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YANG MILLS THEORY-PART TWO SOME RANDOM THOUGHTS

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ABSTRACT. We provide a series of Models for the problems that arise in Yang Mills Theory. No claim is made that the problem is solved. We do factorize the Yang Mills Theory and give a Model for the values of LHS and RHS of the Yang Mills theory. We hope these forms the stepping stone for further factorizations and solutions to the subatomic denominations at Planck's scale. Work also throws light on some important factors like mass acquisition by symmetry breaking, relation between strong interaction and weak interaction, Lagrangian Invariance despite transformations, Gauge field, Noncommutative symmetry group of Gauge Theory and Yang Mills Theory itself. What happens when Yang Mills theory provides a proper explanation of the Physics of nuclei and nuclear constituents, and incorporate this premise in the 36 storey Model built and see what the ramifications on the other variables are. We, feel more erudite scholars would be able to given better physical interpretation which can effectively solve some problems not wholly or in full measure but very substantially.

We take in to consideration the following parameters, processes and concepts:

1. **Transformations Between possible gauges**
2. **Corresponding Lie Groups or Gauge groups**
3. **Gauge**
4. **Redundant degrees of freedom in the Lagrangian**
5. **Algebra of group generators**
6. **Transformable Lie Group**
7. **Inclusion of Gauge Fields in the Lagrangians**
8. **Gauge Invariables**
9. **Yang Mills Theory (We repeat the same Bank's example. Individual debits and Credits are conservative so also the holistic one. Generalized theories are applied to various systems which are parameterized. And we live in 'measurement world'. Classification is done on the parameters of various systems to which the Theory is applied.).**
10. **Physics of Nuclei and Nuclear constituents Yang Mills theory**

$$\mathcal{L}_{\text{gf}} = -\frac{1}{4}\,\text{Tr}(F^2) = -\frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu}$$

11. **Symmetry Breaking**
12. **Acquisition of mass**

TRANSFORMATIONS BETWEEN POSSIBLE GAUGES AND CORRESPOPNDING LIE

GROUP OR GAUGE GROUP:

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF TRANSFOR MATIONS BETWEEN POSSIBLE GAUGES

G_{14} : CATEGORY TWO OF TRANSFORMATIONS BETWEEN POSSIBLE GAUGES

G_{15} : CATEGORY THREE OFTRANSFORMATIONS BETWEEN POSSIBLE GAUGES

T_{13} : CATEGORY ONE OF CORRESPOPNDING LIE GROUPS OR GAUGE GROUPS

T_{14} : CATEGORY TWO OF CORRESPONDING LIE GROUPS OR GAUGE GROUPS

T_{15} :CATEGORY THREE OF CORRESPONDING LIE GROUPS OR GAUGE GROUPS

GAUGES AND REDUNDANT DEGREES OF FREEDOM:

MODULE NUMBERED TWO:

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G_{16} : CATEGORY ONE OF REDUNDANT DEGREES OF FREEDOM IN THE LAGRANGIANS

G_{17} : CATEGORY TWO OF REDUNDANT DEGREES OF FREEDOM IN THE LAGRANGIANS

G_{18} : CATEGORY THREE OF REDUNDANT DEGREES OF FREEDOM IN THE LAGRANGIANS

T_{16} :CATEGORY ONE OF CONCOMITANT GAUGES

T_{17} : CATEGORY TWO OF CONCOMITANT GAUGES

T_{18} : CATEGORY THREE OF CONCOMITANT GAUGES

ALGEBRA OF GROUP GENERATORS AND CORRESPONDING LIE GROUPS:

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF ALGEBRA OF GROUP GENERATORS

G_{21} :CATEGORY TWO OF ALGEBRA OF GROUP GENERATORS

G_{22} : CATEGORY THREE OF ALGEBRA OF GROUP GENERATORS

T_{20} : CATEGORY ONE OF CORRESPONDING LIE GROUPS

T_{21} :CATEGORY TWO OF CORRESPONDING LIE GROUPS

T_{22} : CATEGORY THREE OF CORRESPONDING LIE GROUPS

INCLUSION OF GAUGE FIELDS IN THE LAGRANGIANS AND GAUGE INVARIABLES:

: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OF GAUGE FIELDS IN THE LAGRANGIANS

G_{25} : CATEGORY TWO OF GAUGE FIELDS IN THE LAGRANGIANS

G_{26} : CATEGORY THREE OF GAUGE FIELDS IN THE LAGRANGIANS

T_{24} :CATEGORY ONE OF GAUGE INVARIABLES

T_{25} :CATEGORY TWO OFGAUGE INVARIABLES

T_{26} : CATEGORY THREE OF GAUGE INVARIABLES

YANG MILLS THEORYAND PHYSICS OF NUCLEI AND NUCLEAR CONSTITUENTS:

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF YANG MILLS THEORY

G_{29} : CATEGORY TWO OF YANG MILLS THEORY

G_{30} :CATEGORY THREE OFYANG MILLS THEORY

T_{28} : CATEGORY ONE OFPHYSICS OF NUCLEI AND NUCLEAR CONSTITUENTS (Theory is applied to various subatomic particle systems and the classification is done based on the parametricization of these systems. There is not a single system known which is not characterized by some properties)

T_{29} :CATEGORY TWO OF PHYSICS OF NUCLEI AND NUCLEAR CONSTITUENTS

T_{30} :CATEGORY THREE OF PHYSICS OF NUCLEI AND NUCLEAR CONSTITUENTS (CONSTITUENTS ARE DIFFERENT AND SO ALSO THE PHYSICS, FOR ALL THE VARIOUS ELEMENTS ON THE MENDELEFS TABLE AND ARE APPLICABLE AND STUDIED TO VARIOUS ATOMIC SYSTEMS ,NOTWITHSTANDING THE FACT THAT THERE IS GENERALISATION IN THE FORM OF YANG MILLS THEORY. KIND REFRENCE IS INVITED TO THE BANK'S EXAMPLE STATED IN THE PREVIOUS PAPER. IN ESSENCE WHILE THERE IS CONSERVATIVENESS OF THE TOTAL ASSETS AND LIABILITIES, THIS DOES NOT PREVENT THE INDIVIDUAL DEBITS AND CREDITS WHICH THEMSELVES ARE CONSERVATIVE-THIS MIGHT NOT BE TRUE IN SYSTEMS IN PHYSICS- BUT THAT DOES NOT PREVENT THE HIOLISTIC CONSERVATIVENESS OF THE SYSTEM)

ACQUISITION OF MASS AND SYMMETRY BREAKING :

MODULE NUMBERED SIX:

$$\mathcal{L}_{\text{gf}} = -\frac{1}{4}\,\text{Tr}(F^2) = -\frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu}$$

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G_{32} **: CATEGORY ONE OF SYMMETRY BREAKING**

G_{33} **: CATEGORY TWO OF SYMMETRY BREAKING**

G_{34} **: CATEGORY THREE OF SYMMETRY BREAKING**

T_{32} **: CATEGORY ONE OF ACQUISITION OF MASS**

T_{33} **: CATEGORY TWO OF ACQUISITION OF MASS**

T_{34} **: CATEGORY THREE OF ACQUISITION OF MASS (Theory applied to various characterized systems and the systemic characterizations form the basis for the formulation of the classification).**

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$**:** $(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ , \ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$**,**
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$
$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ , (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

TRANSFORMATIONS BETWEEN POSSIBLE GAUGES AND CORRESPOPNDING LIE GROUP OR GAUGE GROUP: 1

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 2$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 3$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)} + \left(a''_{15}\right)^{(1)}(T_{14},t)\right]G_{15} \quad 4$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 5$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 6$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)} - \left(b''_{15}\right)^{(1)}(G,t)\right]T_{15} \quad 7$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor 8

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

GAUGES AND REDUNDANT DEGREES OF FREEDOM: 9

MODULE NUMBERED TWO

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 10$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 11$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 12$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 13$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 14$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 15$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor 16

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor 17

ALGEBRA OF GROUP GENERATORS AND CORRESPONDING LIE GROUPS: 18

MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 19$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 20$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 21$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 22$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 23$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 24$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b_{20}^{''})^{(3)}(G_{23},t) =$ First detritions factor 25

26

INCLUSION OF GAUGE FIELDS IN THE LAGRANGIANS AND GAUGE INVARIABLES:

: MODULE NUMBERED FOUR:

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The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25},t)\right]G_{24} \quad 27$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\left(a_{25}^{'}\right)^{(4)} + \left(a_{25}^{''}\right)^{(4)}(T_{25},t)\right]G_{25} \quad 28$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25},t)\right]G_{26} \quad 29$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 30$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\left(b_{25}^{'}\right)^{(4)} - \left(b_{25}^{''}\right)^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 31$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 32$$

$+(a_{24}^{''})^{(4)}(T_{25},t) =$ **First augmentation factor** 33

$-(b_{24}^{''})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor** 34

YANG MILLS THEORY AND PHYSICS OF NUCLEI AND NUCLEAR CONSTITUENTS: 35

MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29},t)\right]G_{28} \quad 36$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29},t)\right]G_{29} \quad 37$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29},t)\right]G_{30} \quad 38$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 39$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 40$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 41$$

$+(a_{28}^{''})^{(5)}(T_{29},t) =$ **First augmentation factor** 42

$-(b_{28}^{''})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor** 43

ACQUISITION OF MASS AND SYMMETRY BREAKING : 44

MODULE NUMBERED SIX 45

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 46$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 47$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 48$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 49$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 50$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 51$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor** 52

$-(b''_{32})^{(6)}\left((G_{35}), t\right) =$ **First detritions factor** 53

HOLISTIC CONCATENATE SYTEMAL EQUATIONS HENCEFORTH REFERRED TO AS "GLOBAL EQUATIONS" 54

We take in to consideration the following parameters, processes and concepts:

