Some Properties of Induced Intuitionistic Fuzzy Sets

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***Abstract*:** In this paper we have proved some basic properties, related to union and intersection, of four different types of induced intuitionistic fuzzy sets.

Key words: Degree of membership, degree of non-membership, induced Intuitionistic fuzzy set.

Mathematics Subject Classification: 03B99, 03E72.

I. INTRODUCTION

The notion of Intuitionistic fuzzy set (IFS) was introduced by Atanassov in [1]. Various properties on intuitionistic fuzzy sets were discussed by many authors in [2- 4,9]. The concept of induced intuitionistic fuzzy sets was introduced in [5]. A few relations between induced intuitionistic fuzzy sets and second order induced intuitionistic fuzzy sets were established in [6,7]. Some complement properties of induced intuitionistic fuzzy sets were discussed in [8]. Here, in this paper, we have proved some properties on union and intersection of four different types of induced intuitionistic fuzzy sets.

The section 2 deals with the definitions and notations of intuitionistic fuzzy set and induced intuitionistic fuzzy sets on a set.

In section 3, we have proved some properties related to union and intersection four different types of induced intuitionistic fuzzy sets corresponding to intuitionistic fuzzy sets of a set E.

II. PRELIMINARIES

This section contains some basic definitions and notations which are used through-out the paper.

Definition 2.1 [1-3]: Let E be any non-empty set. An intuitionistic fuzzy set A of E is an object of the form $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \}$, where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ denotes the degree of membership and the non – membership functions respectively and for every $x \in E, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

If A and B are two intuitionistic fuzzy sets of a non - empty set E then the following relations are valid [3]:

$A \subseteq B$ if and only if for all $x \in E$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;

$A = B$ if and only if for all $x \in E$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$;

$A \cap B = \{ (x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) : x \varepsilon E \}$;

$A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) : x \varepsilon E \}$.

Considering the degree of membership $\mu_A(x), \mu_B(x)$ and the non - membership $\nu_A(x), \nu_B(x)$ for each element $x \in E$ of the intuitionistic fuzzy sets A and B respectively, of a non-empty set E, the four different types of induced intuitionistic fuzzy sets are defined as follows:

Definition 2.2 [5]: If A and B are two intuitionistic fuzzy sets of a non - empty set E then

$$A^{\circ}(B) = A^{\circ} = \{ \left(x , \mu_A(x), \min(\nu_A(x), \nu_B(x))\right) : x \in E \};$$

$$A_{\circ}(B) = A_{\circ} = \left\{\left(x , \mu_A(x) , \max(\nu_A(x), \nu_B(x))\right) : x \in E \right\};$$

$$A^{*}(B) = A^{*} = \left\{ (x , \max\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)) : x \in E \right\};$$

$$A_{*}(B) = A_{*} = \left\{(x , \min\left(\mu_A(x), \mu_B(x)\right), \nu_A(x) : x \in E)\right\}.$$

Note 2.3 [5]: It is to be noted that

$$A^{\circ}(B) \neq B^{\circ}(A), A_{\circ}(B) \neq B_{\circ}(A), A^{*}(B) \neq B^{*}(A), A_{*}(B) \neq B_{*}(A).$$

III. SOME PROPERTIES OF INDUCED INTUITIONISTIC FUZZY SETS

Let A_1, A_2 and B be three intuitionistic fuzzy sets of E.Then,

Property 3.1: i) $(A_1 \cup A_2)^{*}(B) = (A_1)^{*}(B) \cup (A_2)^{*}(B)$

ii) $(A_1 \cap A_2)^{*}(B) = (A_1)^{*}(B) \cap (A_2)^{*}(B)$

Proof: i) $(A_1 \cup A_2)^{*}(B) = \left\{\left(x, max\left(\mu_{A_1 \cup A_2}(x), \mu_B(x)\right), \nu_{A_1 \cup A_2}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)\right), min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right)\right) : x \in E\right\} \quad \ldots \ldots \ldots (3.1.1)$$

Also, $(A_1)^{*}(B) \cup (A_2)^{*}(B)$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_B(x)\right), \nu_{A_1}(x)\right) : x \in E\right\} \cup \left\{\left(x, max\left(\mu_{A_2}(x), \mu_B(x)\right), \nu_{A_2}(x)\right) : x \in E\right\} =$$

$$\left\{\left(x, max\left(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)\right), min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right)\right) : x \in E\right\} \quad \ldots \ldots \ldots (3.1.2)$$

