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DIRAC FIELDS, GAUGE FIELDS, AND QUANTUM FIELD THEORIES - AN EPITHALAMIUM

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ABSTRACT: In quantum physics, in order to quantize a gauge theory, like for example Yang-Mills theory, Chern-Simons or BF model, one method is to perform a gauge fixing. This is done in the BRST and Batalin-Vilkovisky formulation. Another is to factor out the symmetry by dispensing with vector potentials altogether (they're not physically observable anyway) and work directly with Wilson loops, Wilson lines contracted with other charged fields at its endpoints and spin networks. Older approaches to quantization for Abelian models use the Gupta-Bleuler formalism with a "semi-Hilbert space" with an indefinite sesquilinear form. However, it is much more elegant to just work with the quotient space of vector field configurations by gauge transformations.An alternative approach using lattice approximations is covered in (Wick rotated) lattice gauge theory. In this paper we construct a new paradigm of relational context, with primary focus and locus of asymptotic freedom and differentially instrumental activity of both QFT and YM Theory. It is to be noted that there exists a qualitative gradient of energy differential ascribed to energy excitation of vacuum. This MIGHT as well be the orthogonal energy state of Vacuum, with solidarity abstraction extant and existential with collective chiral symmetry and causuistral chemistry with transitive states and substantive sub states. We donor enucleate or expatiate upon the subtleties and nuances and nitty grit ties of the various aspects dealt and straight away construct the model with formulator equations .Advanced readers may find the article useful for further research and experimentation.

INTRODUCTION:

What is a gauge?

Gauge is a term which has connotations of being a befuddling fearsome, and perniciously complicated part of mathematics – for instance, playing an important role in quantum field theory, general relativity, geometric PDE, and so forth. Underlying concept is really quite simple: a gauge is nothing more than a "coordinate system" that varies depending on one's "location" with respect to some "base space" or "parameter space", a gauge transform is a change of coordinates applied to each such location, and a gauge theory is a model for some physical or mathematical system to which gauge transforms can be applied (and is typically gauge invariant, in that all physically meaningful quantities are left unchanged (or transform naturally) under gauge transformations). By fixing a gauge (thus breaking or spending the gauge symmetry), the model becomes something easier to analyse mathematically, such as a system of partial differential equations (in classical gauge theories) or a perturbative quantum field theory (in quantum gauge theories), though the tractability of the resulting problem can be heavily dependent on the choice of gauge that one fixed. Deciding exactly how to fix a gauge (or whether one should spend the gauge symmetry at all) is a key question in the analysis of gauge theories, and one that often requires the input of geometric ideas and intuition into that analysis. Quantum gauge theories are the quantization of classical gauge theories and have their own set of conceptual difficulties (coming from quantum field theory).

In early 1954, Chen Ning Yang and Robert Mills extended the concept of gauge theory for abelian groups, e.g. quantum electrodynamics, to nonabelian groups to provide an explanation for strong interactions. The idea by Yang–Mills was criticized by Pauli, as the quanta of the Yang–Mills field must be massless in order to maintain gauge invariance. The idea was set aside until 1960, when the concept of particles acquiring mass through symmetry breaking in massless theories was put forward, initially by Jeffrey Goldstone, Yoichiro Nambu, and Giovanni Jona-Lasinio.

Quantum field theory

Quantum field theory (QFT) **provides a** theoretical framework **for constructing** quantum mechanical models of systems classically parameterized (represented) by an infinite number of degrees of freedom, that is, fields and (in a condensed matter context) many-body systems. Quantum field theory is the natural and quantitative language of particle physics and condensed matter physics. Most theories in modern particle physics, including the Standard Model of elementary particles and their interactions, are formulated as **relativistic quantum field theories**. Quantum field theories are used in many contexts, and are especially vital in elementary particle physics, where the particle count/number **may change over** the course of a reaction. They are also **used in** the description of critical phenomena and quantum phase transitions, such as in the BCS theory of superconductivity.

In perturbative quantum field theory, the forces between particles **are mediated** by other particles. The electromagnetic force between two **electrons is caused** by an exchange of photons. Intermediate vector bosons **mediate the** weak force and gluons **mediate** the strong force. There is currently no complete quantum theory of the remaining fundamental force, gravity, but many of the theories postulate the existence of a graviton particle that mediates it. These force-carrying particles are virtual particles and, by definition, cannot be detected while carrying the force, because such detection will imply that the force is not being carried. In addition, the notion of "force mediating particle" comes from perturbation theory, and thus does not make sense in a context of bound states.

In QFT, photons are not thought of as "little billiard balls" but are rather viewed as field quanta – necessarily chunked ripples in a field, or "excitations", that "look like" particles. Fermions, like the electron, can also be described as ripples/excitations in a field, where each kind of fermion has its own field. In summary, the classical visualization of "everything is particles and fields", in quantum field theory, **resolves into** "everything is particles", which **then resolves into** "everything is fields". In the end, particles **are regarded** as excited states of a field (field quanta). The gravitational field and the electromagnetic field are the only two fundamental fields in Nature that have infinite range and a corresponding classical low-energy limit, which **greatly diminishes and hides** their "particle-like" excitations. Albert Einstein, in 1905, attributed "particle-like" and discrete exchanges of momenta and energy, characteristic of "field quanta", to the electromagnetic field. Originally, his principal motivation was to explain the thermodynamics of radiation. Although it is often claimed that the photoelectric and Compton require a quantum description of the EM field, this is now understood to be untrue, and proper proof of the quantum nature of radiation is now taken up into modern quantum as in the antibunching effect.

In the "low-energy limit", the quantum field-theoretic description of the electromagnetic field, quantum electrodynamics, does not exactly reduce to James Clerk Maxwell's 1864 theory of classical electrodynamics. Small quantum corrections due to virtual electron positron pairs **give rise to** small non-linear corrections to the Maxwell equations, although the "classical limit" of quantum electrodynamics has not been as widely explored as that of quantum mechanics.

Presumably, the as yet unknown correct quantum field-theoretic treatment of the gravitational field will become and "look exactly like" Einstein's general theory of relativity in the "low-energy limit", or, more generally, like the Einstein-Yang-Mills-Dirac System. Indeed, quantum field theory itself is possibly the low-energy-effective-field-theory limit of a more fundamental theory such as superstring theory. Compare in this context the article effective field theory. No definitive and perceptible statements could be made dogmatically about the issue.

Canonical quantization of fields **was extended** to the many-body wave functions of identical particles, a procedure that is sometimes called second quantization. In 1928, Jordan and Eugene found that the quantum field describing electrons, or other fermions, **had to be expanded** using anti-commuting creation and annihilation operators due to the Pauli exclusion principle. This thread of development was incorporated into many-body theory and **strongly influenced** condensed matter physics and nuclear physics

EULEDIAN INVARIANT MEASURES AND QUANTUM FIEL THEORIES(APPLICABLE TO CHARACTERSTIC SYSTEMS AND HENCEFIRTH CLASSIFIABLE BESED ON THE PARAMETRICIZATION OF THE SYTEMS.SPACES WITH GENERALISED FUNCTIONALS:

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF EUCLEDIAN INVARIANT MEASURES AND QUANTUM FIELD THEORIES

G_{14} : CATEGORY TWO OF EUCLEDIAN INVARIANT MEASURES AND QUANTUM FIELD THEORIES

G_{15} : CATEGORY THREE OF EUCLEDIAN INVARIANT MEASURES AND QUANTUM FIELD THEORIES

T_{13} : CATEGORY ONE OF SPACES WITH GENERALISED FUNCTIONALS

T_{14} : CATEGORY TWO OFSPACES WITH GENERALISED FUNCTIONALS

T_{15} :CATEGORY THREE OF SPACES WITH GENERALISED FUNCTIONALS

DIRAC FIELDS AND GAUGE FIELDS AND MEASURES ON SPACES OF FUNCTIONS TAKING VALUES IN A GRASSMAN ALGEBRA:

MODULE NUMBERED TWO:

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G_{16} : CATEGORY ONE OF DIRAC FIELDS AND GAUGE FIELDS

G_{17} : CATEGORY TWO OF DIRAC FIELDS AND GAUGE FIELDS

G_{18} : CATEGORY THREE OF DIRAC FIELDS AND GAUGE FIELDS

T_{16} :CATEGORY ONE OF M,EASURES ON SPACES OF FUNCTIONS TAKING VALUES IN A GRASSMAN ALGEBRA

T_{17} : CATEGORY TWO OF MEASURES ON SPACES OF FUNCTIONS TAKING VALUES IN A GRASSMAN ALGEBRA

T_{18} : CATEGORY THREE OF MEASURES ON SPACES OF FUNCTIONS TAKING VALUES IN A GRASSMAN ALGEBRA

DENORMALIZATION THEORY AND LOCAL SINGULARITIES AND GLOBAL DECAY(INFRA RED REGULARITIES IN QUANTUM FIELD THEORIES:

