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QUANTUM FIELD THEORY AND STATISTICAL MECHANICS- PROBLEM RESIDENCE AND RESOLUTION PRESIDENCE MODEL

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ABSTRACT: In quantum physics, in order to quantize a gauge theory, like for example Yang-Mills theory, Chern-Simons or BF model, one method is to perform a gauge fixing. This is done in the BRST and Batalin-Vilkovisky formulation. Another is to factor out the symmetry by dispensing with vector potentials altogether (they're not physically observable anyway) and work directly with Wilson loops, Wilson lines contracted with other charged fields at its endpoints and spin networks. Older approaches to quantization for Abelian models use the Gupta-Bleuler formalism with a "semi-Hilbert space" with an indefinite sesquilinear form. However, it is much more elegant to just work with the quotient space of vector field configurations by gauge transformations.An alternative approach using lattice approximations is covered in (Wick rotated) lattice gauge theory. In this paper we construct a new paradigm of relational context, with primary focus and locus of asymptotic freedom and differentially instrumental activity of both QFT and YM Theory. It is to be noted that there exists a qualitative gradient of energy differential ascribed to energy excitation of vacuum. This MIGHT as well be the orthogonal energy state of Vacuum, with solidarity abstraction extant and existential with collective chiral symmetry.

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INTRODUCTION:

What is a gauge?

Gauge is a term which has connotations of being a befuddling fearsome, and perniciously complicated part of mathematics – for instance, playing an important role in quantum field theory, general relativity, geometric PDE, and so forth. Underlying concept is really quite simple: a gauge is nothing more than a "coordinate system" that varies depending on one's "location" with respect to some "base space" or "parameter space", a gauge transform is a change of coordinates applied to each such location, and a gauge theory is a model for some physical or mathematical system to which gauge transforms can be applied (and is typically gauge invariant, in that all physically meaningful quantities are left unchanged (or transform naturally) under gauge transformations). By fixing a gauge (thus breaking or spending the gauge symmetry), the model becomes something easier to analyse mathematically, such as a system of partial differential equations (in classical gauge theories) or a perturbative quantum field theory (in quantum gauge theories), though the tractability of the resulting problem can be heavily dependent on the choice of gauge that one fixed. Deciding exactly how to fix a gauge (or whether one should spend the gauge symmetry at all) is a key question in the analysis of gauge theories, and one that often requires the input of geometric ideas and intuition into that analysis. Quantum gauge theories are the quantization of classical gauge theories and have their own set of conceptual difficulties (coming from quantum field theory).

In early 1954, Chen Ning Yang and Robert Mills extended the concept of gauge theory for abelian groups, e.g. quantum electrodynamics, to nonabelian groups to provide an explanation for strong interactions. The idea by Yang–Mills was criticized by Pauli, as the quanta of the Yang–Mills field must be massless in order to maintain gauge invariance. The idea was set aside until 1960, when the concept of particles acquiring mass through symmetry breaking in massless theories was put forward, initially by Jeffrey Goldstone, Yoichiro Nambu, and Giovanni Jona-Lasinio.

PHYSICAL PARTICLE STATES(PROTON,NEUTRON AND PION) AND INVARIANT SU(3),THE PHYSICAL PARAMETER STATES:

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF PHYSICAL PARTCILE STATES

G_{14} : CATEGORY TWO OF PHYSICAL PARTICLE STATES

G_{15} : CATEGORY THREE OF PHYSICAL PARTICLE STATES

T_{13} : CATEGORY ONE OF INVARIANT SU(3), THE PHYSICAL PARAMETER STATES

T_{14} : CATEGORY TWO OF INVARIANT SU(3) THE PJYSICAL PARAMETER STATES

T_{15} :CATEGORY THREE OF INVARIANT SU(3),THE PHYSICAL PARAMETER STATES

CHIRAL SYMMETRY BREAKING AND QUARK BARE MASSES:

MODULE NUMBERED TWO:

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G_{16} : CATEGORY ONE OF QUARK BARE MASSES

G_{17} : CATEGORY TWO OFQUARK BARE MASSES

G_{18} : CATEGORY THREE OF QUARK BARE MASSES

T_{16} :CATEGORY ONE OF CHIRAL SYMMETRY BREAKING

T_{17} : CATEGORY TWO OFCHIRAL SYMMETRY BREAKING

T_{18} : CATEGORY THREE OFCHIRAL SYMMETRY BREAKING

QUANTUM GAUGE THEORY AND FOUR DIMENSIONAL SPACE TIME:

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF QUANTUM GAUGE THEORY(WE HERE SPEAK OF THE CHARACTERISED SYSTEMS FOR WHICH QUANTUM GAUGE THEORY IS APPLICABLE)

G_{21} :CATEGORY TWO OF QUANTUM GAUGE THEORY

G_{22} : CATEGORY THREE OFQUANTUM GAUGE THEORY

T_{20} : CATEGORY ONE OFFOUR DIMENSIONAL SPACE TIME

T_{21} :CATEGORY TWO OF FOUR DIMENCSIONAL SPACE TIME

T_{22} : CATEGORY THREE OF FOUR DIMENSIONAL SPACE TIME

QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARAIBLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS)

: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OF QUANTUM FIELD THEORY(EVALUATIVE PARAMETRICIZATION OF SITUATIONAL ORIENTATIONS AND ESSENTIAL COGNITIVE ORIENTATION AND CHOICE VARIABLES OF THE SYSTEM TO WHICH QFT IS APPLICABLE)

G_{25} : CATEGORY TWO OF QUANTUM FIELD THEORY

G_{26} : CATEGORY THREE OF QUANTUM FIELD THEORY

T_{24} :CATEGORY ONE OF RENORMALIZATION THEORY

T_{25} :CATEGORY TWO OF RENORMALIZATION THEORY(SYSTEMIC INSTRUMENTAL CHARACTERISATIONS AND ACTION ORIENTATIONS AND FUYNCTIONAL IMPERATIVES OF CHANGE MANIFESTED THEREIN)

T_{26} : CATEGORY THREE OF QUANTUM FIELD THEORY

LOCAL SINGULARITIES AND GLOBAL DECAY:

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF LOCAL SINGULARITIES (UV REGULARITIES)

G_{29} : CATEGORY TWO OF LOCAL SINGULARITIES

G_{30} :CATEGORY THREE OFLOCAL SINGULARITIES

T_{28} :CATEGORY ONE OFGLOBAL DECAY(DUE TO RENORMALIZATION)

T_{29} :CATEGORY TWO OF GLOBAL DECAY (DUE TO RENORMALIZATION)

T_{30} :CATEGORY THREE OF GLOBAL DECAY(DUE TO RENORMALIZATION)

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ASSYMETRIC FREEDOM DECISIVE REGULARITIES IN THE CASE WHEN SOBOLEV INEQUALITIES ARE ON THE BORDER LINE OF TRANSITION:

MODULE NUMBERED SIX:

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G_{32} : CATEGORY ONE OF DECISIVE REGULARITIES IN RESPECT OF CASESWHERE SOBOLEV INEQUALITIES ARE IN THE BORDERLINE PHASE TRANSITION

G_{33} : CATEGORY TWO OF DECISIVE REGULARITIES WITH RESPECT TO CASES WHEREIN SOBOLEV INEQUALITIES ARE IN BORDER LINE TRANSITION

G_{34} : CATEGORY THREE OF DECISIVE REGULARITIES WITH RESPECT TOP SYSTEMIC CASES IN REGARD TO WHICH SOBOLEV INEQUALITIES ARE IN BORDER LINE TRANSITION

T_{32} : CATEGORY ONE OF ASYMPTOTIC FREEDOM

T_{33} : CATEGORY TWO OFASYMPTOTIC FREEDOM

T_{34} : CATEGORY THREE OF ASYMPTOTIC FREEDOM

MASS GAP AND ORTHOGONAL ENERGY STATE OF VACUUM:

MODULE NUMBERED SEVEN

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G_{36} : CATEGORY ONE OF ORTHOGONAL ENERGY STATE OF VACUUM(ENERGY DIFFERENTIAL DUE TO EXCITATION OF VACUUM)

G_{37} : CATEGORY TWO OF ORTHOGONAL ENERGY STATE OF VACUUM(ENERGY DIFFERENTIAL DUE TO EXCITATION OF VACUUM)

G_{38} : CATEGORY THREE OF ORTHOGONAL ENERGY STATE OF VACUUM (ENERGY EXCITATION OF THE VACUUM AND CONCOMITANT GENERATION OF ENERGY DIFFERENTIAL-TIME LAG OR INSTANTANEOUSNESSMIGHT EXISTS WHEREBY ACCENTUATION AND ATTRITIONS MODEL MAY ASSUME ZERO POSITIONS)

T_{36} : CATEGORY ONE OF MASS GAP

T_{37} : CATEGORY TWO OF MASS GAP

T_{38} : CATEGORY THREE OF MASS GAP

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}: (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}\ , (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$

$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$, $(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

