

## EVENT, CAUSE, SPACE-TIME AND QUANTUM MEMORY REGISTER-AN AUGMENTATION-ARRONDISSEMENT MODEL

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**ABSTRACT:** We study a consolidated system of event; cause and n Qubit register which makes computation with n Qubits. Model extensively dilates upon systemic properties and analyses the systemic behaviour of the equations together with other concomitant properties. Inclusion of event and cause, we feel enhances the "Quantum ness" of the system holistically and brings out a relevance in the Quantum Computation on par with the classical system, in so far as the analysis is concerned. Additional VARIABLES OF Space Time provide bastion for the quantum space time studies.

### INTRODUCTION:

### EVENT AND ITS VINDICATION:

There definitely is a sense of compunction, contrition, hesitation, regret, remorse, hesitation and reservation to the **acknowledgement** of the fact that there is a personal **relation** to what **happens to** oneself. Louis de Broglie said that the events have already happened and it **shall disclose** to the people **based on** their level of consciousness. So there is destiny to start with! Say I am undergoing some seemingly insurmountable problem, which has hurt my sensibilities, susceptibilities and sentimentalities that I refuse to accept that that event was waiting for me to happen. In fact this is the statement of stoic philosophy which is referred to almost as bookish or abstract. Wound is there; it **had to happen** to me. So I was wounded. Stoics tell us that the wound **existed** before me; I was born **to embody** it. It is the question of consummation, consolidation, concretization, consubstantiation, that of this, that **creates an** "event" in us; thus you have **become a quasi cause** for this wound. For instance, my feeling to **become an** actor made me to behave with such perfectionism everywhere, that people's expectations rose and when I did not come up to them I **fell**; thus the 'wound' was waiting for me and "I" was waiting for the wound! One fellow professor used to say like you are searching for ideas, ideas also searching for you. Thus the wound **possesses in itself** a nature which is "impersonal and preindividual" in character, beyond general and particular, the collective and the private. It is the question of **becoming** universalistic and holistic in your outlook. Unless this fate had not befallen you, the "**grand design**" would not have taken place in its entire entirety. It had to happen. And the concomitant ramifications and pernicious or positive **implications**. Everything is in order **because the** fate befell you. It is not as if the wound had to get something that is best from me or that I am a chosen by God to face the event. As said earlier 'the grand design' would have **been altered**. And it **cannot alter**. You got to play your part and go; there is just no other way. The legacy must go on. You shall be torch bearer and you shall hand over the torch to somebody. This is the name of the game in totalistic and holistic way.

When it comes to ethics, I would say it makes no sense if any obstreperous, obstreperous, ululations, serenading, tintinnabulations are made for the event has happened to me. It means to say that you are unworthy of the fate that has befallen you. To feel that what happened to you was unwarranted and not autonomous, telling the world that you are aggressively iconoclastic, veritably resentful, and volitionally resentful, is choosing the cast of allegation aspersions and accusations at the Grand Design. What is immoral is to invoke the name of god, because some event has **happened to** you. Cursing him is immoral. Realize that it is all "grand design" and you are playing **a part**. Resignation, renunciation, revocation is only one form of resentment. Willing the event is primarily **to release** the eternal truth; in fact you cannot release an event despite the fact everyone tries all ways and means they pray god; they prostrate for others destitution, poverty, penury, misery. But **releasing** an event is something like an "action at a distance" which only super natural power can do.

Here we are face to face with volitional intuition and repetitive transmutation. Like a premeditated skirmisher, **one quarrel** with one self, with others, with god, and finally the accuser **leaves** this world in despair. Now look at this sentence which was quoted by I think Bousquet "if there is a **failure of will**", "I will **substitute a** longing for death" for that shall be apotheosis, a perpetual and progressive glorification

of the will.

## EVENT AND SINGULARITIES IN QUANTUM SYSTEMS:

What is an event? Or for that matter an ideal event? An event **is a** singularity or rather a set of singularities or set of singular points **characterizing a** mathematical curve, a physical state of affairs, a psychological person or a moral person. Singularities are turning points and points of inflection: they are bottle necks, foyers and centers; they are points of fusion; condensation and boiling; points of tears and joy; sickness and health; hope and anxiety; they are so to say "sensitive" points; such singularities should not be confused or confounded, aggravated or exacerbated with personality of a system expressing itself; or the individuality and idiosyncrasies of a system which is designated with a proposition. They should also **not be fused** with the generalizational concept or universalistic axiomatic predications and postulation alcovishness, or the dipsomaniac flageolet dirge of a concept. Possible a concept could be signified by a figurative representation or a schematic configuration. "Singularity is essentially, pre individual, and has no personalized bias in it, or for that matter a prejudice or pre circumspection of a conceptual scheme. It is in this sense **we can define a** "singularity" as being neither affirmative nor non affirmative. It can be positive or negative; it can **create or destroy**. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. They are in that sense "extra-ordinary".

Each singularity is a **source and resource**, the origin, reason and raison d'être of a mathematical series, it could be any series any type, and that is interpolated or extrapolated to the structural location of the **destination of** another singularity. This according to this standpoint, there are different. It can be positive or negative; it can create or destroy. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. There are in that sense "extra-ordinary".

This according to the widely held standpoint, there are different, multifarious, myriad, series IN A structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable **conclusions** that the "singularity" of one system is different from the "other system" in the subterranean realm and ceratoid dualism of comparison and contrast

EPR experiment derived that there exists a communications between two particles. We go a further step to say that there **exists a channel** of communication however slovenly, inept, clumpy, between the two singularities. It is also possible the communication exchange could be one of belligerence, cantankerousness, tempestuousness, astutely truculent, with ensorcelled frenzy. That does not matter. All we are telling is that singularities communicate with each other.

Now, how do find **the reaction** of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics". "intimidation of deriding report", or "cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner roil, eponymous radicalism without and with blitzzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidational **motion in** fluid flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniation and unwarranted(you think so but the system does not!) unrighteous fulminations.

So the point that is made here is "like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to understand their behaviour.

This statement is made in connection to the fact that there shall be **creation or destruction** of particles or complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like "inside traders" they will not put signature at all! How do you find they did it! Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing with systems to. This is accentuation, corroboration, fortification, .fomentatory notes to explain the various coefficients we have used in the model as also the dissipations called for

In the Bank example we have clarified that various systems are individually conservative, and their conservativeness extends holistically too. that one law is universal does not mean there is complete adjudication of **nonexistence of** totality or global or holistic figure. Total always exists and "individual" systems always exist, if we do not bring Kant in to picture! For the time being let us not! Equations would become more eneuretic and frenzied...

Various, myriad, series in a structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable

conclusions that the "singularity" of one system is different from the "other system" in the subterranean realm and ceratoid dualism of comparison and contrast.

### CONSERVATION LAWS:

Conservation laws bears ample testimony ,infallible observatory, and impeccable demonstration to the fact that the essential predications, character constitutions, ontological consonances remain unchanged with evolution despite the system's astute truculence, serenading whimsicality, assymetric disposition or on the other hand anachronistic dispensation ,eponymous radicality, entropic entrepotishness or the subdued ,relationally contributive, diverse parametrisizational, conducive reciprocity to environment, unconventional behaviour, eneuretic nonlinear frenetic ness ,ensorcelled frenzy, abnormal ebulliations, surcharged fulminations , or the inner roil. And that holds well with the evolution with time. We present a model of the generalizational conservation of the theories. A theory of all the conservation theories. That all conservation laws hold and there is no relationship between them is bête noir. We shall on this premise build a 36 storey model that deliberates on various issues, structural, dependent, thematic and discursive,

Note THAT The classification is executed on systemic properties and parameters. And everything that is known to us measurable. We do not know "intangible". Nor we accept or acknowledge that. All laws of conservation must hold. Hence the holistic laws must hold. Towards that end, interrelationships must exist. All science like law wants evidence and here we shall provide one under the premise that for all conservations laws to hold each must be interrelated to the other, lest the very conception is a fricative contretemps. And we live in "Measurement" world.

### QUANTUM REGISTER:

Devices that **harness and explore** the fundamental axiomatic predications of Physics has wide ranging amplitudunial **ramification** with its essence of locus and focus on information processing that outperforms their classical counterparts, and for unconditionally secure communication. However, in particular, implementations **based on** condensed-matter systems face the challenge of short coherence times. Carbon materials, particularly diamond, however, are suitable **for hosting** robust solid-state quantum registers, **owing to** their spin-free lattice and weak spin-orbit **coupling**. Studies with the structurally notched criticism and schizoid fragments of manifestations of historical perspective of diamond hosting quantum register have borne ample testimony and, and at differential and determinate levels have articulated the generalized significations and manifestations of quantum logic elements can **be realized** by exploring long-range magnetic dipolar coupling between individually addressable single electron spins **associated with** separate colour centres in diamond. The strong distance dependence of this coupling was used to characterize the separation of single qubits ( $98 \pm 3 \text{ \AA}$ ) with accuracy close to the value of the crystal-lattice spacing. Coherent **control over** electron spins, **conditional** dynamics, selective readout as well as switchable **interaction** should rip open glittering façade for a prosperous and scintillating irreducible affirmation of open the way towards a viable room-temperature solid-state quantum register. As both electron spins are optically **addressable**, this solid-state quantum device **operating at** ambient conditions **provides a** degree of **control** that is at present available only for a few systems at low temperature (See for instance P. Neumann, R. Kolesov, B. Naydenov, J. Berc F. Remp, M. Steiner V. Jacques,, G. Balasubramanian, M. M. L. Markham,, D. J. Twitchen,, S. Pezzagna,, J. Meijer, J. Twamley, F. Jelezko & J. Wrachtrup)

### CAUSE AND EVENT:

### MODULE NUMBERED ONE

### NOTATION :

$G_{13}$  : CATEGORY ONE OF CAUSE

$G_{14}$  : CATEGORY TWO OF CAUSE

$G_{15}$  : CATEGORY THREE OF CAUSE

$T_{13}$  : CATEGORY ONE OF EVENT

$T_{14}$  : CATEGORY TWO OF EVENT

$T_{15}$  :CATEGORY THREE OF EVENT

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**FIRST TWO CATEGORIES OF QUBITS COMPUTATION:**

**MODULE NUMBERED TWO:**

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$G_{16}$  : CATEGORY ONE OF FIRST SET OF QUBITS

$G_{17}$  : CATEGORY TWO OF FIRST SET OF QUBITS

$G_{18}$  : CATEGORY THREE OF FIRST SET OF QUBITS

$T_{16}$  :CATEGORY ONE OF SECOND SET OF QUBITS

$T_{17}$  : CATEGORY TWO OF SECOND SET OF QUBITS

$T_{18}$  : CATEGORY THREE OF SECOND SET OF QUBITS

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**THIRD SET OF QUBITS AND FOURTH SET OF QUBITS:**

**MODULE NUMBERED THREE:**

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$G_{20}$  : CATEGORY ONE OF THIRD SET OF QUBITS

$G_{21}$  :CATEGORY TWO OF THIRD SET OF QUBITS

$G_{22}$  : CATEGORY THREE OF THIRD SET OF QUBITS

$T_{20}$  : CATEGORY ONE OF FOURTH SET OF QUBITS

$T_{21}$  :CATEGORY TWO OF FOURTH SET OF QUBITS

$T_{22}$  : CATEGORY THREE OF FOURTH SET OF QUBITS

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**FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS**

**: MODULE NUMBERED FOUR:**

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$G_{24}$  : CATEGORY ONE OF FIFTH SET OF QUBITS  
 $G_{25}$  : CATEGORY TWO OF FIFTH SET OF QUBITS  
 $G_{26}$  : CATEGORY THREE OF FIFTH SET OF QUBITS  
 $T_{24}$  :CATEGORY ONE OF SIXTH SET OF QUBITS  
 $T_{25}$  :CATEGORY TWO OF SIXTH SET OF QUBITS  
 $T_{26}$  : CATEGORY THREE OF SIXTH SET OF QUBITS

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**SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS:**