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OFDENORMALIZATION THEORY (WE HERE SPEAK OF THE CHARACTERISED SYSTEMS FOR WHICH QUANTUM GAUGE THEORY IS APPLICABLE)

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G_{21} :CATEGORY TWO OF DENORMALIZATION THEORY

G_{22} : CATEGORY THREE OF DENORMALIZATION THEORY

T_{20} : CATEGORY ONE OF LOCAL SINGULARITIES AND GLOBAL DECAY(INFRA REGULARITIES) IN QUANTUM FIELD THEORIES

T_{21} :CATEGORY TWO OF LOCAL SINGULARITIES AND GLOBAL DECAY(INFRA REGULARITIES) IN QUANTUM FIELD THEORIES

T_{22} : CATEGORY THREE OF LOCAL SINGULARITIES AND GLOBAL DECAY(INFRA REGULARITIES) IN QUANTUM FIELD THEORIES

QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARAIBLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS)

: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OF QUANTUM FIELD THEORY(EVALUATIVE PARAMETRICIZATION OF SITUATIONAL ORIENTATIONS AND ESSENTIAL COGNITIVE ORIENTATION AND CHOICE VARIABLES OF THE SYSTEM TO WHICH QFT IS APPLICABLE)

G_{25} : CATEGORY TWO OF QUANTUM FIELD THEORY

G_{26} : CATEGORY THREE OF QUANTUM FIELD THEORY

T_{24} :CATEGORY ONE OF RENORMALIZATION THEORY

T_{25} :CATEGORY TWO OF RENORMALIZATION THEORY(SYSTEMIC INSTRUMENTAL CHARACTERISATIONS AND ACTION ORIENTATIONS AND FUYNCTIONAL IMPERATIVES OF CHANGE MANIFESTED THEREIN)

T_{26} : CATEGORY THREE OF QUANTUM FIELD THEORY

ASYMPTOTIC FREEDOM AND FOURIER SERIES AND CLASSICAL DYNAMICAL SYSTEMS

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF ASYMPTOTIC FREEDOM

G_{29} : CATEGORY TWO OF ASYMPTOTIC FREEDOM

G_{30} :CATEGORY THREE OF ASYMPTOTIC FREEDOM

T_{28} :CATEGORY ONE OF FOURIER SERIES OF CLASSICAL DYNAMICAL SYSTEMS

T_{29} :CATEGORY TWO OF FOURIER SERIES OF CLASSICAL DYNAMICAL SYSTEMS

T_{30} :CATEGORY THREE OF FOURIER SERIES OF CLASSICAL DYNAMICAL SYSTEMS

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QUANTUN FIELD THEORY AND STATISTICAL MECHANICS AND YANG BAXTER EQUATIONS AND QUANTUM GROUPS AND BOSE FERMION EQUIVALENCE AND RATIONAL CONFORMAL FIELD THEORY:

MODULE NUMBERED SIX:

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G_{32} : CATEGORY ONE OF QUANTUM FIELD THEORY AND STATISTICAL MECHANICS

G_{33} : CATEGORY TWO OF QUANTUM FIELD THEORY AND STATISTICAL MECHANICS

G_{34} : CATEGORY THREE OF QUANTUM FIELD THEORY AND STATISTICAL MECHANICS

T_{32} : CATEGORY ONE OF YANG BAXTER EQUATIONS AND QUANTUM GROUPS AND BOSE EINSTEIN FERMI EQUIVALENCE AND QUANTUM GROUPS AND RATIONAL CONFORMAL FIELD THEORY

T_{33} : CATEGORY TWO OF YANG BAXTER EQUATIONS AND QUANTUM GROUPS AND BOSE EINSTEIN FERMI EQUIVALENCE AND QUANTUM GROUPS AND RATIONAL CONFORMAL FIELD THEORY

T_{34} : CATEGORY THREE OF YANG BAXTER EQUATIONS AND QUANTUM GROUPS AND BOSE EINSTEIN FERMI EQUIVALENCE AND QUANTUM GROUPS AND RATIONAL CONFORMAL FIELD THEORY

LIE ALGEBRAS OF THE GROUP AND TRANSLATIONS IN SPACE TIME

MODULE NUMBERED SEVEN

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G_{36} : CATEGORY ONE OF LIE ALGEBRAS

G_{37} : CATEGORY TWO OF LIE ALGEBRAS

G_{38} : CATEGORY THREE OF LIE ALGEBRAS (ENERGY EXCITATION OF THE VACUUM AND CONCOMITANT GENERATION OF ENERGY DIFFERENTIAL-TIME LAG OR INSTANTANEOUSNESSMIGHT EXISTS WHEREBY ACCENTUATION AND ATTRITIONS MODEL MAY ASSUME ZERO POSITIONS)

T_{36} : CATEGORY ONE OF TRANSLATIONS IN SPACE TIME

T_{37} : CATEGORY TWO OF $TRANSLATIONS\ IN\ SPACE\ TIME$

T_{38} : CATEGORY THREE OF TRANSLATIONS IN SPACE AND TIME

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: $(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$,
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, \left(a'_{15}\right)^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, \left(b'_{15}\right)^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$,
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$, $(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, \left(a'_{25}\right)^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, \left(b'_{25}\right)^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$
$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$, $(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

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EULEDIAN INVARIANT MEASURES AND QUANTUM FIEL THEORIES(APPLICABLE TO CHARACTERSTIC SYSTEMS AND HENCEFIRTH CLASSIFIABLE BESED ON THE PARAMETRICIZATION OF THE SYTEMS.SPACES WITH GENERALISED FUNCTIONALS:

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13}$ 2

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14}$ 3

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)} + \left(a''_{15}\right)^{(1)}(T_{14}, t)\right]G_{15}$ 4

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$ 5

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$ 6

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)} - \left(b''_{15}\right)^{(1)}(G, t)\right]T_{15}$ 7

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor 8

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

DIRAC FIELDS AND GAUGE FIELDS AND MEASURES ON SPACES OF FUNCTIONS TAKING VALUES IN A GRASSMAN ALGEBRA: 9

MODULE NUMBERED TWO:

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17},t)\right]G_{16}$ 10

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17},t)\right]G_{17}$ 11

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17},t)\right]G_{18}$ 12

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 13

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 14

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 15

$+(a_{16}^{''})^{(2)}(T_{17},t) =$ First augmentation factor 16

$-(b_{16}^{''})^{(2)}\left((G_{19}),t\right) =$ First detritions factor 17

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DENORMALIZATION THEORY AND LOCAL SINGULARITIES AND GLOBAL DECAY(INFRA RED REGULARITIES IN QUANTUM FIELD THEORIES:

MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21},t)\right]G_{20}$ 19

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21},t)\right]G_{21}$ 20

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21},t)\right]G_{22}$ 21

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23},t)\right]T_{20}$ 22

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23},t)\right]T_{21}$ 23

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23},t)\right]T_{22}$ 24

$+(a_{20}^{''})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b_{20}^{''})^{(3)}(G_{23},t) =$ First detritions factor 25

QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARAIBLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS) 26

: MODULE NUMBERED FOUR

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25},t)\right]G_{24}$ 27

$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)} (T_{25}, t) \right] G_{25}$ 28

$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)} (T_{25}, t) \right] G_{26}$ 29

$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)} \left((G_{27}), t \right) \right] T_{24}$ 30

$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)} \left((G_{27}), t \right) \right] T_{25}$ 31

$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)} \left((G_{27}), t \right) \right] T_{26}$ 32

$+(a_{24}^{''})^{(4)} (T_{25}, t) =$ **First augmentation factor** 33

$-(b_{24}^{''})^{(4)} \left((G_{27}), t \right) =$ **First detritions factor** 34

ASYMPTOTIC FREEDOM AND FOURIER SERIES AND CLASSICAL DYNAMICAL SYSTEMS 35

MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)} (T_{29}, t) \right] G_{28}$ 36

$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)} (T_{29}, t) \right] G_{29}$ 37

$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)} (T_{29}, t) \right] G_{30}$ 38

$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)} \left((G_{31}), t \right) \right] T_{28}$ 39

$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)} \left((G_{31}), t \right) \right] T_{29}$ 40

$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)} \left((G_{31}), t \right) \right] T_{30}$ 41

$+(a_{28}^{''})^{(5)} (T_{29}, t) =$ **First augmentation factor** 42

$-(b_{28}^{''})^{(5)} \left((G_{31}), t \right) =$ **First detritions factor** 43

QUANTUN FIELD THEORY AND STATISTICAL MECHANICS AND YANG BAXTER EQUATIONS AND QUANTUM GROUPS AND BOSE FERMION EQUIVALENCE AND RATIONAL CONFORMAL FIELD THEORY: 44 45

MODULE NUMBERED SIX:

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The differential system of this model is now (Module numbered Six)