PHYSICAL PARTICLE STATES(PROTON,NEUTRON AND PION) AND INVARIANT SU(3),THE PHYSICAL PARAMETER STATES: 1

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 2$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 3$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)} + \left(a''_{15}\right)^{(1)}(T_{14}, t)\right]G_{15} \quad 4$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 5$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 6$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)} - \left(b''_{15}\right)^{(1)}(G, t)\right]T_{15} \quad 7$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor 8

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

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CHIRAL SYMMETRY BREAKING AND QUARK BARE MASSES:

MODULE NUMBERED TWO:

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 10$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 11$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 12$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 13$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 14$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 15$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor 16

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor 17

QUANTUM GAUGE THEORY AND FOUR DIMENSIONAL SPACE TIME: 18

MODULE NUMBERED THREE:

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 19$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 20$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 21$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 22$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 23$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 24$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor 25

QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARAIBLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS) 26

: MODULE NUMBERED FOUR

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 27$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\left(a'_{25}\right)^{(4)} + \left(a''_{25}\right)^{(4)}(T_{25}, t)\right]G_{25} \quad 28$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 29$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 30$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\left(b'_{25}\right)^{(4)} - \left(b''_{25}\right)^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 31$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 32$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor** 33

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor** 34

LOCAL SINGULARITIES AND GLOBAL DECAY: 35

MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 36$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 37$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 38$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 39$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 40$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 41$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor** 42

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor** 43

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ASSYMETRIC FREEDOM DECISIVE REGULARITIES IN THE CASE WHEN SOBOLEV INEQUALITIES ARE ON THE BORDER LINE OF TRANSITION: 45

MODULE NUMBERED SIX:

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 46$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 47$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 48$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 49$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 50$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 51$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor** 52

MASS GAP AND ORTHOGONAL ENERGY STATE OF VACUUM: 53

MODULE NUMBERED SEVEN

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The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 54$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 55$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 56$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 57$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 58$$

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$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 60$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor** 61

$-(b''_{36})^{(7)}((G_{39}), t) =$ **First detritions factor** 62

FIRST MODULE CONCATENATION:

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14}, t)}\ \boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)}\ \boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25}, t)}\ \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29}, t)}\ \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7)}(T_{37}, t)} \end{array}\right]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{c} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14}, t)}\ \boxed{+(a''_{17})^{(2,2,)}(T_{17}, t)}\ \boxed{+(a''_{21})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25}, t)}\ \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29}, t)}\ \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7)}(T_{37}, t)} \end{array}\right]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14}, t)}\ \boxed{+(a''_{18})^{(2,2,)}(T_{17}, t)}\ \boxed{+(a''_{22})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25}, t)}\ \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29}, t)}\ \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7)}(T_{37}, t)} \end{array}\right]G_{15}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25}, t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29}, t)}$ are fifth augmentation

coefficient for category 1, 2 and 3

$+(a_{32}^{''})^{(6,6,6,6,)}(T_{33},t)$, $+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)$, $+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}^{''})^{(7)}(T_{37},t)$ $+(a_{37}^{''})^{(7)}(T_{37},t)$ $+(a_{38}^{''})^{(7)}(T_{37},t)$ ARESEVENTHAUGMENTATION COEFFICIENTS

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b_{13}^{'})^{(1)} -(b_{16}^{''})^{(1)}(G,t) & -(b_{36}^{''})^{(7,)}(G_{39},t) & -(b_{20}^{''})^{(3,3,)}(G_{23},t) \\ -(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t) & -(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t) & -(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t) \\ & -(b_{36}^{''})^{(7,)}(G_{39},t) & \end{matrix} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{matrix} (b_{14}^{'})^{(1)} -(b_{14}^{''})^{(1)}(G,t) & -(b_{17}^{''})^{(2,2,)}(G_{19},t) & -(b_{21}^{''})^{(3,3,)}(G_{23},t) \\ -(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t) & -(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t) & -(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t) \\ & -(b_{37}^{''})^{(7,)}(G_{39},t) & \end{matrix} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b_{15}^{'})^{(1)} -(b_{15}^{''})^{(1)}(G,t) & -(b_{18}^{''})^{(2,2,)}(G_{19},t) & -(b_{22}^{''})^{(3,3,)}(G_{23},t) \\ -(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t) & -(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t) & -(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t) \\ & -(b_{38}^{''})^{(7,)}(G_{39},t) & \end{matrix} \right] T_{15}$$

Where $-(b_{13}^{''})^{(1)}(G,t)$, $-(b_{14}^{''})^{(1)}(G,t)$, $-(b_{15}^{''})^{(1)}(G,t)$ are first detritions coefficients for category 1, 2 and 3

$-(b_{16}^{''})^{(2,2,)}(G_{19},t)$, $-(b_{17}^{''})^{(2,2,)}(G_{19},t)$, $-(b_{18}^{''})^{(2,2,)}(G_{19},t)$ are second detritions coefficients for category 1, 2 and 3

$-(b_{20}^{''})^{(3,3,)}(G_{23},t)$, $-(b_{21}^{''})^{(3,3,)}(G_{23},t)$, $-(b_{22}^{''})^{(3,3,)}(G_{23},t)$ are third detritions coefficients for category 1, 2 and 3

$-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)$ are fourth detritions coefficients for category 1, 2 and 3

$-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)$ are fifth detritions coefficients for category 1, 2 and 3

$-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)$ are sixth detritions coefficients for category 1, 2 and 3

$-(b_{36}^{''})^{(7,)}(G_{39},t)$ $-(b_{36}^{''})^{(7,)}(G_{39},t)$ $-(b_{36}^{''})^{(7,)}(G_{39},t)$ ARE SEVENTH DETRITION COEFFICIENTS

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b_{15}^{'})^{(1)} -(b_{15}^{''})^{(1)}(G,t) & -(b_{18}^{''})^{(2,2,)}(G_{19},t) & -(b_{22}^{''})^{(3,3,)}(G_{23},t) \\ -(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t) & -(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t) & -(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t) \end{matrix} \right] T_{15} \quad (63)$$

Where $-(b_{13}^{''})^{(1)}(G,t)$, $-(b_{14}^{''})^{(1)}(G,t)$, $-(b_{15}^{''})^{(1)}(G,t)$ are first detrition coefficients for category 1, (64)

2 and 3

$\boxed{-(b_{16}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{20}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

SECOND MODULE CONCATENATION: 65

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c}(a_{16}^{'})^{(2)}\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{13}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{20}^{''})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a_{24}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{28}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{32}^{''})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a_{36}^{''})^{(7,7,)}(T_{37},t)}\end{array}\right]G_{16} \quad (66)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c}(a_{17}^{'})^{(2)}\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{14}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{21}^{''})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a_{25}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{29}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{33}^{''})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a_{37}^{''})^{(7,7,)}(T_{37},t)}\end{array}\right]G_{17} \quad (67)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c}(a_{18}^{'})^{(2)}\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}\boxed{+(a_{15}^{''})^{(1,1,)}(T_{14},t)}\boxed{+(a_{22}^{''})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a_{26}^{''})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{30}^{''})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{34}^{''})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a_{38}^{''})^{(7,7,)}(T_{37},t)}\end{array}\right]G_{18} \quad (68)$$

Where $\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 (69)

$\boxed{+(a_{13}^{''})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}^{''})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}^{''})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3 (70)

$\boxed{+(a_{28}^{''})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{36}^{''})^{(7,7,)}(T_{37},t)}\boxed{+(a_{37}^{''})^{(7,7,)}(T_{37},t)}\boxed{+(a_{38}^{''})^{(7,7,)}(T_{37},t)}$ ARE SEVENTH DETRITION (71)

COEFFICIENTS

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7)}(G_{39},t)} \end{array}\right] T_{16} \quad (72)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{c} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7)}(G_{39},t)} \end{array}\right] T_{17} \quad (73)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39},t)} \end{array}\right] T_{18} \quad (74)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 (75)

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}$ are seventh detrition coefficients

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THIRD MODULE CONCATENATION:

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)} \end{array}\right] G_{20} \quad (76)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)} \end{array}\right] G_{21} \quad (77)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)} \end{array}\right] G_{22} \quad (78)$$

(79)

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3 (80)

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)}\ \boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)}\ \boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)}$ are seventh augmentation coefficient (81)

$$\frac{dT_{20}}{dt} = \quad (82)$$

$$(b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{36})^{(7,7,7)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} \end{array}\right] T_{20}$$

$$\frac{dT_{21}}{dt} = \quad (83)$$

$$(b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} \end{array}\right] T_{21}$$

$$\frac{dT_{22}}{dt} = \quad (84)$$

$$(b_{22})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} \end{array}\right] T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ are first detritions coefficients for category 1, 2 and 3 (85)

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ are third detrition coefficients for

category 1,2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b_{36}^{''})^{(7,7,7)}(G_{39},t)}$ $\boxed{-(b_{37}^{''})^{(7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}^{''})^{(7,7,7)}(G_{39},t)}$ are seventh detritions coefficients 86

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FOURTH MODULE CONCATENATION:

