A Geometric Approach for Determination of Geoidal Height in Akure Environs, Ondo State, Nigeria.

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Abstract

The study attempts to carry out the determination of geoidal height using a geometric approach. This approach involves the use of GPS/levelling to determine the ellipsoidal and orthometric heights. South DGPS was used in acquiring GPS data, which were processed using South GNSS processor software for deriving ellipsoidal heights, while Leica Jogger modern automatic level instrument was used in acquiring leveling data, which were processed to produce orthometric heights. A total of fifty nine (59) points were occupied for both GPS and levelling observations. The geoidal height was determined by computing the difference between the ellipsoidal and orthometric heights. The geoidal map and the geoidal model (3D) were created using Surfer 11 software. The chart for geoidal, ellipsoidal and orthometric heights show the same pattern, which is an indication that all show a true representation of the terrain. The profile produced for geoidal map and the digital geoidal model follows the same slope, which is an indication that both are natural height systems. The geometric approach was successfully used for geoidal height determination in the study area.

Keyword: Geoidal height, GPS, levelling, Orthometric height, Geometric Approach

I. INTRODUCTION

A precise geoid model constitutes one of the most interesting research subjects of geodesy, particularly since 1980s (Essam*et al.* 2015). Geoid modelling deals with the determination of geoidal undulations (N) between the geodetic heights (h) obtained from the Global Navigation Satellite Systems (GNSS) techniques and the orthometric heights (H), relative to the Mean Sea Level (MSL). Thus, geoid models are vital for the application of GNSS (particularly the Global Positioning System, GPS) in civil engineering projects (Essam *et al.* 2015).

The geoid is one of the most fundamental concepts in geodesy.It is defined as an equipotential surface of the earth gravity field which is everywhere perpendicular or at right angle to the direction of gravity at such point of observation which best approximates the mean sea level (MSL) when the mean sea level is undisturbed and extends below the continents. The geoid surface is much smoother than the natural earth surface despite its global undulations (changes) (Aleem*et al*. 2016). In contrast, it is very close to an ellipsoid of revolution, but more irregular, therefore it is approximated by the ellipsoid (Ahmed and Derek, 2011). The geoid height or the geoidal undulation N is described by the separation of the geoid from the ellipsoid of revolution. However due to the spatial irregularities of the geoid surface, it cannot be described by a simple mathematical function. High-resolution geoid model is valuable to geodesy, surveying, geophysics, and several geosciences, because it represents datum to height differences and gravity potential field. More-over, the geoid is very important for connection between local datum and the global datum, for purposes of positioning, levelling, inertial navigation system and geodynamics (Ahmed and Derek, 2011).

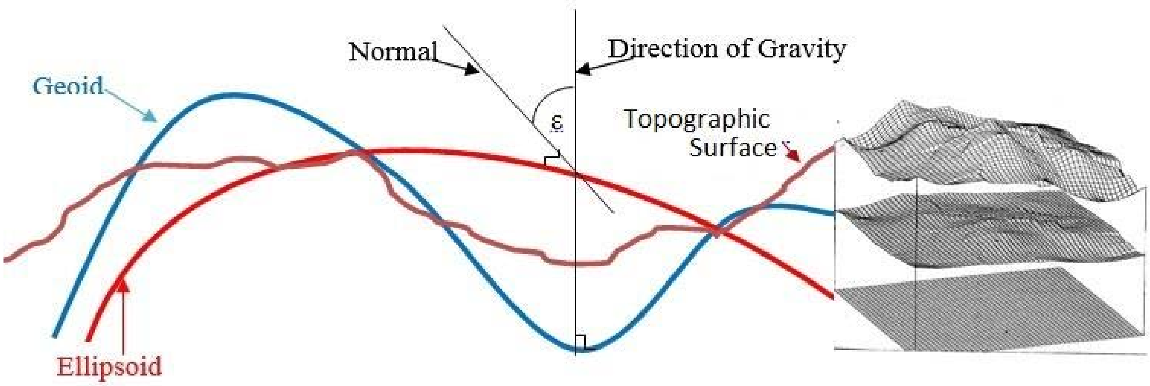

Figure 1: The three geodetic surfaces and their pictorial representation (source: Badejo, *et al*. 2016)

The Topographical Surface

The topographical surface is generally called the Earth’s surface. It is the actual surface of the land and sea. This is the physical surface where all measurements and observations are done. It is an irregular undulating surface, characterised by mountains, valleys, spurs, dunes and other features. Its undulating and irregular nature is caused by uneven distributions of the Earth masses which make it impossible to describe it with any mathematical relation (Vanicek and Krakiwsky, 1986; Uzodinma and Ezenwere, 1993; Vanicek*et al*. 2000; Torge, 2000). Hence, geodetic computation cannot be done on this surface. The surface close to the topographical surface is the geoid.

