Error Analysis in Basic Calculus: A Basis for Improving Teaching Strategies

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Abstract- This study analyzes the errors committed in Calculus and determines appropriate strategies to remediate them. The question that this paper explores is the student's Mathematical Proficiency in Basic Calculus. How are students described in terms of strategic competence, conceptual understanding, and procedural fluency? What are the errors committed by the students? Which among the errors are most prevalent? What teaching strategies can be incorporated to remediate the identified errors? In order to explore these questions, this paper uses the collection of scripts (examination) that was administered to Senior High School Students. In order to answer the question, expert respondents give scores and identified the errors committed. Mathematics competency in the derivative is prone to errors, as can be seen by looking at the performance and common errors made. Errors are probable and are frequently brought on by casual mistakes and computational errors. The most typical mistakes in basic calculus are the exponent, fraction, incorrect derivative, lack of proof, and expanding expression faults. The study emphasizes using general and targeted teaching and learning strategies for teaching and learning Basic Calculus and other Mathematics disciplines. Different methods may be used to motivate students to study and comprehend mathematics. This study suggests emphasizing conceptual understanding and mathematical proficiency, and teachers should take the initiative to call attention to Basic Calculus faults. Additionally, instructors should administer diagnostic tests to gauge students' subject-matter expertise.

Index Terms- education, error analysis, mathematics, teaching strategies.

I. Introduction

Mathematical errors are commonly observed in the problem-solving type of assessment. Collins Dictionary defines a mathematical error as a mistake in a mathematical calculation. The extensive literature on error analysis in mathematics was made with emerging errors. The main concern is to improve the learning of the students and their understanding of Mathematics.

When students consistently commit mistakes, error analysis is a strategy that is frequently used to determine the root of the issue (Cheng, 2012). It involves assessing the work of the pupils and then searching for patterns of misinterpretation. Mathematical error analysis can be factual, procedural, or conceptual, and it can happen for a variety of reasons. Several types of research in error analysis on problem-solving were done, such as fractions (Safriani et al., 2019). (Abdullah et al., 2015) study in solving HOTS problem in Fraction shared several errors such as comprehension, transformation, process skills, and encoding error.

In the Philippines, we cannot deny that we are producing mathematically unequipped students. The 2019 Trends in International Mathematics and Science Studies (TIMSS) showed how far behind the Philippines is from other Asian countries. TIMSS also reveals that the Philippines belongs to the country with poor performance in Mathematics (Mullis et al., 2020). The Philippine average scale score of Grade 4 students in TIMSS is 297, which is below the TIMSS Scale CenterPoint of 500. The Philippines got the lowest average scale score even with countries' participation as benchmarking participants. Compared to the 2003 TIMSS performance of Grade 4, the Philippines' average scale scores dropped from 378 to 297.

Patena and Dinglasan (2013) conducted a study at the Lyceum of the Philippines University about students' performance in mathematics, particularly the performance in the departmental examination. It was revealed that the four colleges obtained poor performance in trigonometry, and three colleges obtained poor performance in Algebra. Some of the causes of poor performance include lack of concentration in class, slow learning, and lack of commitment to the formula.

Capate and Lapinid (2015) showed the mastery level of Grade 8 students in different topics, and half of them are below the average level of mastery. Among the topics that fall below the level of mastery are Special Cases of Factoring, Integral Exponent, Rational Algebraic Expression, Graphs of Linear Equations in Two Variables, Slope of a Line, Equation of a Line, Solving Systems of Linear Equations and Inequalities, Properties of Parallelogram, Measures of Variations.

For mathematical instruction to be more effective and attractive, the student should be able to apply knowledge and skills to any situation. This will help them motivate students to be interested in the subject.

1.1 Aims and Objectives of the Research

This study aims to provide ideas on how to remediate errors in teaching and learning mathematics. It focuses on the errors committed by the students in mathematics, particularly on problem-solving tests.

1.2 Research Questions

In order to understand the different errors committed by the students in calculus, the following questions were answered:

i) How are the respondents may be described in terms of Mathematical Proficiency in Basic Calculus:
   i.i) strategic competence;
   i.ii) conceptual understanding; and
   i.iii) procedural fluency?
ii) What are the errors committed by the students based on the theory of Kilpatrick?
iii) Which among the identified errors are most prevalent?
iv) What teaching strategies can be incorporated to remediate the identified errors?

