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A DEFACTO –DEJURE MODEL FOR POSITRONS,STELLAR NUCLOSYNTHESIS,QUANTUM COHERENCE,SIMULATION,OBJECTIVE REALITY,QUANTUM DECOHERENCE,VIRTUAL PHOTONS ,PHOTON TUNNELLING,ENZYMES ,SPACE TIME ,INCREASE IN SPEED OF CHEMICAL REACTIONS(SVERDUPAND SEMNOV'S NUMBER)AND QUANTUM TUNNELING

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ABSTRACT: There exists differential relations, contiguous similarities; in compassable anti generalities, pre suppositional resemblances, dialectic transformation, portmanteau incompatibilities between different structures of stellar nucleosynthesis that occurs in stars, positrons, and Eulerian kinematic description thereof, calls for the attention of both simulation and Quantum coherence, Quantum Gravity and Objective reality. Despite the crying need for the same, and lack of comprehensive envelope of expression, there seems to be contential staticity and presciential dynamism in so far as these aspects are considered. Atrophied asseveration in stellar nucleosynthesis and environmental decoherence are probably most endearing attributions that call for both rational Leibneizism and Socratic subjectivity and of course discourse relativity. An evolutionist model is expounded with configurational entropy and morphological entity for these variables and we look at the system dispassionately without being disturbed by the state of the system. There is no clamor for participatory seriotological sermonisations or an orientation towards pedagogical pontification. We just state the facts and leave the rest to others to do the divergential affirmation, disjunctive synthesis,. While we do resort to concept formulation, related phenomenological methodologies, transformational minimal conditions we neither resort to glorification or mortification of the thesis. Warts et al are presented without any hesitation, reservation, compunction or contrition, which probably is the testimony for the fact that human knowledge is limited and all the needs to be explored stretches in front like an ocean with all its cacophonous mendacious moorings and thromboses unbenedictory singularities with splashed contours and stigmatized boundaries.

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Parameters taken in to consideration are:

(1) Positrons
(2) Stellar Nucleosynthesis
(3) Simulation
(4) Quantum Coherence
(5) Quantum Gravity
(6) Objective reality
(7) Enzymes
(8) Space-time
(9) Virtual photons
(10) Photonic tunneling(and visibility thereof)
(11) Acceleration in Chemical reactions
(12) Quantum Tunneling

POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF STELLAR NUCLEOSYNTHESIS

G_{14} : CATEGORY TWO OF STELLAR NUCLEOSYNTHESIS

G_{15} : CATEGORY THREE OF STELLAR NUCLEOSYNTHESIS

T_{13} : CATEGORY ONE OF POSITRONS

T_{14} : CATEGORY TWO OF POSITRONS

T_{15} :CATEGORY THREE OF POSITRONS

SIMULATIONS AND QUANTUM COHERENCEMODULE NUMBERED TWO:

Note: Every film is simulation. Every thought is simulation.

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G_{16} : CATEGORY ONE OF SIMULATIONS

G_{17} : CATEGORY TWO OF SIMULATIONS

G_{18} : CATEGORY THREE OF SIMULATIONS

T_{16} :CATEGORY ONE OF QUANTUM COHERENCE

T_{17} : CATEGORY TWO OF QUANTUM COHERENCE

T_{18} : CATEGORY THREE OF QUANTUM COHERENCE

OBJECTIVE REALITY AND QUANTUM DECOHERENCE:MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF OBJECTIVE REALITY

G_{21} :CATEGORY TWO OF OBJECTIVE REALITY

G_{22} : CATEGORY THREE OF OBJECTIVE REALITY

T_{20} :CATEGORY ONE OF QUANTUM DECOHERENCE

T_{21} :CATEGORY TWO OF QUANTUM DECOHERENCE

T_{22} : CATEGORY THREE OFQUANTUM DECOHERENCE

SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OFENZYMES

G_{25} : CATEGORY TWO OF ENZYMES

G_{26} : CATEGORY THREE OF ENZYMES

T_{24} :CATEGORY ONE OF SPACE TIME

T_{25} :CATEGORY TWO OF SPACE TIME

T_{26} : CATEGORY THREE OF SPACETIME

PHOTONIC TUNNELING(VISIBILITY THEREOF) AND VIRTUAL PHOTONS:MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OFPHOTONIC TUNNELING

G_{29} : CATEGORY TWO OF PHOTONIC TUNNELING

G_{30} :CATEGORY THREE OF PHOTONIC TUNNELING

T_{28} :CATEGORY ONE OF VIRTUAL PHOTONS

T_{29} :CATEGORY TWO OFVIRTUAL PHOTONS

T_{30} :CATEGORY THREE OF VIRTUAL PHOTONS

QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION:MODULE NUMBERED SIX:

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G_{32} : CATEGORY ONE OF QUANTUM TUNNELING

G_{33} : CATEGORY TWO OF QUANTUM TUNNELING

G_{34} : CATEGORY THREE OFQUANTUM TUNNELING

T_{32} : CATEGORY ONE OF ACCELERATION IN CHEMICAL REACTIONS

T_{33} : CATEGORY TWO OFACCELERATION IN CHEMICAL REACTIONS

T_{34} : CATEGORY THREE OFACCELERATION IN CHEMICAL REACTIONS

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,

$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

SIMULATIONS AND QUANTUM COHERENCEMODULE NUMBERED TWO

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18}$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18}$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

OBJECTIVE REALITY AND QUANTUM DECOHERENCE:MODULE NUMBERED THREE

The differential system of this model is now

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR

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The differential system of this model is now

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

PHOTONIC TUNNELING(VISIBILITY THEREOF) AND VIRTUAL PHOTONS:MODULE NUMBERED FIVE

The differential system of this model is now

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$

$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}\big((G_{31}), t\big) \right] T_{28}$

$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}\big((G_{31}), t\big) \right] T_{29}$

$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}\big((G_{31}), t\big) \right] T_{30}$

$+(a_{28}^{''})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b_{28}^{''})^{(5)}\big((G_{31}), t\big) =$ **First detritions factor**

QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION:MODULE NUMBERED SIX

The differential system of this model is now

$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}, t) \right] G_{32}$

$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[(a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33}, t) \right] G_{33}$

$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}, t) \right] G_{34}$

$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}\big((G_{35}), t\big) \right] T_{32}$

$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[(b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)}\big((G_{35}), t\big) \right] T_{33}$

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}\big((G_{35}), t\big) \right] T_{34}$

$+(a_{32}^{''})^{(6)}(T_{33}, t) =$ **First augmentation factor**

$-(b_{32}^{''})^{(6)}\big((G_{35}), t\big) =$ **First detritions factor**

HOLISTIC CONCATENATE SYTEMAL EQUATIONS HENCEFORTH REFERRED TO AS "GLOBAL EQUATIONS

POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE

OBJECTIVE REALITY AND QUANTUM DECOHERENCE:MODULE NUMBERED THREE

SIMULATIONS AND QUANTUM COHERENCEMODULE NUMBERED TWO

SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR

PHOTONIC TUNNELING(VISIBILITY THEREOF) AND VIRTUAL PHOTONS:MODULE NUMBERED FIVE

QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION:MODULE NUMBERED SIX

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[\begin{matrix} (a_{13}^{'})^{(1)} \boxed{+(a_{13}^{''})^{(1)}(T_{14}, t)} \boxed{+(a_{16}^{''})^{(2,2,)}(T_{17}, t)} \boxed{+(a_{20}^{''})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,)}(T_{25}, t)} \boxed{+(a_{28}^{''})^{(5,5,5,5,)}(T_{29}, t)} \boxed{+(a_{32}^{''})^{(6,6,6,6,)}(T_{33}, t)} \end{matrix} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{matrix} (a_{14}^{'})^{(1)} \boxed{+(a_{14}^{''})^{(1)}(T_{14},t)} \boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)} \boxed{+(a_{21}^{''})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a_{25}^{''})^{(4,4,4,4,)}(T_{25},t)} \boxed{+(a_{29}^{''})^{(5,5,5,5,)}(T_{29},t)} \boxed{+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)} \end{matrix} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{matrix} (a_{15}^{'})^{(1)} \boxed{+(a_{15}^{''})^{(1)}(T_{14},t)} \boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)} \boxed{+(a_{22}^{''})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a_{26}^{''})^{(4,4,4,4,)}(T_{25},t)} \boxed{+(a_{30}^{''})^{(5,5,5,5,)}(T_{29},t)} \boxed{+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)} \end{matrix} \right] G_{15}$$

