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Fuzzy Shortest Route Algorithm for Telephone Line Connection

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Abstract- In computer science, there are many algorithm that finds a minimum spanning tree for a connected weighted undirected fuzzy graph. This means it finds a subset of the edge-set that forms a fuzzy spanning tree that includes every vertex, where the total weight of all the edges in the fuzzy tree is minimized. Generally algorithm has many applications, such as landline (intercom) phone connection to a particular college in order to minimize the shortest distance, which applies Nearest Neighbor Algorithm, Sorted Edges Algorithm, Kruskal's Algorithm to a randomly weighted fuzzy graph. In this paper, we find the solution for the problem, that A.V.V.M. Sri Pushpam College, (Autonomous), Poondi, Thanjavur District, South India need to connect updated intercom lines connecting all the departments using fuzzy shortest route algorithm. The problem is to minimize the amount of new line using Kruskal's Algorithm with fuzzy graph. These algorithm continuously increases the size of a fuzzy tree, one edge at a time, starting with a fuzzy tree consisting of a single vertex, until it spans all vertices of the fuzzy graph.

Index Terms- Fuzzy spanning tree –Nearest Neighbor Algorithm- Minimum weight link Algorithm-Kruskal's Algorithm.

I. Introduction

In many transportation, routing, communications, economical, and other applications, fuzzy graphs emerge naturally as a mathematical model of the observed real world system. Indeed, many problems can be reformulated as a quest for a path(between two nodes in a fuzzy graph) which is optimal in the sense of a number of preset criteria. Very often, these optimality criteria are evaluated in terms of weights, that is, vectors of real numbers, associated with the links of the fuzzy graph. Numerous algorithms have been developed to ease this and related quests. More recently, fuzzy weighted graphs, along with generalizations of algorithms for finding optimal paths within them, have emerged as an adequate modeling tool for prohibitively complex and/or inherently imprecise systems. We review and formalize these algorithms, paying special attention to the ranking methods used for path comparison. We discussed which criteria must be met for algorithm correctness and present an efficient method, based on defuzzification of fuzzy weights, for finding optimal paths.

The network is a standard application pattern for phone lines to connect them up with each other; with minimum distance and company with different charges for different connection using set of lines that connects with a minimum total cost. It should be a fuzzy spanning tree, since if a network isn't a fuzzy tree.

A less obvious application is that the minimum fuzzy spanning tree can be used to approximately solve the traveling salesman problem. A convenient formal way of defining this problem is to find the shortest path that visits each point exactly once. This problem can be solved by many different algorithms, which are recently developed in research topics. There are several "best" algorithms, depending on the following assumptions:

- A randomized algorithm can solve it in linear expected time.
- It can be solved in worst case time if the weights are large integers.
- Select the minimum weightage unused edge in the fuzzy graph; highlight it.

II. Definition

Definition 2.1: A ***fuzzy graph*** with V as the underlying set is a pair G: (A, Γ) where A: V --→ [0,1] is a fuzzy subset, Γ : VxV ------> [0,1] is a fuzzy relation on the fuzzy subset A, such that $\Gamma(u,v) \leq A(u) \cap A(v)$ for all u,v ϵ V.

Definition 2.2: *A* ***fuzzy Hamiltonian circuit*** is a circuit that visits every vertex in a fuzzy graph once with no repeats, being a fuzzy Hamiltonian circuits must start and end at the same vertex.

Definition 2.3: A ***fuzzy Hamiltonian path*** is a path that passes through each of the vertices in a fuzzy graph exactly once.

