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Comparison of 3-valued Logic using Fuzzy Rules

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***Abstract*:** The information encoded on a computer may have a negative or a positive emphasis. Negative information refers to the statement that some situations are impossible. It is often the case for pieces of background knowledge expressed in a logical format. Positive information refers to observed cases. It is often encountered in data-driven mathematical models, learning, etc. The notion of an "if. . . , then. . . " rule is examined in the context of positive and negative information. We know that the classical logic is two valued TRUE and FALSE. In real life situations the true valued logic can be seen inadequate. We need logics which will allow the other truth values in between TRUE and FALSE. This paper gives an idea of the logic that needs to be put forward beyond classical two valued logic.

Index Terms —Fuzzy rule, antecedent, consequent, fuzzy implications.

1. INTRODUCTION

Fuzzy rules are often considered to be a basic concept in fuzzy logic [1]. Fuzzy rules were meant to represent human knowledge in the design of control systems, when mathematical models were lacking [2]. Topics like data mining, knowledge discovery and various forms of learning techniques have become an important challenge in information technology, due to the huge amount of data stored in information systems. Looking for meaning in data has become again a relevant issue. Even though the classical two-valued logic codifies and explains the human reasoning, it has been felt that it does not reflect whole gamut of our reasoning capabilities.

This leads to the three-valued representation of a rule, according to which a given state of the world is an example of the rule, a counterexample to the rule, or is irrelevant for the rule. This view also sheds light on the typology of fuzzy rules. It explains the difference between a fuzzy rule modeled by a many-valued implication, and expressing negative information, and a fuzzy rule modeled by a conjunction and expressing positive information.

The aim of this paper is:
1. to give a 3-valued logical account of Bochvar's and Heyting's Fuzzy.
2. to show that the typology of fuzzy rules, previously proposed by the authors [18,19] manages to reconcile the knowledge-driven logical tradition and data-driven engineering tradition;

2. PRELIMINARIES

Definition 2.1: Fuzzy Rule

A fuzzy rule is of the form: R: If < x is P > then < y is Q >

where 'x is P' and 'y is Q' are fuzzy propositions.

The meaning of 'x is P', is called the rule antecedent and the meaning of 'y is Q', is called the rule consequent.

Examples:

1. If < temperature is high > then < pressure will be low >
2. If < a tomato is red > then < it is ripe >

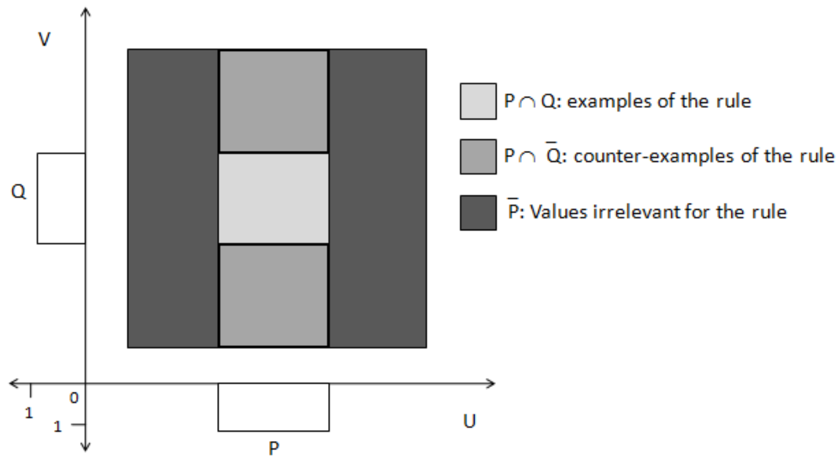

Figure 1: Partition on U x V induced by the rule “If x is P, then y is Q”

Definition2.2. Negative view:

The rule is viewed as a constraint of the form “if x is P , then y *must* be Q”. In other words, if $x \in p$ and $y \notin Q$ then $(x,y) \notin R$, or equivalently $R \subseteq \bar{P} \cup Q$. This view emphasizes only the counter-examples to the rule. It is the implicative form of the rule. Pairs of attribute values in $P \cap \bar{Q}$ are deemed impossible. Other ones remain possible, as shown on Fig

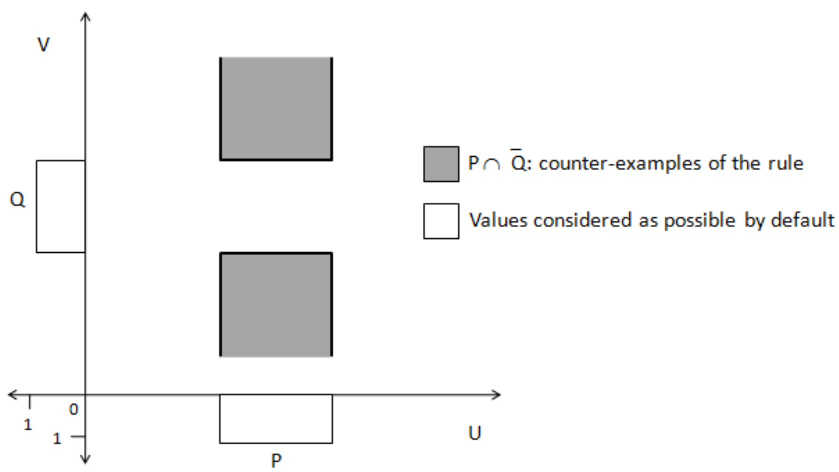

Figure 2: Negative view of the rule “If x is P, then y is Q”

Definition2.3. Positive view:

The rule is viewed as a case of the form “if x is P, then y can be Q”. In other words, if $x \in p$ and $y \in Q$ then $(x,y) \in R$, or equivalently $P \cap Q \subseteq R$. This view emphasizes only the examples to the rule. It is the conjunctive form of the rule. Pairs of attribute values in $P \cap Q$ are guaranteed possible. It is not known as other ones are possible or not, as shown on figure.