**MODULE NUMBERED FIVE:**

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$G_{28}$  : CATEGORY ONE OF SEVENTH SET OF QUBITS  
 $G_{29}$  : CATEGORY TWO OF SEVENTH SET OF QUBITS  
 $G_{30}$  :CATEGORY THREE OF SEVENTH SET OF QUBITS  
 $T_{28}$  :CATEGORY ONE OF EIGHTH SET OF QUBITS  
 $T_{29}$  :CATEGORY TWO OF EIGHTH SET OF QUBITS  
 $T_{30}$  :CATEGORY THREE OF EIGHTH SET OF QUBITS

**(n-1)TH SET OF QUBITS AND nTH SET OF QUBITS :**

**MODULE NUMBERED SIX:**

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$G_{32}$  : CATEGORY ONE OF (n-1)TH SET OF QUBITS  
 $G_{33}$  : CATEGORY TWO OF (n-1)TH SET OF QUBITS  
 $G_{34}$  : CATEGORY THREE OF (N-1)TH SET OF QUBITS  
 $T_{32}$  : CATEGORY ONE OF n TH SET OF QUBITS  
 $T_{33}$  : CATEGORY TWO OF n TH SET OF QUBITS  
 $T_{34}$  : CATEGORY THREE OF n TH SET OF QUBITS

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GLOSSARY OF MODULE NUMBERED SEVEN

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$G_{36}$  : CATEGORY ONE OF TIME

$G_{37}$  : CATEGORY TWO OF TIME

$G_{38}$  : CATEGORY THREE OF TIME

$T_{36}$  : CATEGORY ONE OF SPACE

$T_{37}$  : CATEGORY TWO OF SPACE

$T_{38}$  : CATEGORY THREE OF SPACE

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$   
 $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$   
 $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$   
 $(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$   
 $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$   
 $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$   
 $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

**CAUSE AND EVENT:**

1

**MODULE NUMBERED ONE**

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 2$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 3$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 4$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 5$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 6$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 7$$

$$+(a''_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor} \quad 8$$

$$-(b''_{13})^{(1)}(G, t) = \text{First detritions factor}$$

**FIRST TWO CATEGORIES OF QUBITS COMPUTATION:**

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**MODULE NUMBERED TWO:**

The differential system of this model is now ( Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 10$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 11$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 12$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 13$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 14$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 15$$

$$+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \quad 16$$

$$-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor} \quad 17$$

**THIRD SET OF QUBITS AND FOURTH SET OF QUBITS:** 18

**MODULE NUMBERED THREE**

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 19$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 20$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 21$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 22$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 23$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 24$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor}$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor} \quad 25$$

**FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS** 26

**: MODULE NUMBERED FOUR**

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 27$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 28$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 29$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 30$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 31$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 32$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor} \quad 33$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor} \quad 34$$

**SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS:** 35

**MODULE NUMBERED FIVE**

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 36$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 37$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 38$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 39$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 40$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 41$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor} \quad 42$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor} \quad 43$$

**n-1)TH SET OF QUBITS AND nTH SET OF QUBITS :** 44

**MODULE NUMBERED SIX:** 45

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 46$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 47$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 48$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 49$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 50$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 51$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor} \quad 52$$

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### GOVERNING EQUATIONS:

The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)]G_{36} \quad 54$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)]G_{37} \quad 55$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)]G_{38} \quad 56$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)]T_{36} \quad 57$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)]T_{37} \quad 58$$

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$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)]T_{38} \quad 60$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor} \quad 61$$

$$-(b''_{36})^{(7)}((G_{39}), t) = \text{First detritions factor} \quad 62$$

**FIRST MODULE CONCATENATION:**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \quad | \quad (a''_{16})^{(2,2)}(T_{17}, t) \quad | \quad (a''_{20})^{(3,3)}(T_{21}, t) \\ \hline (a''_{24})^{(4,4,4,4)}(T_{25}, t) \quad | \quad (a''_{28})^{(5,5,5,5)}(T_{29}, t) \quad | \quad (a''_{32})^{(6,6,6,6)}(T_{33}, t) \\ \hline (a''_{36})^{(7)}(T_{37}, t) \end{array} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ \begin{array}{c} (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) \quad | \quad (a''_{17})^{(2,2)}(T_{17}, t) \quad | \quad (a''_{21})^{(3,3)}(T_{21}, t) \\ \hline (a''_{25})^{(4,4,4,4)}(T_{25}, t) \quad | \quad (a''_{29})^{(5,5,5,5)}(T_{29}, t) \quad | \quad (a''_{33})^{(6,6,6,6)}(T_{33}, t) \\ \hline (a''_{37})^{(7)}(T_{37}, t) \end{array} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) \quad | \quad (a''_{18})^{(2,2)}(T_{17}, t) \quad | \quad (a''_{22})^{(3,3)}(T_{21}, t) \\ \hline (a''_{26})^{(4,4,4,4)}(T_{25}, t) \quad | \quad (a''_{30})^{(5,5,5,5)}(T_{29}, t) \quad | \quad (a''_{34})^{(6,6,6,6)}(T_{33}, t) \\ \hline (a''_{38})^{(7)}(T_{37}, t) \end{array} \right] G_{15}$$

Where  $(a''_{13})^{(1)}(T_{14}, t)$ ,  $(a''_{14})^{(1)}(T_{14}, t)$ ,  $(a''_{15})^{(1)}(T_{14}, t)$  are first augmentation coefficients for category 1, 2 and 3

$(a''_{16})^{(2,2)}(T_{17}, t)$ ,  $(a''_{17})^{(2,2)}(T_{17}, t)$ ,  $(a''_{18})^{(2,2)}(T_{17}, t)$  are second augmentation coefficient for category 1, 2 and 3

$(a''_{20})^{(3,3)}(T_{21}, t)$ ,  $(a''_{21})^{(3,3)}(T_{21}, t)$ ,  $(a''_{22})^{(3,3)}(T_{21}, t)$  are third augmentation coefficient for category 1, 2 and 3

$(a''_{24})^{(4,4,4,4)}(T_{25}, t)$ ,  $(a''_{25})^{(4,4,4,4)}(T_{25}, t)$ ,  $(a''_{26})^{(4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficient for category 1, 2 and 3

$(a''_{28})^{(5,5,5,5)}(T_{29}, t)$ ,  $(a''_{29})^{(5,5,5,5)}(T_{29}, t)$ ,  $(a''_{30})^{(5,5,5,5)}(T_{29}, t)$  are fifth augmentation coefficient for category 1, 2 and 3

$(a''_{32})^{(6,6,6,6)}(T_{33}, t)$ ,  $(a''_{33})^{(6,6,6,6)}(T_{33}, t)$ ,  $(a''_{34})^{(6,6,6,6)}(T_{33}, t)$  are sixth augmentation coefficient for category 1, 2 and 3

$(a''_{36})^{(7)}(T_{37}, t)$ ,  $(a''_{37})^{(7)}(T_{37}, t)$ ,  $(a''_{38})^{(7)}(T_{37}, t)$  ARE SEVENTH AUGMENTATION COEFFICIENTS

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{13})^{(1)} - (b''_{16})^{(1)}(G, t) \quad | \quad - (b''_{36})^{(7)}(G_{39}, t) \quad | \quad - (b''_{20})^{(3,3)}(G_{23}, t) \\ \hline - (b''_{24})^{(4,4,4,4)}(G_{27}, t) \quad | \quad - (b''_{28})^{(5,5,5,5)}(G_{31}, t) \quad | \quad - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \\ \hline - (b''_{36})^{(7)}(G_{39}, t) \end{array} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ \begin{array}{c} (b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) \quad | \quad - (b''_{17})^{(2,2)}(G_{19}, t) \quad | \quad - (b''_{21})^{(3,3)}(G_{23}, t) \\ \hline - (b''_{25})^{(4,4,4,4)}(G_{27}, t) \quad | \quad - (b''_{29})^{(5,5,5,5)}(G_{31}, t) \quad | \quad - (b''_{33})^{(6,6,6,6)}(G_{35}, t) \\ \hline - (b''_{37})^{(7)}(G_{39}, t) \end{array} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \quad \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} \quad \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39}, t)} \end{array} \right] T_{15}$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7)}(G_{39}, t)}$   $\boxed{-(b''_{36})^{(7,7)}(G_{39}, t)}$   $\boxed{-(b''_{36})^{(7,7)}(G_{39}, t)}$  ARE SEVENTH DETRITION COEFFICIENTS

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ \begin{array}{ccc} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} & \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{15} \quad 63$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detrition coefficients for category 1, 2 and 3 64

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

**SECOND MODULE CONCATENATION:** 65

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ \begin{array}{ccc} (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} & \boxed{+(a''_{13})^{(1,1)}(T_{14}, t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37}, t)} \end{array} \right] G_{16} \quad 66$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[ \begin{array}{ccc} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) & + (a''_{14})^{(1,1)}(T_{14}, t) & + (a''_{21})^{(3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6,6)}(T_{33}, t) \\ & + (a''_{37})^{(7,7)}(T_{37}, t) & \end{array} \right] G_{17} \quad 67$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[ \begin{array}{ccc} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) & + (a''_{15})^{(1,1)}(T_{14}, t) & + (a''_{22})^{(3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6,6)}(T_{33}, t) \\ & + (a''_{38})^{(7,7)}(T_{37}, t) & \end{array} \right] G_{18} \quad 68$$

Where  $+(a''_{16})^{(2)}(T_{17}, t)$ ,  $+(a''_{17})^{(2)}(T_{17}, t)$ ,  $+(a''_{18})^{(2)}(T_{17}, t)$  are first augmentation coefficients for category 1, 2 and 3 69

$+(a''_{13})^{(1,1)}(T_{14}, t)$ ,  $+(a''_{14})^{(1,1)}(T_{14}, t)$ ,  $+(a''_{15})^{(1,1)}(T_{14}, t)$  are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21}, t)$ ,  $+(a''_{21})^{(3,3,3)}(T_{21}, t)$ ,  $+(a''_{22})^{(3,3,3)}(T_{21}, t)$  are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficient for category 1, 2 and 3 70

$+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)$  are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)$ ,  $+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)$ ,  $+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)$  are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7)}(T_{37}, t)$ ,  $+(a''_{37})^{(7,7)}(T_{37}, t)$ ,  $+(a''_{38})^{(7,7)}(T_{37}, t)$  ARE SEVENTH DETRITION COEFFICIENTS 71

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[ \begin{array}{ccc} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) & - (b''_{13})^{(1,1)}(G, t) & - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6,6)}(G_{35}, t) \\ & - (b''_{36})^{(7,7)}(G_{39}, t) & \end{array} \right] T_{16} \quad 72$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[ \begin{array}{ccc} (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) & - (b''_{14})^{(1,1)}(G, t) & - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{29})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{33})^{(6,6,6,6,6)}(G_{35}, t) \\ & - (b''_{37})^{(7,7)}(G_{39}, t) & \end{array} \right] T_{17} \quad 73$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[ \begin{array}{ccc} (b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) & - (b''_{15})^{(1,1)}(G, t) & - (b''_{22})^{(3,3,3)}(G_{23}, t) \\ - (b''_{26})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{30})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{34})^{(6,6,6,6,6)}(G_{35}, t) \\ & - (b''_{38})^{(7,7)}(G_{39}, t) & \end{array} \right] T_{18} \quad 74$$

where  $-(b''_{16})^{(2)}(G_{19}, t)$ ,  $-(b''_{17})^{(2)}(G_{19}, t)$ ,  $-(b''_{18})^{(2)}(G_{19}, t)$  are first detrition coefficients for category 1, 2 and 3 75

$-(b''_{13})^{(1,1)}(G, t)$ ,  $-(b''_{14})^{(1,1)}(G, t)$ ,  $-(b''_{15})^{(1,1)}(G, t)$  are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3)}(G_{23}, t)$ ,  $-(b''_{21})^{(3,3,3)}(G_{23}, t)$ ,  $-(b''_{22})^{(3,3,3)}(G_{23}, t)$  are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)$ ,  $-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)$ ,  $-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)$  are fourth detritions coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)$ ,  $-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)$ ,  $-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)$  are fifth detritions coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)$ ,  $-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)$ ,  $-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)$  are sixth detritions coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7)}(G_{39}, t)$ ,  $-(b''_{36})^{(7,7)}(G_{39}, t)$ ,  $-(b''_{36})^{(7,7)}(G_{39}, t)$  are seventh detrition coefficients