$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right] G_{32}$ 46

$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right] G_{33}$ 47

$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right] G_{34}$ 48

$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right] T_{32}$ 49

$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right] T_{33}$ 50

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right] T_{34}$ 51

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor** 52

LIE ALGEBRAS OF THE GROUP AND TRANSLATIONS IN SPACE TIME 53

MODULE NUMBERED SEVEN

The differential system of this model is now (SEVENTH MODULE)

$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right] G_{36}$ 54

$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right] G_{37}$ 55

$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right] G_{38}$ 56

$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right] T_{36}$ 57

$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right] T_{37}$ 58

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$\frac{dT_{38}}{dt} = (b_{38})^{(7)} T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right] T_{38}$ 60

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor** 61

$-(b''_{36})^{(7)}\left((G_{39}), t\right) =$ **First detritions factor** 62

FIRST MODULE CONCATENATION:

$$\frac{dG_{13}}{dt}=(a_{13})^{(1)}G_{14}-\left[\begin{matrix}(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}\\ \boxed{+(a''_{36})^{(7)}(T_{37},t)}\end{matrix}\right]G_{13}$$

$$\frac{dG_{14}}{dt}=(a_{14})^{(1)}G_{13}-\left[\begin{matrix}(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}\\ \boxed{+(a''_{37})^{(7)}(T_{37},t)}\end{matrix}\right]G_{14}$$

$$\frac{dG_{15}}{dt}=(a_{15})^{(1)}G_{14}-\left[\begin{matrix}(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}\\ \boxed{+(a''_{38})^{(7)}(T_{37},t)}\end{matrix}\right]G_{15}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{38})^{(7)}(T_{37},t)}$ARESEVENTHAUGMENTATION COEFFICIENTS

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\left[\begin{matrix}(b'_{13})^{(1)}\boxed{-(b''_{16})^{(1)}(G,t)}\boxed{-(b''_{36})^{(7,)}(G_{39},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}\\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}\\ \boxed{-(b''_{36})^{(7,)}(G_{39},t)}\end{matrix}\right]T_{13}$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\left[\begin{matrix}(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}\\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}\\ \boxed{-(b''_{37})^{(7,)}(G_{39},t)}\end{matrix}\right]T_{14}$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\left[\begin{matrix}(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}\\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}\\ \boxed{-(b''_{38})^{(7,)}(G_{39},t)}\end{matrix}\right]T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

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$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \end{matrix}\right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 64

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

SECOND MODULE CONCATENATION: 65

66
$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{matrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,)}(T_{37},t)} \end{matrix}\right] G_{16}$$

67
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{matrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,)}(T_{37},t)} \end{matrix}\right] G_{17}$$

68
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{matrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,)}(T_{37},t)} \end{matrix}\right] G_{18}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 69

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

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$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,)}(T_{37},t)}\boxed{+(a''_{37})^{(7,7,)}(T_{37},t)}\boxed{+(a''_{38})^{(7,7,)}(T_{37},t)}$ARE SEVENTH DETRITION COEFFICIENTS 71

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7)}(G_{39},t)} \end{bmatrix} T_{16}$$ 72

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7)}(G_{39},t)} \end{bmatrix} T_{17}$$ 73

$$\frac{dT_{18}}{dt} = (\mathrm{b}_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39},t)} \end{bmatrix} T_{18}$$ 74

$where$ $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 75

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}$ $are\ seventh\ detrition\ coefficients$

THIRD MODULE CONCATENATION:

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)} \end{bmatrix} G_{20}$$ 76

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)} \end{bmatrix} G_{21}$$ 77

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{matrix} (a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21},t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)} \end{matrix}\right] G_{22}$$ 78

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$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3 80

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)}$ $\boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)}$ are seventh augmentation coefficient 81

$$\frac{dT_{20}}{dt} =$$ 82

$$(b_{20})^{(3)}T_{21} - \left[\begin{matrix} (b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23},t)} \boxed{-(b''_{36})^{(7,7,7)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} \end{matrix}\right] T_{20}$$

$$\frac{dT_{21}}{dt} =$$ 83

$$(b_{21})^{(3)}T_{20} - \left[\begin{matrix} (b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23},t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} \end{matrix}\right] T_{21}$$

$$\frac{dT_{22}}{dt} =$$ 84

$$(b_{22})^{(3)}T_{21} - \left[\begin{matrix} (b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23},t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} \end{matrix}\right] T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ are first detritions coefficients for category 1, 2 and 3 85

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and

3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detritions coefficients 86

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FOURTH MODULE CONCATENATION:

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\left[\begin{matrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\end{matrix}\right]G_{24}$$ 87

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\left[\begin{matrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,)}(T_{37},t)}\end{matrix}\right]G_{25}$$ 88

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\left[\begin{matrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,)}(T_{37},t)}\end{matrix}\right]G_{26}$$ 89

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$Where\ \boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

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$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\ are\ third\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}$ ARE SEVENTH augmentation coefficients

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$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\left[\begin{matrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,,,)}(G_{39},t)}\end{matrix}\right]T_{24}$$ 93

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\left[\begin{matrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,77,,,)}(G_{39},t)}\end{matrix}\right]T_{25}$$ 94

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\left[\begin{array}{c}(b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,,7,,)}(G_{39},t)}\end{array}\right]T_{26}$$ 95

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3 96

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,,)}(G_{39},t)}\boxed{-(b''_{37})^{(7,7,7,7,7,,)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7,7,,)}(G_{39},t)}$ *ARE* SEVENTH DETRITION

COEFFICIENTS

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FIFTH MODULE CONCATENATION: 98

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\left[\begin{array}{c}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,,7,,7,7)}(T_{37},t)}\end{array}\right]G_{28}$$ 99

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\left[\begin{array}{c}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,,,7,,7,7)}(T_{37},t)}\end{array}\right]G_{29}$$ 100

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\left[\begin{array}{c}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,,7,,7,7)}(T_{37},t)}\end{array}\right]G_{30}$$ 101

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3 102

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *a*

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

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$$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[\begin{matrix} (b'_{28})^{(5)} \boxed{-(b''_{28})^{(5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,)}(G_{23},t)} \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,,7,7,7,)}(G_{38},t)} \end{matrix}\right] T_{28}$$ 104

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[\begin{matrix} (b'_{29})^{(5)} \boxed{-(b''_{29})^{(5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,)}(G_{38},t)} \end{matrix}\right] T_{29}$$ 105

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[\begin{matrix} (b'_{30})^{(5)} \boxed{-(b''_{30})^{(5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,)}(G_{38},t)} \end{matrix}\right] T_{30}$$ 106

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3 107

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

SIXTH MODULE CONCATENATION 108

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[\begin{matrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,)}(T_{37},t)} \end{matrix}\right] G_{32}$$ 109

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[\begin{matrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,,7,7,7,)}(T_{37},t)} \end{matrix}\right] G_{33}$$ 110

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[\begin{matrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,)}(T_{37},t)} \end{matrix}\right] G_{34}$$ 111

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$ 112

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ an$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation\ coeffici\ ents\ for\ category\ 1,2\ a$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,,77,7,,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,,7,7,7,)}(T_{37},t)}$ ARE SVENTH AUGMENTATION COEFFICIENTS

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$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{matrix}\right] T_{32}$$ 114

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ -(b''_{37})^{(7,7,7,,7,7,7)}(G_{39},t) \end{matrix}\right] T_{33}$$ 115

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{matrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{matrix}\right] T_{34}$$ 116

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$ 117

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ $are\ third\ detrition\ coefficients\ for\ category\ 1{,}2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

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SEVENTH MODULE CONCATENATION: 119

$\frac{dG_{36}}{dt} =$ 120

$(a_{36})^{(7)}G_{37} -$

$\Big[(a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} + \boxed{(a''_{16})^{(7)}(\mathrm{T}_{17},t)} + \boxed{\boxed{(a''_{20})^{(7)}(T_{21},t)} +} \boxed{(a''_{24})^{(7)}(T_{23},t)G_{36}} +$

$\boxed{(a''_{28})^{(7)}(T_{29},t)} \quad + \quad \boxed{(a''_{32})^{(7)}(T_{33},t)} \quad + \boxed{(a''_{13})^{(7)}(T_{14},t)} \Big] G_{36}$

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$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - \Big[(a'_{37})^{(7)} + \boxed{(a''_{37})^{(7)}(T_{37},t)} \quad + \quad \boxed{(a''_{14})^{(7)}(T_{14},t)} \quad + \quad \boxed{(a''_{21})^{(7)}(T_{21},t)} +$ 122
$\boxed{(a''_{17})^{(7)}(T_{17},t)} \quad + \quad \boxed{(a''_{25})^{(7)}(T_{25},t)} \quad + \boxed{(a''_{33})^{(7)}(T_{33},t)} \; + \; \boxed{(a''_{29})^{(7)}(T_{29},t)} \quad \Big] G_{37}$