Where $\boxed{(a_{13}^{''})^{(1)}(T_{14},t)}$, $\boxed{(a_{14}^{''})^{(1)}(T_{14},t)}$, $\boxed{(a_{15}^{''})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}^{''})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}^{''})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}^{''})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b_{13}^{'})^{(1)} \boxed{-(b_{13}^{''})^{(1)}(G,t)} \boxed{-(b_{16}^{''})^{(2,2,)}(G_{19},t)} \boxed{-(b_{20}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{matrix} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{matrix} (b_{14}^{'})^{(1)} \boxed{-(b_{14}^{''})^{(1)}(G,t)} \boxed{-(b_{17}^{''})^{(2,2,)}(G_{19},t)} \boxed{-(b_{21}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{matrix} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b_{15}^{'})^{(1)} \boxed{-(b_{15}^{''})^{(1)}(G,t)} \boxed{-(b_{18}^{''})^{(2,2,)}(G_{19},t)} \boxed{-(b_{22}^{''})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)} \end{matrix} \right] T_{15}$$

Where $\boxed{-(b_{13}^{''})^{(1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1)}(G,t)}$, $\boxed{-(b_{15}^{''})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{matrix} (a_{16}^{'})^{(2)} \boxed{+(a_{16}^{''})^{(2)}(T_{17},t)} \boxed{+(a_{13}^{''})^{(1,1,)}(T_{14},t)} \boxed{+(a_{20}^{''})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a_{28}^{''})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a_{32}^{''})^{(6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{matrix} (a_{17}^{'})^{(2)} \boxed{+(a_{17}^{''})^{(2)}(T_{17},t)} \boxed{+(a_{14}^{''})^{(1,1,)}(T_{14},t)} \boxed{+(a_{21}^{''})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}^{''})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a_{29}^{''})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a_{33}^{''})^{(6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{matrix} (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{18}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{matrix} (b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1,)}(G,t)} \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \end{matrix} \right] T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{matrix} (b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1,)}(G,t)} \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \end{matrix} \right] T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{matrix} (b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1,)}(G,t)} \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \end{matrix} \right] T_{18}$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{matrix} (a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21},t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{matrix} (a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21},t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{matrix} (a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21},t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \end{matrix} \right] G_{22}$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{matrix}(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}\\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}\end{matrix}\right]T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{matrix}(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}\\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}\end{matrix}\right]T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{matrix}(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}\\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}\end{matrix}\right]T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{matrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{24}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{matrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{25}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{matrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{26}$$

Where $\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{matrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{24}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{matrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{25}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{matrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{26}$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$ *are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$ *are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$ *are sixth detrition coefficients for category* 1, 2 *and* 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{matrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{28}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{matrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{29}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{matrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{30}$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{matrix}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\end{matrix}\right]T_{28}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{matrix}(b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\end{matrix}\right]T_{29}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{matrix}(b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}\end{matrix}\right]T_{30}$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\end{matrix}\right]G_{34}$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix} (b'_{32})^{(6)} -(b''_{32})^{(6)}(G_{35},t) -(b''_{28})^{(5,5,5)}(G_{31},t) -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \end{matrix}\right] T_{32}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix} (b'_{33})^{(6)} -(b''_{33})^{(6)}(G_{35},t) -(b''_{29})^{(5,5,5)}(G_{31},t) -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \end{matrix}\right] T_{33}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{matrix} (b'_{34})^{(6)} -(b''_{34})^{(6)}(G_{35},t) -(b''_{30})^{(5,5,5)}(G_{31},t) -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \end{matrix}\right] T_{34}$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$

$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

<u>Definition of</u> $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14},t) = (p_i)^{(1)}$

$\lim_{G \to \infty} (b''_i)^{(1)} (G,t) = (r_i)^{(1)}$

<u>Definition of</u> $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}$ are positive constants and $i = 13,14,15$

They satisfy Lipschitz condition:

$$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,t)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(1)}(T_{14}^{'},t)$ and $(a_i^{''})^{(1)}(T_{14},t)$. $(T_{14}^{'},t)$ and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)},(\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)}=1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$, the first augmentation coefficient WOULD be absolutely continuous.

<u>Definition of</u> $(\hat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}},\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$$

<u>Definition of</u> $(\hat{P}_{13})^{(1)},(\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)},(\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)},(a_i^{'})^{(1)},(b_i)^{(1)},(b_i^{'})^{(1)},(p_i)^{(1)},(r_i)^{(1)},i=13,14,15,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)}+(a_i^{'})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}]<1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)}+(b_i^{'})^{(1)}+(\hat{B}_{13})^{(1)}+(\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}]<1$$

Where we suppose

(F) $(a_i)^{(2)},(a_i^{'})^{(2)},(a_i^{''})^{(2)},(b_i)^{(2)},(b_i^{'})^{(2)},(b_i^{''})^{(2)}>0,\quad i,j=16,17,18$

(G) The functions $(a_i^{''})^{(2)},(b_i^{''})^{(2)}$ are positive continuous increasing and bounded.

<u>Definition of</u> $(\mathrm{p_i})^{(2)},(\mathrm{r_i})^{(2)}$:

$$(a_i^{''})^{(2)}(T_{17},t)\leq(p_i)^{(2)}\leq(\hat{A}_{16})^{(2)}$$

$$(b_i^{''})^{(2)}(G_{19},t)\leq(r_i)^{(2)}\leq(b_i^{'})^{(2)}\leq(\hat{B}_{16})^{(2)}$$

(H) $\lim_{T_2\to\infty}(a_i^{''})^{(2)}(T_{17},t)=(p_i)^{(2)}$

$\lim_{G\to\infty}(b_i^{''})^{(2)}\big((G_{19}),t\big)=(r_i)^{(2)}$

<u>Definition of</u> $(\hat{A}_{16})^{(2)},(\hat{B}_{16})^{(2)}$:

Where $\boxed{(\hat{A}_{16})^{(2)},(\hat{B}_{16})^{(2)},(p_i)^{(2)},(r_i)^{(2)}}$ are positive constants and $\boxed{i=16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(2)}(T_{17}^{'},t)-(a_i^{''})^{(2)}(T_{17},t)|\leq(\hat{k}_{16})^{(2)}|T_{17}-T_{17}^{'}|e^{-(\hat{M}_{16})^{(2)}t}$$

$$|(b_i^{''})^{(2)}((G_{19})^{'},t)-(b_i^{''})^{(2)}\big((G_{19}),t\big)|<(\hat{k}_{16})^{(2)}||(G_{19})-(G_{19})^{'}||e^{-(\hat{M}_{16})^{(2)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(2)}(T_{17}^{'},t)$ and$(a_i^{''})^{(2)}(T_{17},t)$. $(T_{17}^{'},t)$ and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)},(\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i^{''})^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)}=1$ then the function $(a_i^{''})^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous.