1. **Transformations Between possible gauges**
2. **Corresponding Lie Groups or Gauge groups**
3. **Gauge**
4. **Redundant degrees of freedom in the Lagrangian**
5. **Algebra of group generators**
6. **Transformable Lie Group**
7. **Inclusion of Gauge Fields in the Lagrangians**
8. **Gauge Invariables**
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10. **Physics of Nuclei and Nuclear constituents Yang Mills theory**

$$\mathcal{L}_{\text{gf}} = -\frac{1}{4}\,\text{Tr}(F^2) = -\frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu}$$

11. **Symmetry Breaking**
12. **Acquisition of mass**

$$\mathcal{L}_{\text{gf}} = -\frac{1}{4}\,\text{Tr}(F^2) = -\frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu}$$

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a_{13}^{'})^{(1)}\boxed{+(a_{13}^{''})^{(1)}(T_{14},t)} & \boxed{+(a_{16}^{''})^{(2,2,)}(T_{17},t)} & \boxed{+(a_{20}^{''})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a_{28}^{''})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a_{32}^{''})^{(6,6,6,6,)}(T_{33},t)} \end{array}\right] G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a_{14}^{'})^{(1)}\boxed{+(a_{14}^{''})^{(1)}(T_{14},t)} & \boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)} & \boxed{+(a_{21}^{''})^{(3,3,)}(T_{21},t)} \\ \boxed{+\left(a_{25}^{''}\right)^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a_{29}^{''})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)} \end{array}\right] G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} \left(a_{15}^{'}\right)^{(1)}\boxed{+\left(a_{15}^{''}\right)^{(1)}(T_{14},t)} & \boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)} & \boxed{+(a_{22}^{''})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a_{26}^{''})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a_{30}^{''})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)} \end{array}\right] G_{15} \quad (57)$$

Where $\boxed{(a_{13}^{''})^{(1)}(T_{14},t)}$, $\boxed{(a_{14}^{''})^{(1)}(T_{14},t)}$, $\boxed{\left(a_{15}^{''}\right)^{(1)}(\mathrm{T}_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 (58)

$\boxed{+(a_{16}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3 (59)

$\boxed{+(a_{20}^{''})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}^{''})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+\left(a_{25}^{''}\right)^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}^{''})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,)}(T_{29},\mathrm{t})}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

(60)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b_{13}^{'})^{(1)}\boxed{-(b_{13}^{''})^{(1)}(G,t)} & \boxed{-(b_{16}^{''})^{(2,2,)}(G_{19},t)} & \boxed{-(b_{20}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{array}\right] T_{13} \quad (61)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b_{14}^{'})^{(1)}\boxed{-(b_{14}^{''})^{(1)}(G,t)} & \boxed{-(b_{17}^{''})^{(2,2,)}(G_{19},t)} & \boxed{-(b_{21}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{array}\right] T_{14} \quad (62)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{ccc} (b_{15}^{'})^{(1)}\boxed{-(b_{15}^{''})^{(1)}(G,t)} & \boxed{-(b_{18}^{''})^{(2,2,)}(G_{19},t)} & \boxed{-(b_{22}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{array}\right] T_{15} \quad (63)$$

Where $\boxed{-(b_{13}^{''})^{(1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1)}(G,t)}$, $\boxed{-(b_{15}^{''})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 64

$\boxed{-(b_{16}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{20}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

65

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a_{16}^{'})^{(2)}\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{13}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{20}^{''})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{28}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{32}^{''})^{(6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{16} \quad 66$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a_{17}^{'})^{(2)}\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{14}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{21}^{''})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{29}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{33}^{''})^{(6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{17} \quad 67$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a_{18}^{'})^{(2)}\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{15}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{22}^{''})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{30}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{34}^{''})^{(6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{18} \quad 68$$

Where $\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 69

$\boxed{+(a_{13}^{''})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}^{''})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}^{''})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3 70

$\boxed{+(a_{28}^{''})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

71

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b_{16}^{'})^{(2)}\boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}\ \boxed{-(b_{13}^{''})^{(1,1,)}(G,t)}\ \boxed{-(b_{20}^{''})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b_{24}^{''})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b_{28}^{''})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b_{32}^{''})^{(6,6,6,6,6)}(G_{35},t)} \end{array}\right]T_{16} \quad 72$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{c} (b_{17}^{'})^{(2)}\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}\ \boxed{-(b_{14}^{''})^{(1,1,)}(G,t)}\ \boxed{-(b_{21}^{''})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b_{25}^{''})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b_{29}^{''})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b_{33}^{''})^{(6,6,6,6,6)}(G_{35},t)} \end{array}\right]T_{17} \quad 73$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b_{18}^{'})^{(2)}\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}\ \boxed{-(b_{15}^{''})^{(1,1,)}(G,t)}\ \boxed{-(b_{22}^{''})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b_{26}^{''})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b_{30}^{''})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b_{34}^{''})^{(6,6,6,6,6)}(G_{35},t)} \end{array}\right]T_{18} \quad 74$$

$where\ \boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}\ ,\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}\ ,\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 75

$\boxed{-(b_{13}^{''})^{(1,1,)}(G,t)}, \boxed{-(b_{14}^{''})^{(1,1,)}(G,t)}, \boxed{-(b_{15}^{''})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b_{20}^{''})^{(3,3,3,)}(G_{23},t)}, \boxed{-(b_{21}^{''})^{(3,3,3,)}(G_{23},t)}, \boxed{-(b_{22}^{''})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,4)}(G_{27},t)}, \boxed{-(b_{25}^{''})^{(4,4,4,4,4)}(G_{27},t)}, \boxed{-(b_{26}^{''})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1,2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,5)}(G_{31},t)}, \boxed{-(b_{29}^{''})^{(5,5,5,5,5)}(G_{31},t)}, \boxed{-(b_{30}^{''})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1,2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,6)}(G_{35},t)}, \boxed{-(b_{33}^{''})^{(6,6,6,6,6)}(G_{35},t)}, \boxed{-(b_{34}^{''})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a_{20}^{'})^{(3)}\boxed{+(a_{20}^{''})^{(3)}(T_{21},t)}\ \boxed{+(a_{16}^{''})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a_{13}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{28}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{32}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{20} \quad 76$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a_{21}^{'})^{(3)}\boxed{+(a_{21}^{''})^{(3)}(T_{21},t)}\ \boxed{+(a_{17}^{''})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a_{14}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{25}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{29}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{33}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{21} \quad 77$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a_{22}^{'})^{(3)}\boxed{+(a_{22}^{''})^{(3)}(T_{21},t)}\ \boxed{+(a_{18}^{''})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a_{15}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{26}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{30}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{34}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \end{array}\right]G_{22} \quad 78$$

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$\boxed{+(a_{20}^{''})^{(3)}(T_{21},t)}, \boxed{+(a_{21}^{''})^{(3)}(T_{21},t)}, \boxed{+(a_{22}^{''})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}^{''})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}^{''})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3 80

$\boxed{+(a_{24}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{28}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

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$$\frac{dT_{20}}{dt} = \qquad 82$$

$$(b_{20})^{(3)}T_{21} - \left[\frac{(b_{20}^{'})^{(3)}\boxed{-(b_{20}^{''})^{(3)}(G_{23},t)}\boxed{-(b_{16}^{''})^{(2,2,2)}(G_{19},t)}\boxed{-(b_{13}^{''})^{(1,1,1,)}(G,t)}}{\boxed{-(b_{24}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}}\right]T_{20}$$

$$\frac{dT_{21}}{dt} = \qquad 83$$

$$(b_{21})^{(3)}T_{20} - \left[\frac{(b_{21}^{'})^{(3)}\boxed{-(b_{21}^{''})^{(3)}(G_{23},t)}\boxed{-(b_{17}^{''})^{(2,2,2)}(G_{19},t)}\boxed{-(b_{14}^{''})^{(1,1,1,)}(G,t)}}{\boxed{-(b_{25}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}}\right]T_{21}$$

$$\frac{dT_{22}}{dt} = \qquad 84$$

$$(b_{22})^{(3)}T_{21} - \left[\frac{(b_{22}^{'})^{(3)}\boxed{-(b_{22}^{''})^{(3)}(G_{23},t)}\boxed{-(b_{18}^{''})^{(2,2,2)}(G_{19},t)}\boxed{-(b_{15}^{''})^{(1,1,1,)}(G,t)}}{\boxed{-(b_{26}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}}\right]T_{22}$$

$\boxed{-(b_{20}^{''})^{(3)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3)}(G_{23},t)}$ are first detritions coefficients for category 1, 2 and 3 85

$\boxed{-(b_{16}^{''})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,2)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{13}^{''})^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}^{''})^{(1,1,1,)}(G,t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

86

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{l}(a_{24}^{'})^{(4)}\boxed{+(a_{24}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}^{''})^{(3,3,3,3)}(T_{21},t)}\end{array}\right]G_{24} \quad 87$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{l}(a_{25}^{'})^{(4)}\boxed{+(a_{25}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{33}^{''})^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}^{''})^{(3,3,3,3)}(T_{21},t)}\end{array}\right]G_{25} \quad 88$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{l}(a_{26}^{'})^{(4)}\boxed{+(a_{26}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}^{''})^{(3,3,3,3)}(T_{21},t)}\end{array}\right]G_{26} \quad 89$$

90

$Where\ \boxed{(a_{24}^{''})^{(4)}(T_{25},t)}, \boxed{(a_{25}^{''})^{(4)}(T_{25},t)}, \boxed{(a_{26}^{''})^{(4)}(T_{25},t)}\ are\ first\ augmentation\ coefficients\ fo$

$\boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)}, \boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)}, \boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)}\ are\ second\ augmentation\ coefficien$ 91