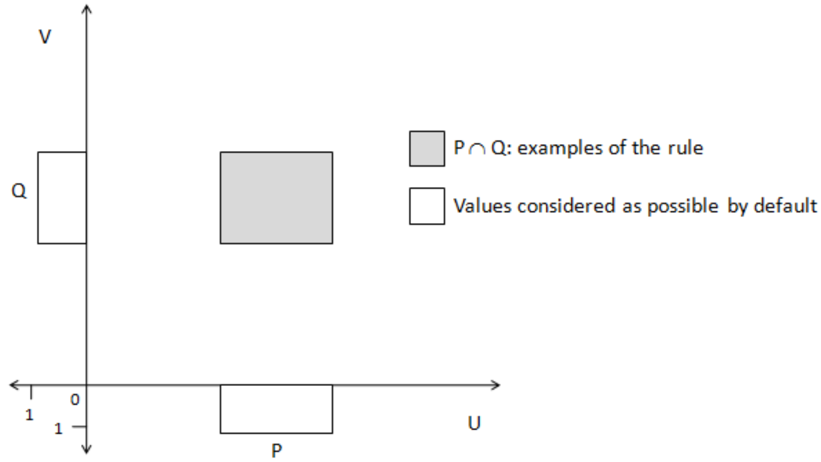

Figure 3: Positive view of the rule "If x is P, then y is Q"

3: Three-valued Logic & IF-THEN rules

(a) The 3- valued logic is same as in 2- valued logic, except that there are truth values: True, May be and False. These linguistic values are represented by 1, ½ and 0. In this 3- valued logic, denoted by L3, the truth value of any statement can be either 1 or ½ or 0.

i.e. $T(p) = 1$ or ½ or 0.

We define these 3 operations on the statements p, q, r, ... denoted by $p \vee q$, $p \wedge q$ and $\neg p$ analogous to the three operations OR, AND and NOT of classical logic. These are defined by their truth values as follows:

$$T(p \vee q) = \max\{T(p), T(q)\}$$

$$T(p \wedge q) = \min\{T(p), T(q)\}$$

$$T(\neg p) = 1 - T(p)$$

(b) Lukasiewicz, also defined the implication operator by

$$T(p \rightarrow q) = 1 - T(p) + T(q) \qquad \text{if } T(p) > T(q)$$

$$= 1 \qquad \text{if } T(p) \leq T(q)$$

Or simply as $(p \rightarrow q) = \min\{1, 1 - T(p) + T(q)\}$

Using this formula we derive the truth table of →.

Table I

→	1	1/2	0
1	1	1/2	0
1/2	1	1	1/2
0	1	1	1

Here the four corner values are same as the condition operator of the classical logic.

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(c) Similarly, we can frame the truth tables of the other three operations. This 3- valued logic has many distinguishing and surprising features. One of them is that WFF [p ∨ (¬p)] is NOT a tautology. This is given in the below Table 2.

Table II

p	¬p	p ∨ (¬p)
1	0	1
1/2	1/2	1/2
0	1	1

Here we see that the last column does not have all entries equal to 1.

The above generalization of the classical logic by Lukasiewicz inspired many other generalizations.

4. COMPARISION OF BOCHVAR'S & HEYTINGS'S 3-VALUED LOGIC

Here we have compared the Bochvar's and Heyting's values in3- valued logic.

Table III: Bochvar's 3- valued Logic

a	0	0	0	1/2	1/2	1/2	1	1	1
b	0	1/2	1	0	1/2	1	0	1/2	1
∧	0	1/2	0	1/2	1/2	1/2	0	1/2	1
∨	0	1/2	1	1/2	1/2	1/2	1	1/2	1
→	1	1/2	1	1/2	1/2	1/2	0	1/2	1

Table IV: Heyting's 3- valued Logic

a	0	0	0	1/2	1/2	1/2	1	1	1
b	0	1/2	1	0	1/2	1	0	1/2	1
∧	0	0	0	0	1/2	1/2	0	1/2	1
∨	0	1/2	1	1/2	1/2	1	1	1	1
→	1	1	1	0	1	1	0	1/2	1

A fuzzy **IF –THEN** rule consists of an IF part (antecedent) and a THEN part (consequent).The antecedent is a combination of terms, where as consequent is exactly one term. In the antecedent the terms can be combined by using Fuzzy Conjunction, Disjunction and Negation.

Conclusion:

This paper has emphasized two complementary types of information called negative and positive information and the idea of 3-valued Logics. Negative information acts as constraints that exclude possible worlds while positive information models observations that enable new possible worlds. It has been shown that "if. . . then. . . " rules convey both kinds of information, through their counter-examples and examples respectively. The existence of if-then rules, whose representation is based either on implications or on conjunctions can be explained by the existence of these two antagonistic views of information, Fuzzy rules and especially conjunctive .T(¬ p) is common for Lukasiewicz and Bochvar's Logic, but it is not true for Heyting's Logic. These 3-valued Logics can be framed as rules where it is combined by using fuzzy conjunction, disjunction and negation.

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