From (3.1.1) and (3.1.2) we have,

$$(A_1 \cup A_2)^{*}(B) = (A_1)^{*}(B) \cup (A_2)^{*}(B).$$

ii) $(A_1 \cap A_2)^{*}(B) = \left\{\left(x, max\left(\mu_{A_1 \cap A_2}(x), \mu_B(x)\right), \nu_{A_1 \cap A_2}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), \mu_B(x)\right), max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right)\right) : x \in E\right\}$$

Therefore, for each $x \in E$,

$$\mu_{(A_1 \cap A_2)^*(B)}(x) = \begin{cases} max\left(\mu_{A_1}(x), \mu_B(x)\right) & for\ \mu_{A_1}(x) \le \mu_{A_2}(x) \\ max\left(\mu_{A_2}(x), \mu_B(x)\right) & for\ \mu_{A_2}(x) < \mu_{A_1}(x) \end{cases}$$

$$= \begin{cases} \mu_B(x) & for\ \mu_B(x) \ge \mu_{A_1}(x), \mu_{A_2}(x) \\ min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) & for\ \mu_B(x) \le \mu_{A_1}(x), \mu_{A_2}(x) \\ \mu_B(x) & for\ \mu_B(x) \in \left[\mu_{A_1}(x), \mu_{A_2}(x)\right] or\ \left[\mu_{A_2}(x), \mu_{A_1}(x)\right] \end{cases} \quad \ldots\ldots\ldots (3.1.3)$$

Also, $(A_1)^*(B) \cap (A_2)^*(B)$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_B(x)\right), \nu_{A_1}(x)\right) : x \in E\right\} \cap \left\{\left(x, max\left(\mu_{A_2}(x), \mu_B(x)\right), \nu_{A_2}(x)\right) : x \in E\right\}$$

$$= \left\{\left(x, min\left(max\left(\mu_{A_1}(x), \mu_B(x)\right), max\left(\mu_{A_2}(x), \mu_B(x)\right)\right), max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right)\right) : x \in E\right\}$$

Therefore, for each $x \in E$,

$$\mu_{(A_1)^*(B) \cap (A_2)^*(B)}(x) = \begin{cases} \mu_B(x) & for\ x \in E\ and\ \mu_B(x) \ge \mu_{A_1}(x), \mu_{A_2}(x) \\ min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) & for\ x \in E\ and\ \mu_B(x) \le \mu_{A_1}(x), \mu_{A_2}(x) \\ \mu_B(x) & for\ x \in E\ and\ \mu_B(x) \in \left[\mu_{A_1}(x), \mu_{A_2}(x)\right] or\ \left[\mu_{A_2}(x), \mu_{A_1}(x)\right] \end{cases} \quad \ldots\ldots\ldots (3.1.4)$$

Also, for each $x \in E$,

$$\nu_{(A_1 \cap A_2)^*(B)}(x) = max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) = \nu_{(A_1)^*(B) \cap (A_2)^*(B)}(x) \quad \ldots\ldots\ldots (3.1.5)$$

From (3.1.3), (3.1.4) and (3.1.5) we have,

$$(A_1 \cap A_2)^*(B) = (A_1)^*(B) \cap (A_2)^*(B).$$

Property 3.2: i) $(A_1 \cup A_2)^\circ(B) = (A_1)^\circ(B) \cup (A_2)^\circ(B)$

ii) $(A_1 \cap A_2)^\circ(B) = (A_1)^\circ(B) \cap (A_2)^\circ(B)$

Proof: i) $(A_1 \cup A_2)^\circ(B) = \left\{\left(x, \mu_{A_1 \cup A_2}(x), min\left(\nu_{A_1 \cup A_2}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), min\left(\nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x)\right)\right) : x \in E\right\} \quad \ldots\ldots\ldots (3.2.1)$$

Also,

$$(A_1)^\circ(B) \cup (A_2)^\circ(B) = \left\{\left(x, \mu_{A_1}(x), min\left(\nu_{A_1}(x), \nu_B(x)\right)\right) : x \in E\right\} \cup \left\{\left(x, \mu_{A_2}(x), min\left(\nu_{A_2}(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), min\left(min\left(\nu_{A_1}(x), \nu_B(x)\right), min\left(\nu_{A_2}(x), \nu_B(x)\right)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), min\left(\nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x)\right)\right) : x \in E\right\} \quad \ldots\ldots\ldots (3.2.2)$$

From (3.2.1) and (3.2.2) we have,

$$(A_1 \cup A_2)^{\circ}(B) = (A_1)^{\circ}(B) \cup (A_2)^{\circ}(B).$$

ii) $(A_1 \cap A_2)^{\circ}(B) = \left\{\left(x, \mu_{A_1 \cap A_2}(x), min\left(\nu_{A_1 \cap A_2}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$$= \left\{\left(x, min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), min\left(max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right), \nu_B(x)\right)\right) : x \in E\right\}.$$