$\boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)}, \boxed{+(a_{33}^{''})^{(6,6,)}(T_{33},t)}, \boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)}\ are\ third\ augmentation\ coefficient$

$\boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a_{16}^{''})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}^{''})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}^{''})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a_{20}^{''})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}^{''})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}^{''})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

92

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{l}(b_{24}^{'})^{(4)}\boxed{-(b_{24}^{''})^{(4)}(G_{27},t)}\ \boxed{-(b_{28}^{''})^{(5,5,)}(G_{31},t)}\ \boxed{-(b_{32}^{''})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b_{13}^{''})^{(1,1,1,1)}(G,t)}\ \boxed{-(b_{16}^{''})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{20}^{''})^{(3,3,3,3)}(G_{23},t)}\end{array}\right]T_{24} \quad 93$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{l}(b_{25}^{'})^{(4)}\boxed{-(b_{25}^{''})^{(4)}(G_{27},t)}\ \boxed{-(b_{29}^{''})^{(5,5,)}(G_{31},t)}\ \boxed{-(b_{33}^{''})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b_{14}^{''})^{(1,1,1,1)}(G,t)}\ \boxed{-(b_{17}^{''})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{21}^{''})^{(3,3,3,3)}(G_{23},t)}\end{array}\right]T_{25} \quad 94$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{l}(b_{26}^{'})^{(4)}\boxed{-(b_{26}^{''})^{(4)}(G_{27},t)}\ \boxed{-(b_{30}^{''})^{(5,5,)}(G_{31},t)}\ \boxed{-(b_{34}^{''})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b_{15}^{''})^{(1,1,1,1)}(G,t)}\ \boxed{-(b_{18}^{''})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{22}^{''})^{(3,3,3,3)}(G_{23},t)}\end{array}\right]T_{26} \quad 95$$

$Where\ \boxed{-(b_{24}^{''})^{(4)}(G_{27},t)}, \boxed{-(b_{25}^{''})^{(4)}(G_{27},t)}, \boxed{-(b_{26}^{''})^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for$ 96

$\boxed{-(b_{28}^{''})^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}^{''})^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}^{''})^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for$

$\boxed{-(b_{32}^{''})^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}^{''})^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}^{''})^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ c$

$\boxed{-(b_{13}^{''})^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}^{''})^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}^{''})^{(1,1,1,1)}(G,t)}$
$are\ fourth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}^{''})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}^{''})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}^{''})^{(2,2,2,2)}(G_{19},t)}$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}^{''})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}^{''})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}^{''})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

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98

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}^{'})^{(5)}\boxed{+(a_{28}^{''})^{(5)}(T_{29},t)}\ \boxed{+(a_{24}^{''})^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}^{''})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}^{''})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}^{''})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}^{''})^{(3,3,3,3,3)}(T_{21},t)} \end{array}\right] G_{28} \quad (99)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}^{'})^{(5)}\boxed{+(a_{29}^{''})^{(5)}(T_{29},t)}\ \boxed{+(a_{25}^{''})^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}^{''})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}^{''})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}^{''})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}^{''})^{(3,3,3,3,3)}(T_{21},t)} \end{array}\right] G_{29} \quad (100)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}^{'})^{(5)}\boxed{+(a_{30}^{''})^{(5)}(T_{29},t)}\ \boxed{+(a_{26}^{''})^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}^{''})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}^{''})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}^{''})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}^{''})^{(3,3,3,3,3)}(T_{21},t)} \end{array}\right] G_{30} \quad (101)$$

Where $\boxed{+(a_{28}^{''})^{(5)}(T_{29},t)}, \boxed{+(a_{29}^{''})^{(5)}(T_{29},t)}, \boxed{+(a_{30}^{''})^{(5)}(T_{29},t)}$ *are first augmentation coefficient* (102)

And $\boxed{+(a_{24}^{''})^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}^{''})^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}^{''})^{(4,4,)}(T_{25},t)}$ *are second augmentation coeffi*

$\boxed{+(a_{32}^{''})^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}^{''})^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}^{''})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficier*

$\boxed{+(a_{13}^{''})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}^{''})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}^{''})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}^{''})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}^{''})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}^{''})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a_{20}^{''})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}^{''})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}^{''})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

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$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{c} (b_{28}^{'})^{(5)}\boxed{-(b_{28}^{''})^{(5)}(G_{31},t)}\ \boxed{-(b_{24}^{''})^{(4,4,)}(G_{27},t)}\ \boxed{-(b_{32}^{''})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}^{''})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b_{16}^{''})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{20}^{''})^{(3,3,3,3,3)}(G_{23},t)} \end{array}\right] T_{28} \quad (104)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{c} (b_{29}^{'})^{(5)}\boxed{-(b_{29}^{''})^{(5)}(G_{31},t)}\ \boxed{-(b_{25}^{''})^{(4,4,)}(G_{27},t)}\ \boxed{-(b_{33}^{''})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}^{''})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b_{17}^{''})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{21}^{''})^{(3,3,3,3,3)}(G_{23},t)} \end{array}\right] T_{29} \quad (105)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{c} (b_{30}^{'})^{(5)}\boxed{-(b_{30}^{''})^{(5)}(G_{31},t)}\ \boxed{-(b_{26}^{''})^{(4,4,)}(G_{27},t)}\ \boxed{-(b_{34}^{''})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}^{''})^{(1,1,1,1,1,)}(G,t)}\ \boxed{-(b_{18}^{''})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{22}^{''})^{(3,3,3,3,3)}(G_{23},t)} \end{array}\right] T_{30} \quad (106)$$

where $\boxed{-(b_{28}^{''})^{(5)}(G_{31},t)}, \boxed{-(b_{29}^{''})^{(5)}(G_{31},t)}, \boxed{-(b_{30}^{''})^{(5)}(G_{31},t)}$ (107)
are first detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b_{24}^{''})^{(4,4,)}(G_{27},t)}, \boxed{-(b_{25}^{''})^{(4,4,)}(G_{27},t)}, \boxed{-(b_{26}^{''})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients fo*

$\boxed{-(b_{32}^{''})^{(6,6,6)}(G_{35},t)}, \boxed{-(b_{33}^{''})^{(6,6,6)}(G_{35},t)}, \boxed{-(b_{34}^{''})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients f*

$\boxed{-(b_{13}^{''})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{15}^{''})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{16}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{20}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

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$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\frac{(a_{32}^{'})^{(6)}\boxed{+(a_{32}^{''})^{(6)}(T_{33},t)}\boxed{+(a_{28}^{''})^{(5,5,5)}(T_{29},t)}\boxed{+(a_{24}^{''})^{(4,4,4,)}(T_{25},t)}}{\boxed{+(a_{13}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{16}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{20}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}}\right]G_{32} \quad (109)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\frac{(a_{33}^{'})^{(6)}\boxed{+(a_{33}^{''})^{(6)}(T_{33},t)}\boxed{+(a_{29}^{''})^{(5,5,5)}(T_{29},t)}\boxed{+(a_{25}^{''})^{(4,4,4,)}(T_{25},t)}}{\boxed{+(a_{14}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{17}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{21}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}}\right]G_{33} \quad (110)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\frac{(a_{34}^{'})^{(6)}\boxed{+(a_{34}^{''})^{(6)}(T_{33},t)}\boxed{+(a_{30}^{''})^{(5,5,5)}(T_{29},t)}\boxed{+(a_{26}^{''})^{(4,4,4,)}(T_{25},t)}}{\boxed{+(a_{15}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{18}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{22}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}}\right]G_{34} \quad (111)$$

$\boxed{+(a_{32}^{''})^{(6)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for c* (112)

$\boxed{+(a_{28}^{''})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficie*

$\boxed{+(a_{24}^{''})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficie*

$\boxed{+(a_{13}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a_{16}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a_{20}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

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$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\frac{(b_{32}^{'})^{(6)}\boxed{-(b_{32}^{''})^{(6)}(G_{35},t)}\boxed{-(b_{28}^{''})^{(5,5,5)}(G_{31},t)}\boxed{-(b_{24}^{''})^{(4,4,4,)}(G_{27},t)}}{\boxed{-(b_{13}^{''})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b_{16}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b_{20}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}}\right]T_{32} \quad (114)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\frac{(b_{33}^{'})^{(6)}\boxed{-(b_{33}^{''})^{(6)}(G_{35},t)}\boxed{-(b_{29}^{''})^{(5,5,5)}(G_{31},t)}\boxed{-(b_{25}^{''})^{(4,4,4,)}(G_{27},t)}}{\boxed{-(b_{14}^{''})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b_{17}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b_{21}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}}\right]T_{33} \quad (115)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{matrix}(b_{34}^{'})^{(6)}\boxed{-(b_{34}^{''})^{(6)}(G_{35},t)}\boxed{-(b_{30}^{''})^{(5,5,5)}(G_{31},t)}\boxed{-(b_{26}^{''})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b_{15}^{''})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b_{18}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b_{22}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}\end{matrix}\right]T_{34} \quad 116$$

$\boxed{-(b_{32}^{''})^{(6)}(G_{35},t)}, \boxed{-(b_{33}^{''})^{(6)}(G_{35},t)}, \boxed{-(b_{34}^{''})^{(6)}(G_{35},t)}$ *are first detrition coefficients for cate* 117

$\boxed{-(b_{28}^{''})^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{29}^{''})^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{30}^{''})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients*

$\boxed{-(b_{24}^{''})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{25}^{''})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{26}^{''})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients f*

$\boxed{-(b_{13}^{''})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{14}^{''})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{15}^{''})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{16}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{20}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

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Where we suppose 119

(A) $(a_i)^{(1)}, (a_i^{'})^{(1)}, (a_i^{''})^{(1)}, (b_i)^{(1)}, (b_i^{'})^{(1)}, (b_i^{''})^{(1)} > 0,$ 120