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{c}(a_{24}^{'})^{(4)}\boxed{+(a_{24}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}^{''})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{24} \quad 87$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{c}(a_{25}^{'})^{(4)}\boxed{+(a_{25}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{33}^{''})^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}^{''})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{25} \quad 88$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{c}(a_{26}^{'})^{(4)}\boxed{+(a_{26}^{''})^{(4)}(T_{25},t)}\ \boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)}\ \boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}^{''})^{(2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}^{''})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{26} \quad 89$$

90

Where $\boxed{(a_{24}^{''})^{(4)}(T_{25},t)}$, $\boxed{(a_{25}^{''})^{(4)}(T_{25},t)}$, $\boxed{(a_{26}^{''})^{(4)}(T_{25},t)}$ are first augmentation coefficients for categ

$\boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)}$ are second augmentation coefficient for 91

$\boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)}$ are third augmentation coefficient for ca

$\boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a_{16}^{''})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a_{20}^{''})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a_{36}^{''})^{(7,7,7,7,)}(T_{37},t)}$ $\boxed{+(a_{36}^{''})^{(7,7,7,7,)}(T_{37},t)}$ $\boxed{+(a_{36}^{''})^{(7,7,7,7,)}(T_{37},t)}$ ARE SEVENTH augmentation

coefficients

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$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{c} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,,,)}(G_{39},t)} \end{array}\right]T_{24}$$ 93

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{c} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,77,,)}(G_{39},t)} \end{array}\right]T_{25}$$ 94

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{c} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,,)}(G_{39},t)} \end{array}\right]T_{26}$$ 95

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ are first detrition coefficients for cate 96

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ are second detrition coefficients for cat

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ are third detrition coefficients for categ

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,,)}(G_{39},t)}$ $\boxed{-(b''_{37})^{(7,7,7,7,7,,)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7,7,,)}(G_{39},t)}$ ARE SEVENTH DETRITION

COEFFICIENTS

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FIFTH MODULE CONCATENATION: 98

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,,7,,7,7)}(T_{37},t)} \end{array}\right]G_{28}$$ 99

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,,,7,,7,7)}(T_{37},t)} \end{array}\right]G_{29}$$ 100

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,,7,,7,7)}(T_{37},t)} \end{array}\right] G_{30} \quad (101)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ are first augmentation coefficients for ((102)

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ are second augmentation coefficien

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ are third augmentation coefficient for

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

(103)

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,)}(G_{23},t)}\ \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,,7,7,7,)}(G_{38},t)} \end{array}\right] T_{28} \quad (104)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{c} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{29} \quad (105)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{30} \quad (106)$$

$where$ $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ (107)
$are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ $are\ second\ detrition\ coefficients\ fo$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients\ f$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients

for category 1,2, and 3

SIXTH MODULE CONCATENATION 108

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)} & \end{array}\right]G_{32} \quad 109$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{37})^{(7,7,7,,7,7,7,)}(T_{37},t)} & \end{array}\right]G_{33} \quad 110$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{38})^{(7,7,7,7,7,7,)}(T_{37},t)} & \end{array}\right]G_{34} \quad 111$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for c* 112

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficie*

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficie*

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,,77,7,,7,)}(\mathrm{T}_{37},t)}\boxed{+(a''_{36})^{(7,7,7,,7,7,7,)}(T_{37},t)}$ ARE SVENTH AUGMENTATION COEFFICIENTS

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$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b''_{36})^{(7,7,7,,7,7,7)}(G_{39},t)} & \end{array}\right]T_{32} \quad 114$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ & -(b''_{37})^{(7,7,7,,7,7,7)}(G_{39},t) & \end{array}\right]T_{33} \quad 115$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{c} (b_{34}^{'})^{(6)}\boxed{-(b_{34}^{''})^{(6)}(G_{35},t)}\ \boxed{-(b_{30}^{''})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b_{26}^{''})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b_{15}^{''})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b_{18}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b_{22}^{''})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{38}^{''})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{array}\right] T_{34} \quad 116$$

$\boxed{-(b_{32}^{''})^{(6)}(G_{35},t)}, \boxed{-(b_{33}^{''})^{(6)}(G_{35},t)}, \boxed{-(b_{34}^{''})^{(6)}(G_{35},t)}$ *are first detrition coefficients for cate* 117

$\boxed{-(b_{28}^{''})^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{29}^{''})^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{30}^{''})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients*

$\boxed{-(b_{24}^{''})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{25}^{''})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{26}^{''})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients f*

$\boxed{-(b_{13}^{''})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{14}^{''})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{15}^{''})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{16}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}^{''})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{20}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}^{''})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{36}^{''})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b_{36}^{''})^{(7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b_{36}^{''})^{(7,7,7,7,7,7)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

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SEVENTH MODULE CONCATENATION: 119

$$\frac{dG_{36}}{dt} = \quad 120$$

$$(a_{36})^{(7)}G_{37} - \left[(a_{36}^{'})^{(7)}\boxed{+(a_{36}^{''})^{(7)}(T_{37},t)} + \boxed{(a_{16}^{''})^{(7)}(T_{17},t)} + \boxed{(a_{20}^{''})^{(7)}(T_{21},t)} + \boxed{(a_{24}^{''})^{(7)}(T_{23},t)G_{36}} + \boxed{(a_{28}^{''})^{(7)}(T_{29},t)} + \boxed{(a_{32}^{''})^{(7)}(T_{33},t)} + \boxed{(a_{13}^{''})^{(7)}(T_{14},t)}\right] G_{36}$$

121

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a_{37}^{'})^{(7)} + \boxed{(a_{37}^{''})^{(7)}(T_{37},t)} + \boxed{(a_{14}^{''})^{(7)}(T_{14},t)} + \boxed{(a_{21}^{''})^{(7)}(T_{21},t)} + \boxed{(a_{17}^{''})^{(7)}(T_{17},t)} + \boxed{(a_{25}^{''})^{(7)}(T_{25},t)} + \boxed{(a_{33}^{''})^{(7)}(T_{33},t)} + \boxed{(a_{29}^{''})^{(7)}(T_{29},t)}\right] G_{37} \quad 122$$

$$\frac{dG_{38}}{dt} = \quad 123$$

$$(a_{38})^{(7)}G_{37} - \quad 124$$

$$\left[(a_{38}^{'})^{(7)} + \boxed{(a_{38}^{''})^{(7)}(T_{37},t)} + \underbrace{\boxed{(a_{15}^{''})^{(7)}(T_{14},t)}}_{\dots} + \boxed{(a_{22}^{''})^{(7)}(T_{21},t) + \boxed{+(a_{18}^{''})^{(7)}(T_{17},t)}} + \boxed{(a_{26}^{''})^{(7)}(T_{25},t)} + \boxed{(a_{34}^{''})^{(7)}(T_{33},t)} + \boxed{(a_{30}^{''})^{(7)}(T_{29},t)}\right] G_{38} \quad 125$$

$$\frac{dT_{36}}{dt} = \quad 126$$

$$(b_{36})^{(7)}T_{37} - \Big[(b_{36}^{'})^{(7)} - \boxed{(b_{36}^{''})^{(7)}((G_{39}),t)} - \boxed{(b_{16}^{''})^{(7)}((G_{19}),t)} - \boxed{(b_{13}^{''})^{(7)}((G_{14}),t)} - \boxed{(b_{20}^{''})^{(7)}((G_{231}),t)} - \boxed{(b_{24}^{''})^{(7)}((G_{27}),t)} - \boxed{(b_{28}^{''})^{(7)}((G_{31}),t)} - \boxed{(b_{32}^{''})^{(7)}((G_{35}),t)}\Big]T_{36}$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \Big[(b_{36}^{'})^{(7)} - \boxed{(b_{37}^{''})^{(7)}((G_{39}),t)} - \boxed{(b_{17}^{''})^{(7)}((G_{19}),t)} - \boxed{(b_{19}^{''})^{(7)}((G_{14}),t)} - \boxed{(b_{21}^{''})^{(7)}((G_{231}),t)} - \boxed{(b_{25}^{''})^{(7)}((G_{27}),t)} - \boxed{(b_{29}^{''})^{(7)}((G_{31}),t)} - \boxed{(b_{33}^{''})^{(7)}((G_{35}),t)}\Big]T_{37} \quad 127$$

Where we suppose

(A) $(a_i)^{(1)}, (a_i^{'})^{(1)}, (a_i^{''})^{(1)}, (b_i)^{(1)}, (b_i^{'})^{(1)}, (b_i^{''})^{(1)} > 0,$
$i, j = 13,14,15$

(B) The functions $(a_i^{''})^{(1)}, (b_i^{''})^{(1)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i^{''})^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i^{''})^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i^{'})^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(C) $lim_{T_2 \to \infty}(a_i^{''})^{(1)}(T_{14},t) = (p_i)^{(1)}$
$lim_{G \to \infty}(b_i^{''})^{(1)}(G,t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants
and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(1)}(T_{14}^{'},t) - (a_i^{''})^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}^{'}|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i^{''})^{(1)}(G^{'},t) - (b_i^{''})^{(1)}(G,T)| < (\hat{k}_{13})^{(1)}||G - G^{'}||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(1)}(T_{14}^{'},t)$ and $(a_i^{''})^{(1)}(T_{14},t)$. $(T_{14}^{'},t)$ and (T_{14},t) are points belonging to the interval $\big[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\big]$. It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$\frac{dT_{38}}{dt} =$ 128

