

THE THEORY OF INTERNAL RESONANCE IN THE COUPLED SPIN SYSTEMS ,CONFLICT BETWEEN QUANTUM GATE OPERATION AND DECOUPLING ,QUANTUM GATES WITH INTEGRATED COUPLING ACCURACY THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION PROCESSING, SCALABLE QUANTUM COMPUTATION , INTEGRATED DIAMOND NETWORKS BASED ON CAVITY COUPLED WITH SPIN IMPURITIES, AND RESONANT EXICTATION- AN ABSOLUTION- SUBTRAHENSION MODEL

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ABSTRACT: What stands out as a verkrampte reactionary with unsettled malleability is the tendentious testament of the experimentation wherein infrared magneto transmission experiments have been performed in magnetic fields up to 32 T on a series of multilayer epitaxial graphene samples. Inharmonious key notes have been struck with many experiments as many have found enthusiastic confidence with the experimenters. Theoretical aspect of the field is not given much important and this work is intended to fill the gap not wholly or in full measure but very substantially Changes in the spectral features and broadening of the main cyclotron transition when the incoming photon energy is in resonance with the lowest Landau level separation and the energy of a *K* point optical phonon, have been detected We develop a theory encompassing various variables like scalable Quantum computation, optical transmission against spectral diffusion, gated control to single nv centres etc.,

The outlay of the paper is as follows:

- I. Introduction**
- II. Formulation of the Problem**
- III. Statement of Governing Equations**
- IV. Obtention of the solution(Body fabric of the paper)**
- V. Conclusions**
- VI. ACKNOLEDGEMENTS**
- VII. REFERENCES**

I. INTRODUCTION:

Following are the systems taken in to consideration:

- (1) Conflict between Quantum Gate operation and Decoupling (Module Numbered one)
- (2) Internal resonance in the coupled spin system (Module numbered one)
- (3) Accuracies threshold for fault tolerant Quantum Information Processing with Solid state devices(Module numbered two)
- (4) Quantum Gates with integrated coupling (Module numbered two)
- (5) Integrated Diamond networks based on cavity coupled with spin impurities (Module numbered three)
- (6) Scalable Quantum Computation (Module numbered three)

- (7) Optical Transition against spectral diffusion (Module numbered four)
- (8) Resonant Excitations (Module Numbered Four)
- (9) Single crystal diamond photonic crystal cavities and other nanophotonics (Module numbered five)
- (10) Gated control to single NV centers (Module numbered five)
- (11) Quantum Coherence of idle Qubit (Module numbered six)
- (12) Dynamical decoupling (Module numbered Six)

II. FORMULATION OF THE PROBLEM:

CONFLICT BETWEEN QUANTUM GATE OPERATION AND DECOUPLING AND INTERNAL RESONANCE IN THE COUPLED SPIN SYSTEM

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF QUANTUM GATE OPERATION AND DYNAMIC DECOUPLING

G_{14} : CATEGORY TWO OF QUANTUM GATE OPERATION AND DYNAMIC DECOUPLING

G_{15} : CATEGORY THREE OF QUANTUM GATE OPERATION AND DYNAMIC DECOUPLING

T_{13} : CATEGORY ONE OF INTERNAL RESONANCE IN THE COUPLED SPIN SYSTEMS

T_{14} : CATEGORY TWO OF INTERNAL RESONANCE IN THE COUPLED SPIN SYSTEMS

T_{15} : CATEGORY THREE OF INTERNAL RESONANCE IN THE COUPLED SPIN SYSTEMS

ACCURACIES THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION PROCESSING WITH SOLID STATE DEVICES AND QUANTUM GATES WITH INTEGRATED COUPLING

MODULE NUMBERED TWO:

G_{16} : CATEGORY ONE OF ACCURACIES THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION

G_{17} : CATEGORY TWO OF ACCURACIES THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION

G_{18} : CATEGORY THREE OF ACCURACIES THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION

T_{16} : CATEGORY ONE OF QUANTUM GATES WITH INTEGRATED COUPLING

T_{17} : CATEGORY TWO OF QUANTUM GATES WITH INTEGRATED COUPLING

T_{18} : CATEGORY THREE OF QUANTUM GATES WITH INTEGRATED COUPLING

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**INTEGRATED DIAMOND NETWORKS BASED ON CAVITY COUPLED WITH SPIN
IMPURITIES -SCALABLE QUANTUM COMPUTATION**

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF INTEGRATED DIAMOND NETWORKS BASED ON CAVITY
COUPLED WITH SPIN IMPURITIES -

G_{21} :CATEGORY TWO OF INTEGRATED DIAMOND NETWORKS BASED ON CAVITY
COUPLED WITH SPIN IMPURITIES -

G_{22} : CATEGORY THREE OF INTEGRATED DIAMOND NETWORKS BASED ON CAVITY
COUPLED WITH SPIN IMPURITIES -

T_{20} : CATEGORY ONE OF SCALABLE QUANTUM COMPUTATION

T_{21} :CATEGORY TWO OF SCALABLE QUANTUM COMPUTATION

T_{22} : CATEGORY THREE OF SCALABLE QUANTUM COMPUTATION

**OPTICAL TRANSITION AGAINST SPECTRAL DIFFUSION AND RESONANT EXCITATION:
MODULE NUMBERED FOUR:**

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G_{24} : CATEGORY ONE OF OPTICAL TRANSITION AGAINST SPECTRAL DIFFUSION

G_{25} : CATEGORY TWO OF OPTICAL TRANSITION AGAINST SPECTRAL DIFFUSION

G_{26} : CATEGORY THREE OF OPTICAL TRANSITION AGAINST SPECTRAL DIFFUSION

T_{24} :CATEGORY ONE OF RESONANT EXCITATION

T_{25} :CATEGORY TWO OF RESONANT EXCITATION

T_{26} : CATEGORY THREE OF RESONANT EXCITATION

**SINGLE CRYSTAL DIAMOND PHOTONIC CRYSTAL CAVITIES AND OTHER
NANOPHOTONICS AND GATED CONTROL TO SINGLE NV CENTERS**

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF SINGLE CRYSTAL DIAMOND PHOTONIC CRYSTAL CAVITIES AND
OTHER NANOPHOTONICS