<u>Definition of</u> $(\hat{M}_{16})^{(2)},(\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

Where we suppose

(J) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(3)} (T_{21}, t) = (p_i)^{(3)}$$

$$\lim_{G \to \infty} (b_i'')^{(3)} (G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)}+(a_i^{'})^{(3)}+\ (\hat{A}_{20})^{(3)}+\ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}]<1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)}+(b_i^{'})^{(3)}+\ (\hat{B}_{20})^{(3)}+\ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}]<1$$

Where we suppose

$$(a_i)^{(4)},(a_i^{'})^{(4)},(a_i^{''})^{(4)},(b_i)^{(4)},(b_i^{'})^{(4)},(b_i^{''})^{(4)}>0,\quad i,j=24,25,26$$

(M) The functions $(a_i^{''})^{(4)},(b_i^{''})^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i^{''})^{(4)}(T_{25},t)\leq(p_i)^{(4)}\leq(\hat{A}_{24})^{(4)}$$

$$(b_i^{''})^{(4)}\big((G_{27}),t\big)\leq\ (r_i)^{(4)}\leq(b_i^{'})^{(4)}\leq(\hat{B}_{24})^{(4)}$$

(N) $lim_{T_2\to\infty}(a_i^{''})^{(4)}(T_{25},t)=(p_i)^{(4)}$
$\lim_{G\to\infty}(b_i^{''})^{(4)}\big((G_{27}),t\big)=\ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)},(\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)},(\hat{B}_{24})^{(4)},(p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i=24,25,26}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(4)}(T_{25}^{'},t)-(a_i^{''})^{(4)}(T_{25},t)|\leq(\hat{k}_{24})^{(4)}|T_{25}-\ T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i^{''})^{(4)}((G_{27})^{'},t)-(b_i^{''})^{(4)}\big((G_{27}),t\big)|<(\hat{k}_{24})^{(4)}||(G_{27})-(G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'},t)$ and$(a_i^{''})^{(4)}(T_{25},t)$. $(T_{25}^{'},t)$ and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)},(\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)}=4$ then the function $(a_i^{''})^{(4)}(T_{25},t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)},(\hat{k}_{24})^{(4)}$:

$(\hat{M}_{24})^{(4)},(\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\ ,\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$$

Definition of $(\hat{P}_{24})^{(4)},(\hat{Q}_{24})^{(4)}$:

(Q) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with
$(\hat{M}_{24})^{(4)},(\hat{k}_{24})^{(4)},(\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants
$(a_i)^{(4)},(a_i^{'})^{(4)},(b_i)^{(4)},(b_i^{'})^{(4)},(p_i)^{(4)},\ (r_i)^{(4)},i=24,25,26,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i^{'})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$

$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i^{'})^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$

Where we suppose

$(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, \quad i,j = 28,29,30$

(S) The functions $(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$(a_i^{''})^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$

$(b_i^{''})^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i^{'})^{(5)} \leq (\hat{B}_{28})^{(5)}$

(T) $lim_{T_2 \to \infty} (a_i^{''})^{(5)}(T_{29}, t) = (p_i)^{(5)}$

$\lim_{G \to \infty} (b_i^{''})^{(5)}(G_{31}, t) = (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28,29,30$

They satisfy Lipschitz condition:

$|(a_i^{''})^{(5)}(T_{29}^{'}, t) - (a_i^{''})^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}^{'}|e^{-(\hat{M}_{28})^{(5)}t}$

$|(b_i^{''})^{(5)}((G_{31})^{'}, t) - (b_i^{''})^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\hat{M}_{28})^{(5)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'}, t)$ and$(a_i^{''})^{(5)}(T_{29}, t)$. $(T_{29}^{'}, t)$ and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i^{''})^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i^{''})^{(5)}(T_{29}, t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$:

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$:

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i^{'})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i^{'})^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$

$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i^{'})^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$

Where we suppose

$(a_i)^{(6)}, (a_i^{'})^{(6)}, (a_i^{''})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (b_i^{''})^{(6)} > 0, \quad i,j = 32,33,34$

(W) The functions $(a_i^{''})^{(6)}, (b_i^{''})^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i^{''})^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i^{''})^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i^{'})^{(6)} \leq (\hat{B}_{32})^{(6)}$$

(X) $lim_{T_2 \to \infty} (a_i^{''})^{(6)} (T_{33}, t) = (p_i)^{(6)}$

$\lim_{G \to \infty} (b_i^{''})^{(6)} \left((G_{35}), t\right) = \ (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(6)}(T_{33}^{'}, t) - (a_i^{''})^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}^{'}|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i^{''})^{(6)}((G_{35})^{'}, t) - (b_i^{''})^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})^{'}||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'}, t)$ and$(a_i^{''})^{(6)}(T_{33}, t)$. $(T_{33}^{'}, t)$ and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i^{''})^{(6)}(T_{33}, t)$, the SIXTH **augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$:

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$:

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i^{'})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (a_i)^{(6)} + (a_i^{'})^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)} \ (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i^{'})^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)} \ (\hat{k}_{32})^{(6)}] < 1$$

Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$

By

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \Big[(a_{13})^{(1)} G_{14}\big(s_{(13)}\big) - \Big((a_{13}^{'})^{(1)} + a_{13}^{''})^{(1)}\big(T_{14}\big(s_{(13)}\big), s_{(13)}\big)\Big) G_{13}\big(s_{(13)}\big)\Big] ds_{(13)}$

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \Big[(a_{14})^{(1)} G_{13}\big(s_{(13)}\big) - \Big((a_{14}^{'})^{(1)} + (a_{14}^{''})^{(1)}\big(T_{14}\big(s_{(13)}\big), s_{(13)}\big)\Big) G_{14}\big(s_{(13)}\big)\Big] ds_{(13)}$

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \Big[(a_{15})^{(1)} G_{14}\big(s_{(13)}\big) - \Big((a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}\big(T_{14}\big(s_{(13)}\big), s_{(13)}\big)\Big) G_{15}\big(s_{(13)}\big)\Big] ds_{(13)}$

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \Big[(b_{13})^{(1)} T_{14}\big(s_{(13)}\big) - \Big((b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}\big(G\big(s_{(13)}\big), s_{(13)}\big)\Big) T_{13}\big(s_{(13)}\big)\Big] ds_{(13)}$

$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \Big[(b_{14})^{(1)} T_{13}\big(s_{(13)}\big) - \Big((b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)}\big(G\big(s_{(13)}\big), s_{(13)}\big)\Big) T_{14}\big(s_{(13)}\big)\Big] ds_{(13)}$

$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \Big[(b_{15})^{(1)} T_{14}\big(s_{(13)}\big) - \Big((b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}\big(G\big(s_{(13)}\big), s_{(13)}\big)\Big) T_{15}\big(s_{(13)}\big)\Big] ds_{(13)}$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$,$T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \Big[(a_{16})^{(2)} G_{17}\big(s_{(16)}\big) - \Big((a_{16}^{'})^{(2)} + a_{16}^{''})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(16)}\big)\Big) G_{16}\big(s_{(16)}\big)\Big] ds_{(16)}$

$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \Big[(a_{17})^{(2)} G_{16}\big(s_{(16)}\big) - \Big((a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(17)}\big)\Big) G_{17}\big(s_{(16)}\big)\Big] ds_{(16)}$

$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \Big[(a_{18})^{(2)} G_{17}\big(s_{(16)}\big) - \Big((a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(16)}\big)\Big) G_{18}\big(s_{(16)}\big)\Big] ds_{(16)}$

$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \Big[(b_{16})^{(2)} T_{17}\big(s_{(16)}\big) - \Big((b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{16}\big(s_{(16)}\big)\Big] ds_{(16)}$

$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \Big[(b_{17})^{(2)} T_{16}\big(s_{(16)}\big) - \Big((b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{17}\big(s_{(16)}\big)\Big] ds_{(16)}$

$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \Big[(b_{18})^{(2)} T_{17}\big(s_{(16)}\big) - \Big((b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{18}\big(s_{(16)}\big)\Big] ds_{(16)}$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$,$T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a_{20}^{'})^{(3)} + a_{20}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$

$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$

$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$

$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$

$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$

$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By

$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}^{'})^{(4)} + a_{24}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$

$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$

$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$

$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$

$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$

$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}\left(s_{(28)}\right) - \left((a_{28}^{'})^{(5)} + a_{28}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{28}\left(s_{(28)}\right) \right] ds_{(28)}$

$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}\left(s_{(28)}\right) - \left((a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{29}\left(s_{(28)}\right) \right] ds_{(28)}$

$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}\left(s_{(28)}\right) - \left((a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{30}\left(s_{(28)}\right) \right] ds_{(28)}$

$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}\left(s_{(28)}\right) - \left((b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)} \left(G\left(s_{(28)}\right), s_{(28)}\right) \right) T_{28}\left(s_{(28)}\right) \right] ds_{(28)}$

$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}\left(s_{(28)}\right) - \left((b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)} \left(G\left(s_{(28)}\right), s_{(28)}\right) \right) T_{29}\left(s_{(28)}\right) \right] ds_{(28)}$