$i, j = 13,14,15$

(B) The functions $(a_i^{''})^{(1)}, (b_i^{''})^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i^{''})^{(1)}(T_{14},t) \le (\mathrm{p}_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i^{''})^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i^{'})^{(1)} \le (\hat{B}_{13})^{(1)} \quad 121$$

(C) $lim_{T_2\to\infty}(a_i^{''})^{(1)}(T_{14},t) = (p_i)^{(1)}$ 122

$lim_{G\to\infty}(b_i^{''})^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 123

$$|(a_i^{''})^{(1)}(T_{14}^{'},t) - (a_i^{''})^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}^{'}|e^{-(\widehat{M}_{13})^{(1)}t} \quad 124$$

$$|(b_i^{''})^{(1)}(G^{'},t) - (b_i^{''})^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G^{'}||e^{-(\widehat{M}_{13})^{(1)}t} \quad 125$$

With the Lipschitz condition, we place a restriction on the behavior of functions 126
$(a_i^{''})^{(1)}(T_{14}^{'},t)$ and$(a_i^{''})^{(1)}(T_{14},t)$. $(T_{14}^{'},t)$ and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\widehat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{13})^{(1)} = 1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$, the first augmentation coefficient WOULD be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 127

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 128

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ 129 130

satisfy the inequalities 131

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \quad 132$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose 134

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$ 135

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded. 136

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$: 137

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 138$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 139$$

(H) $lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$ 140

$$lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 141$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 142

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 143

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 144$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 145$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) And (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient would be absolutely continuous. 146

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 147

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 148

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 149

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i^{'})^{(2)}, (b_i)^{(2)}, (b_i^{'})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i^{'})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (150)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i^{'})^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (151)$$

Where we suppose 152

(J) $(a_i)^{(3)}, (a_i^{'})^{(3)}, (a_i^{''})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (b_i^{''})^{(3)} > 0, \quad i,j = 20,21,22$ 153

The functions $(a_i^{''})^{(3)}, (b_i^{''})^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (r_i)^{(3)}$:

$$(a_i^{''})^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i^{''})^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i^{'})^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty} (a_i^{''})^{(3)} (T_{21}, t) = (p_i)^{(3)} \quad (154)$$

$$lim_{G \to \infty} (b_i^{''})^{(3)} (G_{23}, t) = \ (r_i)^{(3)} \quad (155)$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 156

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 157

$$|(a_i^{''})^{(3)}(T_{21}^{'}, t) - (a_i^{''})^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}^{'}| e^{-(\hat{M}_{20})^{(3)} t} \quad (158)$$

$$|(b_i^{''})^{(3)}(G_{23}^{'}, t) - (b_i^{''})^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}^{'}|| e^{-(\hat{M}_{20})^{(3)} t} \quad (159)$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(3)}(T_{21}^{'}, t)$ and$(a_i^{''})^{(3)}(T_{21}, t)$. $(T_{21}^{'}, t)$ And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i^{''})^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i^{''})^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient, would be absolutely continuous. 160

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 161

(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 162
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants 163
$(a_i)^{(3)}, (a_i^{'})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities 164

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i^{'})^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1 \quad (165)$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i^{'})^{(3)} + \ (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1 \quad 166$$

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Where we suppose 168

$$(a_i)^{(4)}, (a_i^{'})^{(4)}, (a_i^{''})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (b_i^{''})^{(4)} > 0, \quad i,j = 24,25,26 \quad 169$$

(M) The functions $(a_i^{''})^{(4)}, (b_i^{''})^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i^{''})^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i^{''})^{(4)}\big((G_{27}), t\big) \leq \ (r_i)^{(4)} \leq (b_i^{'})^{(4)} \leq (\hat{B}_{24})^{(4)}$$

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(N) $lim_{T_2 \to \infty}(a_i^{''})^{(4)}\,(T_{25}, t) = (p_i)^{(4)}$
$lim_{G \to \infty}(b_i^{''})^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 171

$$|(a_i^{''})^{(4)}(T_{25}^{'}, t) - (a_i^{''})^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i^{''})^{(4)}((G_{27})^{'}, t) - (b_i^{''})^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'}, t)$ 172
and$(a_i^{''})^{(4)}(T_{25}, t)$. $(T_{25}^{'}, t)$ And (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i^{''})^{(4)}(T_{25}, t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous.

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Defi174nition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 174

$(\hat{M}_{24})176^{175(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 175

(Q) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i^{'})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i^{'})^{(4)} + \ (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ \ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose 176

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30$$ 177

(S) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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(T) $lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$
$lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 179

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i'')^{(5)}(T_{29}, t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous. 180

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 181

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants
$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 182

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose 183

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 184

(W) The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i^{''})^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i^{''})^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i^{'})^{(6)} \le (\hat{B}_{32})^{(6)}$$

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(X) $lim_{T_2\to\infty}(a_i^{''})^{(6)}(T_{33},t) = (p_i)^{(6)}$
$lim_{G\to\infty}(b_i^{''})^{(6)}((G_{35}),t) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition: 186

$$|(a_i^{''})^{(6)}(T_{33}^{'},t) - (a_i^{''})^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}^{'}|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i^{''})^{(6)}((G_{35})^{'},t) - (b_i^{''})^{(6)}((G_{35}),t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})^{'}||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'},t)$ and$(a_i^{''})^{(6)}(T_{33},t)$. $(T_{33}^{'},t)$ and (T_{33},t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i^{''})^{(6)}(T_{33},t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. 187

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 188

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants
$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 189

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i^{'})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i^{'})^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i^{'})^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

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Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 191

Definition of $G_i(0), T_i(0)$:

$G_i(t) \le (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)\,, T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

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$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Definition of $G_i(0)\,, T_i(0)$: 196

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)\,, T_i(0)$: 199

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\ T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 200

$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \leq (\hat{P}_{13})^{(1)}\,, T_i^0 \leq (\hat{Q}_{13})^{(1)},$ 201

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 202

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 203

By 204

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a_{13}')^{(1)} + a_{13}''\right)^{(1)} \left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\Big] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)} \quad 205$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)} \quad 206$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)} \quad 207$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)} \quad 208$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)} \quad 209$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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Proof: 211

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)}, \quad 212$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad 213$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad 214$$

By 215

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a_{16}')^{(2)} + a_{16}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)} \quad 216$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)} \quad 217$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b_{16}')^{(2)} - (b_{16}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)} \quad 218$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b_{17}')^{(2)} - (b_{17}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)} \quad 219$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b_{18}')^{(2)} - (b_{18}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)} \quad 220$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 221

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)}, \quad 222$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad 223$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad 224$$

By 225

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)} \quad 226$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)} \quad 227$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)} \quad 228$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)} \quad 229$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)} \quad 230$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ 231
which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)}, \quad 232$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t} \quad 233$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t} \quad 234$$

By 235

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)} \quad 236$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)} \quad 237$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)} \quad 238$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)} \quad 239$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)} \quad 240$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ 241
which satisfy

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$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{28})^{(5)} \text{ , } T_i^0 \le (\hat{Q}_{28})^{(5)}, \quad 243$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)} t} \quad 244$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)} t} \quad 245$$

By 246

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)} \quad 247$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)} \quad 248$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)} \quad 249$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)} \quad 250$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)} \quad 251$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{32})^{(6)} \text{ , } T_i^0 \le (\hat{Q}_{32})^{(6)}, \quad 253$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad 254$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad 255$$

By 256

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)} \quad 257$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)} \quad 258$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)} \quad 259$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)} \quad 260$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)} \quad 261$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

262

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 263

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 264

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$ 265

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 266

$$G_{16}(t) \le G_{16}^{0}+\int_0^t\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)} = \left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+ \quad 267$$
$$\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 268

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$ 269

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 270

$$G_{20}(t) \le G_{20}^{0}+\int_0^t\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 271

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 272

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 273

$$G_{24}(t) \le G_{24}^{0}+\int_0^t\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 274

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 275

$$G_{28}(t) \le G_{28}^{0}+\int_0^t\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 276

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)}+G_{29}^{0}\right)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 277

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 278

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)}+G_{33}^{0}\right)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

279

280

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 281

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

282

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 283$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 284$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself 285

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 286

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote 287

Definition of $\tilde{G}, \tilde{T}$:

$$\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a_{13}^{'})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a_{13}^{''})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 288$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a_{13}^{'})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}^{''})^{(1)}\ and\ (b_{13}^{''})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}\ and\ (\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$. 289

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(1)}\ and\ (b_i^{''})^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 290

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(1)} - (a_i^{''})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0 \qquad 291$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(1)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 292

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}^{'})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}^{'})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}^{'})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is 293

analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(1)}(G(t),t)) = (b_{14}^{'})^{(1)}$ then $T_{14} \to \infty$. 294

Definition of $(m)^{(1)}\ and\ \varepsilon_1$:

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i^{''})^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 295

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right),\quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $lim_{t\to\infty}(b_{15}^{''})^{(1)}(G(t),t) = (b_{15}^{'})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 297

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 298$$

299

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 300

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 301

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 302

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 303

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}^{'})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{-(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}^{''})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\hat{M}_{16})^{(2)}s_{(16)}}e^{(\hat{M}_{16})^{(2)}s_{(16)}} +$$

$G_{16}^{(2)}|(a_{16}^{''})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}^{''})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 304

From the hypotheses it follows

$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$ 305
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a_{16}^{'})^{(2)}+(\widehat{A}_{16})^{(2)}+\right.$
$\left.(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};(G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows 306

Remark 1: The fact that we supposed $(a_{16}^{''})^{(2)}\ and\ (b_{16}^{''})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ and\ (\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 307

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(2)}\ and\ (b_i^{''})^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on $(G_{19})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 308