G_{29} : CATEGORY TWO OF SINGLE CRYSTAL DIAMOND PHOTONIC CRYSTAL CAVITIES AND OTHER NANOPHOTONICS

G_{30} :CATEGORY THREE OF SINGLE CRYSTAL DIAMOND PHOTONIC CRYSTAL CAVITIES AND OTHR NANO PHOTONICS

T_{28} :CATEGORY ONE OF GATED CONTROL TO SINGLE NV CENTERS

T_{29} :CATEGORY TWO OF GATED CONTROL TO SINGLE NV CENTERS

T_{30} :CATEGORY THREE OF GATED CONTROL TO SINGLE NV CENTERS

QUANTUM COHERENCE OF IDLE QUBIT AND DYNAMICAL DECOUPLING

MODULE NUMBERED SIX:

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 G_{32} : CATEGORY ONE OF QUANTUM COHERENCE OF IDLE QUBIT

G_{33} : CATEGORY TWO OF QUANTUM COHERENCE OF IDLE QUBIT

G_{34} : CATEGORY THREE OF QUANTUM COHERENCE OF IDLE QUBIT

T_{32} : CATEGORY ONE OF DYNAMICAL DECOUPLING

T_{33} : CATEGORY TWO OF DYNAMICAL DECOUPLING

T_{34} : CATEGORY THREE OF DYNAMICAL DECOUPLING

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)},$
 $(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
 $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
 $(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
 $, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
 $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}, (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)},$
 $(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

III.STATEMENT OF THE GOVERNING EQUATIONS

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CONFLICT BETWEEN QUANTUM GATE OPERATION AND DECOUPLING AND INTRNAL RESONANCE IN THE COUPLED SPIN SYSTEM:

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15}$$

$$+(a''_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor}$$

$$-(b''_{13})^{(1)}(G, t) = \text{First detritions factor}$$

ACCURACY THRESHOLD FOR FAULT TOLERANT QUANTUM INFORMATION PROCESSING AND QUANTUM GATES WITH INTEGRATED COUPLING

MODULE NUMBERED TWO

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18}$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18}$$

$$+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor}$$

$$-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor}$$

INTEGRATED DIAMOND NETWORKS BASED ON CAVITY COUPLED WITH IMPURITIES AND SCALABLE QUANTUM COMPUTATION:

MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22}$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22}$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor}$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor}$$

**OPTICAL TRANSITION AGAINST SPECTRAL DIFFUSION AND RESONANT EXCITATION:
MODULE NUMBERED FOUR**

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The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26}$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26}$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor}$$

**SINGLE CRYSTAL DIAMOND PHOTONIC CRYSTAL CAVITIES AND OTHER
NANOPHOTONICS PHOTONICS AND GATED CONTROL TO SINGLE NV CENTERS**

MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30}$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30}$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

QUANTUM COHERENCE OF IDLE QUBIT AND DYNAMIC COUPLING

:MODULE NUMBERED SIX

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34}$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34}$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}$$

$$-(b''_{32})^{(6)}((G_{35}), t) = \text{First detritions factor}$$

IV. BODY FABRIC OF THE PAPER: CONCATENATED EQUATIONS,STABILITY ANALYSIS,SOLUTIONAL BEHAVIOUR AND ASYMPTOTIC ANALYSIS

=====HOLISTIC CONCATENATE SYTEMAL EQUATIONS HENCEFORTH REFERRED TO AS "GLOBAL EQUATIONS"

- (1) Conflict between Quantum Gate operation and Decoupling (Module Numbered one)
- (2) Internal resonance in the coupled spin system (Module numbered one)
- (3) Accuracies threshold for fault tolerant Quantum Information Processing with Solid state devices(Module numbered two)
- (4) Quantum Gates with integrated coupling (Module numbered two)
- (5) Integrated Diamond networks based on cavity coupled with spin impurities (Module numbered three)
- (6) Scalable Quantum Computation (Module numbered three)
- (7) Optical Transition against spectral diffusion (Module numbered four)
- (8) Resonant Excitations (Module Numbered Four)
- (9) Single crystal diamond photonic crystal cavities and other nanophotonics(Module numbered five)
- (10) Gated control to single NV centers (Module numbered five)
- (11) Quantum Coherence of idle Qubit(Module numbered six)
- (12) Dynamical decoupling(Module numbered Six)

$-(b''_{24})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) & + (a''_{13})^{(1,1)}(T_{14}, t) & + (a''_{20})^{(3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) & + (a''_{14})^{(1,1)}(T_{14}, t) & + (a''_{21})^{(3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) & + (a''_{15})^{(1,1)}(T_{14}, t) & + (a''_{22})^{(3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{18}$$

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1)}(T_{14}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) & - (b''_{13})^{(1,1)}(G, t) & - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) & - (b''_{14})^{(1,1)}(G, t) & - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4)}(G_{27}, t) & - (b''_{29})^{(5,5,5,5)}(G_{31}, t) & - (b''_{33})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) & - (b''_{15})^{(1,1)}(G, t) & - (b''_{22})^{(3,3,3)}(G_{23}, t) \\ - (b''_{26})^{(4,4,4,4)}(G_{27}, t) & - (b''_{30})^{(5,5,5,5)}(G_{31}, t) & - (b''_{34})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{18}$$

where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[\begin{array}{ccc} \boxed{(a'_{20})^{(3)}} & \boxed{+(a''_{20})^{(3)}(T_{21}, t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} & \end{array} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[\begin{array}{ccc} \boxed{(a'_{21})^{(3)}} & \boxed{+(a''_{21})^{(3)}(T_{21}, t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} & \end{array} \right] G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[\begin{array}{ccc} \boxed{(a'_{22})^{(3)}} & \boxed{+(a''_{22})^{(3)}(T_{21}, t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)} & \end{array} \right] G_{22}$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} & \boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b'_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} & \boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b'_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} & \boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b'_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1,1)}(G, t)}$, $\boxed{-(b'_{14})^{(1,1,1)}(G, t)}$, $\boxed{-(b'_{15})^{(1,1,1)}(G, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{24}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25}, t)} & \boxed{+(a''_{29})^{(5,5)}(T_{29}, t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{25}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25}, t)} & \boxed{+(a''_{30})^{(5,5)}(T_{29}, t)} & \boxed{+(a''_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{26}$$