### THIRD MODULE CONCATENATION:

$$\frac{dG_{20}}{dt} = \tag{76}$$

$$(a_{20})^{(3)} G_{21} - \left[ \begin{array}{ccc} (a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) & + (a''_{16})^{(2,2,2)}(T_{17}, t) & + (a''_{13})^{(1,1,1)}(T_{14}, t) \\ + (a''_{24})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{36})^{(7,7,7)}(T_{37}, t) \end{array} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[ \begin{array}{ccc} (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t) & + (a''_{17})^{(2,2,2)}(T_{17}, t) & + (a''_{14})^{(1,1,1)}(T_{14}, t) \\ + (a''_{25})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{37})^{(7,7,7)}(T_{37}, t) \end{array} \right] G_{21} \tag{77}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[ \begin{array}{ccc} (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t) & + (a''_{18})^{(2,2,2)}(T_{17}, t) & + (a''_{15})^{(1,1,1)}(T_{14}, t) \\ + (a''_{26})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{38})^{(7,7,7)}(T_{37}, t) \end{array} \right] G_{22} \tag{78}$$

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$+(a''_{20})^{(3)}(T_{21}, t)$ ,  $+(a''_{21})^{(3)}(T_{21}, t)$ ,  $+(a''_{22})^{(3)}(T_{21}, t)$  are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17}, t)$ ,  $+(a''_{17})^{(2,2,2)}(T_{17}, t)$ ,  $+(a''_{18})^{(2,2,2)}(T_{17}, t)$  are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1)}(T_{14}, t)$ ,  $+(a''_{14})^{(1,1,1)}(T_{14}, t)$ ,  $+(a''_{15})^{(1,1,1)}(T_{14}, t)$  are third augmentation coefficients for category 1, 2 and 3 80

$+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)$  are fifth augmentation

coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)}$   $\boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)}$   $\boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)}$  are seventh augmentation coefficient 81

$$\frac{dT_{20}}{dt} = \left[ \begin{array}{ccc} (b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} & \boxed{-(b''_{36})^{(7,7,7)}(G_{19}, t)} & \boxed{-(b''_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ & \boxed{-(b''_{36})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{20} \quad 82$$

$$\frac{dT_{21}}{dt} = \left[ \begin{array}{ccc} (b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b''_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ & \boxed{-(b''_{37})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{21} \quad 83$$

$$\frac{dT_{22}}{dt} = \left[ \begin{array}{ccc} (b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b''_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ & \boxed{-(b''_{38})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{22} \quad 84$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$  are first detritions coefficients for category 1, 2 and 3 85

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$  are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39}, t)}$   $\boxed{-(b''_{37})^{(7,7,7)}(G_{39}, t)}$   $\boxed{-(b''_{38})^{(7,7,7)}(G_{39}, t)}$  are seventh detritions coefficients 86

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**FOURTH MODULE CONCATENATION:**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[ \begin{array}{c} (a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{24} \quad 87$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[ \begin{array}{c} (a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{25} \quad 88$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[ \begin{array}{c} (a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{26} \quad 89$$

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Where  $\boxed{(a''_{24})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a''_{25})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a''_{26})^{(4)}(T_{25}, t)}$  are first augmentation coefficients fo  
 $\boxed{+(a''_{28})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5)}(T_{29}, t)}$  are second augmentation coefficient  
 $\boxed{+(a''_{32})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6)}(T_{33}, t)}$  are third augmentation coefficient,  
 $\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$  ARE SEVENTH augmentation coefficients

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$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[ \begin{array}{c} (b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{24} \quad 93$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ \begin{array}{ccc} (b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)} \\ & & \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{25} \quad 94$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[ \begin{array}{ccc} (b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)} \\ & & \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{26} \quad 95$$

Where  $\boxed{-(b''_{24})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4)}(G_{27}, t)}$  are first detrition coefficients for 96

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$  are second detrition coefficients for

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$  are third detrition coefficients for c

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$   
are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$   
are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$   
are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39}, t)}$   $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39}, t)}$   $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39}, t)}$  ARE SEVENTH  
DETRITION

COEFFICIENTS

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**FIFTH MODULE CONCATENATION:**

98

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[ \begin{array}{ccc} (a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ & & \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{28} \quad 99$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[ \begin{array}{ccc} (a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29}, t)} & \boxed{+(a''_{25})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)} \\ & & \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{29} \quad 100$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[ \begin{array}{ccc} (a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29}, t)} & \boxed{+(a''_{26})^{(4,4)}(T_{25}, t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)} \\ & & \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{30} \quad 101$$

Where  $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$  are first augmentation coefficient 102

And  $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$  are second augmentation coeffi

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$  are third augmentation coefficient

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1,2, 3

103

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[ \begin{array}{ccc} \boxed{(b'_{28})^{(5)}} \boxed{-(b''_{28})^{(5)}(G_{31}, t)} & \boxed{-(b''_{24})^{(4,4)}(G_{23}, t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)} \\ & \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{38}, t)} & \end{array} \right] T_{28} \quad 104$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[ \begin{array}{ccc} \boxed{(b'_{29})^{(5)}} \boxed{-(b''_{29})^{(5)}(G_{31}, t)} & \boxed{-(b''_{25})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)} \\ & \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{38}, t)} & \end{array} \right] T_{29} \quad 105$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[ \begin{array}{ccc} \boxed{(b'_{30})^{(5)}} \boxed{-(b''_{30})^{(5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)} \\ & \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{38}, t)} & \end{array} \right] T_{30} \quad 106$$

where  $\boxed{-(b''_{28})^{(5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5)}(G_{31}, t)}$  are first detrition coefficients for category 1, 2 and 3 107

$\boxed{-(b''_{24})^{(4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4)}(G_{27}, t)}$  are second detrition coefficients for

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)}$  are third detrition coefficients for

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)}$  are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)}$  are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)}$  are sixth detrition coefficients for category 1,2, and 3

**SIXTH MODULE CONCATENATION**

108

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[ \begin{array}{ccc} \boxed{(a'_{32})^{(6)}} \boxed{+(a''_{32})^{(6)}(T_{33}, t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ & \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)} & \end{array} \right] G_{32} \quad 109$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} \tag{110}$$

$$- \left[ \begin{array}{ccc} (a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t) & + (a''_{29})^{(5,5,5)}(T_{29}, t) & + (a''_{25})^{(4,4,4)}(T_{25}, t) \\ + (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t) \\ & + (a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t) & \end{array} \right] G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} \tag{111}$$

$$- \left[ \begin{array}{ccc} (a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t) & + (a''_{30})^{(5,5,5)}(T_{29}, t) & + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) \\ & + (a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t) & \end{array} \right] G_{34}$$

$+(a''_{32})^{(6)}(T_{33}, t), +(a''_{33})^{(6)}(T_{33}, t), +(a''_{34})^{(6)}(T_{33}, t)$  are first augmentation coefficients for  $c$  112

$+(a''_{28})^{(5,5,5)}(T_{29}, t), +(a''_{29})^{(5,5,5)}(T_{29}, t), +(a''_{30})^{(5,5,5)}(T_{29}, t)$  are second augmentation coefficients

$+(a''_{24})^{(4,4,4)}(T_{25}, t), +(a''_{25})^{(4,4,4)}(T_{25}, t), +(a''_{26})^{(4,4,4)}(T_{25}, t)$  are third augmentation coefficients

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$  - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$  - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$  sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t), +(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t), +(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t)$  ARE SEVENTH AUGMENTATION COEFFICIENTS

113

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[ \begin{array}{ccc} (b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}, t) & - (b''_{28})^{(5,5,5)}(G_{31}, t) & - (b''_{24})^{(4,4,4)}(G_{27}, t) \\ - (b''_{13})^{(1,1,1,1,1,1)}(G, t) & - (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) \\ & - (b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t) & \end{array} \right] T_{32} \tag{114}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[ \begin{array}{ccc} (b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}, t) & - (b''_{29})^{(5,5,5)}(G_{31}, t) & - (b''_{25})^{(4,4,4)}(G_{27}, t) \\ - (b''_{14})^{(1,1,1,1,1,1)}(G, t) & - (b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t) \\ & - (b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t) & \end{array} \right] T_{33} \tag{115}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[ \begin{array}{ccc} (b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}, t) & - (b''_{30})^{(5,5,5)}(G_{31}, t) & - (b''_{26})^{(4,4,4)}(G_{27}, t) \\ - (b''_{15})^{(1,1,1,1,1,1)}(G, t) & - (b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t) \\ & - (b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t) & \end{array} \right] T_{34} \tag{116}$$

$-(b''_{32})^{(6)}(G_{35}, t)$ ,  $-(b''_{33})^{(6)}(G_{35}, t)$ ,  $-(b''_{34})^{(6)}(G_{35}, t)$  are first detrition coefficients for cate 117

$-(b''_{28})^{(5,5,5)}(G_{31}, t)$ ,  $-(b''_{29})^{(5,5,5)}(G_{31}, t)$ ,  $-(b''_{30})^{(5,5,5)}(G_{31}, t)$  are second detrition coefficients .

$-(b''_{24})^{(4,4,4)}(G_{27}, t)$ ,  $-(b''_{25})^{(4,4,4)}(G_{27}, t)$ ,  $-(b''_{26})^{(4,4,4)}(G_{27}, t)$  are third detrition coefficients f

$-(b''_{13})^{(1,1,1,1,1,1)}(G, t)$ ,  $-(b''_{14})^{(1,1,1,1,1,1)}(G, t)$ ,  $-(b''_{15})^{(1,1,1,1,1,1)}(G, t)$  are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)$ ,  $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)$ ,  $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)$  are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)$ ,  $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)$ ,  $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)$  are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)$ ,  $-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)$ ,  $-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)$  ARE SEVENTH DETRITION COEFFICIENTS

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SEVENTH MODULE CONCATENATION:

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$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) + (a''_{16})^{(7)}(T_{17}, t) + (a''_{20})^{(7)}(T_{21}, t) + (a''_{24})^{(7)}(T_{23}, t)G_{36} + (a''_{28})^{(7)}(T_{29}, t) + (a''_{32})^{(7)}(T_{33}, t) + (a''_{13})^{(7)}(T_{14}, t)] G_{36}$$

120

121

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t) + (a''_{14})^{(7)}(T_{14}, t) + (a''_{21})^{(7)}(T_{21}, t) + (a''_{17})^{(7)}(T_{17}, t) + (a''_{25})^{(7)}(T_{25}, t) + (a''_{33})^{(7)}(T_{33}, t) + (a''_{29})^{(7)}(T_{29}, t)] G_{37}$$

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Type equation here.

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t) + (a''_{15})^{(7)}(T_{14}, t) + (a''_{22})^{(7)}(T_{21}, t) + (a''_{18})^{(7)}(T_{17}, t) + (a''_{26})^{(7)}(T_{25}, t) + (a''_{34})^{(7)}(T_{33}, t) + (a''_{30})^{(7)}(T_{29}, t)] G_{38}$$

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124

125

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{16})^{(7)}((G_{17}), t) - (b''_{13})^{(7)}((G_{14}), t) - ]T_{36}$$

126

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)]T_{37}$$

127

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)]T_{38}$$

128

129  
 130  
 131  
 132  
 134  
 $+(a_{36}^{''})^{(7)}(T_{37}, t) = \text{First augmentation factor}$

(A)  $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16, 17, 18$  135

(B) The functions  $(a_i'')^{(2)}, (b_i'')^{(2)}$  are positive continuous increasing and bounded. 136

**Definition of  $(p_i)^{(2)}, (r_i)^{(2)}$ :** 137

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 138$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 139$$

(C)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$  140

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 141$$

**Definition of  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ :** 142

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16, 17, 18$

They satisfy Lipschitz condition: 143

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17}' - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 144$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G_{19})' - (G_{19})| e^{-(\hat{M}_{16})^{(2)}t} \quad 145$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(2)}(T_{17}', t)$  and  $(a_i'')^{(2)}(T_{17}, t)$ .  $(T_{17}', t)$  and  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a_i'')^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a_i'')^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :** 147

(D)  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ , are positive constants 148

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of  $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$ :** 149

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 150$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 151$$

Where we suppose 152

$$(E) \quad (a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22 \quad 153$$

The functions  $(a_i'')^{(3)}, (b_i'')^{(3)}$  are positive continuous increasing and bounded.

