Decision - Making Model by Applying Fuzzy Numbers in Construction Project

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Abstract- Multi-attribute analysis is a useful tool in many economical, managerial, constructional, etc., problems. There is usually some uncertainty involved in all multi-attribute model inputs. The objective of this work is to demonstrate how simulation can be used to reflect fuzzy inputs, which allows more complete interpretation of model results. A case study is used to demonstrate the concept of general contractor choice of on the basis of multiple attributes of efficiency with fuzzy inputs applying COPRAS method. The work has concluded that the COPRAS method is appropriate to use.

Index Terms- multi-attribute, decision – making model, COPRAS, Fuzzy number, contractor assessment.

I. Introduction

Multiple attribute decision-making problems are encountered under various situations where a number of alternatives and actions or candidates need to be chosen based on a set of attributes when we consider a discrete set of alternatives described by some goals. Determine the optimal alternative with the highest degree of desirability with respect to all relevant goals.

Most approaches in Multi-attribute decision making consist of two stages

The aggregation of the judgements with respect to all goals and per decision alternative.

The rank ordering of the decision alternatives according to the aggregated judgements.

In crisp Multi-attribute decision making models it is usually assumed that the final judgements of the alternatives are expressed as fuzzy numbers. In this case the second stage does not pose any particular problems and suggested algorithms concentrate on the first stage. Fuzzy models are sometimes justified by the argument that the goals or their attainment by the alternatives cannot be defined or judged crisply but only as fuzzy sets. In this case the final judgements are also represented by fuzzy sets which have to be ordered to determine the optimal alternative. Then the second stage is, of course, by far not trivial.

Algorithm for multiple attributes decision – making

Step 1: Establishing system evaluation attributes that relate system capabilities to goals

Step 2: Developing alternative systems for attaining the goals(generating alternatives)

Step 3: Evaluating alternatives interms of attributes(the values of the attributes functions)

Step 4: Applying a normative multiple attributes analysis method.

Step 5: Accepting one alternative as "optimal"(preferred)

Step 6: If the final solution is not accepted, gather new information and go into the next iteration of multiple attributes optimization.

The problem may be

- Choice – select the most appropriate (best)alternative
- Ranking – draw a complete order of the alternatives from the best to the worst ones
- Sorting – select the best k alternatives from the list

Multiple attributes decision provides several powerful and effective tools for confronting sorting problems. Comparing the alternatives is the key of making the decision. However, in case of conflicting alternatives, a decision-maker must also consider imprecise or ambiguous data, which is norm in this type of decision – making problems.

II. Fuzzy Systems and Fuzzy Relational Analysis in Decision Making

The Fuzzy system has been applied in many fields. Noorul Haq and Kannan (2007) developed a hybrid normalized multi – attribute decision making model for evaluation and selecting the vendor using analytical hierarchy process and fuzzy analytical hierarchy process. Experimental results showed that the proposed method presents more precise estimates over the results using the case-based reasoning, classification and regression trees, and artificial neural networks methods.

A Fuzzy number is an generalization of a regular, real number in the sense that is does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. A Fuzzy number is a convex, normalized fuzzy set $\tilde{A} \subseteq \Re$ whose membership function is atleast segmentally continuous and has the functional value $\mu_{\tilde{A}}(x) = 1$ at precisely one element. This can be likened to the funfair game "guess your weight", where someone guesses the contestant's weight, with closer guesses being more correct, and where the guesser"wins"if he or she guesses near enough to the contestant's weight, with the actual weight being completely correct(mapping to 1 by the membership function). A Fuzzy interval is an uncertain set $\tilde{A} \subseteq \Re$ with a mean interval whose

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elements possess the membership function value $\mu_{\tilde{A}}(x) = 1$. As in fuzzy numbers, the membership function must be convex, normalized, atleast segmentally continuous.

The following fuzzy sets are fuzzy numbers

Nearly 6 = $\{(4,0.3),(5,0.7),(6,1),(7,0.8),(8,0.2)\}$

But $\{(4,0.9),(5,1),(6,1),(7,0.8)\}$ is not a fuzzy number because membership function of 5 and 6 is 1.