$(b_{38})^{(7)}T_{37} - \Big[(b'_{38})^{(7)} - \boxed{(b''_{38})^{(7)}((G_{39}),t)} - \boxed{(b''_{18})^{(7)}((G_{19}),t)} - \boxed{(b''_{20})^{(7)}((G_{14}),t)} -$ 129

$\boxed{(b''_{22})^{(7)}((G_{23}),t)} - \boxed{(b''_{26})^{(7)}((G_{27}),t)} - \boxed{(b''_{30})^{(7)}((G_{31}),t)} -$ 130

$\boxed{(b''_{34})^{(7)}((G_{35}),t)} \Big]T_{38}$ 131

132

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor** 134

$(1)(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i,j = 16,17,18$ 135

(F) (2) The functions $(a''_i)^{(2)}, (b''_i)^{(2)}$ are positive continuous increasing and bounded. 136

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$: 137

$(a''_i)^{(2)}(T_{17},t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)}$ 138

$(b''_i)^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b'_i)^{(2)} \le (\hat{B}_{16})^{(2)}$ 139

(G) (3) $lim_{T_2\to\infty}(a''_i)^{(2)}(T_{17},t) = (p_i)^{(2)}$ 140

$lim_{G\to\infty}(b''_i)^{(2)}\big((G_{19}),t\big) = (r_i)^{(2)}$ 141

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 142

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 143

$|(a''_i)^{(2)}(T'_{17},t) - (a''_i)^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T'_{17}|e^{-(\hat{M}_{16})^{(2)}t}$ 144

$|(b''_i)^{(2)}((G_{19})',t) - (b''_i)^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 145

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(2)}(T'_{17},t)$ and$(a''_i)^{(2)}(T_{17},t)$. (T'_{17},t) And (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a''_i)^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a''_i)^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous. 146

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 147

(H) (4) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 148

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 149

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\widehat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 150$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 151$$

Where we suppose 152

(I) (5) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 153

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad 154$$

$$lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)} \quad 155$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 156

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 157

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\widehat{M}_{20})^{(3)}t} \quad 158$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\widehat{M}_{20})^{(3)}t} \quad 159$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\widehat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the THIRD augmentation coefficient, would be absolutely continuous. 160

Definition of $(\widehat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 161

(J) (6) $(\widehat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\widehat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants 162
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ 163
satisfy the inequalities 164

$$\frac{1}{(\widehat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \quad 165$$

$$\frac{1}{(\widehat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \quad 166$$

167

Where we suppose 168

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad 169$$

(L) (7) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i^{''})^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i^{''})^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i^{'})^{(4)} \le (\hat{B}_{24})^{(4)}$$

170

(M) (8) $lim_{T_2\to\infty}(a_i^{''})^{(4)}(T_{25},t) = (p_i)^{(4)}$
$lim_{G\to\infty}(b_i^{''})^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 171

$$|(a_i^{''})^{(4)}(T_{25}^{'},t) - (a_i^{''})^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i^{''})^{(4)}((G_{27})^{'},t) - (b_i^{''})^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'},t)$ and$(a_i^{''})^{(4)}(T_{25},t)$. $(T_{25}^{'},t)$ And (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i^{''})^{(4)}(T_{25},t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous. 172

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Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 174

$(\hat{M}_{24})176^{175(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 175

(P) (9) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i^{'})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i^{'})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i^{'})^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose 176

$(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, \quad i,j = 28,29,30$ 177

(R) (10) The functions $(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i^{''})^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i^{''})^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i^{'})^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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(S) (11) $lim_{T_2\to\infty}(a_i^{''})^{(5)}(T_{29},t) = (p_i)^{(5)}$
$lim_{G\to\infty}(b_i^{''})^{(5)}(G_{31},t) = \ (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28{,}29{,}30}$

They satisfy Lipschitz condition: 179

$$|(a_i^{''})^{(5)}(T_{29}^{'},t) - (a_i^{''})^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}^{'}|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i^{''})^{(5)}((G_{31})^{'},t) - (b_i^{''})^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'},t)$ 180
and$(a_i^{''})^{(5)}(T_{29},t)$. $(T_{29}^{'},t)$ and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i^{''})^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i^{''})^{(5)}(T_{29},t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 181

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants
$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 182

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with
$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants
$(a_i)^{(5)}, (a_i^{'})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28{,}29{,}30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i^{'})^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i^{'})^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose 183

$(a_i)^{(6)}, (a_i^{'})^{(6)}, (a_i^{''})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (b_i^{''})^{(6)} > 0, \quad i,j = 32{,}33{,}34$ 184

(12) The functions $(a_i^{''})^{(6)}, (b_i^{''})^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i^{''})^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i^{''})^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i^{'})^{(6)} \leq (\hat{B}_{32})^{(6)}$$

185

(13) $lim_{T_2\to\infty}(a_i^{''})^{(6)}(T_{33},t) = (p_i)^{(6)}$
$lim_{G\to\infty}(b_i^{''})^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition: 186

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. 187

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 188

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 189

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose 190

(V) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0,$ 191
$i, j = 36,37,38$

(W) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

192

(X) $lim_{T_2\to\infty}\ (a_i^{''})^{(7)}\,(T_{37},t) = (p_i)^{(7)}$
$lim_{G\to\infty}(b_i^{''})^{(7)}\,\big((G_{39}),t\big) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)},\ (r_i)^{(7)}}$ are positive constants
and $\boxed{i = 36{,}37{,}38}$

They satisfy Lipschitz condition: 193

$$|(a_i^{''})^{(7)}(T_{37}^{'},t) - (a_i^{''})^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}^{'}|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i^{''})^{(7)}((G_{39})^{'},t) - (b_i^{''})^{(7)}\big((G_{39}),(T_{39})\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})^{'}||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(7)}(T_{37}^{'},t)$ 194
and$(a_i^{''})^{(7)}(T_{37},t)$. $(T_{37}^{'},t)$ and (T_{37},t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$.
It is to be noted that $(a_i^{''})^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 7$ then the function $(a_i^{''})^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 195

(Y) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 196

(Z) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with
$(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants
$(a_i)^{(7)}, (a_i^{'})^{(7)}, (b_i)^{(7)}, (b_i^{'})^{(7)}, (\mathrm{p}_i)^{(7)},\ (r_i)^{(7)}, i = 36{,}37{,}38,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

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Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0), T_i(0)$: 199

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 200

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 201

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ 202

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ 203

By 204

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)} \quad 205$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)} \quad 206$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)} \quad 207$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (\mathrm{b}''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)} \quad 208$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)} \quad 209$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$
$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)} \left(G(s_{(36)}), s_{(36)} \right) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)}, \quad 212$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad 213$$

$$0 \le T_i(t) - T_i^0 \le (\hat{\mathrm{Q}}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad 214$$

By 215

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a_{16}')^{(2)} + a_{16}'')^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a_{17}')^{(2)} + (a_{17}'')^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)} \quad 216$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a_{18}')^{(2)} + (a_{18}'')^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)} \quad 217$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b_{16}')^{(2)} - (b_{16}'')^{(2)} (G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)} \quad 218$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b_{17}')^{(2)} - (b_{17}'')^{(2)} (G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)} \quad 219$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b_{18}')^{(2)} - (b_{18}'')^{(2)} (G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)} \quad 220$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)}, \quad 222$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad 223$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad 224$$

By 225

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a_{20}')^{(3)} + a_{20}'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}')^{(3)} + (a_{21}'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)} \quad 226$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}')^{(3)} + (a_{22}'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)} \quad 227$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}')^{(3)} - (b_{20}'')^{(3)} (G(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)} \quad 228$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}')^{(3)} - (b_{21}'')^{(3)} (G(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)} \quad 229$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}')^{(3)} - (b_{22}'')^{(3)} \left(G(s_{(20)}), s_{(20)} \right) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 230$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ 231
which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)}, \quad 232$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad 233$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad 234$$

By 235

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}')^{(4)} + a_{24}'')^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}')^{(4)} + (a_{25}'')^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 236$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}')^{(4)} + (a_{26}'')^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 237$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}')^{(4)} - (b_{24}'')^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 238$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}')^{(4)} - (b_{25}'')^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 239$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}')^{(4)} - (b_{26}'')^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 240$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ 241
which satisfy
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$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{28})^{(5)} \text{ , } T_i^0 \le (\hat{Q}_{28})^{(5)}, \quad 243$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad 244$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad 245$$

By 246

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a_{28}')^{(5)} + a_{28}'')^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a_{29}')^{(5)} + (a_{29}'')^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 247$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a_{30}')^{(5)} + (a_{30}'')^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 248$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b_{28}')^{(5)} - (b_{28}'')^{(5)} \left(G(s_{(28)}), s_{(28)} \right) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 249$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b_{29}')^{(5)} - (b_{29}'')^{(5)} \left(G(s_{(28)}), s_{(28)} \right) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 250$$

$$\bar{T}_{30}(t) = T_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b_{30}')^{(5)} - (b_{30}'')^{(5)} \left(G(s_{(28)}), s_{(28)} \right) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 251$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{32})^{(6)} \text{ , } T_i^0 \le (\hat{Q}_{32})^{(6)}, \quad 253$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad 254$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad 255$$

