contest has five rounds, with bye points after intra-divisional games. Considering the descending order of performances, we can give byes, which means the highest performance team, i.e. A1, gets the bye in 1st round and the lowest, i.e. E1, gets the bye in the 5<sup>th</sup> round. Here, we should also order the teams from highest to lowest match points at the beginning of each round. The following figure graphically shows this construction of up to two rounds.

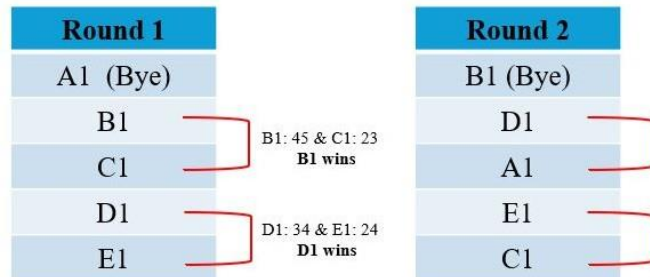


Figure 7: Intra-divisional games construction up to the second round

2. Inter-divisional games

Since there are three divisions and five teams in the division, and it is not possible to have games in the same division teams, we give byes for a team in each division. There are five rounds, and we pair the teams for each round using a unique matching mechanism. Byes are assigned, considering the high to low performance in intra-divisional games. Figure 8 shows the graphical illustration for this.

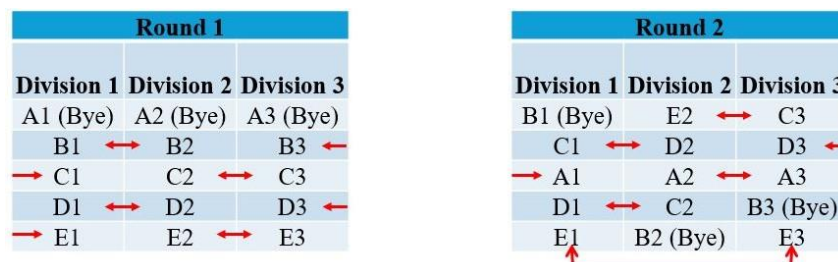


Figure 8: Inter-divisional games within conferences construction up to the second round

3. Inter-conference games

It also uses a matching mechanism similar to the previous game type. But there is no need for byes because there are only two conferences. Similarly, order the teams' high to low performances in each round to perform the points order method. The graphical illustration is as follows.

Round 1			Round 2		
Conference 1	Conference 2		Conference 1	Conference 2	
A1	↔		A1	↔	B4
B1	↔		C1	↔	E4
C1	↔		D1	↔	B5
D1	↔		A2	↔	C5
E1	↔		E2	↔	D5
A2	↔		B3	↔	A6
B2	↔		C3	↔	D6
C2	↔		E3	↔	E6
D2	↔		D3	↔	C6
E2	↔		A3	↔	B6
A3	↔		D2	↔	E5
B3	↔		C2	↔	A5
C3	↔		B2	↔	D4
D3	↔		E1	↔	C4
E3	↔		B1	↔	A4

Assume that round 2 table is in the points order

Figure 9: Inter Conference games construction up to the second round

#### IV. CONCLUSIONS AND RECOMMENDATIONS

##### A. Conclusions

In this research, we explored the application of graph theory in the scheduling of sports tournaments, introducing the Points Order Method as a novel approach to optimize tournament scheduling. The Points Order Method was applied to the FIFA World Cup 2026 and the NBA league to evaluate its effectiveness.

- FIFA World Cup 2026:
  - The proposed method ensures fairness by balancing teams’ strengths across different stages of the tournament.
  - The Preliminary Stage, Points Order Stage, and Extended Elimination Stage were structured to provide equal opportunities for all teams, ensuring competitive balance and reducing logistical challenges.
- NBA League:
  - The Points Order Method was adapted to the NBA’s schedule, demonstrating flexibility and applicability to different sports formats.
  - The scheduling addressed intra-divisional, inter-divisional, and inter-conference games, ensuring a balanced and competitive season.

Through theoretical analysis and simulation studies, the Points Order Method has been shown to provide an efficient, fair, and balanced approach to tournament scheduling. It reduces the number of rounds compared to traditional round-robin formats and ensures that teams have varied and fair matchups.

##### B. Recommendations

Based on the findings of this study, we recommend the following:

###### 1. Adoption of the Points Order Method:

- Sports organizations and tournament organizers should consider adopting the Points Order Method for its efficiency and fairness in scheduling.



## 2. Further Research:

- Further research can be conducted to refine the Points Order Method and explore its application in other sports and competitive events.
- Investigate the impact of different performance metrics on the Points Order Method to enhance its adaptability.

## 3. Software Development:

- Develop software tools that implement the Points Order Method to assist organizers in scheduling tournaments more effectively.

## 4. Feedback from Stakeholders:

- Collect feedback from teams, organizers, and audiences to improve the scheduling method continuously.

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