From 19 to 24 it results

$G_i\,(t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i^{'})^{(2)}-(a_i^{''})^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0 e^{\left(-(b_i^{'})^{(2)}t\right)}>0\quad \text{for } t>0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1,\left((\widehat{M}_{16})^{(2)}\right)_2\ and\ \left((\widehat{M}_{16})^{(2)}\right)_3$: 309

Remark 3: if G_{16} is bounded, the same property have also $G_{17}\ and\ G_{18}$. indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a_{17}^{'})^{(2)}G_{17}$ and by integrating

$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}^{'})^{(2)}$

In the same way , one can obtain

$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}^{'})^{(2)}$ 310

If $G_{17}\ or\ G_{18}$ is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If $G_{16}\ is$ bounded, from below, the same property holds for $G_{17}\ and\ G_{18}$. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 311

Remark 5: If T_{16} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(2)}\,((G_{19})(t),t))=(b_{17}^{'})^{(2)}$ then $T_{17}\ \to\infty$. 312

Definition of $(m)^{(2)}\ and\ \varepsilon_2$:

Indeed let t_2 be so that for $t\ >t_2$

$(b_{17})^{(2)}-(b_i^{''})^{(2)}((G_{19})(t),t)<\varepsilon_2, T_{16}\,(t)>(m)^{(2)}$

Then $\frac{dT_{17}}{dt}\geq(a_{17})^{(2)}(m)^{(2)}-\varepsilon_2 T_{17}$ which leads to 313

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1-e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $lim_{t\to\infty}(b_{18}^{''})^{(2)}\left((G_{19})(t),t\right) = (b_{18}^{'})^{(2)}$ 314

We now state a more precise theorem about the behaviors at infinity of the solutions

315

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 316

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 317$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 318$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i into itself 319

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 320

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 321

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 322

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}^{'})^{(3)}\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a_{20}^{''})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}} + \quad 323$$

$$G_{20}^{(2)}\left|(a_{20}^{''})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}^{''})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t} \leq \quad 324$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}^{'})^{(3)} + (\widehat{A}_{20})^{(3)} + \right.$$
$$\left.(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{20}^{''})^{(3)}\ and\ (b_{20}^{''})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 325

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(3)}\ and\ (b_i^{''})^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t) = 0\ and\ T_i\ (t) = 0$ 326

From 19 to 24 it results

$$G_i\ (t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(3)} - (a_i^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\ (t) \geq T_i^0 e^{(-(b_i^{'})^{(3)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 327

Remark 3: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}^{'})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}^{'})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}^{'})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If $G_{20}\ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 328

Remark 5: If T_{20} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(3)}\left((G_{23})(t), t\right) = (b_{21}^{'})^{(3)}$ then $T_{21} \to \infty$. 329

Definition of $(m)^{(3)}\ and\ \varepsilon_3$: 330

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i^{''})^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\ (t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 331

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $lim_{t\to\infty}(b_{22}^{''})^{(3)}\left((G_{23})(t), t\right) = (b_{22}^{'})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 333

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 334$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 335$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying IN to itself 336

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 337

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a_{24}^{'})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a_{24}^{''})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

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$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 339$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a_{24}^{'})^{(4)}+(\widehat{A}_{24})^{(4)}+\right.$$
$$\left.(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\,(G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{24}^{''})^{(4)}$ and $(b_{24}^{''})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 340

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t) = 0\ and\ T_i\ (t) = 0$ 341

From 19 to 24 it results

$G_i\ (t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(4)} - (a_i^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\ (t) \geq T_i^0 e^{\left(-(b_i^{'})^{(4)} t\right)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 342

Remark 3: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}^{'})^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}^{'})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}^{'})^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If $G_{24}\ \ is$ bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 343

Remark 5: If T_{24} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(4)}\ ((G_{27})(t), t)) = (b_{25}^{'})^{(4)}$ then $T_{25}\ \to \infty$. 344

Definition of $(m)^{(4)}\ and\ \varepsilon_4$:

Indeed let t_4 be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\ (t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 345

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t), t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for $G_{29}\ , G_{30}, T_{28}, T_{29}, T_{30}$

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It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 347

$(\hat{P}_{28})^{(5)}\ and\ (\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 348$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 349$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself 350

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 351

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

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$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 353$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+\right.$$
$$\left.(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a_{28}'')^{(5)}\ and\ (b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 354

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on

$(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 355

From GLOBAL EQUATIONS it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(5)} - (a_i^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(5)} t\right)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 356

Remark 3: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}^{'})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}^{'})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}^{'})^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If $G_{28}\ \ is$ bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 357

Remark 5: If T_{28} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(5)}\ ((G_{31})(t), t)) = (b_{29}^{'})^{(5)}$ then $T_{29} \to \infty$. 358

Definition of $(m)^{(5)}\ and\ \varepsilon_5$:

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i^{''})^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)} \quad 359$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 360

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $lim_{t\to\infty}(b_{30}^{''})^{(5)}\left((G_{31})(t), t\right) = (b_{30}^{'})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for $G_{33}\ , G_{34}, T_{32}, T_{33}, T_{34}$

361

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 362

$(\hat{P}_{32})^{(6)}$ *and* $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{363}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{364}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself (365)

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (366)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$ (367)

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{368}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a_{32}')^{(6)}+(\widehat{A}_{32})^{(6)}+\right.$$
$$\left.(\widehat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\,(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{32}'')^{(6)}$ *and* $(b_{32}'')^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (369)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}, i=32{,}33{,}34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0 \; and \; T_i(t) = 0$ 370

From 69 to 32 it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(6)} - (a_i^{''})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i^{'})^{(6)} t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: 371

Remark 3: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}^{'})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}^{'})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}^{'})^{(6)}$

If $G_{33} \; or \; G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If $G_{32} \; is$ bounded, from below, the same property holds for $G_{33} \; and \; G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 372

Remark 5: If T_{32} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(6)}((G_{35})(t), t)) = (b_{33}^{'})^{(6)}$ then $T_{33} \to \infty$. 373

Definition of $(m)^{(6)} \; and \; \varepsilon_6$:

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i^{''})^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

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Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 375

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $lim_{t\to\infty}(b_{34}^{''})^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}^{'})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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Behavior of the solutions 377

If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}$, $(\sigma_2)^{(1)}$, $(\tau_1)^{(1)}$, $(\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \le -(a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} - (a_{13}^{''})^{(1)}(T_{14}, t) + (a_{14}^{''})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \le -(b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G, t) - (b_{14}^{''})^{(1)}(G, t) \le -(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 378

(b) By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}$,, $(\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 379

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 380

(c) If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$$

$$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$$

and analogously 381

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ 382

are defined respectively

Then the solution satisfies the inequalities 383

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 384$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a_{15}^{'})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a_{15}^{'})^{(1)}t}\right] + G_{15}^0 e^{-(a_{15}^{'})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 385$$

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$ 386

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$ 387

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 388

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions 389

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 390

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$ 391

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \leq -(\tau_1)^{(2)}$ 392

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 393

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 394

(e) of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 395

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 396

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 397

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 398

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 399

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 400

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 401

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 402

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 403

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 404

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 405

and analogously 406

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 407

Then the solution satisfies the inequalities 408

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined 409

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 410

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right.$ 411
$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a_{18}')^{(2)})}[e^{(S_1)^{(2)} t} - e^{-(a_{18}')^{(2)} t}] + G_{18}^0 e^{-(a_{18}')^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 412

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 413

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18}')^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b_{18}')^{(2)} t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)} t} \le T_{18}(t) \le$ 414

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 415

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ 416

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$ 417

$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$

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Behavior of the solutions 419

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a_{20}')^{(3)} + (a_{21}')^{(3)} - (a_{20}'')^{(3)}(T_{21}, t) + (a_{21}'')^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \le -(b_{20}')^{(3)} + (b_{21}')^{(3)} - (b_{20}'')^{(3)}(G, t) - (b_{21}'')^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 420

(b) By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 421

(c) If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 422

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined 423

$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$ 424

$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right.$ 425

$\left.\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$

$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}}$ 426

$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$ 427

$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le$ 428

$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 429

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

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Behavior of the solutions 432

If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 433

(e) By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 434
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By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ 436

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, $\boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, $\boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, $\boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously 437
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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, $\boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

are defined by 59 and 64 respectively

Then the solution satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)}-(p_{24})^{(4)}-(S_2)^{(4)})}\left[e^{((S_1)^{(4)}-(p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)}-(a_{26}')^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a_{26}')^{(4)}t}\right] + G_{26}^0 e^{-(a_{26}')^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b_{26}')^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b_{26}')^{(4)}t}\right] + T_{26}^0 e^{-(b_{26}')^{(4)}t} \le T_{26}(t) \le$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24}')^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24}')^{(4)}$

$(R_2)^{(4)} = (b_{26}')^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a_{28}')^{(5)} + (a_{29}')^{(5)} - (a_{28}'')^{(5)}(T_{29}, t) + (a_{29}'')^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b_{28}')^{(5)} + (b_{29}')^{(5)} - (b_{28}'')^{(5)}\big((G_{31}), t\big) - (b_{29}'')^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$:

(h) By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the

roots of the equations $(a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, $\boldsymbol{if}\ (v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, $\boldsymbol{if}\ (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(5)} < (v_0)^{(5)}$

and analogously 457

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities 458

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 459$$

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$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}\right) \quad 461$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 462$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 463$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 464$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 465

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions 466

If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 467

(k) By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 468

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, $\boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$, 470
and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 471

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities 472

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 473$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 474$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a_{34}^{'})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a_{34}^{'})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a_{34}^{'})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 475$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 476$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}^{'})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b_{34}^{'})^{(6)}t}\right]+T_{34}^{0}e^{-(b_{34}^{'})^{(6)}t}\le T_{34}(t)\le \quad 477$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 478

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a_{32}^{'})^{(6)}$

$$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$$

$$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b_{32}^{'})^{(6)}$$

$$(R_2)^{(6)}=(b_{34}^{'})^{(6)}-(r_{34})^{(6)}$$ 479

Proof : From GLOBAL EQUATIONS we obtain 480

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a_{13}^{'})^{(1)}-(a_{14}^{'})^{(1)}+(a_{13}^{''})^{(1)}(T_{14},t)\right)-\ (a_{14}^{''})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$ 481

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\le\frac{dv^{(1)}}{dt}\le-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)},(v_0)^{(1)}$:-

(a) For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\ge\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

$$it\ follows\ (v_0)^{(1)}\le v^{(1)}(t)\le(v_1)^{(1)}$$

In the same manner , we get 482

$$v^{(1)}(t)\le\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\le v^{(1)}(t)\le(\bar{v}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 483

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 484

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL E486QUATIONS we get easily the result stated in the theorem.