**Definition of  $(p_i)^{(3)}, (r_i)^{(3)}$ :**

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 154$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)} \quad 155$$

**Definition of  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$  :** 156

Where  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$  are positive constants and  $i = 20, 21, 22$

They satisfy Lipschitz condition: 157

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)}t} \quad 158$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} |G_{23} - G_{23}'| e^{-(\hat{M}_{20})^{(3)}t} \quad 159$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(3)}(T_{21}', t)$  and  $(a_i'')^{(3)}(T_{21}, t)$ .  $(T_{21}', t)$  and  $(T_{21}, t)$  are points belonging to the interval  $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$ . It is to be noted that  $(a_i'')^{(3)}(T_{21}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{20})^{(3)} = 1$  then the function  $(a_i'')^{(3)}(T_{21}, t)$ , the THIRD augmentation coefficient, would be absolutely continuous. 160

**Definition of  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$  :** 161

(F)  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ , are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants  $(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  which together with 162

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$  and  $(\hat{B}_{20})^{(3)}$  and the constants 163

$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22,$

satisfy the inequalities 164

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1 \quad 165$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1 \quad 166$$

167

Where we suppose 168

$$(G) \quad (a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24, 25, 26 \quad 169$$

(H) The functions  $(a_i'')^{(4)}, (b_i'')^{(4)}$  are positive continuous increasing and bounded.

**Definition of  $(p_i)^{(4)}, (r_i)^{(4)}$ :**

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

170

(I)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$   
 $\lim_{G \rightarrow \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$

**Definition of**  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$  :

Where  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$  are positive constants and  $i = 24, 25, 26$

They satisfy Lipschitz condition:

171

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25}' - T_{25}| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} |(G_{27})' - (G_{27})| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(4)}(T_{25}', t)$  and  $(a_i'')^{(4)}(T_{25}, t)$ .  $(T_{25}', t)$  and  $(T_{25}, t)$  are points belonging to the interval  $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$ . It is to be noted that  $(a_i'')^{(4)}(T_{25}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{24})^{(4)} = 4$  then the function  $(a_i'')^{(4)}(T_{25}, t)$ , the **FOURTH augmentation coefficient WOULD** be absolutely continuous.

172

173

**Defi174nition of**  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$  :

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ , are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of**  $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$  :

175

(L) There exists two constants  $(\hat{P}_{24})^{(4)}$  and  $(\hat{Q}_{24})^{(4)}$  which together with  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$  and  $(\hat{B}_{24})^{(4)}$  and the constants  $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

176

(N)  $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, i, j = 28, 29, 30$

177

(N) The functions  $(a_i'')^{(5)}, (b_i'')^{(5)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(5)}, (r_i)^{(5)}$ :

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

178

$$(O) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of**  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$  :

Where  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$  are positive constants and  $i = 28, 29, 30$

They satisfy Lipschitz condition:

179

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}(G_{31}', t) - (b_i'')^{(5)}(G_{31}, t)| < (\hat{k}_{28})^{(5)} |(G_{31}) - (G_{31}')| e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(5)}(T_{29}', t)$  and  $(a_i'')^{(5)}(T_{29}, t)$ .  $(T_{29}', t)$  and  $(T_{29}, t)$  are points belonging to the interval  $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$ . It is to be noted that  $(a_i'')^{(5)}(T_{29}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{28})^{(5)} = 5$  then the function  $(a_i'')^{(5)}(T_{29}, t)$ , the **FIFTH augmentation coefficient** attributable would be absolutely continuous.

**Definition of**  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$  :

181

$$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, \text{ are positive constants}$$

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of**  $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$  :

182

There exists two constants  $(\hat{P}_{28})^{(5)}$  and  $(\hat{Q}_{28})^{(5)}$  which together with  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$  and  $(\hat{B}_{28})^{(5)}$  and the constants  $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

183

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32, 33, 34$$

184

(R) The functions  $(a_i'')^{(6)}, (b_i'')^{(6)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(6)}, (r_i)^{(6)}$ :

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}(G_{35}, t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

185

$$(S) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(6)}(G_{35}, t) = (r_i)^{(6)}$$

**Definition of**  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$  :

Where  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$  are positive constants and  $i = 32, 33, 34$

They satisfy Lipschitz condition: 186

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} \|(G_{35})' - (G_{35})\| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(6)}(T_{33}', t)$  187  
 and  $(a_i'')^{(6)}(T_{33}, t)$ .  $(T_{33}', t)$  and  $(T_{33}, t)$  are points belonging to the interval  $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$ . It is  
 to be noted that  $(a_i'')^{(6)}(T_{33}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{32})^{(6)} =$   
 6 then the function  $(a_i'')^{(6)}(T_{33}, t)$ , the **SIXTH augmentation coefficient** would be absolutely  
 continuous.

**Definition of**  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$  : 188

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ , are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of**  $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$  : 189

There exists two constants  $(\hat{P}_{32})^{(6)}$  and  $(\hat{Q}_{32})^{(6)}$  which together with  
 $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$  and  $(\hat{B}_{32})^{(6)}$  and the constants  
 $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$ ,  
 satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

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**Theorem 1:** if the conditions IN THE FOREGOING above are fulfilled, there exists a solution 191  
 satisfying the conditions

Definition of  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

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**Definition of**  $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

194

195

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad T_i(0) = T_i^0 > 0$$

**Definition of**  $G_i(0), T_i(0)$  : 196

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

197

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

198

**Definition of**  $G_i(0), T_i(0)$  :

199

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator  $\mathcal{A}^{(1)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy 200

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 201$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 202$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 203$$

By 204

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13}(1)(T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + a''_{14}(1)(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 205$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + a''_{15}(1)(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \quad 206$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - b''_{13}(1)(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 207$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - b''_{14}(1)(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 208$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - b''_{15}(1)(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 209$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

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**Proof:**

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Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \tag{212}$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \tag{213}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \tag{214}$$

By 215

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(T_{17}(s_{(16)}), s_{(16)})) G_{16}(s_{(16)}) \right) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + a''_{17}(T_{17}(s_{(16)}), s_{(17)})) G_{17}(s_{(16)}) \right) \right] ds_{(16)} \tag{216}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + a''_{18}(T_{17}(s_{(16)}), s_{(16)})) G_{18}(s_{(16)}) \right) \right] ds_{(16)} \tag{217}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - b''_{16}(G(s_{(16)}), s_{(16)})) T_{16}(s_{(16)}) \right) \right] ds_{(16)} \tag{218}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - b''_{17}(G(s_{(16)}), s_{(16)})) T_{17}(s_{(16)}) \right) \right] ds_{(16)} \tag{219}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - b''_{18}(G(s_{(16)}), s_{(16)})) T_{18}(s_{(16)}) \right) \right] ds_{(16)} \tag{220}$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

**Proof:**

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Consider operator  $\mathcal{A}^{(3)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \tag{222}$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \tag{223}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \tag{224}$$

By 225

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20}(T_{21}(s_{(20)}), s_{(20)})) G_{20}(s_{(20)}) \right) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + a''_{21}(T_{21}(s_{(20)}), s_{(20)})) G_{21}(s_{(20)}) \right) \right] ds_{(20)} \tag{226}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + a''_{22}(T_{21}(s_{(20)}), s_{(20)})) G_{22}(s_{(20)}) \right) \right] ds_{(20)} \tag{227}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - b''_{20}(G(s_{(20)}), s_{(20)})) T_{20}(s_{(20)}) \right) \right] ds_{(20)} \tag{228}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - b''_{21}(G(s_{(20)}), s_{(20)})) T_{21}(s_{(20)}) \right) \right] ds_{(20)} \tag{229}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - b''_{22}(G(s_{(20)}), s_{(20)})) T_{22}(s_{(20)}) \right) \right] ds_{(20)} \tag{230}$$

Where  $s_{(20)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(4)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy 231

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)}, \quad 232$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 233$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 234$$

By 235

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)} G_{24}(s_{(24)}) - \left( (a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 236$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 237$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 238$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)} T_{24}(s_{(24)}) - \left( (b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 239$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 240$$

Where  $s_{(24)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(5)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy 241

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 243$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 244$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 245$$

By 246

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 247$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 248$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 249$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 250$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 251$$

Where  $s_{(28)}$  is the integrand that is integrated over an interval  $(0, t)$

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Consider operator  $\mathcal{A}^{(6)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)}, \quad 253$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 254$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 255$$

By 256

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32}(T_{33}(s_{(32)}), s_{(32)})) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + a''_{33}(T_{33}(s_{(32)}), s_{(32)})) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 257$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + a''_{34}(T_{33}(s_{(32)}), s_{(32)})) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 258$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)}(G(s_{(32)}), s_{(32)})) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 259$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)}(G(s_{(32)}), s_{(32)})) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 260$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)}(G(s_{(32)}), s_{(32)})) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 261$$

Where  $s_{(32)}$  is the integrand that is integrated over an interval  $(0, t)$

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(a) The operator  $\mathcal{A}^{(1)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 263  
 Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left( 1 + (a_{13})^{(1)}t \right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left( e^{(\hat{M}_{13})^{(1)}t} - 1 \right)$$

From which it follows that 264

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left( -\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

Analogous inequalities hold also for  $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$  265

(b) The operator  $\mathcal{A}^{(2)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 266  
 Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}s_{(16)}} \right) \right] ds_{(16)} = \left( 1 + (a_{16})^{(2)}t \right) G_{17}^0 +$$

$$\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left( e^{(\hat{M}_{16})^{(2)}t} - 1 \right) \quad 267$$

From which it follows that 268

$$(G_{16}(t) - G_{16}^0)e^{-(\bar{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ ((\hat{P}_{16})^{(2)} + G_{17}^0)e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for  $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$  269

(a) The operator  $\mathcal{A}^{(3)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 270  
 Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left( G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}s(20)} \right) \right] ds_{(20)} =$$

$$\left( 1 + (a_{20})^{(3)}t \right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\bar{M}_{20})^{(3)}} \left( e^{(\bar{M}_{20})^{(3)}t} - 1 \right)$$

From which it follows that 271

$$(G_{20}(t) - G_{20}^0)e^{-(\bar{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ ((\hat{P}_{20})^{(3)} + G_{21}^0)e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for  $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$  272

(b) The operator  $\mathcal{A}^{(4)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 273  
 Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} \left( G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)}s(24)} \right) \right] ds_{(24)} =$$

$$\left( 1 + (a_{24})^{(4)}t \right) G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left( e^{(\bar{M}_{24})^{(4)}t} - 1 \right)$$

From which it follows that 274

$$(G_{24}(t) - G_{24}^0)e^{-(\bar{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\bar{M}_{24})^{(4)}} \left[ ((\hat{P}_{24})^{(4)} + G_{25}^0)e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

(c) The operator  $\mathcal{A}^{(5)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 275  
 Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}s(28)} \right) \right] ds_{(28)} =$$

$$\left( 1 + (a_{28})^{(5)}t \right) G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left( e^{(\bar{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that 276

$$(G_{28}(t) - G_{28}^0)e^{-(\bar{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left[ ((\hat{P}_{28})^{(5)} + G_{29}^0)e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

(d) The operator  $\mathcal{A}^{(6)}$  maps the space of functions satisfying GLOBAL EQUATIONS into itself 277  
 Indeed it is obvious that

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}s(32)} \right) \right] ds_{(32)} =$$

$$(1 + (a_{32})^{(6)}t)G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\bar{M}_{32})^{(6)}}(e^{(\bar{M}_{32})^{(6)}t} - 1)$$

From which it follows that 278

$$(G_{32}(t) - G_{32}^0)e^{-(\bar{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left[ ((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 6

Analogous inequalities hold also for  $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

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It is now sufficient to take  $\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} < 1$  and to choose 281

$(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  large to have 282

$$\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{13})^{(1)} \quad 283$$

$$\frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[ ((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \quad 284$$

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying GLOBAL EQUATIONS into itself 285

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric 286

$$d((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t} \}$$

Indeed if we denote 287

**Definition of  $\tilde{G}, \tilde{T}$  :**

$$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{-(\bar{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} | (a'_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) - (a'_{13})^{(1)}(T_{14}^{(2)}, s_{(13)}) | e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\left| G^{(1)} - G^{(2)} \right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}} \left( (a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)} \right) d \left( (G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \quad 288$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{13})^{(1)}$  and  $(b''_{13})^{(1)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  and  $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  respectively of  $\mathbb{R}_+$ . 289

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(1)}$  and  $(b''_i)^{(1)}$ ,  $i = 13, 14, 15$  depend only on  $T_{14}$  and respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  290

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[ - \int_0^t \{ (a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \} ds_{(13)} \right]} \geq 0 \quad 291$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \quad \text{for } t > 0$$

**Definition of  $((\widehat{M}_{13})^{(1)})_1$ , and  $((\widehat{M}_{13})^{(1)})_3$ :** 292

**Remark 3:** if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$$G_{13} < ((\widehat{M}_{13})^{(1)})_1 \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}$ ,  $G_{15}$  and  $G_{13}$ ,  $G_{14}$  respectively.