Therefore, for each $x \in E$,

$$\nu_{(A_1 \cap A_2)^{\circ}(B)}(x) = \begin{cases} \nu_B(x) & if\ \nu_B(x) \le max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) \\ max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) & if\ \nu_B(x) > max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) \end{cases} \quad \ldots\ldots\ldots (3.2.3)$$

$(A_1)^{\circ}(B) \cap (A_2)^{\circ}(B)$

$$= \left\{\left(x, \mu_{A_1}(x), min\left(\nu_{A_1}(x), \nu_B(x)\right)\right) : x \in E\right\} \cap \left\{\left(x, \mu_{A_2}(x), min\left(\nu_{A_2}(x), \nu_B(x)\right)\right) : x \in E\right\} =$$
$$\left\{\left(x, min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), max\left(min\left(\nu_{A_1}(x), \nu_B(x)\right), min\left(\nu_{A_2}(x), \nu_B(x)\right)\right)\right) : x \in E\right\}$$

Therefore, for each $x \in E$,

$$\nu_{(A_1)^{\circ}(B) \cap (A_2)^{\circ}(B)}(x) = \begin{cases} \nu_B(x) & if\ \nu_B(x) \le max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) \\ max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) & if\ \nu_B(x) > max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) \end{cases} \quad \ldots\ldots\ldots (3.2.4)$$

Hence from (3.2.3) and (3.2.4) we have, for each $x \in E$,

$$\nu_{(A_1 \cap A_2)^{\circ}(B)}(x) = \nu_{(A_1)^{\circ}(B) \cap (A_2)^{\circ}(B)}(x) \quad \ldots\ldots\ldots (3.2.5)$$

Also, for each $x \in E$,

$$\mu_{(A_1 \cap A_2)^{\circ}(B)}(x) = min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) = \mu_{(A_1)^{\circ}(B) \cap (A_2)^{\circ}(B)}(x) \quad \ldots\ldots\ldots (3.2.6)$$

So, by (3.2.5) and (3.2.6) we have,

$$(A_1 \cap A_2)^{\circ}(B) = (A_1)^{\circ}(B) \cap (A_2)^{\circ}(B).$$

Property 3.3: i) $(A_1 \cup A_2)_*(B) = (A_1)_*(B) \cup (A_2)_*(B)$

ii) $(A_1 \cap A_2)_*(B) = (A_1)_*(B) \cap (A_2)_*(B)$

Proof: i) For each $x \in E$,

$$\mu_{(A_1\cup A_2)_*(B)}(x) = min\left(\mu_{A_1\cup A_2}(x), \mu_B(x)\right) = min\left(max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), \mu_B(x)\right)$$

$$= \begin{cases} max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) & when\ \ \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\ \mu_B(x) & elsewhere \end{cases} \quad \ldots\ldots\ldots (3.3.1)$$

and $\mu_{(A_1)_*(B)\cup(A_2)_*(B)}(x) = \max\left(min\left(\mu_{A_1}(x), \mu_B(x)\right), min\left(\mu_{A_2}(x), \mu_B(x)\right)\right)$

$$= \begin{cases} max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) & when\ \ \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\ \mu_B(x) & elsewhere \end{cases} \quad \ldots\ldots\ldots (3.3.2)$$

Hence from (3.3.1) and (3.3.2) we have, for each $x \in E$,

$$\mu_{(A_1\cup A_2)_*(B)}(x) = \mu_{(A_1)_*(B)\cup(A_2)_*(B)}(x) \quad \ldots\ldots\ldots (3.3.3)$$

Also for each $x \in E$,

$$\nu_{(A_1\cup A_2)_*(B)}(x) = \nu_{A_1\cup A_2}(x) = min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) = \nu_{(A_1)_*(B)\cup(A_2)_*(B)}(x) \quad \ldots\ldots\ldots (3.3.4)$$

So, from (3.3.3) and (3.3.4) we have,

$$(A_1 \cup A_2)_*(B) = (A_1)_*(B) \cup (A_2)_*(B).$$

ii) For each $x \in E$,

$$\mu_{(A_1\cap A_2)_*(B)}(x) = min\left(\mu_{A_1\cap A_2}(x), \mu_B(x)\right) = min\left(min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right), \mu_B(x)\right)$$

$$= min\left(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)\right) \quad \ldots\ldots\ldots (3.3.5)$$

and $\mu_{(A_1)_*(B)\cap(A_2)_*(B)}(x) = min\left(min\left(\mu_{A_1}(x), \mu_B(x)\right), min\left(\mu_{A_2}(x), \mu_B(x)\right)\right)$

$$= min\left(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)\right) \quad \ldots\ldots\ldots (3.3.6)$$