The explanation of COPRAS is the multi- attribute convex proportional assessment of alternatives. Fuzzy convex is the membership grade starts at zero, rises to a maximum and then declines to zero again as the domain increases. This method is useful for value evaluation of maximizing as well as minimizing criteria. Ranking alternatives by the COPRAS method assumes direct and proportional dependence of significance and priority of investigated alternatives on a system of criteria . The determination of significance and priority of alternatives, by using COPRAS method. The presented procedure of COPRAS method indicates that it can be easily applied for evaluating the alternatives and selecting the most efficient one, with decision maker being completely aware of the physical meaning of the process.

III. Ranking of the Alternatives by Applying COPRAS Methods

The multi – attribute decision making methods mostly deal with exactly determined information. One of such methods is COPRAS method.

The algorithm of the COPRAS method consists of the steps as is shown in the Fig 1

1. Selection of the available set most important attributes, which describes alternatives.
2. Preparing of the decision – making matrix X:

$$X = \begin{bmatrix} x_{11} & x_{12} & . & . & . & x_{1m} \\ x_{21} & x_{22} & . & . & . & x_{2m} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{n1} & x_{n2} & . & . & . & x_{nm} \end{bmatrix}; \quad j = 1 \, to \, n \;\; and \;\; i = 1 \, to \, m \qquad (1)$$

Multi attribute decision making model by applying Fuzzy numbers

- Decision making matrix (Matrix with values of attributes described in intervals)
- Normalizing decision making matrix
- Weighting normalized decision making matrix
- Calculation minimizing indexes R_j for each alternative(The sums of normalized weighted indexes describing the j-th alternative that must be minimized are calculated)
- Calculation maxmizing indexes P_j for each alternative(The sums of normalized weighted indexes describing the j-th alternative that must be maxmized are calculated)
- Calculating the sums of normalized weighted indexes describing the j-th alternative. The alternatives are described by minimizing indexes
- Determining minimal value of R_j
- Determining significance of alternatives
- Ranking alternatives according to relative significance of each alternative

Fig 1. Ranking of alternatives by applying COPRAS method.

Where attribute ***i*** in the alternative ***j*** of a solution; ***m*** is the number of attributes; ***n*** is the number of the alternatives compared.

Determining weights of the attributes q_i

Normalization of the decision making matrix $\hat{X}$. The normalized values of this matrix are calculated as

$$\bar{x}_{ji} = \frac{x_{ji}}{\sum_{j=1}^{n} x_{ji}}; \quad j = 1 \, to \, n \;\; and \;\; i = 1 \, to \, m \qquad (2)$$

After this step we have normalized decision-making matrix

$$\hat{X} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{12} & . & . & . & \bar{x}_{1m} \\ \bar{x}_{21} & \bar{x}_{22} & . & . & . & \bar{x}_{2m} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \bar{x}_{n1} & \bar{x}_{n2} & . & . & . & \bar{x}_{nm} \end{bmatrix}$$

Calculation of the weighted normalized decision – making matrix $\hat{X}$. The weighted normalized values $\hat{x}_{ji}$ are calculated as

$$\hat{x}_{ji} = \bar{x}_{ji} . q_i; \quad j = 1 \; to \; n \;\; and \;\; i = 1 \, to \, m \qquad (4)$$

In formula (4) q_i is weight of the ith attribute.

After this step we have weighted normalized decision making matrix:

$$\hat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & . & . & . & \hat{x}_{1m} \\ \hat{x}_{21} & \hat{x}_{22} & . & . & . & \hat{x}_{2m} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \hat{x}_{n1} & \hat{x}_{n2} & . & . & . & \hat{x}_{nm} \end{bmatrix}; \quad j = 1 \, to \, n \, \, and \, \, i = 1 \, to \, m \qquad (5)$$

Sums P_j of attributes values which larger values are more preferable(optimization direction is maximization) calculation for each alternative (line of the decision making matrix):

$$P_j = \sum_{i=1}^{k} \hat{x}_{ji} \qquad (6)$$

In formula (6) ***k*** is number of attributes which must to be maximized (it is assumed that in the decision making matrix columns first of all are placed attributes with optimization direction maximum and ones with optimization direction minimum are placed after).