**Remark 4:** If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below. 293

**Remark 5:** If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$  then  $T_{14} \rightarrow \infty$ . 294

**Definition of  $(m)^{(1)}$  and  $\varepsilon_1$ :**

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 295

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_1}$  By taking now  $\varepsilon_1$  sufficiently small one sees that  $T_{14}$  is unbounded. The same property holds for  $T_{15}$  if  $\lim_{t \rightarrow \infty} (b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

296

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\overline{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\overline{M}_{16})^{(2)}} < 1$  and to choose

297

$(\widehat{P}_{16})^{(2)}$  and  $(\widehat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\overline{M}_{16})^{(2)}} \left[ (\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{16})^{(2)}$$

298

299

$$\frac{(b_i)^{(2)}}{(\overline{M}_{16})^{(2)}} \left[ ((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)}$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying

300

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric

301

$$d\left( (G_{19})^{(1)}, (T_{19})^{(1)}, (G_{19})^{(2)}, (T_{19})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\overline{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\overline{M}_{16})^{(2)}t} \right\}$$

302

Indeed if we denote

302

**Definition of  $\widetilde{G}_{19}, \widetilde{T}_{19}$  :**  $(\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

303

$$\begin{aligned} |\widetilde{G}_{16}^{(1)} - \widetilde{G}_{16}^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\overline{M}_{16})^{(2)}s_{(16)}} e^{(\overline{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\overline{M}_{16})^{(2)}s_{(16)}} e^{-(\overline{M}_{16})^{(2)}s_{(16)}} + \\ &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\overline{M}_{16})^{(2)}s_{(16)}} e^{(\overline{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} |(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)})| e^{-(\overline{M}_{16})^{(2)}s_{(16)}} e^{(\overline{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

304

From the hypotheses it follows

$$\begin{aligned} |(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\overline{M}_{16})^{(2)}t} &\leq \\ &\frac{1}{(\overline{M}_{16})^{(2)}} \left( (a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + \right. \\ &\left. (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)} \right) d\left( (G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)} \right) \end{aligned}$$

305

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

306

**Remark 1:** The fact that we supposed  $(a''_{16})^{(2)}$  and  $(b''_{16})^{(2)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{16})^{(2)} e^{(\overline{M}_{16})^{(2)}t}$  and  $(\widehat{Q}_{16})^{(2)} e^{(\overline{M}_{16})^{(2)}t}$  respectively of  $\mathbb{R}_+$ .

307

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then

it suffices to consider that  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$ ,  $i = 16, 17, 18$  depend only on  $T_{17}$  and respectively on  $(G_{19})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  308

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(2)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{16})^{(2)})_1$ ,  $((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 309

**Remark 3:** if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$ . indeed if

$G_{16} < ((\widehat{M}_{16})^{(2)})_1$  it follows  $\frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a_{17}')^{(2)}G_{17}$  and by integrating

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a_{17}')^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a_{18}')^{(2)} \quad 310$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$ ,  $G_{18}$  and  $G_{16}$ ,  $G_{17}$  respectively.

**Remark 4:** If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below. 311

**Remark 5:** If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$  then  $T_{17} \rightarrow \infty$ . 312

**Definition of**  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17}')^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$  which leads to 313

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_2} \text{ By taking now } \varepsilon_2 \text{ sufficiently small one sees that } T_{17} \text{ is} \quad 314$$

unbounded. The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} (b_{18}'')^{(2)}((G_{19})(t), t) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

315

It is now sufficient to take  $\frac{(a_i)^{(3)}}{(M_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(M_{20})^{(3)}} < 1$  and to choose 316

$(\widehat{P}_{20})^{(3)}$  and  $(\widehat{Q}_{20})^{(3)}$  large to have

$$\frac{(a_i)^{(3)}}{(M_{20})^{(3)}} \left[ (\widehat{P}_{20})^{(3)} + ((\widehat{P}_{20})^{(3)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{20})^{(3)} \quad 317$$

$$\frac{(b_i)^{(3)}}{(\overline{M}_{20})^{(3)}} \left[ ((\widehat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)} \right] \leq (\widehat{Q}_{20})^{(3)} \quad 318$$

In order that the operator  $\mathcal{A}^{(3)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself 319

The operator  $\mathcal{A}^{(3)}$  is a contraction with respect to the metric 320

$$d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\overline{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\overline{M}_{20})^{(3)}t} \} \quad 321$$

Indeed if we denote 321

**Definition of  $\widetilde{G}_{23}, \widetilde{T}_{23}$  :**  $((\widetilde{G}_{23}), (\widetilde{T}_{23})) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 322

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\overline{M}_{20})^{(3)}s_{(20)}} e^{(\overline{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\overline{M}_{20})^{(3)}s_{(20)}} e^{-(\overline{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\overline{M}_{20})^{(3)}s_{(20)}} e^{(\overline{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\overline{M}_{20})^{(3)}s_{(20)}} e^{(\overline{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned} \quad 323$$

Where  $s_{(20)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\overline{M}_{20})^{(3)}t} &\leq \\ &\frac{1}{(\overline{M}_{20})^{(3)}} \left( (a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + \right. \\ &\left. (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}); (G_{23})^{(2)}, (T_{23})^{(2)} \right) \end{aligned} \quad 324$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{20})^{(3)}$  and  $(b''_{20})^{(3)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{20})^{(3)} e^{(\overline{M}_{20})^{(3)}t}$  and  $(\widehat{Q}_{20})^{(3)} e^{(\overline{M}_{20})^{(3)}t}$  respectively of  $\mathbb{R}_+$ . 325

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(3)}$  and  $(b''_i)^{(3)}$ ,  $i = 20, 21, 22$  depend only on  $T_{21}$  and respectively on  $(G_{23})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  326

From 19 to 24 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{\left[ - \int_0^t \{ (a'_i)^{(3)} - (a''_i)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \} ds_{(20)} \right]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(3)}t} > 0 \quad \text{for } t > 0 \\ \text{Definition of } &((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2 \text{ and } ((\widehat{M}_{20})^{(3)})_3 : \end{aligned} \quad 327$$

**Remark 3:** if  $G_{20}$  is bounded, the same property have also  $G_{21}$  and  $G_{22}$  . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$  it follows  $\frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21}$  and by integrating

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If  $G_{21}$  or  $G_{22}$  is bounded, the same property follows for  $G_{20}$  ,  $G_{22}$  and  $G_{20}$  ,  $G_{21}$  respectively.

**Remark 4:** If  $G_{20}$  is bounded, from below, the same property holds for  $G_{21}$  and  $G_{22}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{21}$  is bounded from below. 328

**Remark 5:** If  $T_{20}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(3)}((G_{23})(t), t)) = (b'_i)^{(3)}$  then  $T_{21} \rightarrow \infty$ . 329

**Definition of  $(m)^{(3)}$  and  $\varepsilon_3$  :** 330

Indeed let  $t_3$  be so that for  $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then  $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$  which leads to 331

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$

If we take  $t$  such that  $e^{-\varepsilon_3 t} = \frac{1}{2}$  it results

$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_3}$  By taking now  $\varepsilon_3$  sufficiently small one sees that  $T_{21}$  is unbounded. The same property holds for  $T_{22}$  if  $\lim_{t \rightarrow \infty} (b''_{22})^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions 332

It is now sufficient to take  $\frac{(a_i)^{(4)}}{(M_{24})^{(4)}} , \frac{(b_i)^{(4)}}{(M_{24})^{(4)}} < 1$  and to choose 333

$(\widehat{P}_{24})^{(4)}$  and  $(\widehat{Q}_{24})^{(4)}$  large to have

$$\frac{(a_i)^{(4)}}{(M_{24})^{(4)}} \left[ (\widehat{P}_{24})^{(4)} + ((\widehat{P}_{24})^{(4)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{24})^{(4)} \quad 334$$

$$\frac{(b_i)^{(4)}}{(M_{24})^{(4)}} \left[ ((\widehat{Q}_{24})^{(4)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{24})^{(4)} \right] \leq (\widehat{Q}_{24})^{(4)} \quad 335$$

In order that the operator  $\mathcal{A}^{(4)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying IN to itself 336

The operator  $\mathcal{A}^{(4)}$  is a contraction with respect to the metric 337

$$d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t} \}$$

Indeed if we denote

**Definition of**  $(\widehat{G}_{27}), (\widehat{T}_{27}) : ((\widehat{G}_{27}), (\widehat{T}_{27})) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\begin{aligned} |\widetilde{G}_{24}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t \{ (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} | (a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)}) | e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} \} ds_{(24)} \end{aligned}$$

Where  $s_{(24)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

338

$$\begin{aligned} |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\widehat{M}_{24})^{(4)}t} &\leq \\ \frac{1}{(\widehat{M}_{24})^{(4)}} &((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + \\ (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}) &d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}); ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) \end{aligned}$$

339

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{24})^{(4)}$  and  $(b''_{24})^{(4)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$  and  $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$  respectively of  $\mathbb{R}_+$ . 340

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(4)}$  and  $(b''_i)^{(4)}$ ,  $i = 24, 25, 26$  depend only on  $T_{25}$  and respectively on  $(G_{27})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  341

From 19 to 24 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{[-\int_0^t \{ (a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)}) \} ds_{(24)}]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \quad \text{for } t > 0 \end{aligned}$$

**Definition of**  $((\widehat{M}_{24})^{(4)})_1, ((\widehat{M}_{24})^{(4)})_2$  and  $((\widehat{M}_{24})^{(4)})_3$  : 342

**Remark 3:** if  $G_{24}$  is bounded, the same property have also  $G_{25}$  and  $G_{26}$ . indeed if

$$G_{24} < (\widehat{M}_{24})^{(4)} \text{ it follows } \frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)} G_{25} \text{ and by integrating}$$

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)} ((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If  $G_{25}$  or  $G_{26}$  is bounded, the same property follows for  $G_{24}$  ,  $G_{26}$  and  $G_{24}$  ,  $G_{25}$  respectively.