Where $\boxed{(a'_{24})^{(4)}(T_{25}, t)}$, $\boxed{(a'_{25})^{(4)}(T_{25}, t)}$, $\boxed{(a'_{26})^{(4)}(T_{25}, t)}$ are first augmentation coefficients for ca

$\boxed{+(a''_{28})^{(5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5)}(T_{29}, t)}$ are second augmentation coefficient fo

$\boxed{+(a''_{32})^{(6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6)}(T_{33}, t)}$ are third augmentation coefficient for

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$ are fourth augmentation coefficients

for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$ are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)}$ are sixth augmentation coefficients for category 1, 2, and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{ccc} \boxed{(b'_{24})^{(4)}} \boxed{-(b''_{24})^{(4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{24}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{ccc} \boxed{(b'_{25})^{(4)}} \boxed{-(b''_{25})^{(4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{25}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{ccc} \boxed{(b'_{26})^{(4)}} \boxed{-(b''_{26})^{(4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{26}$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27}, t)}$ are first detrition coefficients for ca

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$ are second detrition coefficients for ca

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$ are third detrition coefficients for cate,

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$

are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$

are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$

are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} \boxed{(a'_{28})^{(5)}} \boxed{+(a''_{28})^{(5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{28}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} \boxed{(a'_{29})^{(5)}} \boxed{+(a''_{29})^{(5)}(T_{29}, t)} & \boxed{+(a''_{25})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{29}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} \boxed{(a'_{30})^{(5)}} \boxed{+(a''_{30})^{(5)}(T_{29}, t)} & \boxed{+(a''_{26})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{30}$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$ are first augmentation coefficients f

And $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$ are second augmentation coefficient

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$ are third augmentation coefficient f

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$ are fifth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$ are sixth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} \boxed{(b'_{28})^{(5)} - \boxed{(b''_{28})^{(5)}(G_{31}, t)} - \boxed{(b''_{24})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)} - \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)} - \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{28}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} \boxed{(b'_{29})^{(5)} - \boxed{(b''_{29})^{(5)}(G_{31}, t)} - \boxed{(b''_{25})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)} - \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)} - \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{29}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} \boxed{(b'_{30})^{(5)} - \boxed{(b''_{30})^{(5)}(G_{31}, t)} - \boxed{(b''_{26})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{34})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)} - \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)} - \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)} \end{array} \right] T_{30}$$

where $\boxed{-(b''_{28})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4)}(G_{27}, t)}$ are second detrition coefficients for ca

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)}$ are third detrition coefficients for

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} \boxed{(a'_{32})^{(6)} + \boxed{(a''_{32})^{(6)}(T_{33}, t)} + \boxed{(a''_{28})^{(5,5,5)}(T_{29}, t)} + \boxed{(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)} + \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)} + \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} \boxed{(a'_{33})^{(6)} + \boxed{(a''_{33})^{(6)}(T_{33}, t)} + \boxed{(a''_{29})^{(5,5,5)}(T_{29}, t)} + \boxed{(a''_{25})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)} + \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)} + \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & + (a''_{34})^{(6)}(T_{33}, t) & + (a''_{30})^{(5,5,5)}(T_{29}, t) & + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ \hline + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{34}$$

$+ (a''_{32})^{(6)}(T_{33}, t)$, $+ (a''_{33})^{(6)}(T_{33}, t)$, $+ (a''_{34})^{(6)}(T_{33}, t)$ are first augmentation coefficients for category 1, 2, and 3

$+ (a''_{28})^{(5,5,5)}(T_{29}, t)$, $+ (a''_{29})^{(5,5,5)}(T_{29}, t)$, $+ (a''_{30})^{(5,5,5)}(T_{29}, t)$ are second augmentation coefficients

$+ (a''_{24})^{(4,4,4)}(T_{25}, t)$, $+ (a''_{25})^{(4,4,4)}(T_{25}, t)$, $+ (a''_{26})^{(4,4,4)}(T_{25}, t)$ are third augmentation coefficients

$+ (a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+ (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+ (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$ - are fourth augmentation coefficients

$+ (a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+ (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+ (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$ - fifth augmentation coefficients

$+ (a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+ (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+ (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$ sixth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & - (b''_{32})^{(6)}(G_{35}, t) & - (b''_{28})^{(5,5,5)}(G_{31}, t) & - (b''_{24})^{(4,4,4)}(G_{27}, t) \\ \hline - (b''_{13})^{(1,1,1,1,1,1)}(G, t) & - (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{32}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)} & - (b''_{33})^{(6)}(G_{35}, t) & - (b''_{29})^{(5,5,5)}(G_{31}, t) & - (b''_{25})^{(4,4,4)}(G_{27}, t) \\ \hline - (b''_{14})^{(1,1,1,1,1,1)}(G, t) & - (b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{33}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)} & - (b''_{34})^{(6)}(G_{35}, t) & - (b''_{30})^{(5,5,5)}(G_{31}, t) & - (b''_{26})^{(4,4,4)}(G_{27}, t) \\ \hline - (b''_{15})^{(1,1,1,1,1,1)}(G, t) & - (b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{34}$$

$- (b''_{32})^{(6)}(G_{35}, t)$, $- (b''_{33})^{(6)}(G_{35}, t)$, $- (b''_{34})^{(6)}(G_{35}, t)$ are first detrition coefficients for category 1, 2, and 3

$- (b''_{28})^{(5,5,5)}(G_{31}, t)$, $- (b''_{29})^{(5,5,5)}(G_{31}, t)$, $- (b''_{30})^{(5,5,5)}(G_{31}, t)$ are second detrition coefficients for category 1, 2, and 3