By 256

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a_{32}')^{(6)} + a_{32}'')^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a_{33}')^{(6)} + (a_{33}'')^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 257$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a_{34}')^{(6)} + (a_{34}'')^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 258$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b_{32}')^{(6)} - (b_{32}'')^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 259$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b_{33}')^{(6)} - (b_{33}'')^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 260$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b_{34}')^{(6)} - (b_{34}'')^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 261$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

: if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions 262

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36} \right)^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{36})^{(7)} \text{ , } T_i^0 \le (\hat{Q}_{36})^{(7)}, \quad 263$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad 264$$

$$0 \le T_{\mathrm{i}}(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad 265$$

By 266

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)} \quad 267$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) \mathrm{G}_{38}(s_{(36)})\right] ds_{(36)} \quad 268$$

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$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)} \quad 270$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)} \quad 271$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 272

(a) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 273

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 274

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(b) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 275

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)}\left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 276

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 277

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)}\left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 278

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem1

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying 37,35,36 into itself .Indeed it is obvious that 279

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)}\left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

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From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 281

$(\hat{P}_{13})^{(1)}\ and\ (\hat{Q}_{13})^{(1)}$ large to have

282

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 283$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 284$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself 285

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 286

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 287

Definition of $\tilde{G},\tilde{T}$:

$$(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a_{13}')^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad 288$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)} \; and \; (b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \; and \; (\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 289

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)} \; and \; (b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0 \; and \; T_i(t) = 0$ 290

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0 \quad 291$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, and \; \left((\widehat{M}_{13})^{(1)}\right)_3$: 292

Remark 3: if G_{13} is bounded, the same property have also $G_{14} \; and \; G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14} \; or \; G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13} \; is$ bounded, from below, the same property holds for $G_{14} \; and \; G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 293

Remark 5: If T_{13} is bounded from below and $lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 294

Definition of $(m)^{(1)} \; and \; \varepsilon_1$:

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 295

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 297

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 298$$

299

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 300

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 301

$$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 302

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 303

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}^{'})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}^{''})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)}\left|(a_{16}^{''})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}^{''})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 304

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq \quad 305$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}^{'})^{(2)} + (\widehat{A}_{16})^{(2)} +\right.$$
$$\left.(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 306

Remark 1: The fact that we supposed $(a_{16}^{''})^{(2)}$ and $(b_{16}^{''})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 307

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(2)}$ and $(b_i^{''})^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on

$(\mathrm{G}_{19})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 308

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2\ and\ \left((\widehat{M}_{16})^{(2)}\right)_3$: 309

Remark 3: if G_{16} is bounded, the same property have also $G_{17}\ and\ G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$ 310

If $G_{17}\ or\ G_{18}$ is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If $G_{16}\ \ is$ bounded, from below, the same property holds for $G_{17}\ and\ G_{18}$. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 311

Remark 5: If T_{16} is bounded from below and $lim_{t\to\infty}((b_i'')^{(2)}\,((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 312

Definition of $(m)^{(2)}\ and\ \varepsilon_2$:

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}\,(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 313

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$ 314

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 316

$(\hat{P}_{20})^{(3)}\ and\ (\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)}$$ 317

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 318$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself 319

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 320

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 321

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right)=\mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 322

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a_{20}')^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+ \quad 323$$

$$G_{20}^{(2)}\left|(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a_{20}'')^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 324$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a_{20}')^{(3)}+(\widehat{A}_{20})^{(3)}+\right.$$
$$\left.(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{20}'')^{(3)}\ and\ (b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 325

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}\ and\ (b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 326

From 19 to 24 it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 327

Remark 3: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}^{'})^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}^{'})^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}^{'})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 328

Remark 5: If T_{20} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(3)}\left((G_{23})(t),t\right) = (b_{21}^{'})^{(3)}$ then $T_{21} \to \infty$. 329

Definition of $(m)^{(3)}\ and\ \varepsilon_3$: 330

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i^{''})^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 331

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $lim_{t\to\infty}(b_{22}^{''})^{(3)}\left((G_{23})(t),t\right) = (b_{22}^{'})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 333

$(\hat{P}_{24})^{(4)}\ and\ (\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 334$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 335$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying IN to itself 336

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 337

$d\left(\left((G_{27})^{(1)},(T_{27})^{(1)}\right),\left((G_{27})^{(2)},(T_{27})^{(2)}\right)\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a_{24}^{'})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a_{24}^{''})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

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$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\le \qquad 339$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a_{24}^{'})^{(4)}+(\widehat{A}_{24})^{(4)}+\right.$$
$$\left.(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{24}^{''})^{(4)}\ and\ (b_{24}^{''})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 340

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}\ and\ (b_i^{''})^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 341

From GLOBAL EQUATIONS it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i^{'})^{(4)}-(a_i^{''})^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i^{'})^{(4)}t\right)}>0\quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 342

Remark 3: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\le\left((\widehat{M}_{24})^{(4)}\right)_1-(a_{25}^{'})^{(4)}G_{25}$ and by integrating

$$G_{25}\le\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a_{25}^{'})^{(4)}$$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If $G_{24}\ is$ bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 343

Remark 5: If T_{24} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(4)}\ ((G_{27})(t),t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$. 344

Definition of $(m)^{(4)}\ and\ \varepsilon_4$:

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{\mathrm{d}t} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 345

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t),t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

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It is now sufficient to take $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$ and to choose 347

$(\hat{P}_{28})^{(5)}\ and\ (\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 348$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 349$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself 350

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 351

$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}), (\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}), (\widetilde{T_{31}}) \right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a_{28}^{'})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a_{28}^{''})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a_{28}^{''})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a_{28}^{''})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad 353$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}} \left((a_{28})^{(5)} + (a_{28}^{'})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a_{28}^{''})^{(5)} \; and \; (b_{28}^{''})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t} \; and \; (\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 354

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(5)} \; and \; (b_i^{''})^{(5)}, i = 28{,}29{,}30$ depend only on T_{29} and respectively on $(G_{31})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 355

From GLOBAL EQUATIONS it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(5)} - (a_i^{''})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(5)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{\mathrm{M}}_{28})^{(5)}\right)_2 \; and \; \left((\widehat{M}_{28})^{(5)}\right)_3$: 356

Remark 3: if G_{28} is bounded, the same property have also $G_{29} \; and \; G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}^{'})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)} \left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}^{'})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If $G_{28}\ is$ bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 357

Remark 5: If T_{28} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(5)}\ ((G_{31})(t),t)) = (b_{29}^{'})^{(5)}$ then $T_{29} \to \infty$. 358

Definition of $(m)^{(5)}\ and\ \varepsilon_5$:

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i^{''})^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\ (t) > (m)^{(5)} \quad 359$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 360

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $lim_{t\to\infty}(b_{30}^{''})^{(5)}\left((G_{31})(t),t\right) = (b_{30}^{'})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for $G_{33}\ , G_{34}, T_{32}, T_{33}, T_{34}$

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It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 362

$(\hat{P}_{32})^{(6)}\ and\ (\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 363$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 364$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i\ , T_i$ into itself 365

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 366

$$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a_{32}^{'})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}^{''})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a_{32}^{''})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}^{''})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}\qquad 367$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

(1) $(a_i^{'})^{(1)},(a_i^{''})^{(1)},(b_i)^{(1)},(b_i^{'})^{(1)},(b_i^{''})^{(1)}>0,$
$i,j=13,14,15$

(2)The functions $(a_i^{''})^{(1)},(b_i^{''})^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i^{''})^{(1)}(T_{14},t)\le(p_i)^{(1)}\le(\hat{A}_{13})^{(1)}$$

$$(b_i^{''})^{(1)}(G,t)\le\ (r_i)^{(1)}\le(b_i^{'})^{(1)}\le(\hat{B}_{13})^{(1)}$$

(3) $lim_{T_2\to\infty}(a_i^{''})^{(1)}(T_{14},t)=(p_i)^{(1)}$
$lim_{G\to\infty}(b_i^{''})^{(1)}(G,t)=\ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)},(\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)},(\hat{B}_{13})^{(1)},(p_i)^{(1)},\ (r_i)^{(1)}}$ are positive constants and $\boxed{i=13,14,15}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(1)}(T_{14}^{'},t)-(a_i^{''})^{(1)}(T_{14},t)|\le(\hat{k}_{13})^{(1)}|T_{14}-T_{14}^{'}|e^{-(\widehat{M}_{13})^{(1)}t}$$

$$|(b_i^{''})^{(1)}(G^{'},t)-(b_i^{''})^{(1)}(G,T)|<(\hat{k}_{13})^{(1)}||G-G^{'}||e^{-(\widehat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(1)}(T_{14}^{'},t)$ and$(a_i^{''})^{(1)}(T_{14},t)$. $(T_{14}^{'},t)$ and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)},(\widehat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{13})^{(1)}=1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\widehat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$:

(AA) $(\widehat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}},\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}<1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(BB) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 368

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 7$ and to choose

$(\hat{P}_{36})^{(7)} and\ (\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 369$$

370

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying 37,35,36 into itself 371

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 372

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}), (\widetilde{T_{39}})$:

$$\left((\widetilde{G_{39}}), (\widetilde{T_{39}}) \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$$

It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$

$$\int_0^t \{(a_{36}^{'})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$(a_{36}^{''})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$G_{36}^{(2)} \left|(a_{36}^{''})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}^{''})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a_{36}^{'})^{(7)} + (\widehat{A}_{36})^{(7)} + \right.$$
$$\left. (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d \left(\left((G_{39})^{(1)}, (T_{39})^{(1)}; \; (G_{39})^{(2)}, (T_{39})^{(2)}\right)\right)$$

374

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (37,35,36) the result follows

375

Remark 1: The fact that we supposed $(a_{36}^{''})^{(7)} \; and \; (b_{36}^{''})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(7)} \; and \; (b_i^{''})^{(7)}, i = 36{,}37{,}38$ depend only on T_{37} and respectively on $(G_{39}) (and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

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Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 79 to 36 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(7)} - (a_i^{''})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(7)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 377

Remark 3: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}^{'})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}^{'})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}^{'})^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 378

Remark 5: If T_{36} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(7)}\ ((G_{39})(t), t)) = (b_{37}^{'})^{(7)}$ then 379

$T_{37} \rightarrow \infty$.

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 380

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $lim_{t\rightarrow\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t), t\right) = (b_{38}^{'})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 72

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS AND ITS CONCOMITANT CONDITIONALITIES into itself 381

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The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 383

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$:

$$\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$

$$\int_0^t\{(a_{36}^{'})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$

$$(a_{36}^{''})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$

$$G_{36}^{(2)}\left|(a_{36}^{''})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a_{36}^{''})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

384

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+\ (a_{36}^{'})^{(7)}+(\widehat{A}_{36})^{(7)}+\right.$$
$$\left.(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{36}^{''})^{(7)}\ and\ (b_{36}^{''})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 385

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(7)}\ and\ (b_i^{''})^{(7)}$, i = 36,37,38 depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$ 386

From CONCATENATED GLOBAL EQUATIONS it results

$$G_i\ (t)\geq G_i^0e^{\left[-\int_0^t\{(a_i^{'})^{(7)}-(a_i^{''})^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0e^{\left(-(b_i^{'})^{(7)}t\right)}>0\quad \text{for } t>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 387

Remark 3: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If $G_{37} \; or \; G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If $G_{36} \; is$ bounded, from below, the same property holds for $G_{37} \; and \; G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 388

Remark 5: If T_{36} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 389

Definition of $(m)^{(7)} \; and \; \varepsilon_7$:

Indeed let t_7 be so that for $t > t_7$

$$(\mathrm{b}_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 390

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $lim_{t\to\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$ 391

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right) - (b''_{17})^{(2)}\left((G_{19}),t\right) \leq -(\tau_1)^{(2)}$ 392

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 393

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 394

(a) of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 395

and $(b_{14})^{(2)}\left(u^{(2)}\right)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 396

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 397

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 398

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 399

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 400

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 401

(b) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 402

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, $\boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 403

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 404

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 405

and analogously 406

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, $\boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 407

Then the solution satisfies the inequalities 408

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined 409

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 410

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right.$ 411
$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a_{18}')^{(2)})}[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 412

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 413

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18}')^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \le T_{18}(t) \le$ 414

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 415

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ 416

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$ 417

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

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Behavior of the solutions 419

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 420

(b) By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 421

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, $\boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, $\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 422

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, $\boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, $\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, $\boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined 423

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t}$$ 424

$$\left(\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t}-e^{-(S_2)^{(3)}t}\right]+G_{22}^{0}e^{-(S_2)^{(3)}t}\le G_{22}(t)\le \frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}((S_1)^{(3)}-(a_{22}^{'})^{(3)})}[e^{(S_1)^{(3)}t}-e^{-(a_{22}^{'})^{(3)}t}]+G_{22}^{0}e^{-(a_{22}^{'})^{(3)}t}\right) \quad 425$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 426$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 427$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b_{22}^{'})^{(3)})}\left[e^{(R_1)^{(3)}t}-e^{-(b_{22}^{'})^{(3)}t}\right]+T_{22}^{0}e^{-(b_{22}^{'})^{(3)}t}\le T_{22}(t)\le \quad 428$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 429

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a_{20}^{'})^{(3)}$

$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$

$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b_{20}^{'})^{(3)}$

$(R_2)^{(3)}=(b_{22}^{'})^{(3)}-(r_{22})^{(3)}$

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If we denote and define

Definition of $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)}\le -(a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}-(a_{24}^{''})^{(4)}(T_{25},t)+(a_{25}^{''})^{(4)}(T_{25},t)\le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)}\le -(b_{24}^{'})^{(4)}+(b_{25}^{'})^{(4)}-(b_{24}^{''})^{(4)}\big((G_{27}),t\big)-(b_{25}^{''})^{(4)}\big((G_{27}),t\big)\le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)},(\nu_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},\nu^{(4)},u^{(4)}$: 433

(e) By $(\nu_1)^{(4)}>0$, $(\nu_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0$, $(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2+(\sigma_1)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

Definition of $(\bar{\nu}_1)^{(4)},,(\bar{\nu}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 434
435

By $(\bar{\nu}_1)^{(4)}>0$, $(\bar{\nu}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0$, $(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2+(\sigma_2)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ 436

Definition of $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)},(\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)}=(\nu_0)^{(4)},(m_1)^{(4)}=(\nu_1)^{(4)},$ ***if*** $(\nu_0)^{(4)}<(\nu_1)^{(4)}$

$(m_2)^{(4)}=(\nu_1)^{(4)},(m_1)^{(4)}=(\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)}<(\nu_0)^{(4)}<(\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously 437
438

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined respectively

Then the solution satisfies the inequalities 439
440

$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$ 441
442

where $(p_i)^{(4)}$ is defined 443
444
445

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$ 446
447

$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right.$ 448
$\left.\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$

$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$ 449

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$ 450

$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le$ 451

$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 452

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$ 453

Behavior of the solutions 454
If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a_{28}^{'})^{(5)} + (a_{29}^{'})^{(5)} - (a_{28}^{''})^{(5)}(T_{29}, t) + (a_{29}^{''})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b_{28}^{'})^{(5)} + (b_{29}^{'})^{(5)} - (b_{28}^{''})^{(5)}\big((G_{31}), t\big) - (b_{29}^{''})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 455

(h) By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 456

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 457

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities 458

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 459$$

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$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a_{30}^{'})^{(5)})}\left[e^{(S_1)^{(5)}t} - e^{-(a_{30}^{'})^{(5)}t}\right] + G_{30}^0 e^{-(a_{30}^{'})^{(5)}t}\right) \quad 461$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 462$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 463$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b_{30}^{'})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b_{30}^{'})^{(5)}t}\right]+T_{30}^{0}e^{-(b_{30}^{'})^{(5)}t}\leq T_{30}(t)\leq \quad 464$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 465

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a_{28}^{'})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b_{28}^{'})^{(5)}$

$(R_2)^{(5)}=(b_{30}^{'})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions 466

If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq -(a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}-(a_{32}^{''})^{(6)}(T_{33},t)+(a_{33}^{''})^{(6)}(T_{33},t)\leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq -(b_{32}^{'})^{(6)}+(b_{33}^{'})^{(6)}-(b_{32}^{''})^{(6)}\big((G_{35}),t\big)-(b_{33}^{''})^{(6)}\big((G_{35}),t\big)\leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)},(\nu_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},\nu^{(6)},u^{(6)}$: 467

(k) By $(\nu_1)^{(6)}>0$, $(\nu_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{\nu}_1)^{(6)},,(\bar{\nu}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 468

By $(\bar{\nu}_1)^{(6)}>0$, $(\bar{\nu}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(\nu_0)^{(6)},(m_1)^{(6)}=(\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)}<(\nu_1)^{(6)}$

$(m_2)^{(6)}=(\nu_1)^{(6)},(m_1)^{(6)}=(\bar{\nu}_6)^{(6)},\boldsymbol{if}\ (\nu_1)^{(6)}<(\nu_0)^{(6)}<(\bar{\nu}_1)^{(6)},$ 470

and $\boxed{(\nu_0)^{(6)}=\frac{G_{32}^{0}}{G_{33}^{0}}}$

$(m_2)^{(6)}=(\nu_1)^{(6)},(m_1)^{(6)}=(\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)}<(\nu_0)^{(6)}$

and analogously 471

$(\mu_2)^{(6)}=(u_0)^{(6)},(\mu_1)^{(6)}=(u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)}<(u_1)^{(6)}$

$(\mu_2)^{(6)}=(u_1)^{(6)},(\mu_1)^{(6)}=(\bar{u}_1)^{(6)},\boldsymbol{if}\ (u_1)^{(6)}<(u_0)^{(6)}<(\bar{u}_1)^{(6)},$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities 472