1.3 Scope of the Research

The study was conducted at Rizal Technological University, Mandaluyong City, Philippines because the researcher, at that time of the research, was working as faculty of the Senior High School Department.

1.4 Significance of the Research

The findings of the study were extremely important for Mathematics Teachers because they provided them with new methods for improving mathematical performance using general and targeted learning and teaching strategies. Furthermore, findings were important to students to make them aware of their strengths and weaknesses in that they can exert effort to enhance their knowledge in mathematics, especially in calculus, and improve their attitude towards the subject. The students may also know the importance of the different causes of poor mathematics performance.

II. LITERATURE REVIEW

2.1 Error Analysis

Xie and Jiang (2007) identified four factors of the role of error analysis in teaching and learning. Error occurrence is unavoidable in learning; thus, teachers can accept their student's errors that deal with error analysis. Additionally, learners' errors are helpful feedback, and teachers will develop their teaching strategies centered on that feedback. Errors are essential for the learners themselves to understand the aspects of the target language. Similarly, errors caused by fossilization must be avoided. In line with this, they recommended that teachers should use a variety of error treatment methods that are adaptable to the teaching goals, students' linguistic competence, affective variables, and the efficacy of error correction.

Teachers may value that their students' comprehension is better understood than others when presented with learning on poor work, which they may use to develop their teaching strategy further. Gadgil et al. (2012) conducted a study in which participants who practiced working on faulty work were more prone to develop the proper mentality of correcting their own mistakes and a better understanding of the problem's mechanism than those who only reiterated a perfectly done work. This finding was supported by other researchers (Durkin and Rittle-Johnson, 2012). The learning gaps were filled by instructing students to compare incorrect work with correctly worked exercises. The researchers discovered that when students from elementary mathematics to university undergraduates were provided with correctly worked examples and incorrect examples, they learned more than students who only looked at correctly worked examples.

According to Shalem et al. (2014), as reported in the study by Moru et al. (2014), an error of analysis has elements that include allowing students to be mindful of mistakes. Using the reasons for these errors, or describing the errors several times, will help learners improve their logic about the problem, and this will help teachers diagnose each learner's reasoning about the calculations given. Shalem et al. (2014) stated explicitly that learners and lecturers should be mindful of the mistakes they can encounter in various mathematical problems to solve them and provide explanations.

The points made by Shalem, et al. (2014) were well-explained. However, these theories overlapped with Moru's and Qhobela's theory, which stated that interpreting a mistake from multiple perspectives is necessary because it enables students to experience and learn more about mathematical errors. Aside from that, the same mistake cannot necessarily come from the same source. Furthermore, since students have diverse backgrounds and ways of thinking, they will often have various interpretations.

2.2 Errors in Mathematics

Errors have been occurring in various mathematical problems that students are having trouble in learning. These mistakes the students encountered were and could be analyzed by the lectures in order for them to explain when and how to solve these problems to the students.

According to Cheng (2012), errors in mathematics can be factual, procedural, or conceptual and may occur for several reasons: lack of knowledge, procedural error, factual error, conceptual errors, poor attention, and carelessness. Other possible causes of student error are conceptual, procedural, and carelessness (Oktaviani, 2017). Furthermore, to address this issue, teachers should check before asking the student to illustrate the problem with concrete objects or show and explain the steps used to solve the problem, and teachers should first consider the alignment between the instruction, student ability, and the task.

According to Peng and Luo (2009), as cited by Valtoribio et al. (2018), mathematical error is expressed by misunderstanding definitions and features and ignorance of the conditions of formulas and theorems. They used equivalent transforms, rearranged concepts, misclassified them, and engaged in cycle argumentation, all of which were illogical. They made a tactical mistake by failing to recognize patterns, lacking fundamental concepts, and failing to translate word problems into symbols. Due to inadequate image concepts and a lack of exposure to particular crucial arithmetic tasks, errors persisted despite the students' favorable attitude toward and confidence in mathematics.