$- (b''_{24})^{(4,4,4)}(G_{27}, t)$, $- (b''_{25})^{(4,4,4)}(G_{27}, t)$, $- (b''_{26})^{(4,4,4)}(G_{27}, t)$ are third detrition coefficients for category 1, 2, and 3

$- (b''_{13})^{(1,1,1,1,1,1)}(G, t)$, $- (b''_{14})^{(1,1,1,1,1,1)}(G, t)$, $- (b''_{15})^{(1,1,1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2, and 3

$- (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)$, $- (b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)$, $- (b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2, and 3

$- (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)$, $- (b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)$, $- (b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2, and 3

Where we suppose

(A) $(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0,$

$i, j = 13, 14, 15$

(B) The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$

$\lim_{G \rightarrow \infty} (b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$ are positive constants and $i = 13, 14, 15$

They satisfy Lipschitz condition:

$|(a_i'')^{(1)}(T'_{14}, t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} ||G - G'| e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T'_{14}, t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient WOULD be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$

Where we suppose

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16, 17, 18$

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)}$$

(H) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}(G_{19}, t) = (r_i)^{(2)}$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17}' - T_{17}| e^{-(\hat{M}_{16})^{(2)}t}$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G_{19})' - (G_{19})| e^{-(\hat{M}_{16})^{(2)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

Where we suppose

(J) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$ are positive constants and $i = 20, 21, 22$

They satisfy Lipschitz condition:

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} |G_{23} - G_{23}'| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t) \cdot (T_{21}', t)$. And (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24, 25, 26$$

(M) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}(G_{27}, t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$(N) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \\ \lim_{G \rightarrow \infty} (b_i'')^{(4)}(G_{27}, t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$ are positive constants and $i = 24, 25, 26$

They satisfy Lipschitz condition:

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} \|(G_{27}) - (G_{27})'\| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **FOURTH augmentation coefficient WOULD** be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$:

(P) $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$:

(Q) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$ and $(\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

(R) $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, i, j = 28, 29, 30$

(S) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

(T) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$

$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition:

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29}' - T_{29}| e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} \|(G_{31}) - (G_{31})'\| e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$

then the function $(a_i'')^{(5)}(T_{29}, t)$, the FIFTH **augmentation coefficient** attributable would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$:

-) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants
 $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$:

-) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32, 33, 34$$

- (W) The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

- (X) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$
 $\lim_{G \rightarrow \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$ are positive constants and $i = 32, 33, 34$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T'_{33}, t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T'_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35}) - (G_{35})'| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T'_{33}, t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T'_{33}, t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the SIXTH **augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$:

- $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants
 $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$:

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and $(\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t [(a_{13})^{(1)} G_{14}(s_{(13)}) - ((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})] G_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t [(a_{14})^{(1)} G_{13}(s_{(13)}) - ((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}))] G_{14}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t [(a_{15})^{(1)} G_{14}(s_{(13)}) - ((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}))] G_{15}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t [(b_{13})^{(1)} T_{14}(s_{(13)}) - ((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t [(b_{14})^{(1)} T_{13}(s_{(13)}) - ((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{14}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t [(b_{15})^{(1)} T_{14}(s_{(13)}) - ((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{15}(s_{(13)})] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t [(a_{16})^{(2)} G_{17}(s_{(16)}) - ((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})] G_{16}(s_{(16)})] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t [(a_{17})^{(2)} G_{16}(s_{(16)}) - ((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}))] G_{17}(s_{(16)})] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t [(a_{18})^{(2)} G_{17}(s_{(16)}) - ((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}))] G_{18}(s_{(16)})] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t [(b_{16})^{(2)} T_{17}(s_{(16)}) - ((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}))] T_{16}(s_{(16)})] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20}(s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + a''_{21}(s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + a''_{22}(s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}$$

By

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24}(s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + a''_{25}(s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + a''_{26}(s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}$$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28}(s_{(28)}, s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + a''_{29}(s_{(28)}, s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + a''_{30}(s_{(28)}, s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32}(s_{(32)}, s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + a''_{33}(s_{(32)}, s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{\hat{M}_{13}^{(1)}}} + (\hat{P}_{13})^{(1)} \right]$$

(G_t^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$

$$\left(1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{\hat{M}_{16}^{(2)}}} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{\hat{M}_{20}^{(3)}}} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)}s(24)} \right) \right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)}t \right) G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left(e^{(\bar{M}_{24})^{(4)}t} - 1 \right)$$

From which it follows that

$$(G_{24}(t) - G_{24}^0) e^{-(\bar{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0 \right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0} \right)} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}s(28)} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)}t \right) G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left(e^{(\bar{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that

$$(G_{28}(t) - G_{28}^0) e^{-(\bar{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}s(32)} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)}t \right) G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left(e^{(\bar{M}_{32})^{(6)}t} - 1 \right)$$

From which it follows that

$$(G_{32}(t) - G_{32}^0) e^{-(\bar{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[((\hat{Q}_{13})^{(1)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t} \}$$

Indeed if we denote

Definition of \tilde{G}, \tilde{T} :

$$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$$

It results

$$|\tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)}| \leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{-(\bar{M}_{13})^{(1)}s_{(13)}} + (a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} + G_{13}^{(2)} | (a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)}) | e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\bar{M}_{13})^{(1)}t} \leq \frac{1}{(\bar{M}_{13})^{(1)}} ((a_{13})^{(1)} + (a'_{13})^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}))$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ and $(\hat{Q}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i')^{(1)} - (a_i'')^{(1)})(T_{14}(s_{(13)}), s_{(13)}) ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(1)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{13})^{(1)})_1, ((\widehat{M}_{13})^{(1)})_2$ and $((\widehat{M}_{13})^{(1)})_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < ((\widehat{M}_{13})^{(1)}) \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)}G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)}((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded.}$$

The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose

$(\widehat{P}_{16})^{(2)}$ and $(\widehat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[(\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{16})^{(2)}$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote

Definition of $\widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$\begin{aligned} |\widetilde{G}_{16}^{(1)} - \widetilde{G}_{16}^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{(a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} |(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)})| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)} \end{aligned}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\bar{M}_{16})^{(2)}t} &\leq \\ \frac{1}{(\bar{M}_{16})^{(2)}} &\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)} \right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right); (G_{19})^{(2)}, (T_{19})^{(2)}\right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1, ((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3 :$

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)}) \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)} G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 5: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$ then $T_{17} \rightarrow \infty$.