**Remark 4:** If  $G_{24}$  is bounded, from below, the same property holds for  $G_{25}$  and  $G_{26}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{25}$  is bounded from below. 343

**Remark 5:** If  $T_{24}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$  then  $T_{25} \rightarrow \infty$ . 344

**Definition of  $(m)^{(4)}$  and  $\varepsilon_4$  :**

Indeed let  $t_4$  be so that for  $t > t_4$

$$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then  $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$  which leads to 345

$$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$$

If we take  $t$  such that  $e^{-\varepsilon_4 t} = \frac{1}{2}$  it results

$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_4}$  By taking now  $\varepsilon_4$  sufficiently small one sees that  $T_{25}$  is unbounded. The same property holds for  $T_{26}$  if  $\lim_{t \rightarrow \infty} (b''_{26})^{(4)}((G_{27})(t), t) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for  $G_{29}$  ,  $G_{30}$ ,  $T_{28}$ ,  $T_{29}$ ,  $T_{30}$

346

It is now sufficient to take  $\frac{(a_i)^{(5)}}{(M_{28})^{(5)}}$  ,  $\frac{(b_i)^{(5)}}{(M_{28})^{(5)}} < 1$  and to choose 347

$(\widehat{P}_{28})^{(5)}$  and  $(\widehat{Q}_{28})^{(5)}$  large to have

$$\frac{(a_i)^{(5)}}{(M_{28})^{(5)}} \left[ (\widehat{P}_{28})^{(5)} + ((\widehat{P}_{28})^{(5)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{28})^{(5)}$$
348

$$\frac{(b_j)^{(5)}}{(M_{28})^{(5)}} \left[ ((\widehat{Q}_{28})^{(5)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{28})^{(5)} \right] \leq (\widehat{Q}_{28})^{(5)}$$
349

In order that the operator  $\mathcal{A}^{(5)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself 350

The operator  $\mathcal{A}^{(5)}$  is a contraction with respect to the metric 351

$$d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(M_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(M_{28})^{(5)}t} \right\}$$

Indeed if we denote

**Definition of**  $(\widehat{G}_{31}), (\widehat{T}_{31}) : ((\widehat{G}_{31}), (\widehat{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} ds_{(28)} +$$

$$\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)}$$

Where  $s_{(28)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

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$$|(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widehat{M}_{28})^{(5)}t} \leq$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}} ((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} +$$

$$(\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}) d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}) \right)$$

353

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (35,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{28})^{(5)}$  and  $(b''_{28})^{(5)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$  and  $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$  respectively of  $\mathbb{R}_+$ .

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If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(5)}$  and  $(b''_i)^{(5)}$ ,  $i = 28, 29, 30$  depend only on  $T_{29}$  and respectively on  $(G_{31})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

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From GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(5)} - (a''_i)^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$  and  $((\widehat{M}_{28})^{(5)})_3$  :

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**Remark 3:** if  $G_{28}$  is bounded, the same property have also  $G_{29}$  and  $G_{30}$ . indeed if

$G_{28} < ((\widehat{M}_{28})^{(5)})$  it follows  $\frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)} G_{29}$  and by integrating

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)} ((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If  $G_{29}$  or  $G_{30}$  is bounded, the same property follows for  $G_{28}$  ,  $G_{30}$  and  $G_{28}$  ,  $G_{29}$  respectively.

**Remark 4:** If  $G_{28}$  is bounded, from below, the same property holds for  $G_{29}$  and  $G_{30}$  . The proof is 357  
 analogous with the preceding one. An analogous property is true if  $G_{29}$  is bounded from below.

**Remark 5:** If  $T_{28}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$  then 358  
 $T_{29} \rightarrow \infty$ .

**Definition of  $(m)^{(5)}$  and  $\varepsilon_5$  :**

Indeed let  $t_5$  be so that for  $t > t_5$

$$(b_{29})^{(5)} - (b''_i)^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)} \quad 359$$

Then  $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$  which leads to 360

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$$

If we take  $t$  such that  $e^{-\varepsilon_5 t} = \frac{1}{2}$  it results

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5}$$

By taking now  $\varepsilon_5$  sufficiently small one sees that  $T_{29}$  is unbounded. The same property holds for  $T_{30}$  if  $\lim_{t \rightarrow \infty} (b''_{30})^{(5)}((G_{31})(t), t) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for  $G_{33}$  ,  $G_{34}$ ,  $T_{32}$ ,  $T_{33}$ ,  $T_{34}$

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It is now sufficient to take  $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$  ,  $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$  and to choose 362

$(\widehat{P}_{32})^{(6)}$  and  $(\widehat{Q}_{32})^{(6)}$  large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[ (\widehat{P}_{32})^{(6)} + ((\widehat{P}_{32})^{(6)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{32})^{(6)} \quad 363$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[ ((\widehat{Q}_{32})^{(6)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{32})^{(6)} \right] \leq (\widehat{Q}_{32})^{(6)} \quad 364$$

In order that the operator  $\mathcal{A}^{(6)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself 365

The operator  $\mathcal{A}^{(6)}$  is a contraction with respect to the metric 366

$$d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \}$$

Indeed if we denote

**Definition of**  $(\widehat{G}_{35}), (\widehat{T}_{35}) : ((\widehat{G}_{35}), (\widehat{T}_{35})) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$\begin{aligned} |\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \\ &\int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)} \end{aligned} \tag{367}$$

Where  $s_{(32)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\bar{M}_{32})^{(6)}t} &\leq \\ \frac{1}{(\bar{M}_{32})^{(6)}} &((a_{32})^{(6)} + (a'_{32})^{(6)} + (\hat{A}_{32})^{(6)} + \\ (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}) &d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}); ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) \end{aligned} \tag{368}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{32})^{(6)}$  and  $(b''_{32})^{(6)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$  and  $(\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$  respectively of  $\mathbb{R}_+$ . 369

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(6)}$  and  $(b''_i)^{(6)}$ ,  $i = 32, 33, 34$  depend only on  $T_{33}$  and respectively on  $(G_{35})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  370

From 69 to 32 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{[-\int_0^t \{ (a'_i)^{(6)} - (a''_i)^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \} ds_{(32)}]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(6)}t} > 0 \text{ for } t > 0 \end{aligned}$$

**Definition of**  $((\widehat{M}_{32})^{(6)})_1, ((\widehat{M}_{32})^{(6)})_2$  and  $((\widehat{M}_{32})^{(6)})_3 :$  371

**Remark 3:** if  $G_{32}$  is bounded, the same property have also  $G_{33}$  and  $G_{34}$ . indeed if

$$G_{32} < ((\widehat{M}_{32})^{(6)}) \text{ it follows } \frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a'_{33})^{(6)} G_{33} \text{ and by integrating}$$

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)} ((\widehat{M}_{32})^{(6)})_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a'_{34})^{(6)}$$

If  $G_{33}$  or  $G_{34}$  is bounded, the same property follows for  $G_{32}$  ,  $G_{34}$  and  $G_{32}$  ,  $G_{33}$  respectively.

**Remark 4:** If  $G_{32}$  is bounded, from below, the same property holds for  $G_{33}$  and  $G_{34}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{33}$  is bounded from below. 372

**Remark 5:** If  $T_{32}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(6)}((G_{35})(t), t)) = (b'_{33})^{(6)}$  then  $T_{33} \rightarrow \infty$ . 373

**Definition of  $(m)^{(6)}$  and  $\varepsilon_6$  :**

Indeed let  $t_6$  be so that for  $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)} \tag{374}$$

Then  $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$  which leads to 375

$$T_{33} \geq \left( \frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$$

If we take  $t$  such that  $e^{-\varepsilon_6 t} = \frac{1}{2}$  it results

$$T_{33} \geq \left( \frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_6}$$

By taking now  $\varepsilon_6$  sufficiently small one sees that  $T_{33}$  is unbounded. The same property holds for  $T_{34}$  if  $\lim_{t \rightarrow \infty} (b_{34}'')^{(6)}((G_{35})(t), t(t), t) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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**Behavior of the solutions**

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If we denote and define

**Definition of  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  :**

(a)  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

**Definition of  $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$  :** 378

(b) By  $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$  and respectively  $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of  $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$  :** 379

By  $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$  and respectively  $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition**  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$  :- 380

(c) If we define  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$  by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously 381

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } \boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)} 382$$

are defined respectively

Then the solution satisfies the inequalities 383

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where  $(p_i)^{(1)}$  is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left( \frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)}) - (S_2)^{(1)}} \right) \left[ e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq 384$$

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a_{15})^{(1)})} \left[ e^{(S_1)^{(1)}t} - e^{-a_{15}'^{(1)}t} \right] + G_{15}^0 e^{-a_{15}'^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} 385$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} 386$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b_{15})^{(1)})} \left[ e^{(R_1)^{(1)}t} - e^{-b_{15}'^{(1)}t} \right] + T_{15}^0 e^{-b_{15}'^{(1)}t} \leq T_{15}(t) \leq 387$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)} ((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[ e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition** of  $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$  :- 388

$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a_{13}')^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b_{13}')^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

**Behavior of the solutions** 389

If we denote and define

**Definition of**  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  : 390

(d)  $\sigma_1^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \quad 391$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \quad 392$$

**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  : 393

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots 394

(e) of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  395

and  $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  and 396

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  : 397

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the 398

roots of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  399

and  $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  400

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :- 401

(f) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by 402

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \quad 403$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 404$$

and 
$$(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)} \quad 405$$

and analogously 406

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and 
$$(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 407$$

Then the solution satisfies the inequalities 408

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined 409

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 410$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t} \quad 411$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 412$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 413$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b_{18})^{(2)}t} \leq T_{18}(t) \leq \quad 414$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[ e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$  :- 415

Where  $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$  416

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 417$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

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**Behavior of the solutions** 419

If we denote and define

**Definition of**  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  :

(a)  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

**Definition of**  $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$  : 420

(b) By  $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$  and respectively  $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$  the roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By  $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$  and respectively  $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$  the

roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

**Definition of**  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  :- 421

(c) If we define  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously

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$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$  is defined

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$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

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$$\left( \frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[ e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a_{22})^{(3)})} \left[ e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}$$

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$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}}$$

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$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$$

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$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b_{22})^{(3)})} \left[ e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq$$

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$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[ e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of  $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$ :-**

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$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

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**Behavior of the solutions**

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If we denote and define

**Definition of  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  :**

(d)  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

**Definition**  $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$  : 433

(e) By  $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$  and respectively  $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$  the roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$  and  $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$  and

**Definition**  $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$  : 434

By  $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$  and respectively  $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$  the roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and  $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$  436

**Definition**  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$  :-

(f) If we define  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$  by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

$$\text{and } (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously 437

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

$$\text{and } (u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)}$  where  $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$  are defined by 59 and 64 respectively

Then the solution satisfies the inequalities 439

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where  $(p_i)^{(4)}$  is defined 441

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[ e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$
 449

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 450$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq \quad 451$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})} \left[ e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t} \quad 452$$

**Definition of**  $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$ :- 452

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)} \quad 453$$

**Behavior of the solutions** 454

If we denote and define

**Definition of**  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  :

(g)  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

**Definition of**  $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$  : 455

(h) By  $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$  and respectively  $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$  the roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$  and  $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$  : 456

By  $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$  and respectively  $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$  the roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$  and  $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of**  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$  :-

(i) If we define  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$  by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously 457

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and  $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$ , if  $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$  where  $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$  are defined respectively

Then the solution satisfies the inequalities 458

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where  $(p_i)^{(5)}$  is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 459$$

$$\left( \frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[ e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \right) \leq G_{30}(t) \leq \quad 460$$

$$\frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a_{30})^{(5)})} \left[ e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t} \quad 461$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 462$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 463$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b_{30})^{(5)})} \left[ e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \quad 464$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[ e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

**Definition of  $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$ :-** 465

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

**Behavior of the solutions** 466

If we denote and define

**Definition of  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  :**

(j)  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

**Definition of  $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$  :** 467

(k) By  $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$  and respectively  $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$  and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$  and

**Definition**  $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$  : 468

By  $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$  and respectively  $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition**  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$  :-

(1) If we define  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$  by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)} \tag{470}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously 471

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)}$  where  $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$  are defined respectively

Then the solution satisfies the inequalities 472

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where  $(p_i)^{(6)}$  is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \tag{473}$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq \tag{474}$$

$$\frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a'_{34})^{(6)})} \left[ e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \tag{475}$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \tag{476}$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)} ((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq \tag{477}$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)} ((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition** of  $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$  :- 478

Where  $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

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**Proof :** From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of  $v^{(1)}$  :-** 
$$v^{(1)} = \frac{G_{13}}{G_{14}}$$

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It follows

$$- \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of  $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$  :-**

(a) For  $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

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$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If  $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$  we find like in the previous case,

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$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (\bar{v}_1)^{(1)}$$

(c) If  $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain

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$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of  $v^{(1)}(t)$  :-**

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of  $u^{(1)}(t)$  :-**

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL E486QUATIONS we get easily the result stated in the theorem.