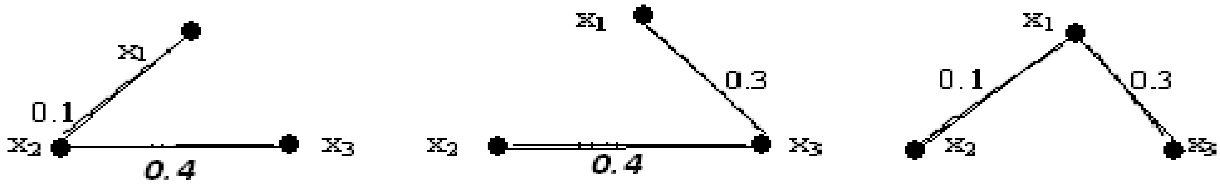
Definition 2.4: A ***fuzzy spanning tree*** is a fuzzy tree which covers all the vertices of a fuzzy graph.

NOTE 2.1:

Fuzzy tress has no circuits, and it is fine to have vertices with degree higher than two.

EXAMPLE 2.1 :

A fuzzy *spanning tree* of a fuzzy graph is just a fuzzy sub-graph that contains all the vertices and is a fuzzy tree. A fuzzy graph may have many fuzzy spanning trees; for instance the complete fuzzy graph on n vertices has n(n-1)/2 fuzzy spanning trees:



Forming the fuzzy spanning tree by finding a fuzzy subgraph -a new fuzzy graph formed using all the vertices and some of the edges from the original fuzzy graph.

Brute Force Algorithm

Step 1: List all possible Fuzzy Hamiltonian circuits .

Step 2: Find the length of each circuit by adding the edge weights.

Step 3: Select the circuit with minimal total weight.

Example:2.2

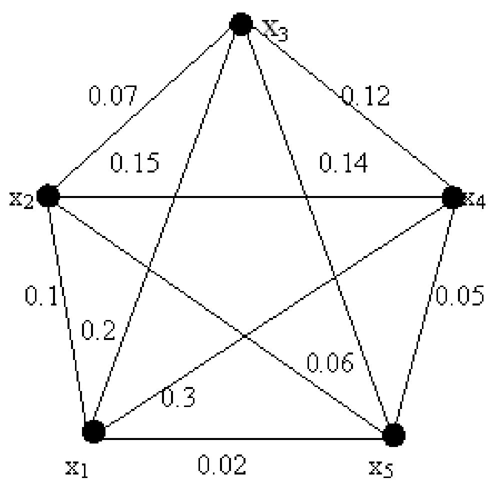


Fig 2.1

SL.NO	Fuzzy Hamiltonian circuits	Weights
1.	$x_1x_2x_3x_4x_5x_1$	0.36
2.	$x_1x_3x_5x_2x_4x_1$	0.85
3.	$x_1x_2x_4x_5x_3\,x_1$	0.64
4.	$x_1x_4x_2x_5x_3\,x_1$	0.85
5.	$x_1x_5x_2x_3x_4\,x_1$	0.57

There are unique circuits on this fuzzy graph. All other circuits are the reverse of the listed ones or start at a different vertex, but result in the same weights. From this we can see that the first circuit, $x_1x_2x_3x_4x_5x_1$, is the fuzzy optimal circuit.

Nearest Neighbor Algorithm (NNA)

Step 1: Select a starting point.

Step 2: Move to the nearest unvisited vertex (the edge with smallest weight).

Step 3: Repeat until the circuit is complete.

Consider Fig 2.1, starting at vertex x_2, the nearest neighbor is vertex x_5 with a weight of 0.06. From x_5, the nearest neighbor is x_1, with a weight of 0.02 . From x_1, the nearest neighbor is x_3, with a weight of 0.2 . From x_3, our only option is to move to vertex x_4, the only unvisited vertex, with a weight of 0.12. From x_4 we return to x_2 with a weight 0.15. The resulting circuit $x_2x_5x_1x_3x_4x_2$ is with a total weight of 0.55 .

Sorted Edges Algorithm (Minimum Weight Link Algorithm)

Step 1: Select the minimum weight unused edge in the fuzzy graph; highlight it.