Therefore from (3.3.5) and (3.3.6), we have,

$$\mu_{(A_1\cap A_2)_*(B)}(x) = \mu_{(A_1)_*(B)\cap(A_2)_*(B)}(x) \quad \ldots\ldots\ldots (3.3.7)$$

Also for each $x \in E$,

$$\nu_{(A_1\cap A_2)_*(B)}(x) = \nu_{A_1\cap A_2}(x) = max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) = \nu_{(A_1)_*(B)\cap(A_2)_*(B)}(x) \quad \ldots\ldots\ldots (3.3.8)$$

Hence from (3.3.7) and (3.3.8), we have,

$$(A_1 \cap A_2)_*(B) = (A_1)_*(B) \cap (A_2)_*(B).$$

Property 3.4: i) $(A_1 \cup A_2)_\circ(B) = (A_1)_\circ(B) \cup (A_2)_\circ(B)$

ii) $(A_1 \cap A_2)_\circ(B) = (A_1)_\circ(B) \cap (A_2)_\circ(B)$

Proof: i) For each $x \in E$,

$$\mu_{(A_1 \cup A_2)_\circ(B)}(x) = \mu_{A_1 \cup A_2}(x) = max\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) = \mu_{(A_1)_\circ(B) \cup (A_2)_\circ(B)}(x) \quad \ldots\ldots\ldots (3.4.1)$$

Also, for each $x \in E$,

$$\nu_{(A_1 \cup A_2)_\circ(B)}(x) = max\left(\nu_{A_1 \cup A_2}(x), \mu_B(x)\right) = max\left(min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right), \mu_B(x)\right)$$

$$= \begin{cases} min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) & when\ \mu_B(x) \le \nu_{A_1}(x), \nu_{A_2}(x) \\ \mu_B(x) & elsewhere \end{cases} \quad \ldots\ldots\ldots (3.4.2)$$

$$\nu_{(A_1)_\circ(B) \cup (A_2)_\circ(B)}(x) = min\left(max\left(\nu_{A_1}(x), \nu_B(x)\right), max\left(\nu_{A_2}(x), \nu_B(x)\right)\right)$$

$$= \begin{cases} min\left(\nu_{A_1}(x), \nu_{A_2}(x)\right) & when\ \mu_B(x) \le \nu_{A_1}(x), \nu_{A_2}(x) \\ \mu_B(x) & elsewhere \end{cases} \quad \ldots\ldots\ldots (3.4.3)$$

From (3.4.2) and (3.4.3) we have, for each $x \in E$,

$$\nu_{(A_1 \cup A_2)_\circ(B)}(x) = \nu_{(A_1)_\circ(B) \cup (A_2)_\circ(B)}(x) \quad \ldots\ldots\ldots (3.4.4)$$

So from (3.4.1) and (3.4.4) we have,

$$(A_1 \cup A_2)_\circ(B) = (A_1)_\circ(B) \cup (A_2)_\circ(B).$$

ii) For each $x \in E$,

$$\mu_{(A_1 \cap A_2)_\circ(B)}(x) = \mu_{A_1 \cap A_2}(x) = min\left(\mu_{A_1}(x), \mu_{A_2}(x)\right) = \mu_{(A_1)_\circ(B) \cap (A_2)_\circ(B)}(x) \quad \ldots\ldots\ldots (3.4.5)$$

Also for each $x \in E$,

$$\nu_{(A_1 \cap A_2)_\circ(B)}(x) = max\left(\nu_{A_1 \cap A_2}(x), \nu_B(x)\right) = max\left(max\left(\nu_{A_1}(x), \nu_{A_2}(x)\right), \nu_B(x)\right)$$

$$= \max\left(\nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x)\right) \quad \ldots\ldots\ldots (3.4.6)$$

and $\nu_{(A_1)_\circ(B) \cap (A_2)_\circ(B)}(x) = \max\left(\nu_{(A_1)_\circ(B)}(x), \nu_{(A_2)_\circ(B)}(x)\right)$

$$= max\left(max\left(\nu_{A_1}(x), \nu_B(x)\right), max\left(\nu_{A_2}(x), \nu_B(x)\right)\right)$$

$$= \max\left(\nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x)\right) \quad \ldots\ldots\ldots (3.4.7)$$

Hence, from (3.4.5), (3.4.6) and (3.5.7) we have,

$$(A_1 \cap A_2)_\circ(B) = (A_1)_\circ(B) \cap (A_2)_\circ(B).$$

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