Geoid

The term *geoid* comes from the word geo which literarily means earth-shaped. The geoid is an empirical approximation of the figure of the earth (minus topographic relief). It is defined as the “equipotential surface of the Earth’s gravity field which best fits, in the least squares sense, of the mean sea level” (Deakin, 1996; Aleem*et al.* 2016). The geoid can also be defined as the “surface which coincides with that surface to which the oceans would conform over the entire Earth, if free to adjust to the combined effect of the earth's mass attraction (gravitational force) and the centrifugal force of the Earth's rotation" (Bomford, 1980). Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere; with respect to gravity. It more or less corresponds to the mean sea level (MSL) over the oceans. It is the surface of an ideal global ocean in the absence of tides and currents, directed and shaped only by gravity. It is a crucial measuring reference for various phenomena such as sea-level change, ocean circulation, and ice dynamics – all affected by climate change(Aleem, 2014).The geoid has a definite physical interpretation, in the sense that it can be fixed by measurements over the ocean with the use of mean sea level. Traditionally, because the sea surface is available worldwide, surveyors, mapmakers and other heights users or professionals have made the task of geoid determination to be simplified by using the average or mean of sea level as the definition of zero elevation. At any point on the geoid, the value of the height is zero, while above is positive and below is negative (Aleem, 2014).

Besides gravimetric approach, precise geoid heights can also be determined geometrically by means of GPS/levelling combination on land area, and altimetric techniques at sea (Banahmed *et al.*2006). If a relatively large number of well distributed points where ellipsoid and orthometric heights are available, the geometric geoid derived by GPS and levelling can be determinedand thecorrection to be applied to the gravimetric geoidcan bemodelled and computed. In this way, the possibility exists to develop an empirical surface (corrector surface) which relates a given gravimetric geoid model to the reference system of GPS ellipsoidal heights, and to the vertical datum of one's orthometric height system (Banahmed *et al.*2006).

Relationship between the Orthometric, Ellipsoidal and Geoidal Heights

The relationship between the three geodetics surfaces (figure 2) can be represented mathematically by:

$$N = h - H \quad (1)$$

$$H = h - N \quad (2)$$

$$h = H + N \quad (3)$$

In equation (1-3), N is the geoidal undulation, H is the orthometric heights, and h is the ellipsoidal height. These equations can be written as:

$$h - H - N = 0 \quad (4)$$

II. STUDY AREA

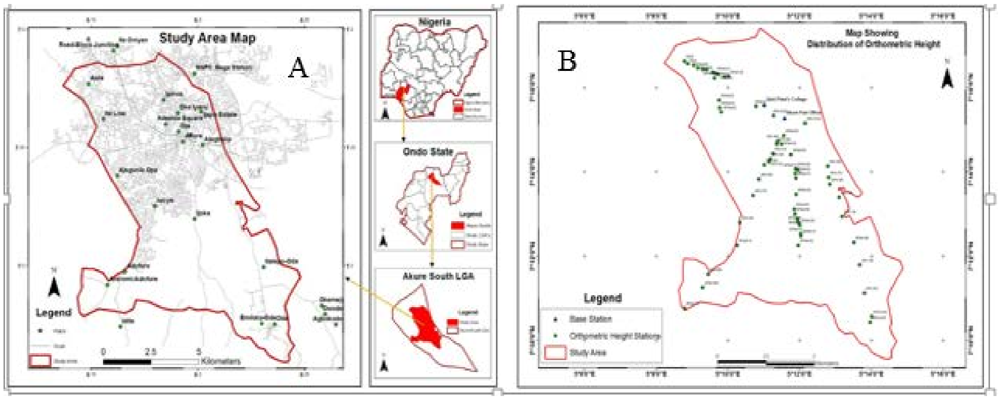

Figure2(A):StudyAreaMap (Akure Environs) and (B) Distribution of GPS/Levelling point used for the study

The study area selected for this research is Akure Environs figure 2 (A) of Ondo State in South-Western part of Nigeria. The geographic location is approximately between Latitudes $07^{\circ}15'$N to $07^{\circ}30'$N and Longitude $05^{\circ}15'$E to $05^{\circ}25'$E.