$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}\left(s_{(28)}\right) - \left((b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)} \left(G\left(s_{(28)}\right), s_{(28)}\right) \right) T_{30}\left(s_{(28)}\right) \right] ds_{(28)}$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$,$T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a_{32}^{'})^{(6)} + a_{32}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{32}\left(s_{(32)}\right) \right] ds_{(32)}$

$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}\left(s_{(32)}\right) - \left((a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{33}\left(s_{(32)}\right) \right] ds_{(32)}$

$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{34}\left(s_{(32)}\right) \right] ds_{(32)}$

$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)} \left(G\left(s_{(32)}\right), s_{(32)}\right) \right) T_{32}\left(s_{(32)}\right) \right] ds_{(32)}$

$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}\left(s_{(32)}\right) - \left((b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)} \left(G\left(s_{(32)}\right), s_{(32)}\right) \right) T_{33}\left(s_{(32)}\right) \right] ds_{(32)}$

$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)} \left(G\left(s_{(32)}\right), s_{(32)}\right) \right) T_{34}\left(s_{(32)}\right) \right] ds_{(32)}$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14}, $G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$
$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for G_{17}, $G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0} \right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for G_{21}, $G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}} \right) \right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1 \right)$$

From which it follows that

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0 \right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0} \right)} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\widehat{M}_{13})^{(1)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\widehat{M}_{13})^{(1)}t}\}$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$:

$$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$$

It results

$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$

$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$

$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$

$G_{13}^{(2)} |(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq$
$\frac{1}{(\widehat{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(\textit{and not on } t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(1)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(1)}(G(t),t)) = (b_{14}^{'})^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i^{''})^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}^{''})^{(1)}(G(t),t) = (b_{15}^{'})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)}$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{Q}_{16})^{(2)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t \{(a_{16}^{'})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} |(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} t} \leq \frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(2)} t\right)} > 0 \quad$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$:

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$.

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)}$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right), \left((G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)}\left|G_{21}^{(1)} - G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)} +$$

$$\int_0^t\{(a_{20}')^{(3)}\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t} \leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ and $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(3)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ and $\left((\widehat{M}_{20})^{(3)}\right)_3$:

Remark 3: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}')^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 5: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$.

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+((\hat{P}_{24})^{(4)}+G_j^0)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)}$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{Q}_{24})^{(4)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying IN to itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a_{24}^{'})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a_{24}^{''})^{(4)}(T_{25}^{(1)},s_{(24)})\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}|(a_{24}^{''})^{(4)}(T_{25}^{(1)},s_{(24)})-(a_{24}^{''})^{(4)}(T_{25}^{(2)},s_{(24)})|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a_{24}^{'})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{24}^{''})^{(4)}$ and $(b_{24}^{''})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ and $T_i\,(t)=0$

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(4)}-(a_i^{''})^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(4)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ and $\left((\widehat{M}_{24})^{(4)}\right)_3$:

Remark 3: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}^{'})^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a_{25}^{'})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a_{26}^{'})^{(4)}$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 5: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(4)}\,((G_{27})(t),t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$.

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1-e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t),t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose

$(\,\widehat{P}_{28}\,)^{(5)}$ and $(\,\widehat{Q}_{28}\,)^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+((\hat{P}_{28})^{(5)}+G_j^0)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{Q}_{28})^{(5)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d\left(\left((G_{31})^{(1)},(T_{31})^{(1)}\right),\left((G_{31})^{(2)},(T_{31})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(\left((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ and $T_i\,(t)=0$

From GLOBAL EQUATIONS it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(5)} - (a_i^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i^{'})^{(5)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$:

Remark 3: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}^{'})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}^{'})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}^{'})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below.

Remark 5: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(5)}((G_{31})(t), t)) = (b_{29}^{'})^{(5)}$ then $T_{29} \to \infty$.

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i^{''})^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}^{''})^{(5)}\left((G_{31})(t), t\right) = (b_{30}^{'})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+((\hat{P}_{32})^{(6)}+G_j^0)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{Q}_{32})^{(6)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d\left(\left((G_{35})^{(1)},(T_{35})^{(1)}\right),\left((G_{35})^{(2)},(T_{35})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(\left((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$

From 69 to 32 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(6)} - (a_i^{''})^{(6)}(T_{33}(s_{(32)}),s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(6)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ and $\left((\widehat{M}_{32})^{(6)}\right)_3$:

Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}^{'})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}^{'})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}^{'})^{(6)}$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below.

Remark 5: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(6)}((G_{35})(t),t)) = (b_{33}^{'})^{(6)}$ then $T_{33} \to \infty$.

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i^{''})^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}^{''})^{(6)}\left((G_{35})(t),t(t),t\right) = (b_{34}^{'})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} - (a_{13}^{''})^{(1)}(T_{14}, t) + (a_{14}^{''})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$:

(b) By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}\ , if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\ , if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined respectively

Then the solution satisfies the inequalities

$G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$

$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}\left((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)}\right)}\left[e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}\left((S_1)^{(1)} - (a'_{15})^{(1)}\right)}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$

$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{\left((R_1)^{(1)} + (r_{13})^{(1)}\right)t}}$

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{\left((R_1)^{(1)} + (r_{13})^{(1)}\right)t}$

$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}\left((R_1)^{(1)} - (b'_{15})^{(1)}\right)}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^0e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:-

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a_{13}^{'})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b_{13}^{'})^{(1)}$

$(R_2)^{(1)}=(b_{15}^{'})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$:

(d) $\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a_{16}^{'})^{(2)}+(a_{17}^{'})^{(2)}-(a_{16}^{''})^{(2)}(T_{17},t)+(a_{17}^{''})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)}$

$-(\tau_2)^{(2)}\le-(b_{16}^{'})^{(2)}+(b_{17}^{'})^{(2)}-(b_{16}^{''})^{(2)}((G_{19}),t)-(b_{17}^{''})^{(2)}((G_{19}),t)\le-(\tau_1)^{(2)}$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$:

By $(\nu_1)^{(2)}>0,(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots

(e) of the equations $(a_{17})^{(2)}(\nu^{(2)})^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$

and $(b_{14})^{(2)}(u^{(2)})^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$:

By $(\bar{\nu}_1)^{(2)}>0,(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the

roots of the equations $(a_{17})^{(2)}(\nu^{(2)})^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$

and $(b_{17})^{(2)}(u^{(2)})^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$

Definition of $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$:-

(f) If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},\ \boldsymbol{if}\ (\nu_0)^{(2)}<(\nu_1)^{(2)}$

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)},\boldsymbol{if}\ (\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)},$

and $\boxed{(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)}<(\nu_0)^{(2)}$

and analogously

$(\mu_2)^{(2)}=(u_0)^{(2)},(\mu_1)^{(2)}=(u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)}<(u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, $\boldsymbol{if}$ $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}$ $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$

Then the solution satisfies the inequalities

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a_{18}')^{(2)})} [e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}] + G_{18}^0 e^{-(a_{18}')^{(2)}t} \right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18}')^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t} \right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:-

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$

$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a_{20}')^{(3)} + (a_{21}')^{(3)} - (a_{20}'')^{(3)}(T_{21}, t) + (a_{21}'')^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b_{20}')^{(3)} + (b_{21}')^{(3)} - (b_{20}'')^{(3)}(G, t) - (b_{21}'')^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

(b) By $(\nu_1)^{(3)} > 0, (\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

<u>Definition of</u> $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}$, $(\mu_2)^{(3)}$:-

(c) If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}$, $(\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}, \ \boldsymbol{if} \ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}, \boldsymbol{if} \ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)},$

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \ \boldsymbol{if} \ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if} \ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \ \boldsymbol{if} \ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

$$\Big(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)} \left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq$$
$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}\left((S_1)^{(3)} - (a_{22}')^{(3)}\right)} \left[e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t}\right] + G_{22}^0 e^{-(a_{22}')^{(3)}t}\Big)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}\left((R_1)^{(3)} - (b_{22}')^{(3)}\right)} \left[e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t}\right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}\left((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)}\right)} \left[e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

<u>Definition of</u> $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \leq -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$:

(e) By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if} \ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if} \ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}, \ \boldsymbol{if} \ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \ \boldsymbol{if} \ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if} \ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if} \ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined by 59 and 64 respectively