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b'_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b'_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(3)}}{(M_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(M_{20})^{(3)}} < 1$ and to choose

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(M_{20})^{(3)}} \left[(\widehat{P}_{20})^{(3)} + ((\widehat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{20})^{(3)}$$

$$\frac{(b_i)^{(3)}}{(M_{20})^{(3)}} \left[((\widehat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{20})^{(3)} \right] \leq (\widehat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$d \left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(M_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(M_{20})^{(3)}t} \}$$

Indeed if we denote

$$\widetilde{G}_{23}, \widetilde{T}_{23} : ((\widetilde{G}_{23}), (\widetilde{T}_{23})) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$$

It results

$$|\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| \leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(M_{20})^{(3)}s} e^{(M_{20})^{(3)}s} ds +$$

$$\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} + (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} + G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{20})^{(3)} t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}} ((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}) d(((G_{23})^{(1)}, (T_{23})^{(1)}); (G_{23})^{(2)}, (T_{23})^{(2)})$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ and $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(3)} - (a''_i)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \} ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(3)} t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{20})^{(3)})_1$, $((\widehat{M}_{20})^{(3)})_2$ and $((\widehat{M}_{20})^{(3)})_3$:

Remark 3: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$$G_{20} < ((\widehat{M}_{20})^{(3)}) \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)} G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)} ((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)} ((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 5: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(3)} ((G_{23})(t), t)) = (b'_{21})^{(3)}$ then $T_{21} \rightarrow \infty$.

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)} ((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_3} \text{ By taking now } \varepsilon_3 \text{ sufficiently small one sees that } T_{21} \text{ is unbounded.}$$

The same property holds for T_{22} if $\lim_{t \rightarrow \infty} (b_{22}'')^{(3)} ((G_{23})(t), t) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} < 1$ and to choose

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[(\hat{P}_{24})^{(4)} + ((\hat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{24})^{(4)}$$

$$\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[((\hat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{24})^{(4)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{24})^{(4)} \right] \leq (\hat{Q}_{24})^{(4)}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying IN to itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d \left(((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t} \}$$

Indeed if we denote

$$\underline{\text{Definition of } (\widehat{G_{27}}), (\widehat{T_{27}})} : (\widehat{G_{27}}), (\widehat{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t \{ (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{-(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} | (a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)}) | e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} \} ds_{(24)} \end{aligned}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\bar{M}_{24})^{(4)}t} \leq$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}} \left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)} \right) d \left(((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)}) \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ and $(b''_i)^{(4)}$, $i = 24, 25, 26$ depend only on T_{25} and respectively on (G_{27}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(4)} - (a''_i)^{(4)}) (T_{25}(s_{(24)}), s_{(24)}) ds_{(24)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{24})^{(4)})_1, ((\widehat{M}_{24})^{(4)})_2$ and $((\widehat{M}_{24})^{(4)})_3$:

Remark 3: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$$G_{24} < ((\widehat{M}_{24})^{(4)})_1 \text{ it follows } \frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)} G_{25} \text{ and by integrating}$$

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)} ((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way, one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)} ((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 5: If T_{24} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(4)} ((G_{27})(t), t)) = (b'_{25})^{(4)}$ then $T_{25} \rightarrow \infty$.

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b''_i)^{(4)} ((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)} (m)^{(4)} - \varepsilon_4 T_{25}$ which leads to

$$T_{25} \geq \left(\frac{(a_{25})^{(4)} (m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$$T_{25} \geq \left(\frac{(a_{25})^{(4)} (m)^{(4)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_4} \text{ By taking now } \varepsilon_4 \text{ sufficiently small one sees that } T_{25} \text{ is unbounded.}$$

The same property holds for T_{26} if $\lim_{t \rightarrow \infty} (b''_{26})^{(4)} ((G_{27})(t), t) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$ and to choose

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[(\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{28})^{(5)}$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[((\hat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G}_{31}), (\widetilde{T}_{31}) : ((\widetilde{G}_{31}), (\widetilde{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$|\tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)}| \leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\bar{M}_{28})^{(5)}t} \leq \frac{1}{(\bar{M}_{28})^{(5)}} \left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)} \right) d\left(((G_{31})^{(1)}, (T_{31})^{(1)}); ((G_{31})^{(2)}, (T_{31})^{(2)}) \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition

necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ and $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28, 29, 30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(5)} t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$ and $((\widehat{M}_{28})^{(5)})_3$:

Remark 3: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$G_{28} < ((\widehat{M}_{28})^{(5)})$ it follows $\frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)} ((\widehat{M}_{28})^{(5)})_1 / (a_{29}')^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)} ((\widehat{M}_{28})^{(5)})_2 / (a_{30}')^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 4: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below.

Remark 5: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \rightarrow \infty$.

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded.}$$

The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}} , \frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} < 1$ and to choose

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[(\hat{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{32})^{(6)}$$

$$\frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d\left((G_{35})^{(1)}, (T_{35})^{(1)}, (G_{35})^{(2)}, (T_{35})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}, \widetilde{T_{35}})$: $(\widetilde{G_{35}}, \widetilde{T_{35}}) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$|\tilde{G}_{32}^{(1)} - \tilde{G}_{32}^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} +$$

$$\int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\bar{M}_{32})^{(6)}t} \leq$$

$$\frac{1}{(\bar{M}_{32})^{(6)}} \left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)} \right) d\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)} \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$ and $(\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32, 33, 34$ depend only on T_{33} and respectively on (G_{35}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 69 to 32 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i')^{(6)} - (a_i'')^{(6)})(T_{33}(s_{(32)}), s_{(32)}) ds_{(32)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(6)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{32})^{(6)})_1, ((\widehat{M}_{32})^{(6)})_2$ and $((\widehat{M}_{32})^{(6)})_3$:

Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$$G_{32} < ((\widehat{M}_{32})^{(6)}) \text{ it follows } \frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a_{33}')^{(6)}G_{33} \text{ and by integrating}$$

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a_{33}')^{(6)}$$

In the same way, one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a_{34}')^{(6)}$$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below.