Errors are valuable for teachers to observe students' learning processes and strategies. The following are pedagogical consequences of error analysis: using the difficulty hierarchy, making use of the contrastive findings, and the value and necessity of remedial services. The development of error-based teaching materials and a syllabus for use in composition classes, implications for individualized instruction, understanding of the learner's methods, and implications for teaching methodology help educators in teaching. Error analysis provides insights into the complex learning development processes and a comprehensive method for defining and explaining students' errors (Jabeen, 2015).

Different factors can affect a senior high school student's grasp of mathematics-related subjects. Junior High School Origin is one of them. Students from STE-specialized junior high schools have better performance in both math and science than non-STEs. In addition, gender and demographics over their performance. In addition, most of the graduates chose STEM over other academic strands (Morados, 2020).

As is commonly known by both students and teachers, mathematics is one of the most commonly feared subjects. Anxiety and other related illnesses can affect the student's performance in the subject. It can decrease the positive disposition towards the subject and inspire fear instead of interest (Tezer & Boskurt, 2015). Salimaco (2020) conducted research at the Davao Oriental State College of Science and Technology in order to identify the relationship between study habits and mathematics anxiety with mathematical achievement. It was found that students themselves must strengthen their mental fortitude and adjust their own study habits.

Teachers have a significant role in improving their students' problem-solving and critical-thinking abilities. Since the K to 12 programs now offer strand specializations, the weight on core subjects of each strand is more pronounced. As such, mathematics professors teaching STEM must enhance students' Proficiency and understanding in these advanced classes. As such, the lack of resources, workforce, and instructional materials makes up for senior high school curriculum flaws (Jaudinez, 2019)

In the study of Syukriani et al. (2017), mathematical performance was more influenced by the tendency to process information; hence there are differences in the strategic competence of students. Differences were seen when formulating, representing, and solving word problems. With these factors in mind, teaching strategies can be improved. There is no such thing as a perfect curriculum, but a beneficial change to both faculty and student performance is highly welcome.

III. METHODOLOGY

3.1 Research Design

The study used a mixed-method design. Qualitative research was used to identify the errors in basic calculus and identify strategies to remediate common errors in Basic Calculus. Quantitative research was used to determine the level of proficiency of the respondents and the frequency of most occurred errors in Basic Calculus.

In this research, an analysis of learners' or document analysis method scripts was used as a research method. Document analysis was used to determine the errors in basic calculus based on the periodical examination of the students. The study described the proficiency level and frequency of errors that occurred in the examination of the students.

3.2 Participants

A total of 54 students from this school took part in the study. Description of the Respondents. The respondents were 54 students from the two sections of Grade 11 STEM students of Rizal Technological University under the researcher's supervision. The respondents are taking the Basic Calculus subject, which is essential in the study. The respondents are taking the subjects Pre-Calculus and Basic Calculus under the Academic Track.

3.3 Data Collection

The research instruments in the study were mainly the transcript (periodical test) of students. The researcher developed and designed the test to determine the errors committed in Basic Calculus and identify the Mathematical Proficiency of the respondents. A scoring rubric in three strands of Mathematical Proficiency is also made to understand the error committed by the learners. Evaluators, who are Master teachers from different schools, validated the test questions and rubrics to identify the mathematical proficiency before test execution.

The respondents were purposively identified. Respondents are crucial in gathering data for the study. The respondents are 54 Grade 11 STEM taking Basic Calculus subject. The next step is to provide an examination to gather data. The study was designed to last for about a semester.

After the validation of the instruments, the examination was given on the day of the scheduled periodical examination. The researcher sought the permission of the Senior High School principal to conduct and collect data. After seeking permission, a consent letter was also sent to the study's respondents. Data of the study were from the Quarterly Examination of the students given by the teacher. The exam is composed of different topics in derivatives. Qualitative research was used in the study, particularly error analysis.

Documents based on their Quarterly examination were analyzed to determine the errors committed and determine the respondents' Mathematical Proficiency. The study data were extracted from the learners' script (performance in quarterly examination) for the first quarter of the second semester. Copies of the learners' scripts were collected from simple classroom tasks to formal examinations. The study covered selected topics from Basic Calculus. The copy of the materials was confidential to protect the respondents' identity. The researcher analyzes the collected learners' scripts that served as a basis for developing strategies.