Then the solution satisfies the inequalities

$G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$

where $(p_i)^{(4)}$ is defined

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$

$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)}-(p_{24})^{(4)}-(S_2)^{(4)})}\left[e^{((S_1)^{(4)}-(p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)}-(a_{26}^{'})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a_{26}^{'})^{(4)}t}\right] + G_{26}^0 e^{-(a_{26}^{'})^{(4)}t}\right)$

$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}$

$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b_{26}^{'})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b_{26}^{'})^{(4)}t}\right] + T_{26}^0 e^{-(b_{26}^{'})^{(4)}t} \leq T_{26}(t) \leq$

$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24}^{'})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24}^{'})^{(4)}$

$(R_2)^{(4)} = (b_{26}^{'})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions
If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \leq -(a_{28}^{'})^{(5)} + (a_{29}^{'})^{(5)} - (a_{28}^{''})^{(5)}(T_{29}, t) + (a_{29}^{''})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$

$-(\tau_2)^{(5)} \leq -(b_{28}^{'})^{(5)} + (b_{29}^{'})^{(5)} - (b_{28}^{''})^{(5)}((G_{31}), t) - (b_{29}^{''})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$:

(h) By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}(\nu^{(5)})^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}(\nu^{(5)})^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)}, (m_1)^{(5)}=(\nu_1)^{(5)},\ \boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)}, (m_1)^{(5)}=(\bar{\nu}_1)^{(5)},\ \boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,

and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)}=(\nu_1)^{(5)}, (m_1)^{(5)}=(\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously

$(\mu_2)^{(5)}=(u_0)^{(5)}, (\mu_1)^{(5)}=(u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)}=(u_1)^{(5)}, (\mu_1)^{(5)}=(\bar{u}_1)^{(5)},\ \boldsymbol{if}\ (u_1)^{(5)}<(u_0)^{(5)}<(\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)}=\frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)}=(u_1)^{(5)}, (\mu_1)^{(5)}=(u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)}<(u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities

$$G_{28}^0 e^{\left((S_1)^{(5)}-(p_{28})^{(5)}\right)t}\le G_{28}(t)\le G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined

$$\frac{1}{(m_5)^{(5)}}G_{28}^0 e^{\left((S_1)^{(5)}-(p_{28})^{(5)}\right)t}\le G_{29}(t)\le \frac{1}{(m_2)^{(5)}}G_{28}^0 e^{(S_1)^{(5)}t}$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^0}{(m_1)^{(5)}\left((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)}-(p_{28})^{(5)}\right)t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^0 e^{-(S_2)^{(5)}t}\le G_{30}(t)\le \frac{(a_{30})^{(5)}G_{28}^0}{(m_2)^{(5)}\left((S_1)^{(5)}-(a'_{30})^{(5)}\right)}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+G_{30}^0 e^{-(a'_{30})^{(5)}t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^0 e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^0 e^{(R_1)^{(5)}t}\le T_{28}(t)\le \frac{1}{(\mu_2)^{(5)}}T_{28}^0 e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}$$

$$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}\left((R_1)^{(5)}-(b'_{30})^{(5)}\right)}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^0 e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le$$

$$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}\left((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)}\right)}\left[e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:-

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$:

(k) By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$:

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities

$$G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$

$\left(\frac{(a_{34})^{(6)}G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^0e^{-(S_2)^{(6)}t}\leq G_{34}(t)\leq\right.$
$\left.\frac{(a_{34})^{(6)}G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)}-(a_{34}')^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a_{34}')^{(6)}t}\right]+G_{34}^0e^{-(a_{34}')^{(6)}t}\right)$

$$\boxed{T_{32}^0e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq T_{32}^0e^{((R_1)^{(6)}+(r_{32})^{(6)})t}}$$

$\frac{1}{(\mu_1)^{(6)}}T_{32}^0e^{(R_1)^{(6)}t}\leq T_{32}(t)\leq\frac{1}{(\mu_2)^{(6)}}T_{32}^0e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$

$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}')^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b_{34}')^{(6)}t}\right]+T_{34}^0e^{-(b_{34}')^{(6)}t}\leq T_{34}(t)\leq$

$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^0e^{-(R_2)^{(6)}t}$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:-

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a_{32}')^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b_{32}')^{(6)}$

$(R_2)^{(6)}=(b_{34}')^{(6)}-(r_{34})^{(6)}$

Proof : From GLOBAL EQUATIONS we obtain

$\frac{d\nu^{(1)}}{dt}=(a_{13})^{(1)}-\left((a_{13}')^{(1)}-(a_{14}')^{(1)}+(a_{13}'')^{(1)}(T_{14},t)\right)-(a_{14}'')^{(1)}(T_{14},t)\nu^{(1)}-(a_{14})^{(1)}\nu^{(1)}$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_2)^{(1)}\nu^{(1)}-(a_{13})^{(1)}\right)\leq\frac{d\nu^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_1)^{(1)}\nu^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

(a) For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}$, $\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$

it follows $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$

In the same manner , we get

$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}$, $\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

<u>Definition of</u> $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

we obtain

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

<u>Definition of</u> $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}\right)\leq\frac{d\nu^{(2)}}{dt}\leq-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(2)},(\nu_0)^{(2)}$:-

(d) For $0<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\nu_1)^{(2)}<(\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t)\geq\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}\quad,\quad\boxed{(C)^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)}\leq\nu^{(2)}(t)\leq(\nu_1)^{(2)}$

In the same manner , we get

$$\nu^{(2)}(t)\leq\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)}\leq\nu^{(2)}(t)\leq(\bar{\nu}_1)^{(2)}$

(e) If $0<(\nu_1)^{(2)}<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\bar{\nu}_1)^{(2)}$ we find like in the previous case,

$$(\nu_1)^{(2)}\leq\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}\leq\nu^{(2)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\leq(\bar{\nu}_1)^{(2)}$$

(f) If $0<(\nu_1)^{(2)}\leq(\bar{\nu}_1)^{(2)}\leq(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(\nu_1)^{(2)}\leq\nu^{(2)}(t)\leq\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\leq(\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:-

$$(m_2)^{(2)}\leq\nu^{(2)}(t)\leq(m_1)^{(2)},\quad\boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)}\leq u^{(2)}(t)\leq(\mu_1)^{(2)},\quad\boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

.

Particular case :

If $(a''_{16})^{(2)}=(a''_{17})^{(2)}, then\ (\sigma_1)^{(2)}=(\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)}=(\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)}=(\nu_1)^{(2)}$ then $\nu^{(2)}(t)=(\nu_0)^{(2)}$ and as a consequence $G_{16}(t)=(\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)}=(b''_{17})^{(2)}, then\ (\tau_1)^{(2)}=(\tau_2)^{(2)}$ and then

$(u_1)^{(2)}=(\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)}=(u_1)^{(2)}$ then $T_{16}(t)=(u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case,

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

(c) If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

: From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case,

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

<u>Definition of</u> $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$**.**

From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

<u>Definition of</u> $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

<u>Definition of</u> $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_0)^{(5)}\right)t\right]}}{5+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{5+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case,

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{28}^{''})^{(5)} = (a_{29}^{''})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}^{''})^{(5)} = (b_{29}^{''})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

we obtain

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case,

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$**.**

We can prove the following

Theorem 3: If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with$ $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined, then the system

If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 ,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

$with$ $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0 ,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

$with$ $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied , then the system

If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)}-(b_{24})^{(4)}(b_{25})^{(4)}>0$,

$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)}-(b_{24})^{(4)}(b_{25})^{(4)}-(b_{24}^{'})^{(4)}(r_{25})^{(4)}-(b_{25}^{'})^{(4)}(r_{25})^{(4)}+(r_{24})^{(4)}(r_{25})^{(4)}<0$

$with\ \ (p_{24})^{(4)}$, $(r_{25})^{(4)}$ as defined are satisfied , then the system

If $(a_i^{''})^{(5)} and\ (b_i^{''})^{(5)}$ are independent on t , and the conditions

$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)}-(a_{28})^{(5)}(a_{29})^{(5)}<0$