Remark 5: If T_{32} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(6)}((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \rightarrow \infty$.

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}$$

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right), t = \log \frac{2}{\varepsilon_6} \text{ By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is unbounded.}$$

The same property holds for T_{34} if $\lim_{t \rightarrow \infty} (b_{34}'')^{(6)}((G_{35})(t), t(t), t) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

(b) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations
 $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations
 $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

are defined respectively

Then the solution satisfies the inequalities

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \right) \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-a'_{15}(1)t} \right] + G_{15}^0 e^{-a'_{15}(1)t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

(d) $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$$

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:-

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)},$$

and $(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)}$$

and analogously

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}$$

Then the solution satisfies the inequalities

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:-

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

(b) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations

$$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a_{22})^{(3)}t}$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}}$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{21}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b_{22})^{(3)}t} \leq T_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$$

$$\begin{aligned} (S_2)^{(3)} &= (a_{22})^{(3)} - (p_{22})^{(3)} \\ (R_1)^{(3)} &= (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)} \\ (R_2)^{(3)} &= (b'_{22})^{(3)} - (r_{22})^{(3)} \end{aligned}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$\begin{aligned} -(\sigma_2)^{(4)} &\leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)} \\ -(\tau_2)^{(4)} &\leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)} \end{aligned}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$:

(e) By $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$$\begin{aligned} (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} &= 0 \\ \text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} &= 0 \text{ and} \end{aligned}$$

Definition of $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

By $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$\begin{aligned} (m_2)^{(4)} &= (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)} \\ (m_2)^{(4)} &= (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}, \\ \text{and } (v_0)^{(4)} &= \frac{G_{24}^0}{G_{25}^0} \end{aligned}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

$$\begin{aligned} (\mu_2)^{(4)} &= (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)} \\ (\mu_2)^{(4)} &= (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}, \\ \text{and } (u_0)^{(4)} &= \frac{T_{24}^0}{T_{25}^0} \end{aligned}$$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, if $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined by 59 and 64 respectively

Then the solution satisfies the inequalities

$$G_{24}^0 e^{(S_1)^{(4)} - (p_{24})^{(4)} t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} - (p_{24})^{(4)} t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} (S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)}} \left[e^{(S_1)^{(4)} - (p_{24})^{(4)} t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \right) \leq G_{26}(t) \leq \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} (S_1)^{(4)} - (a_{26})^{(4)}} \left[e^{(S_1)^{(4)} t} - e^{-(a_{26}')^{(4)} t} \right] + G_{26}^0 e^{-(a_{26}')^{(4)} t}$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \leq T_{24}(t) \leq T_{24}^0 e^{(R_1)^{(4)} + (r_{24})^{(4)} t}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} + (r_{24})^{(4)} t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} (R_1)^{(4)} - (b_{26}')^{(4)}} \left[e^{(R_1)^{(4)} t} - e^{-(b_{26}')^{(4)} t} \right] + T_{26}^0 e^{-(b_{26}')^{(4)} t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[e^{((R_1)^{(4)} + (r_{24})^{(4)}) t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a_{24}')^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b_{24}')^{(4)}$$

$$(R_2)^{(4)} = (b_{26}')^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a_{28}')^{(5)} + (a_{29}')^{(5)} - (a_{28}'')^{(5)} (T_{29}, t) + (a_{29}'')^{(5)} (T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b_{28}')^{(5)} + (b_{29}')^{(5)} - (b_{28}'')^{(5)} ((G_{31}), t) - (b_{29}'')^{(5)} ((G_{31}), t) \leq -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$:

(h) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations

$$(a_{29})^{(5)} (v^{(5)})^2 + (\sigma_1)^{(5)} v^{(5)} - (a_{28})^{(5)} = 0$$

$$\text{and } (b_{29})^{(5)} (u^{(5)})^2 + (\tau_1)^{(5)} u^{(5)} - (b_{28})^{(5)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the

roots of the equations $(a_{29})^{(5)} (v^{(5)})^2 + (\sigma_2)^{(5)} v^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)} (u^{(5)})^2 + (\tau_2)^{(5)} u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } (u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \right) \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:-

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$:

(k) By $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations

$$(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$:

By $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a_{34})^{(6)})} \left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})} \left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})} \left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:-

$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Proof : From GLOBAL EQUATIONS we obtain

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $v^{(1)} = \frac{G_{13}}{G_{14}}$

It follows

$$- \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(a) For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

$$\text{it follows } (v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$$

In the same manner , we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

$$\text{From which we deduce } (v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$$

(b) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

$$\begin{aligned}
 (v_1)^{(1)} &\leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}} \leq v^{(1)}(t) \leq \\
 &\frac{(\bar{v}_1)^{(1)} + (C)^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (\bar{v}_1)^{(1)}
 \end{aligned}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (C)^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

we obtain

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$- \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

(d) For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

(e) If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case,

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(f) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:-

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)} \quad , \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)} \quad , \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Particular case :

If $(a_{16}''^{(2)}) = (a_{17}''^{(2)})$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}''^{(2)}) = (b_{17}''^{(2)})$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

From GLOBAL EQUATIONS we obtain

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) - (a''_{21})^{(3)}(T_{21}, t) \right) v^{(3)} - (a_{21})^{(3)} v^{(3)}$$

Definition of $v^{(3)}$:-
$$v^{(3)} = \frac{G_{20}}{G_{21}}$$

It follows

$$- \left((a_{21})^{(3)} (v^{(3)})^2 + (\sigma_2)^{(3)} v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)} (v^{(3)})^2 + (\sigma_1)^{(3)} v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)} (v_2)^{(3)} e^{[-(a_{21})^{(3)} (v_1)^{(3)} - (v_0)^{(3)}] t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)} (v_1)^{(3)} - (v_0)^{(3)}] t}}, \quad (C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get

$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)} (\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}}, \quad (\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case,

$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)} (v_2)^{(3)} e^{[-(a_{21})^{(3)} (v_1)^{(3)} - (v_2)^{(3)}] t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)} (v_1)^{(3)} - (v_2)^{(3)}] t}} \leq v^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)} (\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}} \leq (\bar{v}_1)^{(3)}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)} (\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)} (\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}] t}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{20}''^{(3)}) = (a_{21}''^{(3)})$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}''^{(3)}) = (b_{21}''^{(3)})$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

: From GLOBAL EQUATIONS we obtain

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}''^{(4)})(T_{25}, t) \right) - (a_{25}''^{(4)})(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

(d) For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case,

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (\bar{v}_1)^{(4)}$$

(f) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (C)^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$ **this also defines** $(v_0)^{(4)}$ **for the special case**.