$\frac{dG_{38}}{dt} =$ 123
$(a_{38})^{(7)} G_{37} -$ 124
$\Big[(a'_{38})^{(7)} + \boxed{(a''_{38})^{(7)}(T_{37},t)} \quad + \quad \underbrace{\boxed{(a''_{15})^{(7)}(T_{14},t)}} \; + \; \boxed{(a''_{22})^{(7)}(T_{21},t) +} \boxed{+(a''_{18})^{(7)}(T_{17},t)} \quad +$ 125
$\boxed{(a''_{26})^{(7)}(T_{25},t)} \quad + \quad \boxed{(a''_{34})^{(7)}(T_{33},t)} \quad + \; \boxed{(a''_{30})^{(7)}(T_{29},t)} \Big] G_{38}$

$\frac{dT_{36}}{dt} =$ 126
$(b_{36})^{(7)} T_{37} - \Big[(b'_{36})^{(7)} - \boxed{(b''_{36})^{(7)}\big((G_{39}),t\big)} \; - \boxed{(b''_{16})^{(7)}\big((G_{19}),t\big)} \quad - \boxed{(b''_{13})^{(7)}\big((G_{14}),t\big)} \quad -$
$\boxed{(b''_{20})^{(7)}\big((G_{231}),t\big)} \quad - \quad \boxed{(b''_{24})^{(7)}\big((G_{27}),t\big)} \quad - \boxed{(b''_{28})^{(7)}\big((G_{31}),t\big)} \quad -$
$\boxed{(b''_{32})^{(7)}\big((G_{35}),t\big)} \qquad \Big] T_{36}$

$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} -$ 127
$\Big[(b'_{36})^{(7)} - \boxed{(b''_{37})^{(7)}\big((G_{39}),t\big)} \; - \boxed{(b''_{17})^{(7)}\big((G_{19}),t\big)} \quad - \boxed{(b''_{19})^{(7)}\big((G_{14}),t\big)} \quad -$
$\boxed{(b''_{21})^{(7)}\big((G_{231}),t\big)} \quad - \quad \boxed{(b''_{25})^{(7)}\big((G_{27}),t\big)} \quad - \boxed{(b''_{29})^{(7)}\big((G_{31}),t\big)} \quad -$
$\boxed{(b''_{33})^{(7)}\big((G_{35}),t\big)} \qquad \Big] T_{37}$

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$
$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(1)}, \; (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G,t) \leq \; (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14},t) = (p_i)^{(1)}$
$lim_{G \to \infty} (b''_i)^{(1)} (G,t) = \; (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants
and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(1)}(T_{14}^{'},t) - (a_i^{''})^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}^{'}|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i^{''})^{(1)}(G^{'},t) - (b_i^{''})^{(1)}(G,T)| < (\hat{k}_{13})^{(1)}||G - G^{'}||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(1)}(T_{14}^{'},t)$ and $(a_i^{''})^{(1)}(T_{14},t)$. $(T_{14}^{'},t)$ and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and \ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i^{'})^{(1)}, (b_i)^{(1)}, (b_i^{'})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i^{'})^{(1)} + \ (\hat{A}_{13})^{(1)} + \ (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i^{'})^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)}\ \ (\hat{k}_{13})^{(1)}] < 1$$

$\frac{dT_{38}}{dt} =$ 128

$(b_{38})^{(7)}T_{37} - \Big[(b_{38}^{'})^{(7)} - \boxed{(b_{38}^{''})^{(7)}\big((G_{39}),t\big)} - \boxed{(b_{18}^{''})^{(7)}\big((G_{19}),t\big)} - \boxed{(b_{20}^{''})^{(7)}\big((G_{14}),t\big)} -$ 129

$\boxed{(b_{22}^{''})^{(7)}\big((G_{23}),t\big)} - \boxed{(b_{26}^{''})^{(7)}\big((G_{27}),t\big)} - \boxed{(b_{30}^{''})^{(7)}\big((G_{31}),t\big)} -$ 130

$\boxed{(b_{34}^{''})^{(7)}\big((G_{35}),t\big)} \Big] T_{38}$ 131

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$+(a_{36}^{''})^{(7)}(T_{37},t) =$ **First augmentation factor** 134

$(1)(a_i)^{(2)}, (a_i^{'})^{(2)}, (a_i^{''})^{(2)}, (b_i)^{(2)}, (b_i^{'})^{(2)}, (b_i^{''})^{(2)} > 0, \quad i,j = 16,17,18$ 135

(F) (2) The functions $(a_i^{''})^{(2)}, (b_i^{''})^{(2)}$ are positive continuous increasing and bounded. 136

Definition of $(p_i)^{(2)}, \ (r_i)^{(2)}$: 137

$(a_i^{''})^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$ 138

$(b_i^{''})^{(2)}(G_{19},t) \leq (r_i)^{(2)} \leq (b_i^{'})^{(2)} \leq (\hat{B}_{16})^{(2)}$ 139

(G) (3) $lim_{T_2 \to \infty}(a_i^{''})^{(2)}(T_{17},t) = (p_i)^{(2)}$ 140

$lim_{G \to \infty}(b_i^{''})^{(2)}\big((G_{19}),t\big) = (r_i)^{(2)}$ 141

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 142

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 143

$|(a_i^{''})^{(2)}(T_{17}^{'},t) - (a_i^{''})^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}^{'}|e^{-(\hat{M}_{16})^{(2)}t}$ 144

$|(b_i^{''})^{(2)}((G_{19})^{'},t) - (b_i^{''})^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})^{'}||e^{-(\hat{M}_{16})^{(2)}t}$ 145

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(2)}(T_{17}^{'},t)$ and$(a_i^{''})^{(2)}(T_{17},t)$. $(T_{17}^{'},t)$ And (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i^{''})^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i^{''})^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous. 146

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 147

(H) (4) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 148

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 149

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i^{'})^{(2)}, (b_i)^{(2)}, (b_i^{'})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i^{'})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 150

$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i^{'})^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ 151

Where we suppose 152

(I) (5) $(a_i)^{(3)}, (a_i^{'})^{(3)}, (a_i^{''})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (b_i^{''})^{(3)} > 0, \quad i,j = 20,21,22$ 153

The functions $(a_i^{''})^{(3)}, (b_i^{''})^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$(a_i^{''})^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$(b_i^{''})^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i^{'})^{(3)} \leq (\hat{B}_{20})^{(3)}$

$lim_{T_2 \to \infty}(a_i^{''})^{(3)}(T_{21}, t) = (p_i)^{(3)}$ 154

$lim_{G \to \infty}(b_i^{''})^{(3)}(G_{23}, t) = (r_i)^{(3)}$ 155

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 156

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 157

$|(a_i^{''})^{(3)}(T_{21}^{'}, t) - (a_i^{''})^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}^{'}|e^{-(\hat{M}_{20})^{(3)}t}$ 158

$|(b_i^{''})^{(3)}(G_{23}^{'}, t) - (b_i^{''})^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}^{'}||e^{-(\hat{M}_{20})^{(3)}t}$ 159

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(3)}(T_{21}^{'}, t)$ and$(a_i^{''})^{(3)}(T_{21}, t)$. $(T_{21}^{'}, t)$ And (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i^{''})^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i^{''})^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient, would be absolutely continuous. 160

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 161

(J) (6) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i^{'})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 162

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$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i^{'})^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$ 165

$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i^{'})^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$ 166

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Where we suppose 168

$(a_i)^{(4)}, (a_i^{'})^{(4)}, (a_i^{''})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (b_i^{''})^{(4)} > 0, \quad i,j = 24,25,26$ 169

(L) (7) The functions $(a_i^{''})^{(4)}, (b_i^{''})^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i^{''})^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i^{''})^{(4)}\big((G_{27}), t\big) \leq \ (r_i)^{(4)} \leq (b_i^{'})^{(4)} \leq (\hat{B}_{24})^{(4)}$$

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(M) (8) $lim_{T_2\to\infty}(a_i^{''})^{(4)}\,(T_{25}, t) = (p_i)^{(4)}$
$lim_{G\to\infty}(b_i^{''})^{(4)}\big((G_{27}), t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 171

$$|(a_i^{''})^{(4)}(T_{25}^{'}, t) - (a_i^{''})^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i^{''})^{(4)}((G_{27})^{'}, t) - (b_i^{''})^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'}, t)$ and$(a_i^{''})^{(4)}(T_{25}, t)$. $(T_{25}^{'}, t)$ And (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i^{''})^{(4)}(T_{25}, t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous. 172

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Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 174

$(\hat{M}_{24})176^{175\,(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 175

(P) (9) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i^{'})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i^{'})^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i^{'})^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ \ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose 176

$(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, \quad i, j = 28{,}29{,}30$ 177

(R) (10) The functions $(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i^{''})^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$(b_i^{''})^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i^{'})^{(5)} \leq (\ \hat{B}_{28}\ )^{(5)}$

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(S) (11) $lim_{T_2\to\infty}(a_i^{''})^{(5)}\,(T_{29},t) = (p_i)^{(5)}$
$lim_{G\to\infty}(b_i^{''})^{(5)}\,(G_{31},t) = \ (r_i)^{(5)}$