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Particular case :

If  $(a''_{13})^{(1)} = (a''_{14})^{(1)}$ , then  $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$  and in this case  $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$  if in addition  $(v_0)^{(1)} = (v_1)^{(1)}$  then  $v^{(1)}(t) = (v_0)^{(1)}$  and as a consequence  $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$  this also defines  $(v_0)^{(1)}$  for the special case

Analogously if  $(b''_{13})^{(1)} = (b''_{14})^{(1)}$ , then  $(\tau_1)^{(1)} = (\tau_2)^{(1)}$  and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$  if in addition  $(u_0)^{(1)} = (u_1)^{(1)}$  then  $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$  This is an important consequence of the relation between  $(v_1)^{(1)}$  and  $(\bar{v}_1)^{(1)}$ , and definition of  $(u_0)^{(1)}$ .

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we obtain

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$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left( (a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

**Definition of  $v^{(2)}$  :-**

$$\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$$

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It follows

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$$- \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

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**Definition of  $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$  :-**

(d) For  $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (\bar{C})^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_0)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_0)^{(2)}]t}}, \quad \boxed{(\bar{C})^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get

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$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}, \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

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(e) If  $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$  we find like in the previous case, 493

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (C)^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(f) If  $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 494

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (C)^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c), we have

**Definition of  $v^{(2)}(t)$  :-** 495

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 496

**Definition of  $u^{(2)}(t)$  :-**

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

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**Particular case :** 498

If  $(a_{16}''^{(2)}) = (a_{17}''^{(2)})$ , then  $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$  and in this case  $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$  if in addition  $(v_0)^{(2)} = (v_1)^{(2)}$  then  $v^{(2)}(t) = (v_0)^{(2)}$  and as a consequence  $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if  $(b_{16}''^{(2)}) = (b_{17}''^{(2)})$ , then  $(\tau_1)^{(2)} = (\tau_2)^{(2)}$  and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$  if in addition  $(u_0)^{(2)} = (u_1)^{(2)}$  then  $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$  This is an important consequence of the relation between  $(v_1)^{(2)}$  and  $(\bar{v}_1)^{(2)}$

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From GLOBAL EQUATIONS we obtain 500

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left( (a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t) \right) - (a_{21}'')^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

**Definition of  $v^{(3)}$  :-** 501

$$\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$$

It follows

$$- \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

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From which one obtains

(a) For  $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}, \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner, we get

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$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}, \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

**Definition of  $(\bar{v}_1)^{(3)}$  :-**

From which we deduce  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If  $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$  we find like in the previous case,

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$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}} \leq (\bar{v}_1)^{(3)}$$

(c) If  $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain

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$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (C)^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

**Definition of  $v^{(3)}(t)$  :-**

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of  $u^{(3)}(t)$  :-**

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If  $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}$ , then  $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$  and in this case  $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$  if in addition  $(v_0)^{(3)} = (v_1)^{(3)}$  then  $v^{(3)}(t) = (v_0)^{(3)}$  and as a consequence  $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if  $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}$ , then  $(\tau_1)^{(3)} = (\tau_2)^{(3)}$  and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$  if in addition  $(u_0)^{(3)} = (u_1)^{(3)}$  then  $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$  This is an important consequence of the relation between  $(v_1)^{(3)}$  and  $(\bar{v}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left( (a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of**  $v^{(4)}$  :-

$$v^{(4)} = \frac{G_{24}}{G_{25}}$$

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It follows

$$- \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$  :-

(d) For  $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get

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$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If  $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$  we find like in the previous case,

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$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (\bar{v}_1)^{(4)}$$

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(f) If  $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$  , we obtain

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$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(4)}(t)$  :-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)} , \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(4)}(t)$  :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

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If  $(a_{24}'' )^{(4)} = (a_{25}'' )^{(4)}$ , then  $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$  and in this case  $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$  if in addition  $(\nu_0)^{(4)} = (\nu_1)^{(4)}$  then  $\nu^{(4)}(t) = (\nu_0)^{(4)}$  and as a consequence  $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$  **this also defines  $(\nu_0)^{(4)}$  for the special case .**

Analogously if  $(b_{24}'' )^{(4)} = (b_{25}'' )^{(4)}$ , then  $(\tau_1)^{(4)} = (\tau_2)^{(4)}$  and then  $(u_1)^{(4)} = (\bar{u}_1)^{(4)}$  if in addition  $(u_0)^{(4)} = (u_1)^{(4)}$  then  $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$  This is an important consequence of the relation between  $(\nu_1)^{(4)}$  and  $(\bar{\nu}_1)^{(4)}$ , **and definition of  $(u_0)^{(4)}$ .**

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From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left( (a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of  $\nu^{(5)}$  :-**  $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$- \left( (a_{29})^{(5)}(\nu^{(5)})^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left( (a_{29})^{(5)}(\nu^{(5)})^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

**Definition of  $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$  :-**

(g) For  $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{[-(a_{29})^{(5)}((\nu_1)^{(5)} - (\nu_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((\nu_1)^{(5)} - (\nu_0)^{(5)})t]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}$$

it follows  $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get

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$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)})t]}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)})t]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}$$

From which we deduce  $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If  $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$  we find like in the previous case,

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$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{[-(a_{29})^{(5)}((\nu_1)^{(5)} - (\nu_2)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((\nu_1)^{(5)} - (\nu_2)^{(5)})t]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)})t]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If  $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 518

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (C)^{(5)} (\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)} ((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}) t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)} ((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}) t]}} \leq (v_0)^{(5)} \quad 519$$

And so with the notation of the first part of condition (c), we have

**Definition of  $v^{(5)}(t)$  :-**

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

**Definition of  $u^{(5)}(t)$  :-**

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a_{28}'' )^{(5)} = (a_{29}'' )^{(5)}$ , then  $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$  and in this case  $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$  if in addition  $(v_0)^{(5)} = (v_5)^{(5)}$  then  $v^{(5)}(t) = (v_0)^{(5)}$  and as a consequence  $G_{28}(t) = (v_0)^{(5)} G_{29}(t)$  **this also defines  $(v_0)^{(5)}$  for the special case .**

Analogously if  $(b_{28}'' )^{(5)} = (b_{29}'' )^{(5)}$ , then  $(\tau_1)^{(5)} = (\tau_2)^{(5)}$  and then  $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$  if in addition  $(u_0)^{(5)} = (u_1)^{(5)}$  then  $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$  This is an important consequence of the relation between  $(v_1)^{(5)}$  and  $(\bar{v}_1)^{(5)}$ , **and definition of  $(u_0)^{(5)}$ .**

we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left( (a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of  $v^{(6)}$  :-**  $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$- \left( (a_{33})^{(6)} (v^{(6)})^2 + (\sigma_2)^{(6)} v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left( (a_{33})^{(6)} (v^{(6)})^2 + (\sigma_1)^{(6)} v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

**Definition of  $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$  :-**

(j) For  $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)} (v_2)^{(6)} e^{[-(a_{33})^{(6)} ((v_1)^{(6)} - (v_0)^{(6)}) t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)} ((v_1)^{(6)} - (v_0)^{(6)}) t]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get 522

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}}$$
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From which we deduce  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If  $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$  we find like in the previous case, 524

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)} (v_2)^{(6)} e^{[-(a_{33})^{(6)} (v_1)^{(6)} - (v_2)^{(6)}] t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)} (v_1)^{(6)} - (v_2)^{(6)}] t}} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}} \leq (\bar{v}_1)^{(6)}$$
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(l) If  $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$  , we obtain

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of  $v^{(6)}(t)$  :-**

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)} \quad , \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of  $u^{(6)}(t)$  :-**

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)} \quad , \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{32})^{(6)} = (a''_{33})^{(6)}$ , then  $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$  and in this case  $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$  if in addition  $(v_0)^{(6)} = (v_1)^{(6)}$  then  $v^{(6)}(t) = (v_0)^{(6)}$  and as a consequence  $G_{32}(t) = (v_0)^{(6)} G_{33}(t)$  **this also defines  $(v_0)^{(6)}$  for the special case .**

Analogously if  $(b''_{32})^{(6)} = (b''_{33})^{(6)}$ , then  $(\tau_1)^{(6)} = (\tau_2)^{(6)}$  and then  $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$  if in addition  $(u_0)^{(6)} = (u_1)^{(6)}$  then  $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$  This is an important consequence of the relation between  $(v_1)^{(6)}$  and  $(\bar{v}_1)^{(6)}$ , **and definition of  $(u_0)^{(6)}$ .**

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We can prove the following

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**Theorem 3:** If  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  are independent on  $t$  , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 \quad ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with  $(p_{13})^{(1)}, (r_{14})^{(1)}$  as defined, then the system

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If  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  are independent on  $t$ , and the conditions

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0,$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

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with  $(p_{16})^{(2)}, (r_{17})^{(2)}$  as defined are satisfied, then the system

If  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  are independent on  $t$ , and the conditions

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$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with  $(p_{20})^{(3)}, (r_{21})^{(3)}$  as defined are satisfied, then the system

If  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$  are independent on  $t$ , and the conditions

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$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with  $(p_{24})^{(4)}, (r_{25})^{(4)}$  as defined are satisfied, then the system

If  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$  are independent on  $t$ , and the conditions

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$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with  $(p_{28})^{(5)}, (r_{29})^{(5)}$  as defined satisfied, then the system

If  $(a_i'')^{(6)}$  and  $(b_i'')^{(6)}$  are independent on  $t$ , and the conditions

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$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0, \quad 539$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{32})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with  $(p_{32})^{(6)}, (r_{33})^{(6)}$  as defined are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 540$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 541$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 542$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 543$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 544$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 545$$

has a unique positive solution, which is an equilibrium solution for the system 546

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 547$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 548$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 549$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 550$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 551$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 552$$

has a unique positive solution, which is an equilibrium solution for 553

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 554$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 555$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 556$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 557$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 558$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 559$$

has a unique positive solution, which is an equilibrium solution 560

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 561$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 563$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 564$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27})]T_{24} = 0 \quad 565$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27})]T_{25} = 0 \quad 566$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27})]T_{26} = 0 \quad 567$$

has a unique positive solution , which is an equilibrium solution for the system 568

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 569$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 570$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 571$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 572$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 573$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 574$$

has a unique positive solution , which is an equilibrium solution for the system 575

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 576$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 577$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 578$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 579$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 580$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 584$$

has a unique positive solution , which is an equilibrium solution for the system 582

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(a) Indeed the first two equations have a nontrivial solution  $G_{13}, G_{14}$  if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

585

(a) Indeed the first two equations have a nontrivial solution  $G_{16}, G_{17}$  if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) +$$

$$(a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0 \quad 586$$

587

(a) Indeed the first two equations have a nontrivial solution  $G_{20}, G_{21}$  if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

588

(a) Indeed the first two equations have a nontrivial solution  $G_{24}, G_{25}$  if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

589

(a) Indeed the first two equations have a nontrivial solution  $G_{28}, G_{29}$  if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

560

(a) Indeed the first two equations have a nontrivial solution  $G_{32}, G_{33}$  if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Definition and uniqueness of  $T_{14}^*$  :-**

561

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(1)}(T_{14})$  being increasing, it follows that there exists a unique  $T_{14}^*$  for which  $f(T_{14}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

**Definition and uniqueness of  $T_{17}^*$  :-**

562

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(2)}(T_{17})$  being increasing, it follows that there exists a unique  $T_{17}^*$  for which  $f(T_{17}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 563$$

**Definition and uniqueness of  $T_{21}^*$  :-**

564

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(3)}(T_{21})$  being increasing, it follows that there exists a unique  $T_{21}^*$  for which  $f(T_{21}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