The topography of the Basement Complex terrain of Akure is generally undulating with a virtually rugged terrain consisting of hills and valleys with field recorded elevation varying between 330m above mean sea level in the southwestern border (Nigeria Army barracks) and399m in the north eastern border (Shagari Estate) (Michael and Franklin 2017).

III. METHODOLOGY

In carrying out the determination of geoidal height for Akure environs using geometric approach, methods and mode of operation were carefully chosen, based on some fundamental principles. These procedures include GPS observation and levelling data acquisition. A total of 59 common points were observed for both GPS and levelling. Processing strategy adopted includes, program development to enable the computation of geoidal height and orthometric height alongside deducing levelling data using Microsoft Excel spread sheet and processing of GPS observed data using South GNSS Processor.

Data Acquisition

Data acquisition is the process of gathering data or information from the field or from any other sources. The process involved; capturing, collecting, extracting, and surveys etc and then transforming these data into a reality. For the purpose of this study only the primary data was used.

Primary data

Primary data are the first hand information observed directly from the field by the researcher, which has not been used for any purpose or been transformed into any format. The primary data used are as:

1. The reduced level (orthometric height) of the stations.
2. The geodetic coordinates of the various control points (φ, λ, h)

Spirit levelling operation

Spirit levelling is the process of determining the difference in heights between two or more points on the earth surface. It is based on the fact that if a graduated staff can be placed vertically on the two ground points and the instrument, called a level, is set to the middle, and levelled, on bisecting a staff, the instrument will define a horizontal line of sight that will intersect the two staff reading at two different points, which depends on difference in elevation between the two points. For this operation to be performed, a two peg test must be carried out to determine if the quality and accuracy of the instrument really meet the standard for the operation. The collimation error was found to be 0.002mm which indicated that the instrument is in good order and can be used in carrying out observations.

In this study, the operation was carried out in closed loop levelling nets in order to obtain the height differences between the points. The Leica Jogger 20/24/28/32 modern automatic level instrument was set-up at a convenient point and the elevation of the survey control point AKU 001which is of second order accuracy was observed (back sight reading) with the help of the levelling staff held over the GPS control point and another levelling staff was held over another AKU 002control point for foresight reading and later closed back on AKU 001 figure 3.

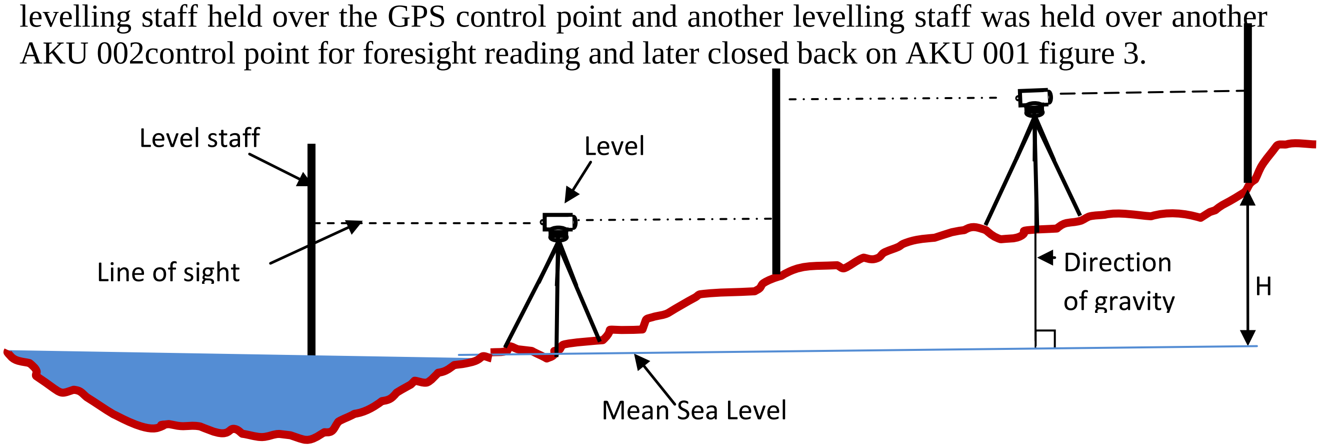

Figure 3: Levelling procedure to determine difference in height. (Source: Badejo, *et al*. 2016)