Definition of $(\ \hat{A}_{28}\ )^{(5)}, (\ \hat{B}_{28}\ )^{(5)}$:

Where $\boxed{(\ \hat{A}_{28}\ )^{(5)}, (\ \hat{B}_{28}\ )^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 179

$|(a_i^{''})^{(5)}(T_{29}^{'},t) - (a_i^{''})^{(5)}(T_{29},t)| \leq (\ \hat{k}_{28}\ )^{(5)}|T_{29} - \ T_{29}^{'}|e^{-(\ \hat{M}_{28}\ )^{(5)}t}$

$|(b_i^{''})^{(5)}((G_{31})^{'},t) - (b_i^{''})^{(5)}\big((G_{31}),\mathrm{t}\big)| < (\ \hat{k}_{28}\ )^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\ \hat{M}_{28}\ )^{(5)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'},t)$ and$(a_i^{''})^{(5)}(T_{29},t)$. $(T_{29}^{'},t)$ and (T_{29},t) are points belonging to the interval $\big[(\ \hat{k}_{28}\ )^{(5)}, (\ \hat{M}_{28}\ )^{(5)}\big]$. It is to be noted that $(a_i^{''})^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\ \hat{M}_{28}\ )^{(5)} = 5$ then the function $(a_i^{''})^{(5)}(T_{29},t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous. 180

Definition of $(\ \hat{M}_{28}\ )^{(5)}, (\ \hat{k}_{28}\ )^{(5)}$: 181

$(\ \hat{M}_{28}\ )^{(5)}, (\ \hat{k}_{28}\ )^{(5)},$ are positive constants
$\frac{(a_i)^{(5)}}{(\ \hat{M}_{28}\ )^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\ \hat{M}_{28}\ )^{(5)}} < 1$

Definition of $(\ \hat{P}_{28}\ )^{(5)}, (\ \hat{Q}_{28}\ )^{(5)}$: 182

There exists two constants $(\ \hat{P}_{28}\ )^{(5)}$ and $(\ \hat{Q}_{28}\ )^{(5)}$ which together with $(\ \hat{M}_{28}\ )^{(5)}, (\ \hat{k}_{28}\ )^{(5)}, (\hat{A}_{28})^{(5)} and\ (\ \hat{B}_{28}\ )^{(5)}$ and the constants $(a_i)^{(5)}, (a_i^{'})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\ \hat{M}_{28}\ )^{(5)}}[\ (a_i)^{(5)} + (a_i^{'})^{(5)} + \ (\ \hat{A}_{28}\ )^{(5)} + \ (\ \hat{P}_{28}\ )^{(5)}\ (\ \hat{k}_{28}\ )^{(5)}] < 1$$

$$\frac{1}{(\ \hat{M}_{28}\ )^{(5)}}[\ (b_i)^{(5)} + (b_i^{'})^{(5)} + \ (\ \hat{B}_{28}\ )^{(5)} + \ (\ \hat{Q}_{28}\ )^{(5)}\ \ (\ \hat{k}_{28}\ )^{(5)}] < 1$$

Where we suppose 183

$(a_i)^{(6)}, (a_i^{'})^{(6)}, (a_i^{''})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (b_i^{''})^{(6)} > 0, \quad i,j = 32,33,34$ 184
(12) The functions $(a_i^{''})^{(6)}, (b_i^{''})^{(6)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(6)},\ (r_i)^{(6)}$:

$(a_i^{''})^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\ \hat{A}_{32}\ )^{(6)}$

$(b_i^{''})^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i^{'})^{(6)} \leq (\ \hat{B}_{32}\ )^{(6)}$

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(13) $lim_{T_2\to\infty}(a_i^{''})^{(6)}\,(T_{33},t) = (p_i)^{(6)}$
$lim_{G\to\infty}(b_i^{''})^{(6)}\,\big((G_{35}),t\big) = \ (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition: 186

$$|(a_i^{''})^{(6)}(T_{33}^{'}, t) - (a_i^{''})^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}^{'}|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i^{''})^{(6)}((G_{35})^{'}, t) - (b_i^{''})^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})^{'}||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'}, t)$ and$(a_i^{''})^{(6)}(T_{33}, t)$. $(T_{33}^{'}, t)$ and (T_{33}, t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i^{''})^{(6)}(T_{33}, t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. 187

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 188

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 189

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i^{'})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (a_i)^{(6)} + (a_i^{'})^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i^{'})^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose 190

(V) $(a_i)^{(7)}, (a_i^{'})^{(7)}, (a_i^{''})^{(7)}, (b_i)^{(7)}, (b_i^{'})^{(7)}, (b_i^{''})^{(7)} > 0,$ 191
$i, j = 36,37,38$

(W) The functions $(a_i^{''})^{(7)}, (b_i^{''})^{(7)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i^{''})^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i^{''})^{(7)}(G, t) \leq \ (r_i)^{(7)} \leq (b_i^{'})^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(X) $lim_{T_2\to\infty}\ (a_i^{''})^{(7)}\ (T_{37},t) = (p_i)^{(7)}$
$lim_{G\to\infty}(b_i^{''})^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\ \hat{A}_{36}\ )^{(7)}, (\ \hat{B}_{36}\ )^{(7)}$:

Where $\boxed{(\ \hat{A}_{36}\ )^{(7)}, (\ \hat{B}_{36}\ )^{(7)}, (p_i)^{(7)},\ (r_i)^{(7)}}$ are positive constants
and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 193

$$|(a_i^{''})^{(7)}(T_{37}^{'},t) - (a_i^{''})^{(7)}(T_{37},t)| \leq (\ \hat{k}_{36}\ )^{(7)}|T_{37} - T_{37}^{'}|e^{-(\ \hat{M}_{36}\ )^{(7)}t}$$

$$|(b_i^{''})^{(7)}((G_{39})^{'},t) - (b_i^{''})^{(7)}\left((G_{39}),(T_{39})\right)| < (\ \hat{k}_{36}\ )^{(7)}||(G_{39}) - (G_{39})^{'}||e^{-(\ \hat{M}_{36}\ )^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(7)}(T_{37}^{'},t)$ 194
and$(a_i^{''})^{(7)}(T_{37},t)$. $(T_{37}^{'},t)$ and (T_{37},t) are points belonging to the interval $\left[(\ \hat{k}_{36}\ )^{(7)}, (\ \hat{M}_{36}\ )^{(7)}\right]$. It is to be noted that $(a_i^{''})^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\ \hat{M}_{36}\ )^{(7)} = 7$ then the function $(a_i^{''})^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\ \hat{M}_{36}\ )^{(7)}, (\ \hat{k}_{36}\ )^{(7)}$: 195

(Y) $(\ \hat{M}_{36}\ )^{(7)}, (\ \hat{k}_{36}\ )^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\ \hat{M}_{36}\ )^{(7)}}\ ,\frac{(b_i)^{(7)}}{(\ \hat{M}_{36}\ )^{(7)}} < 1$$

Definition of $(\ \hat{P}_{36}\ )^{(7)}, (\ \hat{Q}_{36}\ )^{(7)}$: 196

(Z) There exists two constants $(\ \hat{P}_{36}\ )^{(7)}$ and $(\ \hat{Q}_{36}\ )^{(7)}$ which together with $(\ \hat{M}_{36}\ )^{(7)}, (\ \hat{k}_{36}\ )^{(7)}, (\hat{A}_{36})^{(7)} and\ (\ \hat{B}_{36}\ )^{(7)}$ and the constants $(a_i)^{(7)}, (a_i^{'})^{(7)}, (b_i)^{(7)}, (b_i^{'})^{(7)}, (p_i)^{(7)},\ (r_i)^{(7)}, i = 36,37,38,$
satisfy the inequalities

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$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

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Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0), T_i(0)$: 199

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 200

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 201

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 202

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 203

By 204

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}^{'})^{(1)} + a_{13}^{''})^{(1)} \big(T_{14}(s_{(13)}), s_{(13)} \big) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}^{'})^{(1)} + (a_{14}^{''})^{(1)} \big(T_{14}(s_{(13)}), s_{(13)} \big) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$ 205

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)} \big(T_{14}(s_{(13)}), s_{(13)} \big) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$ 206

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)} \big(G(s_{(13)}), s_{(13)} \big) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$ 207

$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)} \big(G(s_{(13)}), s_{(13)} \big) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$ 208

$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)} \big(G(s_{(13)}), s_{(13)} \big) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$ 209

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By

$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a_{36}^{'})^{(7)} + a_{36}^{''})^{(7)} \big(T_{37}(s_{(36)}), s_{(36)} \big) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$

$\bar{G}_{37}(t) = G_{37}^0 +$
$\int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a_{37}^{'})^{(7)} + (a_{37}^{''})^{(7)} \big(T_{37}(s_{(36)}), s_{(36)} \big) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$

$\bar{G}_{38}(t) = G_{38}^0 +$
$\int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)} \big(T_{37}(s_{(36)}), s_{(36)} \big) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$