Particular case : 485

If $(a_{13}^{''})^{(1)} = (a_{14}^{''})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}^{''})^{(1)} = (b_{14}^{''})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

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we obtain 487

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}^{'})^{(2)} - (a_{17}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17},t)\right) - (a_{17}^{''})^{(2)}(T_{17},t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 488

It follows 489

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 490

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

$$it\ follows\ (\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$$

In the same manner , we get 491

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 492

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 493

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 494

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 495

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 496

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

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Particular case : 498

If $(a_{16}^{''})^{(2)} = (a_{17}^{''})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}^{''})^{(2)} = (b_{17}^{''})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

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From GLOBAL EQUATIONS we obtain 500

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 501

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \leq \frac{dv^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

$it\ follows\ (v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get 503

$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 504

$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \leq v^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \leq (\bar{v}_1)^{(3)}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 505

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{20}^{''})^{(3)} = (a_{21}^{''})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b_{20}^{''})^{(3)} = (b_{21}^{''})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain 507

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}^{'})^{(4)} - (a_{25}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}, t)\right) - (a_{25}^{''})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$ 508

It follows

$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

$it\ follows\ (\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 509

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (C)^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 510

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)}+(\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)}-(\bar{v}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)}-(\bar{v}_2)^{(4)}\right)t\right]}} \leq (\bar{v}_1)^{(4)}$$

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(f) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)}+(\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)}-(\bar{v}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)}-(\bar{v}_2)^{(4)}\right)t\right]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{24}^{''})^{(4)} = (a_{25}^{''})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition 513
$(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$ **this also defines** $(v_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}^{''})^{(4)} = (b_{25}^{''})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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From GLOBAL EQUATIONS we obtain 515

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}^{'})^{(5)} - (a_{29}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29}, t)\right) - (a_{29}^{''})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

Definition of $v^{(5)}$:- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(g) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)}+(C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)}-(v_0)^{(5)}\right)t\right]}}{5+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)}-(v_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)}-(v_0)^{(5)}}{(v_0)^{(5)}-(v_2)^{(5)}}}$$

$it\ follows\ (\nu_0)^{(5)} \le \nu^{(5)}(t) \le (\nu_1)^{(5)}$

In the same manner , we get 516

$$\nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{5+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \le \nu^{(5)}(t) \le (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 517

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 518

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)} \quad 519$$

And so with the notation of the first part of condition (c) , we have
Definition of $\nu^{(5)}(t)$:-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain
Definition of $u^{(5)}(t)$:-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{28}^{''})^{(5)} = (a_{29}^{''})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}^{''})^{(5)} = (b_{29}^{''})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

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we obtain 521

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}^{'})^{(6)} - (a_{33}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}, t)\right) - (a_{33}^{''})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)\leq\frac{d\nu^{(6)}}{dt}\leq-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)},(\nu_0)^{(6)}$:-

(j) For $0<\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}<(\nu_1)^{(6)}<(\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t)\geq\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}\quad,\quad\boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

$it\ follows\ (\nu_0)^{(6)}\leq\nu^{(6)}(t)\leq(\nu_1)^{(6)}$

In the same manner , we get 522

$$\nu^{(6)}(t)\leq\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}\qquad(523)$$

From which we deduce $(\nu_0)^{(6)}\leq\nu^{(6)}(t)\leq(\bar{\nu}_1)^{(6)}$

(k) If $0<(\nu_1)^{(6)}<(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}<(\bar{\nu}_1)^{(6)}$ we find like in the previous case, 524

$$(\nu_1)^{(6)}\leq\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}\leq\ \nu^{(6)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\leq(\bar{\nu}_1)^{(6)}$$

(l) If $0<(\nu_1)^{(6)}\leq(\bar{\nu}_1)^{(6)}\leq\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 525

$$(\nu_1)^{(6)}\leq\ \nu^{(6)}(t)\leq\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\leq(\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)}\leq\ \nu^{(6)}(t)\leq(m_1)^{(6)},\quad\boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)}\leq\ u^{(6)}(t)\leq(\mu_1)^{(6)},\quad\boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{32}^{''})^{(6)}=(a_{33}^{''})^{(6)},then\ (\sigma_1)^{(6)}=(\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)}=(\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)}=(\nu_1)^{(6)}$ then $\nu^{(6)}(t)=(\nu_0)^{(6)}$ and as a consequence $G_{32}(t)=(\nu_0)^{(6)}G_{33}(t)$ **this also**

defines $(v_0)^{(6)}$ **for the special case** .
Analogously if $(b_{32}^{''})^{(6)} = (b_{33}^{''})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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We can prove the following 528

Theorem 3: If $(a_i^{''})^{(1)} and\ (b_i^{''})^{(1)}$ are independent on t , and the conditions

$(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a_{14}^{'})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13}^{'})^{(1)}(r_{14})^{(1)} - (b_{14}^{'})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined, then the system

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If $(a_i^{''})^{(2)} and\ (b_i^{''})^{(2)}$ are independent on t , and the conditions 530.

$(a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 531

$(a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a_{17}^{'})^{(2)}(\mathrm{p}_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 532

$(b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 533

$(b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16}^{'})^{(2)}(r_{17})^{(2)} - (b_{17}^{'})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 534

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

If $(a_i^{''})^{(3)} and\ (b_i^{''})^{(3)}$ are independent on t , and the conditions 535

$(a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a_{21}^{'})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20}^{'})^{(3)}(r_{21})^{(3)} - (b_{21}^{'})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied , then the system

If $(a_i^{''})^{(4)} and\ (b_i^{''})^{(4)}$ are independent on t , and the conditions 536

$(a_{24}^{'})^{(4)}(a_{25}^{'})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a_{24}^{'})^{(4)}(a_{25}^{'})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a_{25}^{'})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b_{24}^{'})^{(4)}(r_{25})^{(4)} - (b_{25}^{'})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined are satisfied , then the system

If $(a_i^{''})^{(5)} and\ (b_i^{''})^{(5)}$ are independent on t , and the conditions 537

$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a_{29}^{'})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b_{28}^{'})^{(5)}(r_{29})^{(5)} - (b_{29}^{'})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined satisfied , then the system

If $(a_i^{''})^{(6)} and\ (b_i^{''})^{(6)}$ are independent on t , and the conditions 538

$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a_{33}^{'})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32}^{'})^{(6)}(r_{33})^{(6)} - (b_{33}^{'})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$ 539

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a_{13}^{'})^{(1)} + (a_{13}^{''})^{(1)}(T_{14})\right]G_{13} = 0$ 540

$(a_{14})^{(1)}G_{13} - \left[(a_{14}^{'})^{(1)} + (a_{14}^{''})^{(1)}(T_{14})\right]G_{14} = 0$ 541

$(a_{15})^{(1)}G_{14} - \left[(a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}(T_{14})\right]G_{15} = 0$ 542

$(b_{13})^{(1)}T_{14} - [(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G)\ ]T_{13} = 0$ 543

$(b_{14})^{(1)}T_{13} - [(b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)}(G)\ ]T_{14} = 0$ 544

$(b_{15})^{(1)}T_{14} - [(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G)\ ]T_{15} = 0$ 545

has a unique positive solution , which is an equilibrium solution for the system 546

$(a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17})\right]G_{16} = 0$ 547

$(a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17})\right]G_{17} = 0$ 548

$(a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17})\right]G_{18} = 0$ 549

$(b_{16})^{(2)}T_{17} - [(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}(G_{19})\ ]T_{16} = 0$ 550

$(b_{17})^{(2)}T_{16} - [(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}(G_{19})\ ]T_{17} = 0$ 551

$(b_{18})^{(2)}T_{17} - [(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}(G_{19})\ ]T_{18} = 0$ 552

has a unique positive solution , which is an equilibrium solution for 553

$(a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})\right]G_{20} = 0$ 554

$(a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})\right]G_{21} = 0$ 555

$(a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})\right]G_{22} = 0$ 556

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 557$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 558$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 559$$

has a unique positive solution , which is an equilibrium solution 560

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 561$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 563$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 564$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 565$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 566$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 567$$

has a unique positive solution , which is an equilibrium solution for the system 568

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 569$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 570$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 571$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 572$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 573$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 574$$

has a unique positive solution , which is an equilibrium solution for the system 575

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 576$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 577$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 578$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 579$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 580$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 584$$

has a unique positive solution , which is an equilibrium solution for the system 582

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584

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}^{'})^{(1)}(a_{14}^{''})^{(1)}(T_{14}) + (a_{14}^{'})^{(1)}(a_{13}^{''})^{(1)}(T_{14}) + (a_{13}^{''})^{(1)}(T_{14})(a_{14}^{''})^{(1)}(T_{14}) = 0$$

585

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16}^{'})^{(2)}(a_{17}^{''})^{(2)}(T_{17}) + (a_{17}^{'})^{(2)}(a_{16}^{''})^{(2)}(T_{17}) + (a_{16}^{''})^{(2)}(T_{17})(a_{17}^{''})^{(2)}(T_{17}) = 0$$ 586