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}$, then $(\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

From GLOBAL EQUATIONS we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t) \right) - (a_{29}'')^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

Definition of $v^{(5)}$:- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$- \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(g) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner, we get

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (C)^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}, \quad \boxed{(C)^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (C)^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (\bar{v}_1)^{(5)}$$

(i) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (C)^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines** $(v_0)^{(5)}$ **for the special case .**

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

we obtain

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$- \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

$$(j) \text{ For } 0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}, \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}, \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case,

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (\bar{v}_1)^{(6)}$$

(l) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{32}'')^{(6)} = (a_{33}'')^{(6)}$, then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case .**

Analogously if $(b_{32}'')^{(6)} = (b_{33}'')^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

We can prove the following

Theorem 3: If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined, then the system

If $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ are independent on t , and the conditions

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied, then the system

If $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ are independent on t , and the conditions

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied, then the system

If $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ are independent on t , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with $(p_{24})^{(4)}, (r_{25})^{(4)}$ as defined are satisfied, then the system

If $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ are independent on t , and the conditions

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{28})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined satisfied, then the system

If $(a''_i)^{(6)}$ and $(b''_i)^{(6)}$ are independent on t , and the conditions

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{32})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with $(p_{32})^{(6)}, (r_{33})^{(6)}$ as defined are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0$$

has a unique positive solution, which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0$$

has a unique positive solution, which is an equilibrium solution for

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0$$

has a unique positive solution, which is an equilibrium solution

$$\begin{aligned} (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} &= 0 \\ (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} &= 0 \\ (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} &= 0 \\ (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27})]T_{24} &= 0 \\ (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27})]T_{25} &= 0 \\ (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27})]T_{26} &= 0 \end{aligned}$$

has a unique positive solution , which is an equilibrium solution for the system

$$\begin{aligned} (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} &= 0 \\ (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} &= 0 \\ (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} &= 0 \\ (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} &= 0 \\ (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} &= 0 \\ (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} &= 0 \end{aligned}$$

has a unique positive solution , which is an equilibrium solution for the system

$$\begin{aligned} (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} &= 0 \\ (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} &= 0 \\ (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} &= 0 \\ (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} &= 0 \\ (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} &= 0 \\ (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} &= 0 \end{aligned}$$

has a unique positive solution , which is an equilibrium solution for the system

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) +$$

$$(a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Definition and uniqueness of T_{14}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

Definition and uniqueness of T_{21}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - [(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - [(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{17}^* such that $\varphi((G_{19})^*) = 0$

(g) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - [(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(h) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*) = 0, T_{14}^*$ given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} , T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0, T_{17}^*$ given by $f(T_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^*)]}$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{13}}{dt} &= -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^*T_{14} \\ \frac{dG_{14}}{dt} &= -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^*T_{14} \\ \frac{dG_{15}}{dt} &= -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^*T_{14} \\ \frac{dT_{13}}{dt} &= -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*G_j \\ \frac{dT_{14}}{dt} &= -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*G_j \\ \frac{dT_{15}}{dt} &= -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*G_j \end{aligned}$$

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of G_i, T_i :-

$$\begin{aligned} G_i &= G_i^* + G_i, \quad T_i = T_i^* + T_i \\ \frac{\partial (a''_{17})^{(2)}}{\partial T_{17}}(T_{17}^*) &= (q_{17})^{(2)}, \quad \frac{\partial (b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \end{aligned}$$

taking into account equations (global)and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{16}}{dt} &= -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \\ \frac{dG_{17}}{dt} &= -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \\ \frac{dG_{18}}{dt} &= -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \\ \frac{dT_{16}}{dt} &= -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*G_j \\ \frac{dT_{17}}{dt} &= -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j \\ \frac{dT_{18}}{dt} &= -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j \end{aligned}$$

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of G_i, T_i :-

$$\begin{aligned} G_i &= G_i^* + G_i, \quad T_i = T_i^* + T_i \\ \frac{\partial (a''_{21})^{(3)}}{\partial T_{21}}(T_{21}^*) &= (q_{21})^{(3)}, \quad \frac{\partial (b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij} \end{aligned}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21}$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21}$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21}$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)}) T_{20}^* G_j$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)}) T_{21}^* G_j$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)}) T_{22}^* G_j$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}} (T_{25}^*) = (q_{25})^{(4)}, \quad \frac{\partial (b_i'')^{(4)}}{\partial G_j} ((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^* T_{25}$$

$$\frac{dG_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^* T_{25}$$

$$\frac{dG_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^* T_{25}$$

$$\frac{dT_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)}) T_{24}^* G_j$$

$$\frac{dT_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)}) T_{25}^* G_j$$

$$\frac{dT_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)}) T_{26}^* G_j$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}} (T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial (b_i'')^{(5)}}{\partial G_j} ((G_{31})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{28}}{dt} &= -((a'_{28})^{(5)} + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^* T_{29} \\ \frac{dG_{29}}{dt} &= -((a'_{29})^{(5)} + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^* T_{29} \\ \frac{dG_{30}}{dt} &= -((a'_{30})^{(5)} + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^* T_{29} \\ \frac{dT_{28}}{dt} &= -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^* G_j \\ \frac{dT_{29}}{dt} &= -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^* G_j \\ \frac{dT_{30}}{dt} &= -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^* G_j \end{aligned}$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of G_i, T_i :-