The tests of the respondents were checked by the teacher to determine what errors were committed by the respondents. The errors were analyzed based on the concept and process of solving particular test items concerning the topic selected. With the error committed, it determined what strand of Kilpatrick's mathematical Proficiency needs to develop to have high performance in Basic Calculus.

Calculation error, Procedural error, and Symbolic error were the three encountered errors in mathematics. Some specific errors in computations are basic operations, Division by zero, bad/lost/assumed parenthesis, improper distribution, canceling error, improper use of factorization, conjugate, and rationalization, and rounding numbers.

IV. RESULT ANALYSIS AND DISCUSSION

1.1 Mathematical Proficiency of the Respondents in Basic Calculus

The description of error types is obtained by identifying the students' answers to the Basic Calculus questions given. The following are the results:

1.1.1 Strategic Competence

Table 1: Mathematical Proficiency in Basic Calculus in Terms of Strategic Competence

Competency	Mean Scores	SD	Proficiency
Derivative of a Constant	3.364	0.866	Excellent
Derivative of a Product	2.848	0.539	Good
Derivative of a Quotient	2.559	0.563	Good
Derivative of a Power	2.649	0.846	Good
Derivative of Sum and Difference	3.292	0.782	Excellent
Cain Rule	3.222	0.586	Good
Overall Mean	**2.997**	**0.453**	**Good**

According to the table, the students are excellent at deriving constant, and Sum and Differences with mean scores of 3.364 and 3.292 respectively. The overall mean is 2.997 with a standard Deviation of 0.453 interpreted as Good. This also implies that the students in the competencies presented are familiar with the Derivative of Constant and Derivative of Sum and Difference, yet they still commit mathematical errors. It also implies that most respondents can formulate, represent and solve mathematical problems.

1.1.2 Conceptual Understanding

Table 2: Mathematical Proficiency in Basic Calculus in Terms of Conceptual Understanding

Competency	Mean Scores	SD	Proficiency
Derivative of a Constant	3.438	0.745	Excellent
Derivative of a Product	2.994	0.48	Good
Derivative of a Quotient	2.747	0.534	Good
Derivative of a Power	2.84	0.76	Good
Derivative of Sum and Difference	3.42	0.747	Excellent
Cain Rule	3.324	0.565	Excellent
Overall Mean	**3.127**	**0.415**	**Good**

According to the table, the students are excellent in 3 competencies, deriving Constant, Sum and Differences, and Chain Rule with mean scores of 3.438, 3.42, and 3.324 respectively. The overall mean is 3.127 with a standard Deviation of 0.415 interpreted as Good. This implies that students are familiar with the Derivative of Constant, Derivative of Sum, and Difference and Chain Rule among the competencies presented. It also implies that most respondents understand the mathematical concept, operations, and procedures, but they still find mathematics difficult.

1.1.3 Procedural Fluency

Table 3: Mathematical Proficiency in Basic Calculus in Terms of Procedural Fluency

Competency	Mean Scores	SD	Proficiency
Derivative of a Constant	3.29	0.932	Excellent
Derivative of a Product	2.683	0.579	Good
Derivative of a Quotient	2.407	0.559	Fair
Derivative of a Power	2.478	0.902	Fair
Derivative of Sum and Difference	3.16	0.828	Good
Cain Rule	3.123	0.615	Good
Overall Mean	**2.857**	**0.46**	**Good**

According to the table, the students are excellent in 1 competency, deriving Constant with mean scores of 3.29. Fair proficiency can be seen in the derivative of Quotient and Power with mean scores of 2.407 and 2.478 respectively. The overall mean is 2.857 with a standard Deviation of 0.46 interpreted as Good. This implies that the students among the competencies presented have difficulty in the Derivative of Quotient and Derivative of Power. It also implies that most respondents have fair difficulty carrying out procedures flexibly, accurately, efficiently, and appropriately in the Derivative of quotient and power, maybe because of the process of finding the derivatives of the said competencies

1.2 Errors Committed by the Students Based on the Theory of Kilpatrick

When asked what errors are associated with the different theories, student's answers revealed the following:

1.2.1 Errors Associated with Conceptual Understanding

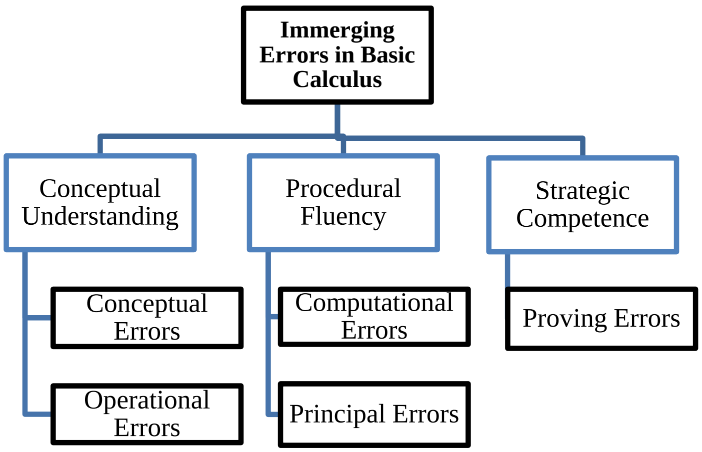
1.2.2 Errors Associated with Procedural Fluency

1.2.3 Errors Associated with Strategic Competence

Strategic Competence is generally associated with proving and reasoning errors. The student gave the correct answer but did now show the solution to prove the claim. They were able to give an example that corresponds to the given question but did not show the process of proving it. The solution or process must be presented in order for the student to prove and have a conclusion. This is basically some of the errors being committed by students in proving a certain given problem. As for reasoning, the student did not claim whether the same process could be applied or not.

Based on the figure presented, there are five types of errors: conceptual error, operational error, computational error, principal error, and proving error. Some of the Conceptual errors were incorrectly used of signs, exponents, and variables—incorrect use of operation and combining like terms were some of the errors committed in Operational Error. Computational Errors, simplifying and expanding expressions, fraction, and exponents were committed. The incorrect process that leads to a wrong answer is an error committed for Principal Errors. For Proving Errors, the process of proving is incorrect or no proof of the given claim.

Figure 1: Immerging Errors in Basic Calculus



1.3 Most Prevalent Errors in Basic Calculus

When asked what errors are most prevalent to occur in basic Calculus, the result revealed that errors in the Laws of Exponent rank number 1.

Table 4: Frequency Distribution of Prevalent Errors in Basic Calculus

Errors	Frequency	%	Rank
Misidentified Sign	12	22.22	9
Error on Laws of Exponent	35	64.81	1
Rewriting Expression with Exponent	19	35.19	6
Missing Variables	8	14.81	12
Operation Rule	10	18.52	11
Combining Terms	11	20.37	10
Simplifying Fraction	32	59.26	2
Expanding Expression	20	37.04	5
Using the Wrong Formula	14	55.56	7
Wrong Process of Derivatives	27	50	3
No Proof	25	46.30	4
No Claim/No Conclusion	13	24.07	8

As can be seen, Errors on Laws of Exponents tops the most errors committed by the students with a percentage of 64.81. The most common errors in Basic Calculus are Errors on Laws of exponent, with 35 respondents who committed the error equivalent of 64.81%. The most committed error is Simplifying Fractions with a frequency of 32, equivalent to 59.26%. Half of the respondents committed errors in the process of derivatives which is the third most common error in Basic Calculus, equivalent to 50%. The first two common errors, in general, are the topics that students are having difficulty with. Twenty-five (25) respondents committed mistakes in no proof error, equivalent to 46.30%, the fourth most common error in Basic Calculus. In expanding expression, 20 students committed the error, equivalent to 37.04%, the fifth most common error. The sixth common error is Rewriting Expression with exponent where 19 students committed an error and are equivalent to 35.19%. The seventh common error is using the wrong formula, where 14 students committed the error, which is equivalent to 55.56%. The rest of the errors are no claim/no conclusion (13 students), which is 24.07%, misidentified sign (12 students), which is 22.22%, combining terms (11 students) which is 20.37%, operation rule (10 students) which is 18.52% and missing variables (8 students) which is 14.71% of the total respondents. The rest of the errors stated are the easy concepts in Basic calculus that the student learned.