$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)} \big(G(s_{(36)}), s_{(36)} \big) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$

$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b_{37}^{'})^{(7)} - (b_{37}^{''})^{(7)} \big(G(s_{(36)}), s_{(36)} \big) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$

$$\bar{T}_{38}(t) = T_{38}^{0} + \int_0^t \left[(b_{38})^{(7)} T_{37}\left(s_{(36)}\right) - \left((b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)} \left(G\left(s_{(36)}\right), s_{(36)} \right) \right) T_{38}\left(s_{(36)}\right) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$, 212

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 213

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 214

By 215

$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a_{16}^{'})^{(2)} + a_{16}^{''})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$

$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$ 216

$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$ 217

$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$ 218

$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$ 219

$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$ 220

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$, 222

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 223

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 224

By 225

$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a_{20}^{'})^{(3)} + a_{20}^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$

$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$ 226

$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$ 227

$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$ 228

$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$ 229

$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}\left(G(s_{(20)}), s_{(20)}\right)\right) T_{22}(s_{(20)})\right] ds_{(20)}$ 230

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 231

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$, 232

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 233

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 234

By 235

$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}^{'})^{(4)} + a_{24}^{''})^{(4)}\left(T_{25}(s_{(24)}), s_{(24)}\right)\right) G_{24}(s_{(24)})\right] ds_{(24)}$

$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}\left(T_{25}(s_{(24)}), s_{(24)}\right)\right) G_{25}(s_{(24)})\right] ds_{(24)}$ 236

$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}\left(T_{25}(s_{(24)}), s_{(24)}\right)\right) G_{26}(s_{(24)})\right] ds_{(24)}$ 237

$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\left(G(s_{(24)}), s_{(24)}\right)\right) T_{24}(s_{(24)})\right] ds_{(24)}$ 238

$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}\left(G(s_{(24)}), s_{(24)}\right)\right) T_{25}(s_{(24)})\right] ds_{(24)}$ 239

$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\left(G(s_{(24)}), s_{(24)}\right)\right) T_{26}(s_{(24)})\right] ds_{(24)}$ 240

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 241

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$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$, 243

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 244

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 245

By 246

$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a_{28}^{'})^{(5)} + a_{28}^{''})^{(5)}\left(T_{29}(s_{(28)}), s_{(28)}\right)\right) G_{28}(s_{(28)})\right] ds_{(28)}$

$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}\left(T_{29}(s_{(28)}), s_{(28)}\right)\right) G_{29}(s_{(28)})\right] ds_{(28)}$ 247

$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}\left(T_{29}(s_{(28)}), s_{(28)}\right)\right) G_{30}(s_{(28)})\right] ds_{(28)}$ 248

$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}\left(G(s_{(28)}), s_{(28)}\right)\right) T_{28}(s_{(28)})\right] ds_{(28)}$ 249

$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}\left(G(s_{(28)}), s_{(28)}\right)\right) T_{29}(s_{(28)})\right] ds_{(28)}$ 250

$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}\left(s_{(28)}\right) - \left((b_{30}')^{(5)} - (b_{30}'')^{(5)}\left(G\left(s_{(28)}\right), s_{(28)}\right)\right) T_{30}\left(s_{(28)}\right)\right] ds_{(28)}$ 251

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$, 253

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 254

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 255

By 256

$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a_{32}')^{(6)} + a_{32}'')^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{32}\left(s_{(32)}\right)\right] ds_{(32)}$

$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}\left(s_{(32)}\right) - \left((a_{33}')^{(6)} + (a_{33}'')^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{33}\left(s_{(32)}\right)\right] ds_{(32)}$ 257

$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a_{34}')^{(6)} + (a_{34}'')^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{34}\left(s_{(32)}\right)\right] ds_{(32)}$ 258

$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b_{32}')^{(6)} - (b_{32}'')^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{32}\left(s_{(32)}\right)\right] ds_{(32)}$ 259

$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}\left(s_{(32)}\right) - \left((b_{33}')^{(6)} - (b_{33}'')^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{33}\left(s_{(32)}\right)\right] ds_{(32)}$ 260

$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b_{34}')^{(6)} - (b_{34}'')^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{34}\left(s_{(32)}\right)\right] ds_{(32)}$ 261

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

: if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions 262

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$, 263

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ 264

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ 265

By 266

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a_{36}')^{(7)} + a_{36}'')^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

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$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a_{37}')^{(7)} + (a_{37}'')^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a_{38}')^{(7)} + (a_{38}'')^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$ 268

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$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b_{36}')^{(7)} - (b_{36}'')^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b_{37}')^{(7)} - (b_{37}'')^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$ 270

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b_{38}')^{(7)} - (b_{38}'')^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$ 271

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 272

(a) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 273

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 274

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(b) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 275

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 276

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 277

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 278

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem1

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying 37,35,36 into itself .Indeed it is obvious that 279

$$G_{36}(t) \leq G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

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From which it follows that

$$(G_{36}(t)-G_{36}^0)e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)}+G_{37}^0\right)e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^0}{G_{37}^0}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 281

$(\hat{P}_{13})^{(1)}$ *and* $(\hat{Q}_{13})^{(1)}$ large to have

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283

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)}$$

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$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself 285

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 286

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 287

Definition of $\tilde{G},\tilde{T}$:

$$\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a_{13}')^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq$ 288
$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ *and* $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 289

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 290

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$ 291

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)}t\right)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1$, *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 292

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 293

Remark 5: If T_{13} is bounded from below and $lim_{t\to\infty}((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 294

Definition of $(m)^{(1)}$ *and* ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 295

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $lim_{t\to\infty}(b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 297

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{16})^{(2)}$$ 298

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$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \le (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 300

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 301

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 302

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 303

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t \{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 304

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \le$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} +\right.$$
$$\left.(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$ 305

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows 306

Remark 1: The fact that we supposed $(a_{16}'')^{(2)}$ *and* $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ *and* $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 307

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ *and* $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on

$(G_{19})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 308

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(2)} - (a_i^{''})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i^{'})^{(2)} t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2\ and\ \left((\widehat{M}_{16})^{(2)}\right)_3$: 309

Remark 3: if G_{16} is bounded, the same property have also $G_{17}\ and\ G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}^{'})^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}^{'})^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}^{'})^{(2)}$ 310

If $G_{17}\ or\ G_{18}$ is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If $G_{16}\ \ is$ bounded, from below, the same property holds for $G_{17}\ and\ G_{18}$. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 311

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(2)}\,((G_{19})(t), t)) = (b_{17}^{'})^{(2)}$ then $T_{17} \to \infty$. 312

Definition of $(m)^{(2)}\ and\ \varepsilon_2$:

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i^{''})^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}\,(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 313

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}^{''})^{(2)}\left((G_{19})(t), t\right) = (b_{18}^{'})^{(2)}$ 314

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 316

$(\hat{P}_{20})^{(3)}\ and\ (\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)}$$ 317

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$ 318

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself 319

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 320

$$d\left(\left((G_{23})^{(1)},(T_{23})^{(1)}\right),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 321

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right)=\mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 322

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a_{20}')^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$ 323

$$G_{20}^{(2)}|(a_{20}'')^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a_{20}'')^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$ 324

$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a_{20}')^{(3)}+(\widehat{A}_{20})^{(3)}+\right.$$

$$\left.(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{20}'')^{(3)}$ *and* $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 325

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 326

From 19 to 24 it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad\text{for } t>0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 327

Remark 3: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 328

Remark 5: If T_{20} is bounded from below and $lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 329

Definition of $(m)^{(3)}$ *and* ε_3 **:** 330

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 331

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 333

$(\hat{P}_{24})^{(4)}$ *and* $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)}$$ 334

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)}$$ 335

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying IN to itself 336

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 337

$d\left(\left((G_{27})^{(1)},(T_{27})^{(1)}\right),\left((G_{27})^{(2)},(T_{27})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\widehat{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\widehat{M}_{24})^{(4)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{27})}, \widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})} \right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq$ 339
$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} +\right.$
$\left.(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{24})^{(4)}$ *and* $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 340

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ *and* $(b''_i)^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 341

From GLOBAL EQUATIONS it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(4)}t\right)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: 342

Remark 3: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a_{26}^{'})^{(4)}$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 343

Remark 5: If T_{24} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(4)}((G_{27})(t),t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$. 344

Definition of $(m)^{(4)}$ *and* ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 345

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t),t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

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It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 347

$(\hat{P}_{28})^{(5)}$ *and* $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)}$$ 348

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)}$$ 349

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself 350

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 351

$d\left(\left((G_{31})^{(1)},(T_{31})^{(1)}\right),\left((G_{31})^{(2)},(T_{31})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})} \right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \, ds_{(28)} +$