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 473$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t}\right] + G_{34}^0 e^{-(S_2)^{(6)} t} \le G_{34}(t) \le \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)} t}\right) \quad 474$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 475$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 476$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \le T_{34}(t) \le \quad 477$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t}\right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 478

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$ 479

If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(m) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying
$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$

$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \le -(\tau_1)^{(7)}$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 480

(n) By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)} v^{(7)} - (a_{36})^{(7)} = 0$ and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_1)^{(7)} u^{(7)} - (b_{36})^{(7)} = 0$ and 481

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 482

By $(\bar{v}_1)^{(7)} > 0$, $(\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}$, $(\mu_2)^{(7)}$, $(v_0)^{(7)}$:-

(o) If we define $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}$, $(\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, $\boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, $\boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 483

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, $\boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, $\boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined respectively

Then the solution satisfies the inequalities 484

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined

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$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 486$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})} \left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 487$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)}-(a_{38}^{'})^{(7)})} [e^{(S_1)^{(7)}t} - e^{-(a_{38}^{'})^{(7)}t}] + G_{38}^0 e^{-(a_{38}^{'})^{(7)}t} \right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 488$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 489$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)}-(b_{38}^{'})^{(7)})} \left[e^{(R_1)^{(7)}t} - e^{-(b_{38}^{'})^{(7)}t} \right] + T_{38}^0 e^{-(b_{38}^{'})^{(7)}t} \le T_{38}(t) \le \quad 490$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})} \left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 491

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}^{'})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}^{'})^{(7)}$

$(R_2)^{(7)} = (b_{38}^{'})^{(7)} - (r_{38})^{(7)}$

From GLOBAL EQUATIONS we obtain 492

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}^{'})^{(7)} - (a_{37}^{'})^{(7)} + (a_{36}^{''})^{(7)}(T_{37}, t) \right) - (a_{37}^{''})^{(7)}(T_{37}, t) v^{(7)} - (a_{37})^{(7)} v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)} \left(v^{(7)}\right)^2 + (\sigma_2)^{(7)} v^{(7)} - (a_{36})^{(7)} \right) \le \frac{dv^{(7)}}{dt} \le$$

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

(a) For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

$it\ follows\ (\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 493

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

(b) If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 494

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\ \nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

(c) If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 495

$$(\nu_1)^{(7)}\leq\ \nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{36}^{''})^{(7)} = (a_{37}^{''})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}^{''})^{(7)} = (b_{37}^{''})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then

$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

We can prove the following 496

If $(a_i^{''})^{(7)} and\ (b_i^{''})^{(7)}$ are independent on t , and the conditions 496A

$$(a_{36}^{'})^{(7)}(a_{37}^{'})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0 \quad 496B$$

$$(a_{36}^{'})^{(7)}(a_{37}^{'})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a_{37}^{'})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0 \quad 496C$$

497C

497D

$$(b_{36}^{'})^{(7)}(b_{37}^{'})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0\ ,$$

497E

497F

$$(b_{36}^{'})^{(7)}(b_{37}^{'})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b_{36}^{'})^{(7)}(r_{37})^{(7)} - (b_{37}^{'})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0 \quad 497G$$

$with\ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined are satisfied , then the system WITH THE SATISFACTION OF

THE FOLLOWING PROPERTIES HAS A SOLUTION AS DERIVED BELOW.

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Particular case :

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If $(a_{16}^{''})^{(2)} = (a_{17}^{''})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}^{''})^{(2)} = (b_{17}^{''})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

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From GLOBAL EQUATIONS we obtain

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$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}^{'})^{(3)} - (a_{21}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}, t)\right) - (a_{21}^{''})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$

501

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

$it\ follows\ (\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get

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$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case,

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$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)}+(C)^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}} \leq (\bar{v}_1)^{(3)}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 505

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)}+(C)^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{20}^{''})^{(3)} = (a_{21}^{''})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}^{''})^{(3)} = (b_{21}^{''})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain 507

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}^{'})^{(4)} - (a_{25}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}, t)\right) - (a_{25}^{''})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$ 508

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \leq \frac{dv^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

(d) For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)}=\frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

$it\ follows\ (\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 509

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)}=\frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 510

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)}=\frac{G_{24}^0}{G_{25}^0}}$, we obtain 512

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t)=\frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t)=\frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{24}^{''})^{(4)}=(a_{25}^{''})^{(4)}, then\ (\sigma_1)^{(4)}=(\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)}=(\bar{\nu}_1)^{(4)}$ if in addition 513
$(\nu_0)^{(4)}=(\nu_1)^{(4)}$ then $\nu^{(4)}(t)=(\nu_0)^{(4)}$ and as a consequence $G_{24}(t)=(\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}^{''})^{(4)}=(b_{25}^{''})^{(4)}, then\ (\tau_1)^{(4)}=(\tau_2)^{(4)}$ and then
$(u_1)^{(4)}=(\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)}=(u_1)^{(4)}$ then $T_{24}(t)=(u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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From GLOBAL EQUATIONS we obtain 515

$$\frac{d\nu^{(5)}}{dt}=(a_{28})^{(5)}-\left((a_{28}^{'})^{(5)}-(a_{29}^{'})^{(5)}+(a_{28}^{''})^{(5)}(T_{29},t)\right)-(a_{29}^{''})^{(5)}(T_{29},t)\nu^{(5)}-(a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

$it\ follows\ (\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 516

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 517

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 518

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)} \quad 519$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{28}^{''})^{(5)} = (a_{29}^{''})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}^{''})^{(5)} = (b_{29}^{''})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

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we obtain 521

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}^{'})^{(6)} - (a_{33}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}, t)\right) - (a_{33}^{''})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

$it\ follows\ (\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 522

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}} \qquad 523$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 524

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 525

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have
Definition of $v^{(6)}(t)$:-

$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain
Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{32}^{''})^{(6)} = (a_{33}^{''})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $v^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b_{32}^{''})^{(6)} = (b_{33}^{''})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Behavior of the solutions 527

If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(p) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a_{36}^{'})^{(7)} + (a_{37}^{'})^{(7)} - (a_{36}^{''})^{(7)}(T_{37}, t) + (a_{37}^{''})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b_{36}^{'})^{(7)} + (b_{37}^{'})^{(7)} - (b_{36}^{''})^{(7)}\big((G_{39}), t\big) - (b_{37}^{''})^{(7)}\big((G_{39}), t\big) \leq -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 528

(q) By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)} v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)} u^{(7)} - (b_{36})^{(7)} = 0$ and

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Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 530.

By $(\bar{v}_1)^{(7)} > 0$, $(\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the

roots of the equations $(a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}$, $(\mu_2)^{(7)}$, $(\nu_0)^{(7)}$:-

(r) If we define $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}$, $(\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, $\boldsymbol{if}$ $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, $\boldsymbol{if}$ $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 531

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, $\boldsymbol{if}$ $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$, $\boldsymbol{if}$ $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}$ $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined by 59 and 67 respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 532

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)}-(p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 533$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)}\right)} \left[e^{\left((S_1)^{(7)}-(p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \right. \quad 534$$
$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)}-(a_{38}^{'})^{(7)}\right)} \left[e^{(S_1)^{(7)}t} - e^{-(a_{38}^{'})^{(7)}t} \right] + G_{38}^0 e^{-(a_{38}^{'})^{(7)}t} \right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t}} \quad 535$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t} \quad 536$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)}-(b_{38}^{'})^{(7)}\right)} \left[e^{(R_1)^{(7)}t} - e^{-(b_{38}^{'})^{(7)}t} \right] + T_{38}^0 e^{-(b_{38}^{'})^{(7)}t} \leq T_{38}(t) \leq \quad 537$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)}\right)} \left[e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 538

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}^{'})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$
$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}^{'})^{(7)} \quad 539$$
$$(R_2)^{(7)} = (b_{38}^{'})^{(7)} - (r_{38})^{(7)}$$

From CONCATENATED GLOBAL EQUATIONS we obtain 540

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}^{'})^{(7)} - (a_{37}^{'})^{(7)} + (a_{36}^{''})^{(7)}(T_{37}, t) \right) -$$
$$(a_{37}^{''})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_2)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)\leq\frac{d\nu^{(7)}}{dt}\leq$$

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

(m) For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

$it\ follows\ (\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 541

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

(n) If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 542

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

(o) If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 543

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{36}^{''})^{(7)} = (a_{37}^{''})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}^{''})^{(7)} = (b_{37}^{''})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then