$$\begin{aligned} G_i &= G_i^* + G_i, \quad T_i = T_i^* + T_i \\ \frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}} (T_{33}^*) &= (q_{33})^{(6)}, \quad \frac{\partial (b_i'')^{(6)}}{\partial G_j} ((G_{35})^*) = s_{ij} \end{aligned}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{32}}{dt} &= -((a'_{32})^{(6)} + (p_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^* T_{33} \\ \frac{dG_{33}}{dt} &= -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^* T_{33} \\ \frac{dG_{34}}{dt} &= -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^* T_{33} \\ \frac{dT_{32}}{dt} &= -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(32)(j)})T_{32}^* G_j \\ \frac{dT_{33}}{dt} &= -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34} (s_{(33)(j)})T_{33}^* G_j \\ \frac{dT_{34}}{dt} &= -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(34)(j)})T_{34}^* G_j \end{aligned}$$

The characteristic equation of this system is

$$\begin{aligned} &((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ &\left[((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\ &((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \\ &+ ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \} \end{aligned}$$

$$\begin{aligned}
 & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\
 & \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & \left(((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & + \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\
 & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^* \right) \\
 & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \} = 0 \\
 & + \\
 & \left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)} \right) \{ (\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)} \} \\
 & \left[\left(((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \right] \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\
 & + \left(((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\
 & \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & \left(((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & + \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) \left((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \} = 0 \\
 & + \\
 & \left((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) \{ (\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \} \\
 & \left[\left(((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \right] \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T_{21}^* + (b_{21})^{(3)}s_{(20),(21)}T_{21}^* \right) \\
 & + \left(((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^* \right) \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T_{21}^* + (b_{21})^{(3)}s_{(20),(20)}T_{20}^* \right) \\
 & \left(((\lambda)^{(3)})^2 + ((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) (\lambda)^{(3)} \right) \\
 & \left(((\lambda)^{(3)})^2 + ((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}) (\lambda)^{(3)} \right)
 \end{aligned}$$

$$\begin{aligned}
 &+ \left((\lambda)^{(3)} \right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22} \\
 &+ \left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left((a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \\
 &\left(\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right) \} = 0 \\
 &+ \\
 & \left((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \{ (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \} \\
 & \left[\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right] \\
 & \left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \right) \\
 & + \left(\left((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \right) \\
 & \left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \right) \\
 & \left((\lambda)^{(4)} \right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \\
 & \left(\left((\lambda)^{(4)} \right)^2 + \left((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \right) \\
 & + \left((\lambda)^{(4)} \right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26} \\
 & + \left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) \left((a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right) \\
 & \left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(26)} T_{25}^* + (b_{25})^{(4)} s_{(24),(26)} T_{24}^* \right) \} = 0 \\
 &+ \\
 & \left((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)} \right) \{ (\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)} \} \\
 & \left[\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right] \\
 & \left(\left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(29)} T_{29}^* + (b_{29})^{(5)} s_{(28),(29)} T_{29}^* \right) \\
 & + \left(\left((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)} \right) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \right) \\
 & \left(\left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(28)} T_{29}^* + (b_{29})^{(5)} s_{(28),(28)} T_{28}^* \right) \\
 & \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} \\
 & \left(\left((\lambda)^{(5)} \right)^2 + \left((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} \right) (\lambda)^{(5)} \right) \\
 & + \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} (q_{30})^{(5)} G_{30} \\
 & + \left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) \left((a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^* + (b_{29})^{(5)}s_{(28),(30)}T_{28}^* \right) = 0 \\
 & + \\
 & \left((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)} \right) \left\{ (\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)} \right. \\
 & \left[\left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right] \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(33)} T_{33}^* + (b_{33})^{(6)} s_{(32),(33)} T_{33}^* \\
 & + \left((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)} \right) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(32)} T_{33}^* + (b_{33})^{(6)} s_{(32),(32)} T_{32}^* \\
 & \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & \left((\lambda)^{(6)} \right)^2 + \left((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & + \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\
 & + \left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) \left((a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left. \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(34)} T_{33}^* + (b_{33})^{(6)} s_{(32),(34)} T_{32}^* \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

(V) CONCLUSIONS:

One of the notable works in recent years pertaining to the subject matter in question is that of Gurvits, L. , Shortn, R. ; Mason, O. where it was conjectured that the Hurwitz stability of the convex hull of a set of Metzler matrices is a necessary and sufficient condition for the asymptotic stability of the associated switched linear system under arbitrary switching. They show that (1) this conjecture is true for systems constructed from a pair of second-order Metzler matrices; (2) the conjecture is true for systems constructed from an arbitrary finite number of second-order Metzler matrices; and (3) the conjecture is in general false for higher order systems. . For a class of differential inclusions, to which many of the practically important control systems can be reduced, necessary and sufficient conditions for asymptotic stability of the zero solution are established by the method of Lyapunov functions. Similar results are obtained for the corresponding class of difference inclusions. Asymptotic stability; exponential stability; differential inclusions; difference inclusions; Lyapunov functions are some of the stability analyses that are made in the context of quantum systems. Some of the stability problems prove extremely difficult in the analysis of the Solutional behaviour and one has to make lot of assumptions denovo, and suo moto, to obtain solution. We have in our paper not resorted to any such communicating singularities, counteractualisation assumptions, such as exogenous influences and endogenous applications. We have made the view of essentially holistic without any singularly superficial, or perpetually hackneyed key assumptions that violates the very body fabric of the thesis. Be it equidistributional random matrices of the system, Schofield’s cavitations tensor, Quantum complementarity, jet stream atmospheric, or endogenous pressure field, the system is allowed to take the variables and reach its own destination. We also discuss Solutional behavior and stability analysis of the system

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The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's L:etters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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1. .

(22)[^] Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)[^] Assuming a 90/10 alloy of Pt/Ir by weight, a C_p of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average C_p of 25.8, 5.134 moles of metal, and 132 J.K^{-1} for the prototype. A variation of ± 1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ± 2 micrograms.

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