$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)}-(a_{28})^{(5)}(a_{29})^{(5)}+(a_{28})^{(5)}(p_{28})^{(5)}+(a_{29}^{'})^{(5)}(p_{29})^{(5)}+(p_{28})^{(5)}(p_{29})^{(5)}>0$

$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)}-(b_{28})^{(5)}(b_{29})^{(5)}>0$,

$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)}-(b_{28})^{(5)}(b_{29})^{(5)}-(b_{28}^{'})^{(5)}(r_{29})^{(5)}-(b_{29}^{'})^{(5)}(r_{29})^{(5)}+(r_{28})^{(5)}(r_{29})^{(5)}<0$

$with\ \ (p_{28})^{(5)}$, $(r_{29})^{(5)}$ as defined satisfied , then the system

If $(a_i^{''})^{(6)} and\ (b_i^{''})^{(6)}$ are independent on t , and the conditions

$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)}-(a_{32})^{(6)}(a_{33})^{(6)}<0$

$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)}-(a_{32})^{(6)}(a_{33})^{(6)}+(a_{32})^{(6)}(p_{32})^{(6)}+(a_{33}^{'})^{(6)}(p_{33})^{(6)}+(p_{32})^{(6)}(p_{33})^{(6)}>0$

$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)}-(b_{32})^{(6)}(b_{33})^{(6)}>0$,

$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)}-(b_{32})^{(6)}(b_{33})^{(6)}-(b_{32}^{'})^{(6)}(r_{33})^{(6)}-(b_{33}^{'})^{(6)}(r_{33})^{(6)}+(r_{32})^{(6)}(r_{33})^{(6)}<0$

$with\ \ (p_{32})^{(6)}$, $(r_{33})^{(6)}$ as defined are satisfied , then the system

$(a_{13})^{(1)}G_{14}-\left[(a_{13}^{'})^{(1)}+(a_{13}^{''})^{(1)}(T_{14})\right]G_{13}=0$

$(a_{14})^{(1)}G_{13}-\left[(a_{14}^{'})^{(1)}+(a_{14}^{''})^{(1)}(T_{14})\right]G_{14}=0$

$(a_{15})^{(1)}G_{14}-\left[(a_{15}^{'})^{(1)}+(a_{15}^{''})^{(1)}(T_{14})\right]G_{15}=0$

$(b_{13})^{(1)}T_{14}-[(b_{13}^{'})^{(1)}-(b_{13}^{''})^{(1)}(G)\ ]T_{13}=0$

$(b_{14})^{(1)}T_{13}-[(b_{14}^{'})^{(1)}-(b_{14}^{''})^{(1)}(G)\ ]T_{14}=0$

$(b_{15})^{(1)}T_{14}-[(b_{15}^{'})^{(1)}-(b_{15}^{''})^{(1)}(G)\ ]T_{15}=0$

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17}-\left[(a_{16}^{'})^{(2)}+(a_{16}^{''})^{(2)}(T_{17})\right]G_{16}=0$

$(a_{17})^{(2)}G_{16}-\left[(a_{17}^{'})^{(2)}+(a_{17}^{''})^{(2)}(T_{17})\right]G_{17}=0$

$(a_{18})^{(2)}G_{17}-\left[(a_{18}^{'})^{(2)}+(a_{18}^{''})^{(2)}(T_{17})\right]G_{18}=0$

$(b_{16})^{(2)}T_{17}-[(b_{16}^{'})^{(2)}-(b_{16}^{''})^{(2)}(G_{19})\ ]T_{16}=0$

$(b_{17})^{(2)}T_{16}-[(b_{17}^{'})^{(2)}-(b_{17}^{''})^{(2)}(G_{19})\ ]T_{17}=0$

$(b_{18})^{(2)}T_{17}-[(b_{18}^{'})^{(2)}-(b_{18}^{''})^{(2)}(G_{19})\ ]T_{18}=0$

has a unique positive solution , which is an equilibrium solution for

$$(a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})\right]G_{20} = 0$$

$$(a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})\right]G_{21} = 0$$

$$(a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})\right]G_{22} = 0$$

$$(b_{20})^{(3)}T_{21} - [(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23})\ ]T_{20} = 0$$

$$(b_{21})^{(3)}T_{20} - [(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23})\ ]T_{21} = 0$$

$$(b_{22})^{(3)}T_{21} - [(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23})\ ]T_{22} = 0$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25})\right]G_{24} = 0$$

$$(a_{25})^{(4)}G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25})\right]G_{25} = 0$$

$$(a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25})\right]G_{26} = 0$$

$$(b_{24})^{(4)}T_{25} - [(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\left((G_{27})\right)\ ]T_{24} = 0$$

$$(b_{25})^{(4)}T_{24} - [(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}\left((G_{27})\right)\ ]T_{25} = 0$$

$$(b_{26})^{(4)}T_{25} - [(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\left((G_{27})\right)\ ]T_{26} = 0$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29})\right]G_{28} = 0$$

$$(a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29})\right]G_{29} = 0$$

$$(a_{30})^{(5)}G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29})\right]G_{30} = 0$$

$$(b_{28})^{(5)}T_{29} - [(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}(G_{31})\ ]T_{28} = 0$$

$$(b_{29})^{(5)}T_{28} - [(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}(G_{31})\ ]T_{29} = 0$$

$$(b_{30})^{(5)}T_{29} - [(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}(G_{31})\ ]T_{30} = 0$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{32})^{(6)}G_{33} - \left[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33})\right]G_{32} = 0$$

$$(a_{33})^{(6)}G_{32} - \left[(a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33})\right]G_{33} = 0$$

$$(a_{34})^{(6)}G_{33} - \left[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33})\right]G_{34} = 0$$

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0$

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0$

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0$

has a unique positive solution , which is an equilibrium solution for the system

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Definition and uniqueness of T_{14}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a_{13}^{'})^{(1)}+(a_{13}^{''})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a_{15}^{'})^{(1)}+(a_{15}^{''})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a_{16}^{'})^{(2)}+(a_{16}^{''})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a_{18}^{'})^{(2)}+(a_{18}^{''})^{(2)}(T_{17}^*)]}$$

Definition and uniqueness of T_{21}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a_{20}^{'})^{(3)}+(a_{20}^{''})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a_{22}^{'})^{(3)}+(a_{22}^{''})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a_{24}^{'})^{(4)}+(a_{24}^{''})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a_{26}^{'})^{(4)}+(a_{26}^{''})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a_{28}^{'})^{(5)}+(a_{28}^{''})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a_{30}^{'})^{(5)}+(a_{30}^{''})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a_{32}^{'})^{(6)}+(a_{32}^{''})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a_{34}^{'})^{(6)}+(a_{34}^{''})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$$\varphi(G) = (b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G) + (b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(G)\right]+(b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if

$$\varphi(G_{19}) = (b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b_{16}^{'})^{(2)}(b_{17}^{''})^{(2)}(G_{19}) + (b_{17}^{'})^{(2)}(b_{16}^{''})^{(2)}(G_{19})\right] + (b_{16}^{''})^{(2)}(G_{19})(b_{17}^{''})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$

(g) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if

$$\varphi(G_{23}) = (b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b_{20}^{'})^{(3)}(b_{21}^{''})^{(3)}(G_{23}) + (b_{21}^{'})^{(3)}(b_{20}^{''})^{(3)}(G_{23})\right] + (b_{20}^{''})^{(3)}(G_{23})(b_{21}^{''})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(h) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if

$$\varphi(G_{27}) = (b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b_{24}^{'})^{(4)}(b_{25}^{''})^{(4)}(G_{27}) + (b_{25}^{'})^{(4)}(b_{24}^{''})^{(4)}(G_{27})\right] + (b_{24}^{''})^{(4)}(G_{27})(b_{25}^{''})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if

$$\varphi(G_{31}) = (b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b_{28}^{'})^{(5)}(b_{29}^{''})^{(5)}(G_{31}) + (b_{29}^{'})^{(5)}(b_{28}^{''})^{(5)}(G_{31})\right] + (b_{28}^{''})^{(5)}(G_{31})(b_{29}^{''})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if

$$\varphi(G_{35}) = (b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b_{32}^{'})^{(6)}(b_{33}^{''})^{(6)}(G_{35}) + (b_{33}^{'})^{(6)}(b_{32}^{''})^{(6)}(G_{35})\right] + (b_{32}^{''})^{(6)}(G_{35})(b_{33}^{''})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]}$, $G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$