$\int_0^t \{(a_{28}^{'})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a_{28}^{''})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} |(a_{28}^{''})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a_{28}^{''})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 353
$\frac{1}{(\widehat{M}_{28})^{(5)}} \left((a_{28})^{(5)} + (a_{28}^{'})^{(5)} + (\widehat{A}_{28})^{(5)} +\right.$
$\left.(\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a_{28}^{''})^{(5)} \; and \; (b_{28}^{''})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t} \; and \; (\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 354

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(5)} \; and \; (b_i^{''})^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31}) (and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 355

From GLOBAL EQUATIONS it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(5)} - (a_i^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(5)} t\right)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2 \; and \; \left((\widehat{M}_{28})^{(5)}\right)_3$: 356

Remark 3: if G_{28} is bounded, the same property have also $G_{29} \; and \; G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}^{'})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)} \left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}^{'})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}^{'})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 357

Remark 5: If T_{28} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(5)}((G_{31})(t),t)) = (b_{29}^{'})^{(5)}$ then $T_{29} \to \infty$. 358

Definition of $(m)^{(5)}$ *and* ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i^{''})^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)} \quad 359$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 360

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $lim_{t\to\infty}(b_{30}^{''})^{(5)}\left((G_{31})(t),t\right) = (b_{30}^{'})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

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It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 362

$(\hat{P}_{32})^{(6)}$ *and* $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 363$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 364$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i into itself 365

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 366

$d\left(\left((G_{35})^{(1)},(T_{35})^{(1)}\right),\left((G_{35})^{(2)},(T_{35})^{(2)}\right)\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$ 367

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

(1) $(a_i')^{(1)},(a_i'')^{(1)},(b_i)^{(1)},(b_i')^{(1)},(b_i'')^{(1)}>0$,
$i,j=13,14,15$

(2)The functions $(a_i'')^{(1)},(b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)},\ (r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t)\leq(p_i)^{(1)}\leq(\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t)\leq\ (r_i)^{(1)}\leq(b_i')^{(1)}\leq(\hat{B}_{13})^{(1)}$$

(3) $lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t)=(p_i)^{(1)}$
$lim_{G\to\infty}(b_i'')^{(1)}(G,t)=\ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)},(\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)},(\hat{B}_{13})^{(1)},(p_i)^{(1)},\ (r_i)^{(1)}}$ are positive constants and $\boxed{i=13,14,15}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(1)}(T_{14}',t)-(a_i'')^{(1)}(T_{14},t)|\leq(\hat{k}_{13})^{(1)}|T_{14}-T_{14}'|e^{-(\widehat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t)-(b_i'')^{(1)}(G,T)|<(\hat{k}_{13})^{(1)}||G-G'||e^{-(\widehat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and$(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)},(\widehat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{13})^{(1)}=1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\widehat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$:

(AA) $(\widehat{M}_{13})^{(1)},(\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\ ,\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}<1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(BB) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 368

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 7$ and to choose

$(\hat{P}_{36})^{(7)}\ and\ (\hat{Q}_{36})^{(7)}$ large to have

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$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)}$$

370

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying 37,35,36 into itself 371

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 372

$$d\left(\left((G_{39})^{(1)}, (T_{39})^{(1)}\right), \left((G_{39})^{(2)}, (T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$:

$$\left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}),(T_{39}))$$

It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \, ds_{(36)} +$$

$$\int_0^t \{(a_{36}^{'})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$(a_{36}^{''})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$G_{36}^{(2)} |(a_{36}^{''})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}^{''})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + \; (a_{36}^{'})^{(7)} + (\widehat{A}_{36})^{(7)} + \right.$$
$$\left. (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)} \right) d \left(\left((G_{39})^{(1)}, (T_{39})^{(1)}; \; (G_{39})^{(2)}, (T_{39})^{(2)} \right) \right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis (37,35,36) the result follows 374

Remark 1: The fact that we supposed $(a_{36}^{''})^{(7)} \; and \; (b_{36}^{''})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 375

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(7)} \; and \; (b_i^{''})^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39}) (and \; not \; on \; \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

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Remark 2: There does not exist any t where $G_i\,(t) = 0$ and $T_i\,(t) = 0$

From 79 to 36 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ and $\left((\widehat{M}_{36})^{(7)}\right)_3$: 377

Remark 3: if G_{36} is bounded, the same property have also G_{37} and G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If G_{37} or G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If G_{36} is bounded, from below, the same property holds for G_{37} and G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 378

Remark 5: If T_{36} is bounded from below and $lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t), t)) = (b_{37}')^{(7)}$ then 379

$T_{37} \to \infty$.

Definition of $(m)^{(7)}$ *and* ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 380

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $lim_{t\to\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t), t\right) = (b_{38}^{'})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 72

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS AND ITS CONCOMITANT CONDITIONALITIES into itself 381

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The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 383

$$d\left(\left((G_{39})^{(1)}, (T_{39})^{(1)}\right), \left((G_{39})^{(2)}, (T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$:

$$\left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}),(T_{39}))$$

It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$

$$\int_0^t \{(a_{36}^{'})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$

$$G_{36}^{(2)} |(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}^{''})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)|\; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a_{36}^{'})^{(7)} + (\widehat{A}_{36})^{(7)} + \right.$$
$$\left.(\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(\left((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{36}^{''})^{(7)}$ and $(b_{36}^{''})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ and $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 385

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(7)}$ and $(b_i^{''})^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0$ and $T_i\,(t) = 0$ 386

From CONCATENATED GLOBAL EQUATIONS it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(7)} - (a_i^{''})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(7)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ and $\left((\widehat{M}_{36})^{(7)}\right)_3$: 387

Remark 3: if G_{36} is bounded, the same property have also G_{37} and G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}^{'})^{(7)}G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}^{'})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}^{'})^{(7)}$

If G_{37} or G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If G_{36} is bounded, from below, the same property holds for G_{37} and G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 388

Remark 5: If T_{36} is bounded from below and $lim_{t\to\infty}((b_i^{''})^{(7)}((G_{39})(t),t)) = (b_{37}^{'})^{(7)}$ then $T_{37} \to \infty$. 389

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 390

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $lim_{t\to\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t),t\right) = (b_{38}^{'})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$-(\sigma_2)^{(2)} \leq -(a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} - (a_{16}^{''})^{(2)}(T_{17}\,,t) + (a_{17}^{''})^{(2)}(T_{17}\,,t) \leq -(\sigma_1)^{(2)}$ 391

$-(\tau_2)^{(2)} \leq -(b_{16}^{'})^{(2)} + (b_{17}^{'})^{(2)} - (b_{16}^{''})^{(2)}\left((G_{19}),t\right) - (b_{17}^{''})^{(2)}\left((G_{19}),t\right) \leq -(\tau_1)^{(2)}$ 392

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 393

By $(\nu_1)^{(2)} > 0\,, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0\,, (u_2)^{(2)} < 0$ the roots 394

(a) of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 395

and $(b_{14})^{(2)}\left(u^{(2)}\right)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 396

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 397

By $(\bar{\nu}_1)^{(2)} > 0\,, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0\,, (\bar{u}_2)^{(2)} < 0$ the 398 roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 399

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 400

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 401

(b) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 402

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \ \boldsymbol{if} \ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 403

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if} \ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)},$ 404

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 405

and analogously 406

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if} \ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if} \ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ \boldsymbol{if} \ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 407

Then the solution satisfies the inequalities 408

$$G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined 409

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \qquad 410$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}\left((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)}\right)}\left[e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \right. \qquad 411$$
$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}\left((S_1)^{(2)} - (a_{18}')^{(2)}\right)}\left[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}\right] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t}} \qquad 412$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t} \qquad 413$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}\left((R_1)^{(2)} - (b_{18}')^{(2)}\right)}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \leq T_{18}(t) \leq \qquad 414$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}\left((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)}\right)}\left[e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 415

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ 416

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$ 417

$(R_2)^{(2)} = (b_{18}^{'})^{(2)} - (r_{18})^{(2)}$

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Behavior of the solutions 419

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \leq -(a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} - (a_{20}^{''})^{(3)}(T_{21}, t) + (a_{21}^{''})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \leq -(b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G, t) - (b_{21}^{''})^{(3)}\big((G_{23}), t\big) \leq -(\tau_1)^{(3)}$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 420

(b) By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 421

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)},\ \boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)},\ \boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 422

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)},\ \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined 423

$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$ 424

$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \right.$ 425

$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a_{22}')^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t}] + G_{22}^0 e^{-(a_{22}')^{(3)}t} \right)$

$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}}$ 426

$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}$ 427

$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b_{22}')^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t}\right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq$ 428

$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 429

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{20}')^{(3)}$

$(R_2)^{(3)} = (b_{22}')^{(3)} - (r_{22})^{(3)}$

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If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a_{24}')^{(4)} + (a_{25}')^{(4)} - (a_{24}'')^{(4)}(T_{25}, t) + (a_{25}'')^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \leq -(b_{24}')^{(4)} + (b_{25}')^{(4)} - (b_{24}'')^{(4)}\left((G_{27}), t\right) - (b_{25}'')^{(4)}\left((G_{27}), t\right) \leq -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 433