Step 2: Repeat step 1, adding the cheapest unused edge to the fuzzy graph, unless:

a. Addition of edge would create a circuit that doesn't contain all vertices, or

b. Addition of edge would give a vertex degree 3.

3) Repeat until a fuzzy Hamiltonian circuit containing all vertices is formed.

Consider example:2.2, we have five vertex fuzzy graph, the minimum weight edge is x_1x_5, with a weight of 0.02. The next fuzzy shortest edge is x_5x_4, with a weight of 0.05. The next fuzzy shortest edge is x_4x_3, with a weight of 0.12. The next fuzzy shortest edge is x_3x_2, with a weight of 0.07. The next fuzzy shortest edge is x_2x_4, but that edge would create a fuzzy Hamiltonian circuit $x_1x_5x_4x_3x_2x_4$.

EXAMPLE 2.3: The telephone line connection to plan and connect an efficient fuzzy route for A.V.V.M. Sri Pushpam College(Autonomous), Poondi, Thanjavur District for intercom landline to reach it all the department exactly once and return to starting location .(eg. Office room). Consider each department as vertex such as x_1 –office room, x_2- mathematics, x_3 – economics, x_4 – History, x_5 – Computer Science, x_6 – Library, x_7 – Physics, x_8 – Chemistry, x_9 – Botany and x_{10} – Physical education. The distance between them are represented as fuzzy weights in the tabular column 2.1.

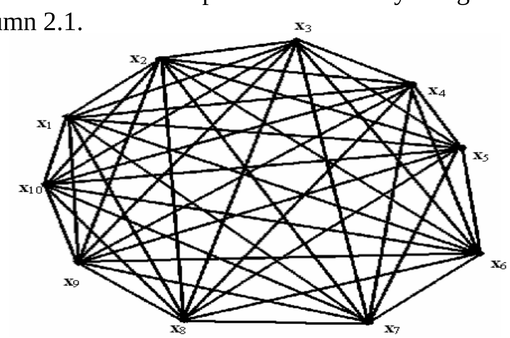


Fig.2.2

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-	x_1	X_2	X_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	-	0.37	0.1	0.22	0.07	0.17	0.25	0.28	0.24	0.35
x_2	0.37	-	0.25	0.16	0.5	0.19	0.07	0.1	0.13	0.01
x_3	0.1	0.25	-	0.12	0.27	0.12	0.09	0.16	0.13	0.24
x_4	0.22	0.16	0.12	-	0.43	0.03	0.04	0.08	0.02	0.15
x_5	0.07	0.5	0.27	0.43	-	0.45	0.47	0.34	0.38	0.42
x_6	0.17	0.19	0.12	0.03	0.45	-	0.06	0.11	0.06	0.18
x_7	0.25	0.07	0.09	0.04	0.47	0.06	-	0.11	0.08	0.11
x_8	0.28	0.1	0.16	0.08	0.34	0.11	0.11	-	0.03	0.05
x_9	0.24	0.13	0.13	0.02	0.38	0.06	0.08	0.03	-	0.11
x_{10}	0.35	0.01	0.24	0.15	0.42	0.18	0.11	0.05	0.11	-

Table 2.1

Using NNA starting at x_8 (Nearest Neighbor Algorithm) with a large number of departments, it is helpful to markoff the departments as they are connected to keep from accidently connecting them again. Looking in the row for x_8, the smallest distance is 0.03, to x_9. Following that idea, our circuit will be:

x_8 to x_9 0.03
x_9 to x_4 0.02
x_4 to x_6 0.03
x_6 to x_7 0.06
x_7 to x_{10} 0.11
x_{10} to x_2 0.01
x_2 to x_3 0.25
x_3 to x_1 0.1
x_1 to x_5 0.07
x_5 to x_8 0.34

Therefore, total connecting length is 1 (approximately).