$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G^*)]}$, $T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and

$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}^*)]}$, $G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}^*)]}$

$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b_{16}')^{(2)} - (b_{16}'')^{(2)}((G_{19})^*)]}$, $T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b_{18}')^{(2)} - (b_{18}'')^{(2)}((G_{19})^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]}$, $G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$

$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}{}^*)]}$, $T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}{}^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}^*)]}$, $G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}^*)]}$

$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b_{24}')^{(4)} - (b_{24}'')^{(4)}((G_{27})^*)]}$, $T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b_{26}')^{(4)} - (b_{26}'')^{(4)}((G_{27})^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}^*)]}$, $G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}^*)]}$

$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]}$, $T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]}$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(\mathrm{a}''_i)^{(2)}$ and $(\mathrm{b}''_i)^{(2)}$ Belong to $\mathrm{C}^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$$\mathrm{G}_i = \mathrm{G}_i^* + \mathbb{G}_i \qquad , \mathrm{T}_i = \mathrm{T}_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{17})^{(2)}}{\partial \mathrm{T}_{17}}(\mathrm{T}_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b''_i)^{(2)}}{\partial \mathrm{G}_j}((G_{19})^*) = s_{ij}$$

taking into account equations (global)and neglecting the terms of power 2, we obtain

$$\frac{\mathrm{d}\mathbb{G}_{16}}{\mathrm{dt}} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} \mathrm{G}_{16}^* \mathbb{T}_{17}$$

$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^{*}\mathbb{T}_{17}$

$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^{*}\mathbb{T}_{17}$

$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}^{'})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^{*}\mathbb{G}_j\right)$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}$ *and* $(b_i^{''})^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^{*}) = (q_{21})^{(3)} \quad , \frac{\partial (b_i^{''})^{(3)}}{\partial G_j}(\,(G_{23})^{*}\,) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21}$

$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21}$

$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}^{'})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21}$

$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}^{'})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right)$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(4)}$ *and* $(b_i^{''})^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}^{''})^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial (b_i^{''})^{(4)}}{\partial G_j}((G_{27})^{*}\ ) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25}$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25}$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}^{'})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25}$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}^{'})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}$ and $(b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a_{29}^{''})^{(5)}}{\partial T_{29}}(T_{29}^{*}) = (q_{29})^{(5)} \quad , \frac{\partial (b_i^{''})^{(5)}}{\partial G_j}\left((G_{31})^{*} \right) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29}$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}^{'})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^{*}\mathbb{T}_{29}$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}^{'})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^{*}\mathbb{T}_{29}$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^{*}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}^{'})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^{*}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}^{'})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^{*}\mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}$ and $(b_i^{''})^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a_{33}^{''})^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial (b_i^{''})^{(6)}}{\partial G_j}\left((G_{35})^{*} \right) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}^{'})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33}$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}^{'})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33}$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}^{'})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33}$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}^{'})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_{j}\right)$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}^{'})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_{j}\right)$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}^{'})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_{j}\right)$$

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(14)}T_{14}^{*}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^{*} + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^{*}\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(13)}T_{13}^{*}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2} + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right) (\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2} + \left((b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right) (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2} + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right) (\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(15)}T_{13}^{*}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^{*} + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^{*} + (b_{17})^{(2)}s_{(16),(17)}T_{17}^{*}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^{*} + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^{*}\right)$$

$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T_{17}^{*} + (b_{17})^{(2)}s_{(16),(16)}T_{16}^{*}\right)$$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right)$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((b_{16}^{'})^{(2)} + (b_{17}^{'})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right) (\lambda)^{(2)}\right)$

$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$

$+\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}_{17}^{*} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}_{16}^{*}\right)$

$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}_{17}^{*} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}_{16}^{*}\right)\} = 0$

$+$

$\left((\lambda)^{(3)} + (b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\{\left((\lambda)^{(3)} + (a_{22}^{'})^{(3)} + (p_{22})^{(3)}\right)$

$\left[\left(\left((\lambda)^{(3)} + (a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)(q_{21})^{(3)}G_{21}^{*} + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)\right]$

$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(21)}T_{21}^{*}\right)$

$+\left(\left((\lambda)^{(3)} + (a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G_{20}^{*} + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^{*}\right)$

$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(20)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(20)}T_{20}^{*}\right)$

$\left(\left((\lambda)^{(3)}\right)^2 + \left((a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right) (\lambda)^{(3)}\right)$

$\left(\left((\lambda)^{(3)}\right)^2 + \left((b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right) (\lambda)^{(3)}\right)$

$+\left(\left((\lambda)^{(3)}\right)^2 + \left((a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right) (\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$

$+\left((\lambda)^{(3)} + (a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^{*} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)$

$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(22)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(22)}T_{20}^{*}\right)\} = 0$

$+$

$\left((\lambda)^{(4)} + (b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\{\left((\lambda)^{(4)} + (a_{26}^{'})^{(4)} + (p_{26})^{(4)}\right)$

$\left[\left(\left((\lambda)^{(4)} + (a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right)(q_{25})^{(4)}G_{25}^{*} + (a_{25})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)\right]$

$\left(\left((\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(25)}T_{25}^{*} + (b_{25})^{(4)}s_{(24),(25)}T_{25}^{*}\right)$

$+\left(\left((\lambda)^{(4)} + (a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)(q_{24})^{(4)}G_{24}^{*} + (a_{24})^{(4)}(q_{25})^{(4)}G_{25}^{*}\right)$

$\left(\left((\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(24)}T_{25}^{*} + (b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)$

$\left(\left((\lambda)^{(4)}\right)^2 + \left((a_{24}^{'})^{(4)} + (a_{25}^{'})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right) (\lambda)^{(4)}\right)$

$$\left(\left((\lambda)^{(4)}\right)^2 + \left((b_{24}^{'})^{(4)} + (b_{25}^{'})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}\right) (\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2 + \left((a_{24}^{'})^{(4)} + (a_{25}^{'})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right) (\lambda)^{(4)}\right) (q_{26})^{(4)} G_{26}$$

$$+\left((\lambda)^{(4)} + (a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right) \left((a_{26})^{(4)}(q_{25})^{(4)} G_{25}^{*} + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)} G_{24}^{*}\right)$$

$$\left(\left((\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right) s_{(25),(26)} T_{25}^{*} + (b_{25})^{(4)} s_{(24),(26)} T_{24}^{*}\right)\} = 0$$

+

$$\left((\lambda)^{(5)} + (b_{30}^{'})^{(5)} - (r_{30})^{(5)}\right)\{\left((\lambda)^{(5)} + (a_{30}^{'})^{(5)} + (p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)} + (a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right)(q_{29})^{(5)} G_{29}^{*} + (a_{29})^{(5)}(q_{28})^{(5)} G_{28}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(5)} + (b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right) s_{(29),(29)} T_{29}^{*} + (b_{29})^{(5)} s_{(28),(29)} T_{29}^{*}\right)$$

$$+\left(\left((\lambda)^{(5)} + (a_{29}^{'})^{(5)} + (p_{29})^{(5)}\right)(q_{28})^{(5)} G_{28}^{*} + (a_{28})^{(5)}(q_{29})^{(5)} G_{29}^{*}\right)$$

$$\left(\left((\lambda)^{(5)} + (b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right) s_{(29),(28)} T_{29}^{*} + (b_{29})^{(5)} s_{(28),(28)} T_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2 + \left((a_{28}^{'})^{(5)} + (a_{29}^{'})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right) (\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2 + \left((b_{28}^{'})^{(5)} + (b_{29}^{'})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}\right) (\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2 + \left((a_{28}^{'})^{(5)} + (a_{29}^{'})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right) (\lambda)^{(5)}\right) (q_{30})^{(5)} G_{30}$$

$$+\left((\lambda)^{(5)} + (a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right) \left((a_{30})^{(5)}(q_{29})^{(5)} G_{29}^{*} + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)} G_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)} + (b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right) s_{(29),(30)} T_{29}^{*} + (b_{29})^{(5)} s_{(28),(30)} T_{28}^{*}\right)\} = 0$$