Sums R_j of attributes values which smaller values are more preferable (optimization direction is minimization) calculation for each alternative (line of the decision making matrix)

$$R_j = \sum_{i=k+1}^{m} \hat{x}_{ji} \qquad (7)$$

Determining the minimal value of R_j

$$R_{\min} = \min_j R_j; \qquad j = 1 \, to \, n \qquad (8)$$

Calculation of the relative weight of each alternative Q_j

$$Q_j = P_j + \frac{R_{\min} \sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} \frac{R_{\min}}{R_j}} \qquad (9)$$

Formula (9) can to be written as follows

$$Q_j = P_j + \frac{\sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} \frac{1}{R_j}} \qquad (9^*)$$

Determination of the optimality criterion K

$$K = \max_j Q_j \quad ; \quad j = 1 \, to \, n \qquad (10)$$

of the priority of the project. The greater weight (relative weight of alternative) Q_j, the higher is priority(rank) of the project. In the case of $Q_{\max}$ the satisfaction degree is the highest.

Calculation of the utility degree of each alternative

$$N_j = \frac{Q_j}{Q_{\max}} 100\% \qquad (11)$$

Where Q_j and $Q_{\max}$ are the weight of projects obtained from eqn(9*).

IV. Case Study of Contractors Selection

Contractor's selecting in construction problem was selected to illustrate newly proposed method. Construction projects are complicated, unique and built only once. The designing and construction process always deals with uncertainty, risk and risk management. The right selection of a qualified contractor depends on many attributes and gives confidence to the stakeholder that the selected contractor can achieve the project goals. The efficiency of a construction process is often associated with the successful choice of a contractor. The construction industry contractors should employ solid and reliable strategies to establish their profit and risk margins for their offers. The importance of non-price attributes is well recognized in the literature. In order to select the most appropriate contractor for the project and prepare the most realistic and accurate bid proposal, general contractors have to know all financial, technical and general information about these contractors.

V. Problem Solution

The set of attributes and initial values of attributes are determined on the basis of expert, normative and calculation methods. The set of alternatives $X = \{x_1, x_2, x_3\}$ and the Goals be given as

$$\tilde{G}_1(x_i) = \{(x_1, 0.7), (x_2, 0.5), (x_3, 0.4)\}$$
$$\tilde{G}_2(x_i) = \{(x_1, 0.3), (x_2, 0.8), (x_3, 0.6)\}$$
$$\tilde{G}_3(x_i) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.8)\}$$
$$\tilde{G}_4(x_i) = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.2)\}$$

The weights of the goals have been established as: $(q_1, q_2, q_3, q_4) = (0.15, 0.45, 0.15, .25)$ then we find the optimal alternative.

The decision making matrix X

$$X = \begin{bmatrix} 0.7 & 0.3 & 0.2 & 0.5 \\ 0.5 & 0.8 & 0.3 & 0.1 \\ 0.4 & 0.6 & 0.8 & 0.2 \end{bmatrix}$$

The normalized decision making matrix as

$$\overline{X} = \begin{bmatrix} 0.438 & 0.177 & 0.154 & 0.625 \\ 0.313 & 0.471 & 0.231 & 0.125 \\ 0.25 & 0.353 & 0.615 & 0.25 \end{bmatrix}$$

And the weighted normalized decision making matrix

$$\hat{X} = \begin{bmatrix} 0.066 & 0.0708 & 0.0308 & 0.156 \\ 0.047 & 0.1884 & 0.0462 & 0.0313 \\ 0.0375 & 0.1412 & 0.123 & 0.063 \end{bmatrix}$$

Here $(p_1, p_2, p_3) = (0.1368, 0.2354, 0.1787)$

and $(R_1, R_2, R_3) = (0.1868, 0.0775, 0.186)$

$R_{\min} = 0.0775 = R_2$

$(Q_1, Q_2, Q_3) = (0.2388, 0.4813, 0.2811)$

The utility degree of each alternative is $(N_1, N_2, N_3) = (50\%, 100\%, 58\%)$

Thus the best second alternative is selected according to COPRAS method. The corresponding alternative rank as follows $x_2 > x_3 > x_1$.

VI. Conclusions

In real life multi attribute modeling of multi alternative assessment problems attribute values, which deals with the future, can to be expressed in fuzzy intervals. COPRAS is developed method for assessment of alternatives by multiple attribute values determined in fuzzy intervals. This approach is intended to support the decision making process and increase the efficiency of the resolution process. COPRAS method can be applied to the solution of wide range discrete multi attribute assessment problems in construction.

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