GPS Observation

South DGPS was used for acquisition of the 3D coordinatesof the existing GPS control points located within the study area. The instrument was mounted on AKU 001 control point as the base station and was levelled. The necessary settings of the parameters needed for the observation were performed (base or rover) such as the ID station, antenna height, epoch for streaming of data, mask angle, mode of observation e.t.c. After the setting operation, GPS was allowed to track not less than 4 satellites for data streaming.
The observations were done in post-processing static mode with the base station at AKU 001 and the rover moving round from pillar-to-pillar after carrying out all the necessary settings. In all the observation (base or rover) the PDOP (Positional Dilution of Precision) was ensured to be consistently less than 2.00. From the GPS observations, ellipsoidal heights were derived.

IV. RESULT AND DISCUSSION

Table 1: Ellipsoidal, Orthometric and Geoidal Heights

STATIONS	Latitude [Degrees]	Longitude [Degrees]	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Geoidal Heights N = h -H [m]
AKU001	7.2707993	5.1670482	359.9130	346.4700	13.44304
AKU002	7.2707205	5.1651083	358.6052	345.1465	13.45874
AKU003	7.2718627	5.162073	352.0484	338.3880	13.66040
AKU004	7.2725468	5.1597576	350.3126	336.6660	13.64660
AKU005	7.2737013	5.1568474	348.2843	334.6510	13.63330
AKU006	7.2736669	5.1551974	351.0061	337.3650	13.64110
AKU007	7.2743554	5.1527432	356.3205	342.5380	13.78250
AKU008	7.2754062	5.150703	359.4789	345.8310	13.64790
AKU009	7.2759909	5.1481422	359.5066	345.8170	13.68960
AKU010	7.2769842	5.1466017	353.7181	340.1150	13.60310
AKU011	7.2723335	5.1607864	351.9264	338.2150	13.71140
AKU012	7.2702406	5.1606406	347.8124	334.1020	13.71040
AKU013	7.2615421	5.1625064	363.9258	349.7650	14.16080
AKU014	7.258587	5.162796	360.0313	345.8400	14.19130
AKU015	7.256818	5.1636238	353.4029	339.1970	14.20590
MEAN			**347.7476**	**334.3930**	**13.3885**
STDEV			**10.95886**	**10.56190788**	**0.671478388**