Using ***SORTED EDGES ALGORITHM*** it is helpful to draw an empty fuzzy graph, perhaps by drawing vertices in a circular pattern. Adding edges to the fuzzy graph and select them with help of any fuzzy circuits or vertices with degree 3.

x_{10} to x_2 0.01
x_4 to x_9 0.02
x_8 to x_9 0.03
x_4 to x_6 0.03

The fuzzy graph after adding these edges is shown in the following figure 2.3.

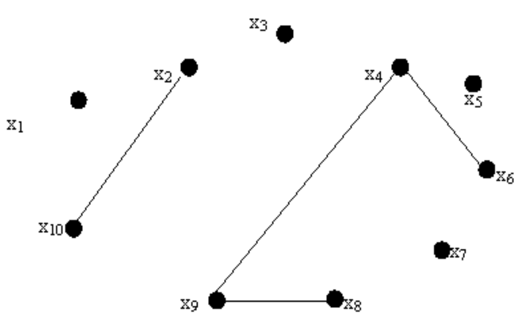

Fig.2.3

The next shortest edge is from x_4 to x_7 at 0.04, but adding that edge would give x_4 degree 3. Continuing on, and skip over any edge pair that contains x_9 to x_4, since they both already have degree 2.

x_8 to x_{10} 0.05
x_6 to x_7 0.06
x_8 to x_2 (reject – it forms fuzzy circuit)
x_1 to x_5 0.07

The fuzzy graph after adding these edges is shown in the following figure 2.4. At this point, skip over any edge pair that contains x_9, x_{10}, x_6, x_8 or x_4 since they already have degree 2.

x_7 to x_2 (reject – it forms fuzzy circuit)
x_7 to x_3 0.09
x_3 to x_1 0.1

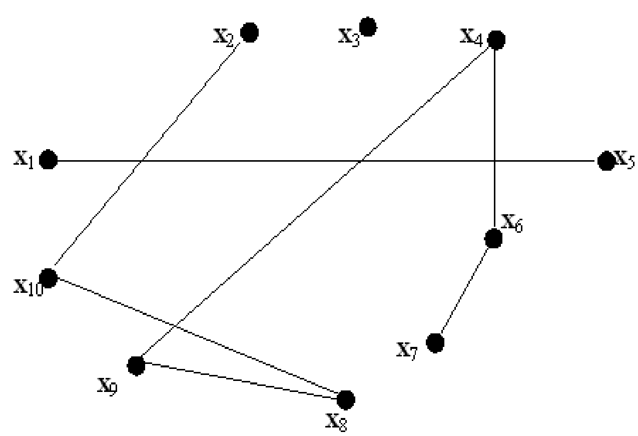

Fig.2.4

At this point the only way to complete the circuit is to add:
x_5 to x_2 0.5

Therefore, total trip length is 0.97

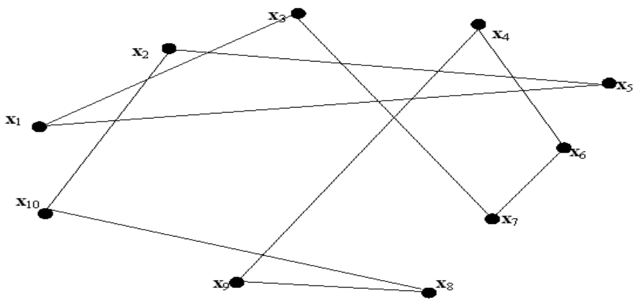

Fig.2.5

Another fuzzy route can make the tour $x_1x_5x_3x_9x_8x_2x_{10}x_7x_4x_6x_1$ is 0.96 .

III. KRUSKAL'S ALGORITHM

Step 1: Select the minimum weight unused edge in the fuzzy graph; highlight it.
Step 2: Repeat step 1, addition of minimum weight unused edge to the fuzzy graph, unless:
Addition of edge would create a fuzzy circuit.
Step 3: Repeat until a fuzzy spanning tree is formed.