1.4 Teaching Strategies Incorporated to Remediate the Identified Errors

There shall be a Comprehensive and in-depth discussion of the topics in Laws of Exponents and Fractions. A diagnosis in the very first introduction of mathematical concepts must be taken also into consideration to unlock this difficulty. More often than not, expert respondents agree that most of the common errors in higher mathematics are associated with the lack of mastery of simple mathematical concepts and procedures. These mathematical concepts and procedures were actually being traced back from their lower mathematics but not being addressed accordingly. As it may be true in other mathematical concepts other than the laws of exponents and fractions, diagnosis of errors at an earlier stage and appropriately addressing it is highly recommended.

The validator posited that the root first and second in the rank of errors implies, in general, that errors are associated with a lack of conceptual understanding. This becomes the root cause of all errors which further leads to another form of error. For instance, a lack of basic understanding of laws of simplifying fractions leads to wrong procedures, wrong steps, and ultimately wrong proofs and answers.

Most mathematics teachers, after giving assessments whether in the form of formative or summative assessments, are focused on scores, especially when students attained a passing score, they tend to go to the next topic without addressing other errors in the test. These errors then become evident in the next assessment because the same errors are committed, this becomes a routine and a pattern until they reach future mathematics subjects.

More specifically, the following lists are suggested teaching strategies to avoid all other forms of errors noted in this study:

1. *Employ Various Opportunities to Introducing Mathematical Concepts*. In teaching a specific lesson, the teacher must always ensure that the concepts have been explained thoroughly so that the students truly understand the concepts before moving on to the next lesson. Students must learn the basic concepts given so that they can process them and subsequently apply them to a given problem. Exploring new opportunities can help students understand more about math. Teachers can see what ways are effective for giving students new concepts in learning math and being motivated to study the subject. Incorporating multiple instructional tactics encourages students to become more actively involved in learning. It has the potential to pique their interest during the class. It can also be used to determine which method of teaching mathematics is best for the students.
2. *Prioritize Conceptual Understanding in Assessment*. We cannot deny that concepts and procedures work hand-in-hand in the teaching and learning process. Understanding the concept is more essential in mathematics than memorizing concepts. When students memorized a process, it does not

necessarily mean that they understood it, and as a result, it will be more difficult for them to learn the later topics. Prioritizing conceptual understanding in assessment would help the learners understand the concept of the lesson further and make sure that learning the topic is the priority. This helps pupils understand the importance of a mathematical topic and how it may be used in a variety of contexts. Their organization of the material makes it possible for them to learn new ideas by connecting them to what they currently understand.