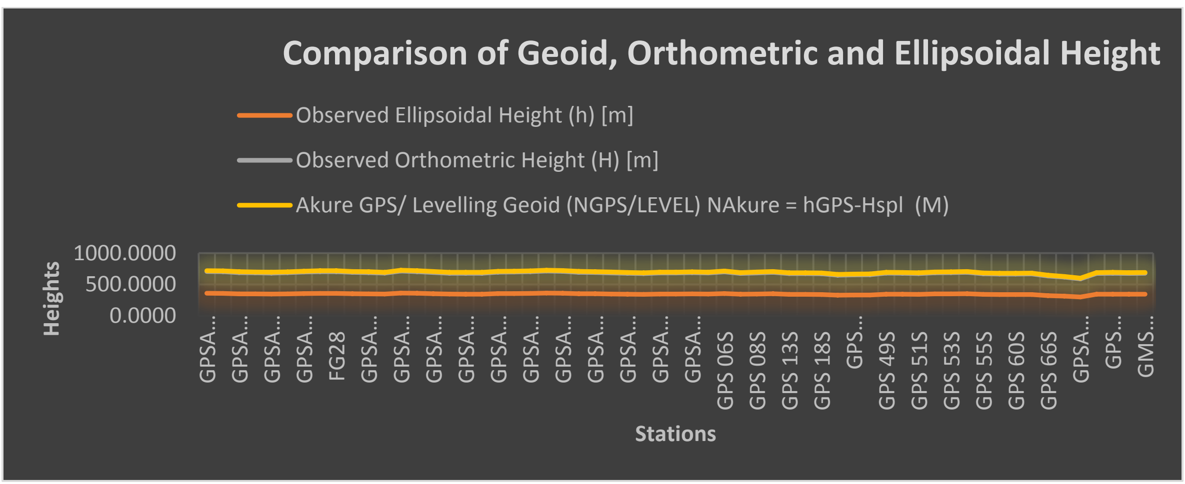

Figure 4: graphical Comparison of Geoidal, Orthometric, and Ellipsoidal heights

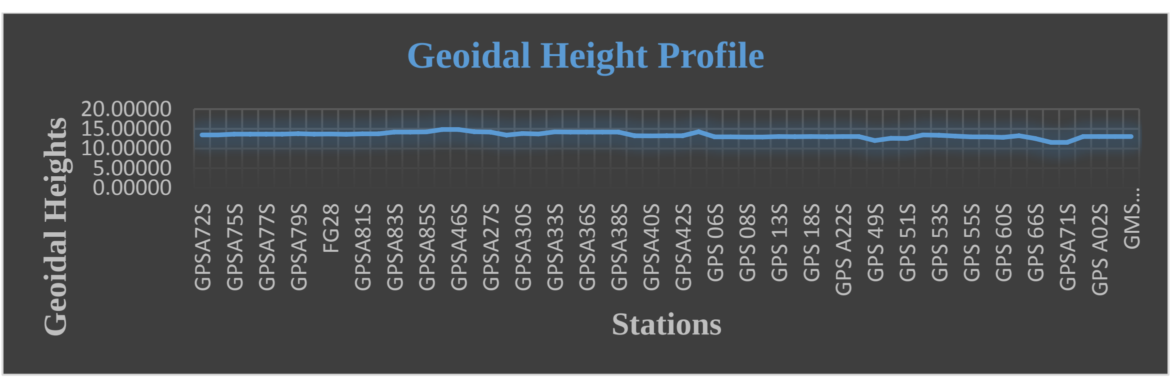

Figure 5: chart showing geoidal height profile

Table 2: Ellipsoidal and orthometric height differences

STATIONS	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Δh (m)	ΔH (m)	Δh -ΔH (m)
AKU001	359.9130	346.4700			
AKU002	358.6052	345.1465	1.3078	1.3235	-0.0157
AKU003	352.0484	338.3880	6.5568	6.7585	-0.2017
AKU004	350.3126	336.6660	1.7358	1.7220	0.0138
AKU005	348.2843	334.6510	2.0283	2.0150	0.0133
AKU006	351.0061	337.3650	-2.7218	-2.7140	-0.0078
AKU007	356.3205	342.5380	-5.3144	-5.1730	-0.1414
AKU008	359.4789	345.8310	-3.1584	-3.2930	0.1346
AKU009	359.5066	345.8170	-0.0277	0.0140	-0.0417
AKU010	353.7181	340.1150	5.7885	5.7020	0.0865
AKU011	351.9264	338.2150	1.7917	1.9000	-0.1083
AKU012	347.8124	334.1020	4.1140	4.1130	0.0010
AKU013	363.9258	349.7650	-16.1134	-15.6630	-0.4504
AKU014	360.0313	345.8400	3.8945	3.9250	-0.0305
AKU015	353.4029	339.1970	6.6284	6.6430	-0.0146

Table 3: Statistical result from change in ellipsoidal and orthometric height

STATIONS	Δh (m)	ΔH (m)	Δh -ΔH (m)	Mean Square Error (MSE) [m] $(\Delta h - \Delta H)^2$(m)
AKU001				
AKU002	1.3078	1.3235	-0.0157	0.000246
AKU003	6.5568	6.7585	-0.2017	0.040665
AKU004	1.7358	1.7220	0.0138	0.000190
AKU005	2.0283	2.0150	0.0133	0.000177
AKU006	-2.7218	-2.7140	-0.0078	0.000061
AKU007	-5.3144	-5.1730	-0.1414	0.019994
AKU008	-3.1584	-3.2930	0.1346	0.018117
AKU009	-0.0277	0.0140	-0.0417	0.001739
AKU010	5.7885	5.7020	0.0865	0.007482
AKU011	1.7917	1.9000	-0.1083	0.011729
AKU012	4.1140	4.1130	0.0010	0.000001
AKU013	-16.1134	-15.6630	-0.4504	0.202860
AKU014	3.8945	3.9250	-0.0305	0.000930
AKU015	6.6284	6.6430	-0.0146	0.000213
MEAN	**0.2363**	**0.2291**	**0.0071**	**0.2612**
STDEV	**8.4658**	**8.4134**	**0.5154**	**0.6779**

The change in elevation between each point for both the ellipsoidal and orthometric heights were computed using equation (6)

$$\left(h_i - h_{i+1}\right) - \left(H_i + H_{i+1}\right) = \left(N_i - N_{i+1}\right) \quad (6)$$

h_i is the ellipsoidal height of station *i*
h_{i+1} is the ellipsoidal height of points prior station *i*
H_i is the orthometric height of station *i*
H_{i+1} is the orthometric height of points prior station *i*
N_i is the geoidal undulation of station *i*
N_{i+1} is the Geoidal Undulation of points prior station

Root Mean Square Error:

The Root Mean Square Error (RMSE) of the orthometric height differences and ellipsoidal height differences were computed by squaring the difference in heights usingequation (7)

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(\Delta h - \Delta H)^2}, \text{ where,} \quad (7)$$

n is the number of the point used for the accuracy confirmation and k is the residual sequence.