587

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20}^{'})^{(3)}(a_{21}^{''})^{(3)}(T_{21}) + (a_{21}^{'})^{(3)}(a_{20}^{''})^{(3)}(T_{21}) + (a_{20}^{''})^{(3)}(T_{21})(a_{21}^{''})^{(3)}(T_{21}) = 0$$

588

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a_{24}^{'})^{(4)}(a_{25}^{'})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24}^{'})^{(4)}(a_{25}^{''})^{(4)}(T_{25}) + (a_{25}^{'})^{(4)}(a_{24}^{''})^{(4)}(T_{25}) + (a_{24}^{''})^{(4)}(T_{25})(a_{25}^{''})^{(4)}(T_{25}) = 0$$

589

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28}^{'})^{(5)}(a_{29}^{''})^{(5)}(T_{29}) + (a_{29}^{'})^{(5)}(a_{28}^{''})^{(5)}(T_{29}) + (a_{28}^{''})^{(5)}(T_{29})(a_{29}^{''})^{(5)}(T_{29}) = 0$$

560

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32}^{'})^{(6)}(a_{33}^{''})^{(6)}(T_{33}) + (a_{33}^{'})^{(6)}(a_{32}^{''})^{(6)}(T_{33}) + (a_{32}^{''})^{(6)}(T_{33})(a_{33}^{''})^{(6)}(T_{33}) = 0$$

Definition and uniqueness of T_{14}^{*} :- 561

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^{*} for which $f(T_{14}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a_{13}^{'})^{(1)}+(a_{13}^{''})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a_{15}^{'})^{(1)}+(a_{15}^{''})^{(1)}(T_{14}^{*})]}$$

Definition and uniqueness of T_{17}^{*} :- 562

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)} G_{17}}{[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)} G_{17}}{[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17}^*)]} \qquad 563$$

Definition and uniqueness of T_{21}^* :- 564

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}^*)]}$$

565

Definition and uniqueness of T_{25}^* :- 566

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 567

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 568

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 569

$$\varphi(G) = (b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G) + (b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(G)\right] + (b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 570

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 571

(g) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if 572

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$ 573

(h) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if 574

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if 575

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if 578

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$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

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$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

581

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94 582

G_{14}^* *given by* $\varphi(G^*) = 0$, T_{14}^* *given by* $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G^{*})]}$, $T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G^{*})]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

G_{17}^{*} *given by* $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} *given by* $f(T_{17}^{*}) = 0$ and 584

$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17}^{*})]}$, $G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17}^{*})]}$ 585

$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}((G_{19})^{*})]}$, $T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}((G_{19})^{*})]}$ 586

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution 588

G_{21}^{*} *given by* $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} *given by* $f(T_{21}^{*}) = 0$ and

$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}^{*})]}$, $G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}^{*})]}$

$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23}{}^{*})]}$, $T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 589

G_{25}^{*} *given by* $\varphi(G_{27}) = 0$, T_{25}^{*} *given by* $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}^{*})]}$, $G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}((G_{27})^{*})]}$ 590

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 591

G_{29}^{*} *given by* $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} *given by* $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}((G_{31})^{*})]}$ 592

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 593

G_{33}^{*} *given by* $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} *given by* $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}^{*})]}$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}((G_{35})^{*})]} \qquad 594$$

Obviously, these values represent an equilibrium solution

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(1)}\ and\ (b_i^{''})^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$
$$\frac{\partial (a_{14}^{''})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i^{''})^{(1)}}{\partial G_j}(G^{*}) = s_{ij} \qquad 596$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \qquad 598$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \qquad 599$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \qquad 600$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \qquad 601$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}^{'})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \qquad 602$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \qquad 603$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(2)}\ and\ (b_i^{''})^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable 604

Denote 605

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i \qquad 606$$

$$\frac{\partial (a_{17}^{''})^{(2)}}{\partial T_{17}}(T_{17}^{*}) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b_i^{''})^{(2)}}{\partial G_j}((G_{19})^{*}) = s_{ij} \qquad 607$$

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^{*}\mathbb{T}_{17} \qquad 609$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^{*}\mathbb{T}_{17} \qquad 610$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^{*}\mathbb{T}_{17} \qquad 611$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)} T_{16}^{*}\mathbb{G}_j\right) \qquad 612$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}^{'})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^{*}\mathbb{G}_j\right) \quad 613$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^{*}\mathbb{G}_j\right) \quad 614$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}\ and\ (b_i^{''})^{(3)}$ 615
Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^{*}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i^{''})^{(3)}}{\partial G_j}((G_{23})^{*}) = s_{ij}$$

616

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21} \quad 618$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21} \quad 619$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}^{'})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21} \quad 6120$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^{*}\mathbb{G}_j\right) \quad 621$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}^{'})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 622$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 623$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(4)}\ and\ (b_i^{''})^{(4)}$ 624
Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 625

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}^{''})^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i^{''})^{(4)}}{\partial G_j}((G_{27})^{*}) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 627$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 628$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}^{'})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 629$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 630$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 631$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 632$$

633

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}$ and $(b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 634

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}^{''})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i^{''})^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 636$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 637$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 638$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 639$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 640$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 641$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}$ and $(b_i^{''})^{(6)}$ 642
Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 643

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}^{''})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i^{''})^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 645$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 646$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 647$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 648$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 649$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 650$$

651

The characteristic equation of this system is 652

$$\begin{aligned}
&\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)\\
&\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]\\
&\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)\\
&+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)\\
&\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)\\
&\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}\\
&+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\\
&\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0
\end{aligned} \quad 653$$

+

$$\begin{aligned}
&\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)\\
&\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G^*_{16}\right)\right]\\
&\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T^*_{17} + (b_{17})^{(2)}s_{(16),(17)}T^*_{17}\right)\\
&+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G^*_{17}\right)\\
&\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T^*_{17} + (b_{17})^{(2)}s_{(16),(16)}T^*_{16}\right)\\
&\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)\\
&\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)\\
&+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}G_{18}
\end{aligned}$$

$$+\left((\lambda)^{(2)}+(a_{16}')^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^{*}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^{*}\right)$$

$$\left(\left((\lambda)^{(2)}+(b_{16}')^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}T_{17}^{*}+(b_{17})^{(2)}s_{(16),(18)}T_{16}^{*}\right)\}=0$$

+

$$\left((\lambda)^{(3)}+(b_{22}')^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a_{22}')^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a_{20}')^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b_{20}')^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(21)}T_{21}^{*}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a_{21}')^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G_{20}^{*}+(a_{20})^{(3)}(q_{21})^{(1)}G_{21}^{*}\right)$$

$$\left(\left((\lambda)^{(3)}+(b_{20}')^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(20)}T_{20}^{*}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a_{20}')^{(3)}+(a_{21}')^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b_{20}')^{(3)}+(b_{21}')^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a_{20}')^{(3)}+(a_{21}')^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a_{20}')^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)$$

$$\left(\left((\lambda)^{(3)}+(b_{20}')^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(22)}T_{20}^{*}\right)\}=0$$

+

$$\left((\lambda)^{(4)}+(b_{26}')^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a_{26}')^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a_{24}')^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G_{25}^{*}+(a_{25})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b_{24}')^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(25)}T_{25}^{*}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a_{25}')^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G_{24}^{*}+(a_{24})^{(4)}(q_{25})^{(4)}G_{25}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}+(b_{24}')^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}')^{(4)}+(a_{25}')^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b_{24}')^{(4)}+(b_{25}')^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}')^{(4)}+(a_{25}')^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a_{24}')^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^{*}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}+(b_{24}')^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(26)}T_{24}^{*}\right)\}=0$$

+

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

+

$$\left(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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The introduction is a collection of information from various articles, Books, News Paper

reports, Home Pages Of authors, Journal Reviews, Nature 's L:etters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidiation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

REFERENCES

1. Dr K N Prasanna Kumar, Prof B S Kiranagi, Prof C S Bagewadi - MEASUREMENT DISTURBS EXPLANATION OF QUANTUM MECHANICAL STATES-A HIDDEN VARIABLE THEORY - published at: "International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition".

2. DR K N PRASANNA KUMAR, PROF B S KIRANAGI and PROF C S BAGEWADI - CLASSIC 2 FLAVOUR COLOR SUPERCONDUCTIVITY AND ORDINARY NUCLEAR MATTER-A NEW PARADIGM STATEMENT - published at: "International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition".

3. A HAIMOVICI: "On the growth of a two species ecological system divided on age groups". Tensor, Vol 37 (1982),Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday

4. FRTJOF CAPRA: "The web of life" Flamingo, Harper Collins See "Dissipative structures" pages 172-188

5. HEYLIGHEN F. (2001): "The Science of Self-organization and Adaptivity", in L. D. Kiel, (ed) . Knowledge Management, Organizational Intelligence and Learning, and Complexity, in: The Encyclopedia of Life Support Systems ((EOLSS), (Eolss Publishers, Oxford) [http://www.eolss.net

6. MATSUI, T, H. Masunaga, S. M. Kreidenweis, R. A. Pielke Sr., W.-K. Tao, M. Chin, and Y. J Kaufman (2006), "Satellite-based assessment of marine low cloud variability associated with aerosol, atmospheric stability, and the diurnal cycle", J. Geophys. Res., 111, D17204, doi:10.1029/2005JD006097

7. STEVENS, B, G. Feingold, W.R. Cotton and R.L. Walko, "Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus" J. Atmos. Sci., 53, 980-1006

8. FEINGOLD, G, Koren, I; Wang, HL; Xue, HW; Brewer, WA (2010), "Precipitation-generated oscillations in open cellular cloud fields" Nature, 466 (7308) 849-852, doi: 10.1038/nature09314, Published 12-Aug 2010

(9)^ [a] [b] [c] Einstein, A. (1905), "Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?", Annalen der Physik **18**:
639 Bibcode 1905AnP...323..639E,DOI:10.1002/andp.19053231314. See also the English translation.