$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

$$(b_{14})^{(1)} T_{13} - [(b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)}(G)\,] T_{14} = 0 \qquad 544$$

$$(b_{15})^{(1)} T_{14} - [(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G)\,] T_{15} = 0 \qquad 545$$

has a unique positive solution , which is an equilibrium solution for the system 546

$$(a_{16})^{(2)} G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17})\right] G_{16} = 0 \qquad 547$$

$$(a_{17})^{(2)} G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17})\right] G_{17} = 0 \qquad 548$$

$$(a_{18})^{(2)} G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17})\right] G_{18} = 0 \qquad 549$$

$$(b_{16})^{(2)} \mathrm{T}_{17} - [(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}(G_{19})\,] T_{16} = 0 \qquad 550$$

$$(b_{17})^{(2)} T_{16} - [(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}(G_{19})\,] T_{17} = 0 \qquad 551$$

$$(b_{18})^{(2)} T_{17} - [(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}(G_{19})\,] T_{18} = 0 \qquad 552$$

has a unique positive solution , which is an equilibrium solution for 553

$$(a_{20})^{(3)} G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})\right] G_{20} = 0 \qquad 554$$

$$(a_{21})^{(3)} G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})\right] G_{21} = 0 \qquad 555$$

$$(a_{22})^{(3)} G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})\right] G_{22} = 0 \qquad 556$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 557$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 558$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 559$$

has a unique positive solution , which is an equilibrium solution 560

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 561$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 563$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 564$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 565$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 566$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 567$$

has a unique positive solution , which is an equilibrium solution for the system 568

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 569$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 570$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 571$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 572$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 573$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 574$$

has a unique positive solution , which is an equilibrium solution for the system 575

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 576$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 577$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 578$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 579$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 580$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 584$$

has a unique positive solution , which is an equilibrium solution for the system 582

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 583$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 584$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 585$$

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$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 587$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(\mathrm{G}_{39})\,]T_{37} = 0 \quad 588$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 589$$

has a unique positive solution , which is an equilibrium solution for the system 560

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Definition and uniqueness of T^*_{37} :- 561

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

(e) By the same argument, the equations(SOLUTIONAL) admit solutions G_{36}, G_{37} if

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi(G^*) = 0$ 562

Finally we obtain the unique solution OF THE SYSTEM

$G_{37}^{*}\ given\ by\ \varphi((G_{39})^{*})=0$, $T_{37}^{*}\ given\ by\ f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a_{36}^{'})^{(7)}+(a_{36}^{''})^{(7)}(T_{37}^{*})]}\quad,\quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a_{38}^{'})^{(7)}+(a_{38}^{''})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b_{36}^{'})^{(7)}-(b_{36}^{''})^{(7)}((G_{39})^{*})]}\quad,\quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b_{38}^{'})^{(7)}-(b_{38}^{''})^{(7)}((G_{39})^{*})]} \qquad 563$$

Definition and uniqueness of T_{21}^{*} :- 564

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^{*} for which $f(T_{21}^{*})=0$. With this value , we obtain from the three first equations

$$G_{20}=\frac{(a_{20})^{(3)}G_{21}}{[(a_{20}^{'})^{(3)}+(a_{20}^{''})^{(3)}(T_{21}^{*})]}\quad,\quad G_{22}=\frac{(a_{22})^{(3)}G_{21}}{[(a_{22}^{'})^{(3)}+(a_{22}^{''})^{(3)}(T_{21}^{*})]}$$

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Definition and uniqueness of T_{25}^{*} :- 566

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^{*} for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a_{24}^{'})^{(4)}+(a_{24}^{''})^{(4)}(T_{25}^{*})]}\quad,\quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a_{26}^{'})^{(4)}+(a_{26}^{''})^{(4)}(T_{25}^{*})]}$$

Definition and uniqueness of T_{29}^{*} :- 567

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^{*} for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a_{28}^{'})^{(5)}+(a_{28}^{''})^{(5)}(T_{29}^{*})]}\quad,\quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a_{30}^{'})^{(5)}+(a_{30}^{''})^{(5)}(T_{29}^{*})]}$$

Definition and uniqueness of T_{33}^{*} :- 568

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a_{32}^{'})^{(6)}+(a_{32}^{''})^{(6)}(T_{33}^{*})]}\quad,\quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a_{34}^{'})^{(6)}+(a_{34}^{''})^{(6)}(T_{33}^{*})]}$$

(f) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 569

$$\varphi(G)=(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G)+(b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(\mathrm{G})\right]+(b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G)=0$$

Where in $G(G_{13},G_{14},G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0)>0$, $\varphi(\infty)<0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*})=0$

(g) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 570

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that 571
φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$

(a) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if 572

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that
φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$ 573

(b) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if 574

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that
φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(c) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if 575

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that
φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(d) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if 578

579

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

580

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

581

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94 582

G_{14}^* *given by* $\varphi(G^*) = 0$, T_{14}^* *given by* $f(T_{14}^*) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a_{13}^{'})^{(1)} + (a_{13}^{''})^{(1)} (T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)} (T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)} (G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)} (G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

G_{17}^{*} *given by* $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} *given by* $f(T_{17}^{*}) = 0$ and 584

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)} (\mathrm{T}_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)} (T_{17}^{*})]} \qquad 585$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)} ((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)} ((G_{19})^{*})]} \qquad 586$$

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution 588

G_{21}^{*} *given by* $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} *given by* $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)} (T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)} (T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)} (G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)} (G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 589

G_{25}^{*} *given by* $\varphi(G_{27}) = 0$, T_{25}^{*} *given by* $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)} (T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)} (T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)} ((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)} ((G_{27})^{*})]} \qquad 590$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 591

G_{29}^{*} *given by* $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} *given by* $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)} (T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)} (T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)} ((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)} ((G_{31})^{*})]} \qquad 592$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 593

G_{33}^{*} *given by* $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} *given by* $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)} (T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)} (T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)} ((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)} ((G_{35})^{*})]} \qquad 594$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 595

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(1)}\ and\ (b_i^{''})^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$
$$\frac{\partial (a_{14}^{''})^{(1)}}{\partial T_{14}} (T_{14}^{*}) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i^{''})^{(1)}}{\partial G_j} (G^{*}) = s_{ij} \qquad 596$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*} \mathbb{T}_{14} \qquad 598$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*} \mathbb{T}_{14} \qquad 599$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} \mathrm{G}_{15}^{*} \mathbb{T}_{14} \qquad 600$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*} \mathbb{G}_j\right) \qquad 601$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}^{'})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*} \mathbb{G}_j\right) \qquad 602$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*} \mathbb{G}_j\right) \qquad 603$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(2)}\ and\ (b_i^{''})^{(2)}$ 604
Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote 605

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i \qquad 606$$

$$\frac{\partial (a_{17}^{''})^{(2)}}{\partial T_{17}} (T_{17}^{*}) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b_i^{''})^{(2)}}{\partial G_j} ((G_{19})^{*}) = s_{ij} \qquad 607$$

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^{*} \mathbb{T}_{17} \qquad 609$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^{*} \mathbb{T}_{17} \qquad 610$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^{*} \mathbb{T}_{17} \qquad 611$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 612$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 613$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 614$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}\ and\ (b_i^{''})^{(3)}$ 615
Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i^{''})^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

616

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 618$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 619$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 6120$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 621$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 622$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 623$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(4)}\ and\ (b_i^{''})^{(4)}$ 624
Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 625

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}^{''})^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i^{''})^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 627$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 628$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 629$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 630$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 631$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 632$$

633

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}\ and\ (b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 634

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}^{''})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i^{''})^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 636$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 637$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 638$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 639$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 640$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 641$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}\ and\ (b_i^{''})^{(6)}$ 642
Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 643

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}^{''})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i^{''})^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 645$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 646$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 647$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 648$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 649$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 650$$

Obviously, these values represent an equilibrium solution of 79,20,36,22,23, 651

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(7)}\ and\ (b_i^{''})^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 652

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad 653$$

$$\frac{\partial(a_{37}^{''})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i^{''})^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain 654

655

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (\mathrm{a}_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 656$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 657$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 658$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 659$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 660$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 661$$

2. 662

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

+

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T^*_{17} + (b_{17})^{(2)}s_{(16),(17)}T^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T^*_{17} + (b_{17})^{(2)}s_{(16),(16)}T^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}G_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}T^*_{17} + (b_{17})^{(2)}s_{(16),(18)}T^*_{16}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(29)}T_{29}^{*}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a_{29}^{'})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G_{28}^{*}+(a_{28})^{(5)}(q_{29})^{(5)}G_{29}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(28)}T_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b_{28}^{'})^{(5)}+(b_{29}^{'})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^{*}+(\mathrm{a}_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(30)}T_{28}^{*}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b_{34}^{'})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a_{34}^{'})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a_{33}^{'})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G_{32}^{*}+(a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b_{32}^{'})^{(6)}+(b_{33}^{'})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b_{38}^{'})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a_{38}^{'})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a_{36}^{'})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(37)}T_{37}^{*}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a_{37}^{'})^{(7)}+(p_{37})^{(7)}\right)(\mathrm{q}_{36})^{(7)}\mathrm{G}_{36}^{*}+(\mathrm{a}_{36})^{(7)}(\mathrm{q}_{37})^{(7)}\mathrm{G}_{37}^{*}\right)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37} + (b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37} + (a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37} + (b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\} = 0$$

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≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a C_p of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average C_p of 25.8, 5.134 moles of metal, and 132 $J.K^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

(24)^ [3] Article on Earth rotation energy. Divided by c^2.

(25)^ **a b** Earth's gravitational self-energy is 4.6×10^{-10} that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.

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Acknowledgments:

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