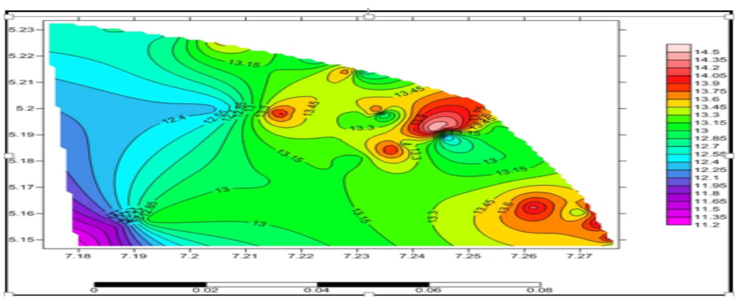

Figure 6: Contour plot of geoidal mapof the study area

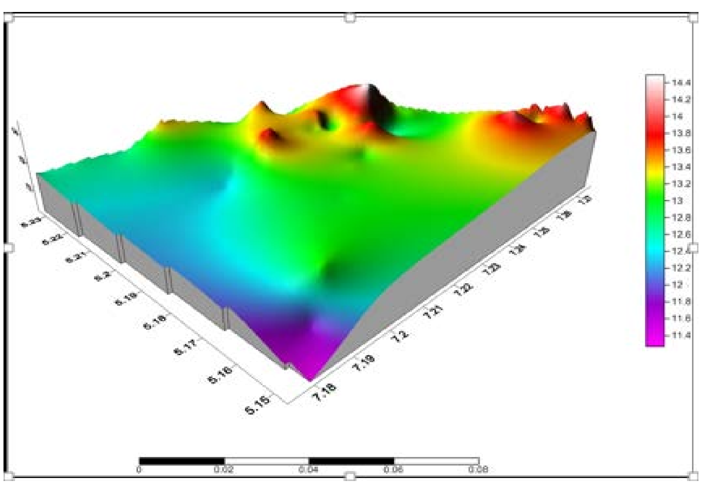

Figure 7: Digital geoidal model of the study area

V. DISCUSSION OF RESULT

The geographical coordinate and ellipsoidal heights (h) were obtained from GPS post –processed observation using South GNSS processor software Table 1. The adjusted orthometric heights (H) were extracted from the deduced levelling field book Table 1. The geoidal height was determine from the difference between the ellipsoidal and orthometric height Table 1. Figure 4 is a graphical comparison between the geoidal, orthometric and ellipsoidal heights. Figure 5 is the geoidal height profile to depict the true nature of the terrain. The differences between the ellipsoidal and orthometric height were computed and the result is shown in Table 2. Table 3 shows the statistical result for the changes in elevation between the two height systems. The result produce the standard deviation value as 8.4558 for ellipsoidal heights and 8.4134 for orthometric height respectively while the RMSE value was 0.51107. Figure 6 and 7 are the contour plot of geoidal map and digital geoidal model (3D) plotted using Surfer 11 software adopting kriging gridding method at 0.15m contour interval. The chart comparison for geoidal, ellipsoidal and orthometric height followed the same pattern which is an indication that both are the true representation of the terrain. The geoidal map and the digital geoidal model follows the same slope toward a specific direction which is an indication that both are natural height systems.

VI. CONCLUSION.

In order to determine the geoidal height of the study area, a geometric method was adopted. The geometric method involves the use of GPS/Levelling data. The data were acquired for orthometric and ellipsoidal heights using South DGPS and Leica Jogger modern automatic level instrument. The difference between ellipsoidal and orthometric height was used forgeoidal height determination. The height difference between the ellipsoidal and orthometric heights were computed. The geoidal height computed was used for the production of geoidal map and geoidal model of the study area.The geoidal map and the digital geoidal modelshowthe same slope toward a specific direction,which is an indication that both are natural height systems.The geometric approach was successfully used for geoidal height determination in the study area.

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