565

**Definition and uniqueness of  $T_{25}^*$  :-**

566

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(4)}(T_{25})$  being increasing, it follows that there exists a unique  $T_{25}^*$  for which  $f(T_{25}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a_{24}')^{(4)}+(a_{24}'')^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a_{26}')^{(4)}+(a_{26}'')^{(4)}(T_{25}^*)]}$$

**Definition and uniqueness of  $T_{29}^*$  :-**

567

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(5)}(T_{29})$  being increasing, it follows that there exists a unique  $T_{29}^*$  for which  $f(T_{29}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a_{28}')^{(5)}+(a_{28}'')^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a_{30}')^{(5)}+(a_{30}'')^{(5)}(T_{29}^*)]}$$

**Definition and uniqueness of  $T_{33}^*$  :-**

568

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(6)}(T_{33})$  being increasing, it follows that there exists a unique  $T_{33}^*$  for which  $f(T_{33}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a_{32}')^{(6)}+(a_{32}'')^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a_{34}')^{(6)}+(a_{34}'')^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations 92,93 admit solutions  $G_{13}, G_{14}$  if

569

$$\varphi(G) = (b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - [(b_{13}')^{(1)}(b_{14}'')^{(1)}(G) + (b_{14}')^{(1)}(b_{13}'')^{(1)}(G)] + (b_{13}'')^{(1)}(G)(b_{14}'')^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions  $G_{16}, G_{17}$  if

570

$$\varphi(G_{19}) = (b_{16}')^{(2)}(b_{17}')^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - [(b_{16}')^{(2)}(b_{17}'')^{(2)}(G_{19}) + (b_{17}')^{(2)}(b_{16}'')^{(2)}(G_{19})] + (b_{16}'')^{(2)}(G_{19})(b_{17}'')^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi((G_{19})^*) = 0$

571

(g) By the same argument, the concatenated equations admit solutions  $G_{20}, G_{21}$  if

572

$$\varphi(G_{23}) = (b_{20}')^{(3)}(b_{21}')^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - [(b_{20}')^{(3)}(b_{21}'')^{(3)}(G_{23}) + (b_{21}')^{(3)}(b_{20}'')^{(3)}(G_{23})] + (b_{20}'')^{(3)}(G_{23})(b_{21}'')^{(3)}(G_{23}) = 0$$

Where in  $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{21}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows

that there exists a unique  $G_{21}^*$  such that  $\varphi((G_{23})^*) = 0$  573

(h) By the same argument, the equations of modules admit solutions  $G_{24}, G_{25}$  if 574

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in  $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{25}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{25}^*$  such that  $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations (modules) admit solutions  $G_{28}, G_{29}$  if 575

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in  $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{29}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{29}^*$  such that  $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations (modules) admit solutions  $G_{32}, G_{33}$  if 578

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$
581

Where in  $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$  must be replaced by their values It is easy to see that  $\varphi$  is a decreasing function in  $G_{33}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{33}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94 582

$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} , T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0, T_{17}^*$  given by  $f(T_{17}^*) = 0$  and 584

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} 585$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} 586$$

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution

588

$G_{21}^*$  given by  $\varphi((G_{23})^*) = 0$ ,  $T_{21}^*$  given by  $f(T_{21}^*) = 0$  and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

589

$G_{25}^*$  given by  $\varphi(G_{27}) = 0$ ,  $T_{25}^*$  given by  $f(T_{25}^*) = 0$  and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} , T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]}$$

590

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

591

$G_{29}^*$  given by  $\varphi((G_{31})^*) = 0$ ,  $T_{29}^*$  given by  $f(T_{29}^*) = 0$  and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} , T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]}$$

592

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

593

$G_{33}^*$  given by  $\varphi((G_{35})^*) = 0$ ,  $T_{33}^*$  given by  $f(T_{33}^*) = 0$  and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} , G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^*)]} , T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^*)]}$$

594

Obviously, these values represent an equilibrium solution

#### ASYMPTOTIC STABILITY ANALYSIS

595

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  belong to  $C^{(1)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + G_i , T_i = T_i^* + T_i$$

596

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^*T_{14} \quad \text{598}$$

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^*T_{14} \quad \text{599}$$

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^*T_{14} \quad \text{600}$$

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(13)(j)})T_{13}^*G_j \quad \text{601}$$

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15}(s_{(14)(j)})T_{14}^*G_j \quad \text{602}$$

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(15)(j)})T_{15}^*G_j \quad \text{603}$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  belong to  $C^{(2)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable 604

Denote 605

**Definition of  $G_i, T_i$  :-**

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \quad \text{606}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij} \quad \text{607}$$

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \quad \text{609}$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \quad \text{610}$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \quad \text{611}$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18}(s_{(16)(j)})T_{16}^*G_j \quad \text{612}$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18}(s_{(17)(j)})T_{17}^*G_j \quad \text{613}$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18}(s_{(18)(j)})T_{18}^*G_j \quad \text{614}$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  belong to  $C^{(3)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable 615

Denote

**Definition of  $G_i, T_i$  :-**

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij} \tag{616}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \tag{618}$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21} \tag{619}$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \tag{6120}$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^*G_j \tag{621}$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^*G_j \tag{622}$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^*G_j \tag{623}$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$  belong to  $C^{(4)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable 624

Denote

**Definition of  $G_i, T_i$  :-** 625

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \quad \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$$\frac{dG_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^*T_{25} \tag{627}$$

$$\frac{dG_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^*T_{25} \tag{628}$$

$$\frac{dG_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^*T_{25} \tag{629}$$

$$\frac{dT_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^*G_j \tag{630}$$

$$\frac{dT_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^*G_j \tag{631}$$

$$\frac{dT_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^*G_j \tag{632}$$

633

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$  belong to  $C^{(5)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of  $G_i, T_i$  :-** 634

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}''^{(5)})}{\partial T_{29}} (T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial (b_i''^{(5)})}{\partial G_j} ((G_{31})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$$\frac{d\mathbb{G}_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 636$$

$$\frac{d\mathbb{G}_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 637$$

$$\frac{d\mathbb{G}_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 638$$

$$\frac{d\mathbb{T}_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*\mathbb{G}_j \quad 639$$

$$\frac{d\mathbb{T}_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*\mathbb{G}_j \quad 640$$

$$\frac{d\mathbb{T}_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*\mathbb{G}_j \quad 641$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(6)}$  and  $(b_i'')^{(6)}$  belong to  $C^{(6)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable 642

Denote

**Definition of  $\mathbb{G}_i, \mathbb{T}_i$  :-** 643

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}''^{(6)})}{\partial T_{33}} (T_{33}^*) = (q_{33})^{(6)}, \quad \frac{\partial (b_i''^{(6)})}{\partial G_j} ((G_{35})^*) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 645$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 646$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 647$$

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34} (s_{(32)(j)})T_{32}^*\mathbb{G}_j \quad 648$$

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34} (s_{(33)(j)})T_{33}^*\mathbb{G}_j \quad 649$$

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34} (s_{(34)(j)})T_{34}^*\mathbb{G}_j \quad 650$$

651

The characteristic equation of this system is 652

$$((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)})\} \\ \left[ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right]$$

$$\begin{aligned}
 & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\
 & + \left( ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\
 & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\
 & \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & \left( ((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & + \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\
 & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*) \\
 & \left. \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \right\} = 0 \\
 & + \\
 & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}) \{ ((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\
 & \left[ ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\
 & + \left( ((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\
 & \left( ((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & \left( ((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & + \left( ((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) ((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^*) \\
 & \left. \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \right\} = 0 \\
 & + \\
 & ((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}) \{ ((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \\
 & \left[ ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right] \\
 & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T_{21}^* + (b_{21})^{(3)}s_{(20),(21)}T_{21}^* \right) \\
 & + \left( ((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(3)}G_{21}^* \right) \\
 & \left. \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T_{21}^* + (b_{21})^{(3)}s_{(20),(20)}T_{20}^* \right) \right\}
 \end{aligned}$$

$$\begin{aligned} & \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \\ & \left( (\lambda)^{(3)} \right)^2 + \left( (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda)^{(3)} \\ & + \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22} \\ & + \left( (\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left( (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \\ & \left. \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right\} = 0 \end{aligned}$$

+

$$\begin{aligned} & \left( (\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \left\{ \left( (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \right) \right. \\ & \left. \left[ \left( (\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right] \right. \\ & \left. \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \right) \\ & + \left( (\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \\ & \left. \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \right) \\ & \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \\ & \left( (\lambda)^{(4)} \right)^2 + \left( (b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \\ & + \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26} \\ & + \left( (\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) \left( (a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right) \\ & \left. \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(26)} T_{25}^* + (b_{25})^{(4)} s_{(24),(26)} T_{24}^* \right\} = 0 \end{aligned}$$

+

$$\begin{aligned} & \left( (\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)} \right) \left\{ \left( (\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)} \right) \right. \\ & \left. \left[ \left( (\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right] \right. \\ & \left. \left( (\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(29)} T_{29}^* + (b_{29})^{(5)} s_{(28),(29)} T_{29}^* \right) \\ & + \left( (\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)} \right) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \\ & \left. \left( (\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(28)} T_{29}^* + (b_{29})^{(5)} s_{(28),(28)} T_{28}^* \right) \\ & \left( (\lambda)^{(5)} \right)^2 + \left( (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} \\ & \left( (\lambda)^{(5)} \right)^2 + \left( (b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} \right) (\lambda)^{(5)} \end{aligned}$$

$$\begin{aligned}
 &+ \left( (\lambda^{(5)})^2 + (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda^{(5)}) (q_{30})^{(5)} G_{30} \\
 &+ \left( (\lambda^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) \left( (a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \right. \\
 &\left. \left( (\lambda^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) s_{(29),(30)} T_{29}^* + (b_{29})^{(5)} s_{(28),(30)} T_{28}^* \right) \right\} = 0 \\
 &+ \\
 & \left( (\lambda^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)}) \left\{ (\lambda^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}) \right. \right. \\
 & \left. \left[ \left( (\lambda^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \right] \right. \\
 & \left. \left( (\lambda^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) s_{(33),(33)} T_{33}^* + (b_{33})^{(6)} s_{(32),(33)} T_{33}^* \right) \right. \\
 & \left. + \left( (\lambda^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)}) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \right) \right. \\
 & \left. \left( (\lambda^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) s_{(33),(32)} T_{33}^* + (b_{33})^{(6)} s_{(32),(32)} T_{32}^* \right) \right. \\
 & \left. \left( (\lambda^{(6)})^2 + (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda^{(6)}) \right. \\
 & \left. \left( (\lambda^{(6)})^2 + (b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda^{(6)}) \right. \\
 & \left. + \left( (\lambda^{(6)})^2 + (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda^{(6)}) (q_{34})^{(6)} G_{34} \right. \\
 & \left. + \left( (\lambda^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) \left( (a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \right. \right. \\
 & \left. \left. \left( (\lambda^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) s_{(33),(34)} T_{33}^* + (b_{33})^{(6)} s_{(32),(34)} T_{32}^* \right) \right\} \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's L:etters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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(13)<sup>a</sup> Note that the relativistic mass, in contrast to the rest mass  $m_0$ , is not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity  $\mathbf{dx}$ , where  $\mathbf{dx}$  is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between  $d\tau$  and  $dt$ .

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(20)<sup>a</sup> [2] Cockcroft-Walton experiment

(21)<sup>a b c</sup> Conversions used: 1956 International (Steam) Table (IT) values where one calorie  $\equiv 4.1868$  J and one BTU  $\equiv 1055.05585262$  J. Weapons designers' conversion value of one gram TNT

≡ 1000 calories used.

(22)<sup>^</sup> Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)<sup>^</sup> Assuming a 90/10 alloy of Pt/Ir by weight, a  $C_p$  of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average  $C_p$  of 25.8, 5.134 moles of metal, and  $132 \text{ J.K}^{-1}$  for the prototype. A variation of  $\pm 1.5$  picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are  $\pm 2$  micrograms.

(24)<sup>^</sup> [3] Article on Earth rotation energy. Divided by  $c^2$ .

(25)<sup>^ a b</sup> Earth's gravitational self-energy is  $4.6 \times 10^{-10}$  that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)<sup>^</sup> There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.

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