Example: 2.4

The telephone line connection to plan and connect an efficient fuzzy route for A.V.V.M. Sri Pushpam College, (Autonomous), Poondi, Thanjavur District, Tamil Nadu, South India. Consider each department as vertex such as x_1 –office room, x_2- mathematics, x_3 – economics, x_4 – History, x_5 – Computer Science, x_6 – Library, x_7– Physics, x_8 – Chemistry, x_9 – Botany and x_{10} – Physical education. The distance between them are represented as fuzzy weights shown in the Tabular column 2.1.

We find the solution for the problem A.V.V.M Sri Pushpam College, (Autonomous), Poondi, Thanjavur District, South India needs to connect updated intercom lines connecting all the departments exactly once. The problem is to minimize the amount of new line using Kruskal's Algorithm with fuzzy graph.

Solution: Selecting the cheapest unused edge in the fuzzy graph, we have

x_{10} to x_2 0.01
x_4 to x_9 0.02
x_8 to x_9 0.03
x_4 to x_6 0.03
x_4 to x_7 0.04
x_9 to x_6 (reject – it forms fuzzy circuit)
x_8 to x_{10} 0.05
x_7 to x_9 (reject – it forms fuzzy circuit)
x_4 to x_8 (reject – it forms fuzzy circuit)
x_6 to x_7 (reject – it forms fuzzy circuit)
x_8 to x_2 (reject – it forms fuzzy circuit)
x_1 to x_5 0.**07**
x_6 to x_8 (reject – it forms fuzzy circuit)
x_7 to x_8 (reject – it forms fuzzy circuit)
x_7 to x_{10} (reject – it forms fuzzy circuit)
x_9 to x_{10} (reject – it forms fuzzy circuit)
x_3 to x_6 0.12
x_3 to x_9 (reject – it forms fuzzy circuit)
x_2 to x_7 (reject – it forms fuzzy circuit)
x_9 to x_2 (reject – it forms fuzzy circuit)
x_4 to x_{10} (reject – it forms fuzzy circuit)
x_8 to x_3 (reject – it forms fuzzy circuit)
x_2 to x_4 (reject – it forms fuzzy circuit)
x_6 to x_1 0.17

Accepted path is connected and shown in the following figure 2.6.

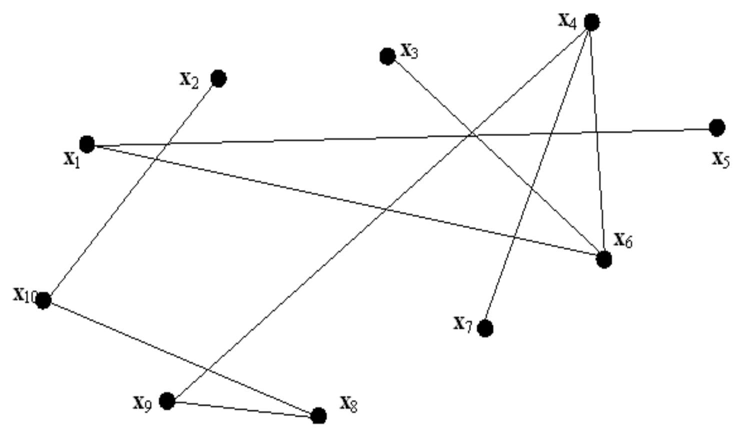

Fig.2.6

This connects the fuzzy graph. The total length of cable to lay would be 0.54

IV. CONCLUSION

The result on comparing with heuristic algorithm such as Nearest Neighbor Algorithm, Sorted Edges Algorithm and Kruskal's Algorithm for the problem that connecting distances to plan and visit an efficient fuzzy route for A.V.V.M Sri Pushpam College, (Autonomous), Poondi, Thanjavur District, South India intercom landline(phone) to reach it all the department exactly once is Kruskal's Algorithm. So can we conclude that Kruskal's Algorithm is the best to adopt for these types of problems.

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