(10)^ [a] [b] Paul Allen Tipler, Ralph A. Llewellyn (2003-01), Modern Physics, W. H. Freeman and Company, pp. 87–88, ISBN 0-7167-4345-0

(11)^ [a] [b] Rainville, S. et al. World Year of Physics: A direct test of E=mc2. Nature 438, 1096-1097 (22 December 2005) | doi: 10.1038/4381096a; Published online 21 December 2005.

(12)^ In F. Fernflores. The Equivalence of Mass and Energy. Stanford Encyclopedia of Philosophy

(13)^ Note that the relativistic mass, in contrast to the rest mass m_0, is not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity , where is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between **dτ** and **dt**.

(14)^ Relativity DeMystified, D. McMahon, Mc Graw Hill (USA), 2006, ISBN 0-07-145545-0

(15)^ Dynamics and Relativity, J.R. Forshaw, A.G. Smith, Wiley, 2009, ISBN 978-0-470-01460-8

(16)^ Hans, H. S.; Puri, S. P. (2003). Mechanics (2 ed.). Tata McGraw-Hill. p. 433. ISBN 0-07-047360-9., Chapter 12 page 433

(17)^ E. F. Taylor and J. A. Wheeler, **Spacetime Physics**, W.H. Freeman and Co., NY. 1992.ISBN 0-7167-2327-1, see pp. 248-9 for discussion of mass remaining constant after detonation of nuclear bombs, until heat is allowed to escape.

(18)^ Mould, Richard A. (2002). Basic relativity (2 ed.). Springer. p. 126. ISBN 0-387-95210-1., Chapter 5 page 126

(19)^ Chow, Tail L. (2006). Introduction to electromagnetic theory: a modern perspective. Jones & Bartlett Learning. p. 392. ISBN 0-7637-3827-1., Chapter 10 page 392

(20)^ [2] Cockcroft-Walton experiment

(21)^ [a] [b] [c] Conversions used: 1956 International (Steam) Table (IT) values where one calorie ≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers' conversion value of one gram TNT

≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a C_p of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average C_p of 25.8, 5.134 moles of metal, and 132 $J.K^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

(24)^ [3] Article on Earth rotation energy. Divided by c^2.

(25)^ **a b** Earth's gravitational self-energy is 4.6×10^{-10} that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.

(27)^ G. 't Hooft, "Computation of the quantum effects due to a four-dimensional pseudoparticle", Physical Review D14:3432–3450 (1976).

(28)^ A. Belavin, A. M. Polyakov, A. Schwarz, Yu. Tyupkin, "Pseudoparticle Solutions to Yang Mills Equations", Physics Letters 59B:85 (1975).

(29)^ F. Klinkhammer, N. Manton, "A Saddle Point Solution in the Weinberg Salam Theory", Physical Review D 30:2212.

(30)^ Rubakov V. A. "Monopole Catalysis of Proton Decay", Reports on Progress in Physics 51:189–241 (1988).

(31)^ S.W. Hawking "Black Holes Explosions?" Nature 248:30 (1974).

(32)^ Einstein, A. (1905), "Zur Elektrodynamik bewegter Körper." (PDF), Annalen der Physik **17**: 891–921, Bibcode 1905AnP...322...891E,DOI:10.1002/andp.19053221004. English translation.

(33)^ See e.g. Lev B.Okun, The concept of Mass, Physics Today **42** (6), June 1969, p. 31–36, http://www.physicstoday.org/vol-42/iss-6/vol42no6p31_36.pdf

(34)^ Max Jammer (1999), Concepts of mass in contemporary physics and philosophy, Princeton University Press, p. 51, ISBN 0-691-01017-X

(35)^ Eriksen, Erik; Vøyenli, Kjell (1976), "The classical and relativistic concepts of mass",Foundations of Physics (Springer) **6**: 115–124, Bibcode 1976FoPh....6..115E,DOI:10.1007/BF00708670

(36)^ **a b** Jannsen, M., Mecklenburg, M. (2007), From classical to relativistic mechanics: Electromagnetic models of the electron., in V. F. Hendricks, et al., , Interactions: Mathematics, Physics and Philosophy (Dordrecht: Springer): 65–134

(37)^ **a b** Whittaker, E.T. (1951–1953), 2. Edition: A History of the theories of aether and electricity, vol. 1: The classical theories / vol. 2: The modern theories 1900–1926, London: Nelson

(38)^ Miller, Arthur I. (1981), Albert Einstein's special theory of relativity. Emergence (1905) and early interpretation (1905–1911), Reading: Addison–Wesley, ISBN 0-201-04679-2

(39)^ **a b** Darrigol, O. (2005), "The Genesis of the theory of relativity." (PDF), Séminaire Poincaré **1**: 1–22

(40)^ Philip Ball (Aug 23, 2011). "Did Einstein discover E = mc2?" Physics World.

(41)^ Ives, Herbert E. (1952), "Derivation of the mass-energy relation", Journal of the Optical Society of America **42** (8): 540–543, DOI:10.1364/JOSA.42.000540

(42)^ Jammer, Max (1961/1997). Concepts of Mass in Classical and Modern Physics. New York: Dover. ISBN 0-486-29998-8.

(43)^ Stachel, John; Torretti, Roberto (1982), "Einstein's first derivation of mass-energy equivalence", American Journal of Physics **50** (8): 760–763, Bibcode1982AmJPh..50..760S, DOI:10.1119/1.12764

(44)^ Ohanian, Hans (2008), "Did Einstein prove E=mc2?", Studies In History and Philosophy of Science Part B **40** (2): 167–173, arXiv:0805.1400,DOI:10.1016/j.shpsb.2009.03.002

(45)^ Hecht, Eugene (2011), "How Einstein confirmed E0=mc2", American Journal of Physics **79** (6): 591–600, Bibcode 2011AmJPh..79..591H, DOI:10.1119/1.3549223

(46)^ Rohrlich, Fritz (1990), "An elementary derivation of E=mc2", American Journal of Physics **58** (4): 348–349, Bibcode 1990AmJPh..58..348R, DOI:10.1119/1.16168

(47) (1996). Lise Meitner: A Life in Physics. California Studies in the History of Science. **13**. Berkeley: University of California Press. pp. 236–237. ISBN 0-520-20860-

---

(48)^ UIBK.ac.at

(49)^ J. J. L. Morton; et al. (2008). "Solid-state quantum memory using the ^{31}P nuclear spin". Nature **455** (7216): 1085–1088. Bibcode 2008Natur.455.1085M.DOI:10.1038/nature07295.

(50)^ S. Weisner (1983). "Conjugate coding". Association of Computing Machinery, Special Interest Group in Algorithms and Computation Theory **15**: 78–88.

(51)^ A. Zelinger, Dance of the Photons: From Einstein to Quantum Teleportation, Farrar, Straus & Giroux, New York, 2010, pp. 189, 192, ISBN 0374239665

(52)^ B. Schumacher (1995). "Quantum coding". Physical Review A **51** (4): 2738–2747. Bibcode 1995PhRvA..51.2738S. DOI:10.1103/PhysRevA.51.2738.

(53)^ **a b** Straumann, N (2000). "On Pauli's invention of non-abelian Kaluza-Klein Theory in 1953". ArXiv: gr-qc/0012054 [gr-qc].

(54)^ See Abraham Pais' account of this period as well as L. Susskind's "Superstrings, Physics World on the first non-abelian gauge theory" where Susskind wrote that Yang–Mills was "rediscovered" only because Pauli had chosen not to publish.

(55)^ Reifler, N (2007). "Conditions for exact equivalence of Kaluza-Klein and Yang-Mills theories". ArXiv: gr-qc/0707.3790 [gr-qc].\

(56)^ Yang, C. N.; Mills, R. (1954). "Conservation of Isotopic Spin and Isotopic Gauge Invariance". Physical Review **96** (1): 191–195. Bibcode 1954PhRv...96...191Y.DOI:10.1103/PhysRev.96.191.

(57)^ Caprini, I.; Colangelo, G.; Leutwyler, H. (2006). "Mass and width of the lowest resonance in QCD". Physical Review Letters **96** (13): 132001. ArXiv: hep-ph/0512364.Bibcode 2006PhRvL..96m2001C. DOI:10.1103/PhysRevLett.96.132001.

(58)^ Yndurain, F. J.; Garcia-Martin, R.; Pelaez, J. R. (2007). "Experimental status of the ππ isoscalar S wave at low energy: f_0 (600) pole and scattering length". Physical Review D**76** (7): 074034. ArXiv:hep-ph/0701025. Bibcode 2007PhRvD..76g4034G.DOI:10.1103/PhysRevD.76.074034.

(59)^ Novikov, V. A.; Shifman, M. A.; A. I. Vainshtein, A. I.; Zakharov, V. I. (1983). "Exact Gell-Mann-Low Function of Supersymmetric Yang-Mills Theories From Instanton Calculus". Nuclear **229** (2): 381–393. Bibcode 1983NuPhB.229..381N.DOI:10.1016/0550-3213(83)90338-3.

(60)^ Ryttov, T.; Sannino, F. (2008). "Super symmetry Inspired QCD Beta Function". Physical Review D **78** (6): 065001. Bibcode 2008PhRvD..78f5001R.DOI:10.1103/PhysRevD.78.065001

(61)^ Bogolubsky, I. L.; Ilgenfritz, E.-M.; A. I. Müller-Preussker, M.; Sternbeck, A. (2009). "Lattice gluodynamics computation of Landau-gauge Green's functions in the deep infrared". Physics Letters B **676** (1-3): 69–73. Bibcode 2009PhLB..676...69B.DOI:10.1016/j.physletb.2009.04.076.

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