3. *Use Historical Notes/Research/Journals*. The use of historical notes, research, and journals is a big help since most topics in mathematics were established and discovered a very long time ago. It helps the student develop a deeper understanding of Mathematics and how it evolved over time. Using historical notes, research, and journals would give the instructor more background and a basis for the lesson. Using certain past studies, we can know the ups and downs of teaching math. From this, we can improve what is lacking before and avoid the most common mistakes that are still seen today.
4. *Allow Use of Available Technology*. Though most teachers do not still embrace technology to aid in performing operations, the use of technology now is, in fact, encouraged in the k-12 curriculum. For the learning to be more innovative, we can suggest using different kinds of learning instruments such as, but not limited to technology tools, gadgets, and online applications to do and check student's work. Since we are in a pandemic time, they can utilize online apps and resources for more exploration of solutions and formulas.
5. *Give Opportunities for Student to Check Their Process*. The most reliable transfer of learning is that the students can/or be able to check their work and provide and display solutions and formulas at their own pace. We can verify that they acquired the concept given because they can manage on their own by checking their own work. There is a need for the student to check their process to understand what lesson they lack knowledge of. Knowing this, they know what part of the lesson they need to improve more. The student needs to check their process for it helps them assess their own performance and develop their motivation to learn. It allows them to see how far they have gone and check for themselves the knowledge they gained throughout the course. This will also give way to understanding and correcting their mistakes. They would be aware of it and have more chance of getting it right the next time they encounter such a problem.
6. *Avoid the Careless Process*. Most of the students are not fast learners, and teachers should take extra steps to ensure that all of the students are actually learning. The teacher must teach step by step to avoid a careless process so that the learners can actually keep up with the lesson. Allow students to follow a particular style or way of solving to arrive at the answer with a much more precise and concise method. Teachers must constantly remind students to be careful when doing math equations. Solving math equations is a continuous rechecking of solutions and formulas until verified multiple times. Teaching with precision will enable students to learn more quickly and clearly. Let us not settle for less and take shortcuts when it comes to solving math equations.
7. *Encourage Students to Develop their Style of Solving*. As teachers, it is one of our tasks to inculcate to the students to explore and find different solutions, especially in math. It is one way to discover their potential in creating more ways to solve the problems given. Exploration is not insufficient until we find it new and exciting to do. Different institutions have diverse cultures; therefore, students have different approaches to solving problems. Therefore, they have different approaches to absorbing and processing the given task in their distinctive way.
8. *Enhance Peer Teaching*. To boost more students' learning, enhancing peer teaching is necessary to develop teamwork and camaraderie. Not everyone in the class can follow every topic discussed in math, and there will be students who will have a hard time understanding specific lessons. The teacher cannot constantly monitor who in the class cannot follow up on the lessons taught, so the best way is to encourage the students to practice peer teaching. Peer teaching aids the development of essential qualities in both fast and slow learners. Students who teach each other mathematical ideas improve their knowledge of those subjects.
9. *Cooperative Learning Strategies*. Cooperative learning also allows students to strategize and brainstorm on how to tackle problems, which allows many ideas to be formed and thrown around to solve a problem successfully. Cooperative learning also helps develop positive relationships among students, which is vital in creating a learning community that values diversity. Since there are shreds of evidence that Cooperative learning is beneficial to students, it should be practiced by Mathematics teachers in schools, and it should be accepted as a significant learning technique in Mathematics to produce students to produce more exemplary students
10. *Teacher must be aware of the error to be highlighted in teaching.* No individual is perfect in any manner. As teachers, we are also not perfect, whether in giving instructions, relationships with students, or our manner of teaching. We can say the same thing for teachers doing math. It is not about being perfect in solving problems but about accepting the reality that sometimes we commit mistakes and errors. Teachers should be aware of the frequently committed errors to be able to address them and highlight them the right way so the students would not be confused and make the same mistakes again. This is also to correct their students in advance and give them the necessary knowledge on what to avoid when solving the given problem. This can also help the student improve the area where most errors occur.
11. *Discuss with the student what are the errors*. It is essential to discuss the students' errors in both conceptual and procedural knowledge. Identifying students' errors is the first step to providing correct instructions. Teachers sometimes focus on just the basic facts and forget to check on error patterns (Riccomini, 2005). It is essential to discuss the errors with the students, especially in solving mathematical problems, so that they know where the committed mistakes

are and fix them so that they will not make the same mistake next time. Discussing the errors with the students would make them understand them more and the reason why it becomes an error.

12. *Learn to examine the Assessment*. Assessment is one key factor in the transfer of learning, but it does not end by just conducting an assessment but also reviewing if the given assessment is effective and reliable. We study the pros and cons of the assessment given. We anticipate positive and negative outcomes. We accept the result of the assessment, and in one way, we can improve and improvise more in the tryout. It will benefit both the students and the teachers.

V. CONCLUSION

The findings provided the basis for the following conclusions:

i) The students have "Good" mathematics proficiency in derivatives however they are prone to error;
ii) The errors are likely to occur usually associated due to careless mistakes and computational error;
iii) The most common error in basic calculus are errors in-laws of exponent, simplifying fractions, wrong derivative process, no proof, and expanding expression.
iv) The topics of exponent and fraction need to have an in-depth discussion and highlight common errors as specific strategies in teaching Basic Calculus.

The paper emphasizes adopting general and specific strategies in teaching and learning Basic Calculus across different Mathematics subjects. Students may be motivated to study and understand Mathematics using different strategies. This paper recommends giving focus on Conceptual Understanding and Mathematical proficiency and teachers must take the initiative in highlighting errors in Basic Calculus. Teachers should also carry out diagnostic test assessments of learners' knowledge of the topic

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