+

$$\left((\lambda)^{(6)} + (b_{34}^{'})^{(6)} - (r_{34})^{(6)}\right)\{\left((\lambda)^{(6)} + (a_{34}^{'})^{(6)} + (p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)} + (a_{32}^{'})^{(6)} + (p_{32})^{(6)}\right)(q_{33})^{(6)} G_{33}^{*} + (a_{33})^{(6)}(q_{32})^{(6)} G_{32}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(6)} + (b_{32}^{'})^{(6)} - (r_{32})^{(6)}\right) s_{(33),(33)} T_{33}^{*} + (b_{33})^{(6)} s_{(32),(33)} T_{33}^{*}\right)$$

$$+\left(\left((\lambda)^{(6)} + (a_{33}^{'})^{(6)} + (p_{33})^{(6)}\right)(q_{32})^{(6)} G_{32}^{*} + (a_{32})^{(6)}(q_{33})^{(6)} G_{33}^{*}\right)$$

$$\left(\left((\lambda)^{(6)} + (b_{32}^{'})^{(6)} - (r_{32})^{(6)}\right) s_{(33),(32)} T_{33}^{*} + (b_{33})^{(6)} s_{(32),(32)} T_{32}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2 + \left((a_{32}^{'})^{(6)} + (a_{33}^{'})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}\right) (\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2 + \left((b_{32}^{'})^{(6)} + (b_{33}^{'})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)}\right) (\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}\right) (\lambda)^{(6)}\right) (q_{34})^{(6)} G_{34}$$

$$+\left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}\right) \left((a_{34})^{(6)}(q_{33})^{(6)} G^*_{33} + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)} G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}\right) s_{(33),(34)} T^*_{33} + (b_{33})^{(6)} s_{(32),(34)} T^*_{32}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's L:etters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidiation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

REFERENCES

1. Dr K N Prasanna Kumar, Prof B S Kiranagi, Prof C S Bagewadi - MEASUREMENT DISTURBS EXPLANATION OF QUANTUM MECHANICAL STATES-A HIDDEN VARIABLE THEORY - published at: "*International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition*".
2. DR K N PRASANNA KUMAR, PROF B S KIRANAGI and PROF C S BAGEWADI -CLASSIC 2 FLAVOUR COLOR SUPERCONDUCTIVITY AND ORDINARY NUCLEAR MATTER-A NEW PARADIGM STATEMENT - published at: "*International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition*".
3. A HAIMOVICI: "On the growth of a two species ecological system divided on age groups". Tensor, Vol 37 (1982),Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday
4. FRTJOF CAPRA: "The web of life" Flamingo, Harper Collins See "Dissipative structures" pages 172-188
5. HEYLIGHEN F. (2001): "The Science of Self-organization and Adaptivity", in L. D. Kiel, (ed) . Knowledge Management, Organizational Intelligence and Learning, and Complexity, in: The Encyclopedia of Life Support Systems ((EOLSS), (Eolss Publishers, Oxford) [http://www.eolss.net
6. MATSUI, T, H. Masunaga, S. M. Kreidenweis, R. A. Pielke Sr., W.-K. Tao, M. Chin, and Y. J Kaufman (2006), "Satellite-based assessment of marine low cloud variability associated with aerosol, atmospheric stability, and the diurnal cycle", J. Geophys. Res., 111, D17204, doi:10.1029/2005JD006097
7. STEVENS, B, G. Feingold, W.R. Cotton and R.L. Walko, "Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus" J. Atmos. Sci., 53, 980-1006

8. FEINGOLD, G, Koren, I; Wang, HL; Xue, HW; Brewer, WA (2010), "Precipitation-generated oscillations in open cellular cloud fields" *Nature*, *466* (7308) 849-852, doi: 10.1038/nature09314, Published 12-Aug 2010

9. R WOOD "The rate of loss of cloud droplets by coalescence in warm clouds" J.Geophys. Res., 111, doi: 10.1029/2006JD007553, 2006

10. H. RUND, "The Differential Geometry of Finsler Spaces", Grund. Math. Wiss. Springer-Verlag, Berlin, 1959

11. A. Dold, "Lectures on Algebraic Topology", 1972, Springer-Verlag

12. S LEVIN "Some Mathematical questions in Biology vii ,Lectures on Mathematics in life sciences, vol 8" The American Mathematical society, Providence , Rhode island 1976

(13) Arndt, M. et al., Wave-particle duality of C60 molecules, Nature 401, 680-682 (1999)

(14) Yacoby, A., Heiblum, M., Mahalu, D., and Shtrikman, H., Coherence and phase sensitive measurements in a quantum dot, Phys. Rev. Lett. 74, 4047-4050 (1995).

(15) Gustavsson, S., Leturcq, R., Studer, M., Ihn, T., and Ensslin, K., Time-Resolved Detection of Single-Electron Interference, Nano Lett. 8, 2547-2550 (2008).

(16)^ F. Hoyle (1946). "The synthesis of the elements from hydrogen". *Monthly Notices of the Royal Astronomical Society* **106**: 343–383. Bibcode 1946MNRAS.106..343H.

(17)^ E. M. Burbidge, G. R. Burbidge, W. A. Fowler, F. Hoyle (1957). "Synthesis of the Elements in Stars". *Reviews of Modern Physics* **29** (4): 547–650. Bibcode1957RvMP...29...547B. DOI:10.1103/RevModPhys.29.547.

(18)^ Jones, Lauren V. (2009), *Stars and galaxies*, Greenwood guides to the universe, ABC-CLIO, p. 65–67, ISBN 0-313-34075-7

(19)^ [a b c d] Böhm-Vitense, Erika (1992), *Introduction to Stellar Astrophysics*, **3**, Cambridge University Press, pp. 93–100, ISBN 0-521-34871-4

(20)^ Reiners, A.; Basri, G. (March 2009). "On the magnetic topology of partially and fully convective stars". *Astronomy and Astrophysics* **496** (3): 787–790. Bibcode2009A&A...496...787R. DOI: 10.1051/0004-6361:200811450.

(21)^ [a b] de Loore, Camiel W. H.; Doom, C. (1992), *Structure and evolution of single and binary stars*, Astrophysics and space science library, **179**, Springer, pp. 200–214,ISBN 0-7923-1768-8

(22)^ [a b c] Jeffrey, C. Simon (2010), "Stellar structure and evolution: an introduction", in Go swami, A.; Reddy, B. E., *Principles and Perspectives in Cosmo chemistry*, Springer, pp. 64–66, ISBN 3-642-10368-5

(23)^ Karttunen, Hannu; Oja, Heikki (2007), *Fundamental astronomy* (5th ed.), Springer, p. 247, ISBN 3-540-34143-9

(24)^ Reid, I. Neill; Suzanne L., Hawley (2005), *New light on dark stars: red dwarfs, low-mass stars, brown dwarfs*, Springer-Praxis books in astrophysics and astronomy (2nd ed.), Springer, p. 108, ISBN 3-540-25124-3

(26)^ Salaris, Maurizio; Cassisi, Santi (2005), *Evolution of stars and stellar populations*, John Wiley and Sons, pp. 119–123, ISBN 0-470-09220-3

(27)^ Schuler, S. C.; King, J. R.; The, L.-S. (2009), "Stellar Nucleosynthesis in the Hyades Open Cluster", *The Astrophysical Journal* **701** (1): 837–49, arXiv:0906.4812,Bibcode 2009ApJ...701..837S, DOI:10.1088/0004-637X/701/1/837

(28)^ Goupil, M. J.; Lebreton, Y.; Marques, J. P.; Samadi, R.; Baudin, F. (January 2011), "Open issues in probing interiors of solar-like oscillating main sequence stars 1. From the Sun to nearly suns", *Journal of Physics: Conference Series* **271** (1): 012031, Bibcode 2011JPhCS.271a2031G, DOI:10.1088/1742-6596/271/1/012031.

(29)Wolf, S. A. et al., Spintronics: A spin-based electronic vision for the future, Science 294, 1488-1495 (2001).

(30)Awschalom, D. D., and Flatt´e, M. E., Challenges for semiconductor spintronics, Nature Phys. 3 153-159 (2007).

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