(e) By $(\nu_1)^{(4)} > 0, (\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 434

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By $(\bar{\nu}_1)^{(4)} > 0, (\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ 436

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if} \ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if} \ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously 437
438

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}\ , \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined respectively

Then the solution satisfies the inequalities 439
440

$$G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$ 441
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where $(p_i)^{(4)}$ is defined 443
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$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$ 446
447

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}\left((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)}\right)}\left[e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}\left((S_1)^{(4)} - (a'_{26})^{(4)}\right)}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$ 448

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}}$$ 449

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}$$ 450

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}\left((R_1)^{(4)} - (b'_{26})^{(4)}\right)}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le$$ 451

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}\left((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)}\right)}\left[e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 452

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$ 453

Behavior of the solutions 454
If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$

$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \leq -(\tau_1)^{(5)}$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 455

(h) By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 456

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)},\ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}\ ,\boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$
and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 457

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}\ ,\boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$
and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities 458

$G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$

where $(p_i)^{(5)}$ is defined

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$ 459

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$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}\left((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}\left((S_1)^{(5)} - (a'_{30})^{(5)}\right)}\left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}\right)$

$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{\left((R_1)^{(5)} + (r_{28})^{(5)}\right)t}}$ 462

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{\left((R_1)^{(5)} + (r_{28})^{(5)}\right)t}$ 463

$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b_{30}')^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b_{30}')^{(5)}t}\right]+T_{30}^0 e^{-(b_{30}')^{(5)}t} \leq T_{30}(t) \leq$ 464

$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^0 e^{-(R_2)^{(5)}t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 465

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a_{28}')^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b_{28}')^{(5)}$

$(R_2)^{(5)} = (b_{30}')^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions 466

If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)} \leq -(a_{32}')^{(6)} + (a_{33}')^{(6)} - (a_{32}'')^{(6)}(T_{33}, t) + (a_{33}'')^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)} \leq -(b_{32}')^{(6)} + (b_{33}')^{(6)} - (b_{32}'')^{(6)}((G_{35}), t) - (b_{33}'')^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 467

(k) By $(\nu_1)^{(6)} > 0, (\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}(\nu^{(6)})^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 468

By $(\bar{\nu}_1)^{(6)} > 0, (\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}(\nu^{(6)})^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if} \ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

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$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if} \ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 471

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if} \ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if} \ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$, and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities 472

$$G_{32}^0 e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$ 473

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)}-(a_{34}')^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a_{34}')^{(6)}t}\right] + G_{34}^0 e^{-(a_{34}')^{(6)}t}\right)$$ 474

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}}$$ 475

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$ 476

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}')^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b_{34}')^{(6)}t}\right] + T_{34}^0 e^{-(b_{34}')^{(6)}t} \leq T_{34}(t) \leq$$ 477

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 478

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32}')^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32}')^{(6)}$$

$$(R_2)^{(6)} = (b_{34}')^{(6)} - (r_{34})^{(6)}$$

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If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(m) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a_{36}')^{(7)} + (a_{37}')^{(7)} - (a_{36}'')^{(7)}(T_{37}, t) + (a_{37}'')^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b_{36}')^{(7)} + (b_{37}')^{(7)} - (b_{36}'')^{(7)}((G_{39}), t) - (b_{37}'')^{(7)}((G_{39}), t) \leq -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 480

(n) By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$ 481
and $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 482

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the

roots of the equations $(a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

(o) If we define $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)},\ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}\ , \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 483

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}\ , \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined respectively

Then the solution satisfies the inequalities 484

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined

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$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t}$ 486

$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})} \left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \right.$ 487

$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)}-(a_{38}')^{(7)})} \left[e^{(S_1)^{(7)}t} - e^{-(a_{38}')^{(7)}t} \right] + G_{38}^0 \mathrm{e}^{-(a_{38}')^{(7)}t} \right)$

$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}}$ 488

$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}$ 489

$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)}-(b_{38}')^{(7)})} \left[e^{(R_1)^{(7)}t} - e^{-(b_{38}')^{(7)}t} \right] + T_{38}^0 e^{-(b_{38}')^{(7)}t} \leq T_{38}(t) \leq$ 490

$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})} \left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 491

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}')^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}')^{(7)}$

$(R_2)^{(7)} = (b_{38}')^{(7)} - (r_{38})^{(7)}$

From GLOBAL EQUATIONS we obtain 492

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}')^{(7)} - (a_{37}')^{(7)} + (a_{36}'')^{(7)}(T_{37}, t) \right) - (a_{37}'')^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$-\left((a_{37})^{(7)} \left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} \right) \leq \frac{d\nu^{(7)}}{dt} \leq$

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

(a) For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

$it\ follows\ (\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 493

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

(b) If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 494

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\ \nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

(c) If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 495

$$(\nu_1)^{(7)}\leq\ \nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then

$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

We can prove the following 496

If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions 496A

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$ 496B

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$ 496C

497C

497D

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

497E

497F

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$ 497G

$with$ $(p_{36})^{(7)}, (r_{37})^{(7)}$ as defined are satisfied , then the system WITH THE SATISFACTION OF

THE FOLLOWING PROPERTIES HAS A SOLUTION AS DERIVED BELOW.

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Particular case : 498

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

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From GLOBAL EQUATIONS we obtain 500

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 501

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

$it\ follows\ (\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 503

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 504

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$\frac{(\bar{\nu}_1)^{(3)}+(C)^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$

(c) If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 505

$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(C)^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain 507

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$ 508

It follows

$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

$it\ follows\ (\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 509

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 510

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** . 513

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$**.**

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From GLOBAL EQUATIONS we obtain 515

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right) - (a''_{29})^{(5)}(T_{29},t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

$it\ follows\ (\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 516

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

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(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case,

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

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And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

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we obtain 521

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

$it\ follows\ (\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 522

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$ 523

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 524

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 525

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have
Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain
Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.
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Behavior of the solutions 527

If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(p) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \leq -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 528

(q) By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

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Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 530.

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$ and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

(r) If we define $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if} \ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if} \ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 531

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if} \ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if} \ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if} \ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$ are defined by 59 and 67 respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 532

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)}-(p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t}$$ 533

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)}-(p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \right.$$ 534

$$\left.\frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)}-(a_{38}')^{(7)}\right)}\left[e^{(S_1)^{(7)}t} - e^{-(a_{38}')^{(7)}t}\right] + G_{38}^0 e^{-(a_{38}')^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t}}$$ 535

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t}$$ 536

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)}-(b_{38}')^{(7)}\right)}\left[e^{(R_1)^{(7)}t} - e^{-(b_{38}')^{(7)}t}\right] + T_{38}^0 e^{-(b_{38}')^{(7)}t} \leq T_{38}(t) \leq$$ 537

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)}+(r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 538

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}')^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}')^{(7)}$$ 539

$$(R_2)^{(7)} = (b_{38}')^{(7)} - (r_{38})^{(7)}$$

From CONCATENATED GLOBAL EQUATIONS we obtain 540

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}')^{(7)} - (a_{37}')^{(7)} + (a_{36}'')^{(7)}(T_{37},t)\right) - (a_{37}'')^{(7)}(T_{37},t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_2)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)\leq\frac{d\nu^{(7)}}{dt}\leq$$

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

(m) For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

$it\ follows\ (\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 541

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

(n) If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 542

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\ \nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

(o) If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 543

$$(\nu_1)^{(7)}\leq\ \nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then

$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

$(b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ] T_{14} = 0$ 544

$(b_{15})^{(1)} T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ] T_{15} = 0$ 545

has a unique positive solution , which is an equilibrium solution for the system 546

$(a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right] G_{16} = 0$ 547

$(a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right] G_{17} = 0$ 548

$(a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right] G_{18} = 0$ 549

$(b_{16})^{(2)} T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ] T_{16} = 0$ 550

$(b_{17})^{(2)} T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ] T_{17} = 0$ 551

$(b_{18})^{(2)} T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ] T_{18} = 0$ 552

has a unique positive solution , which is an equilibrium solution for 553

$(a_{20})^{(3)} G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right] G_{20} = 0$ 554

$(a_{21})^{(3)} G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right] G_{21} = 0$ 555

$(a_{22})^{(3)} G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right] G_{22} = 0$ 556

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 557

$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0$ 558

$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0$ 559

has a unique positive solution , which is an equilibrium solution 560

$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0$ 561

$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0$ 563

$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0$ 564

$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0$ 565

$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0$ 566

$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0$ 567

has a unique positive solution , which is an equilibrium solution for the system 568

$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0$ 569

$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0$ 570

$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0$ 571

$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\ ]T_{28} = 0$ 572

$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\ ]T_{29} = 0$ 573

$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\ ]T_{30} = 0$ 574

has a unique positive solution , which is an equilibrium solution for the system 575

$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0$ 576

$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0$ 577

$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0$ 578

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\ ]T_{32} = 0$ 579

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\ ]T_{33} = 0$ 580

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\ ]T_{34} = 0$ 584 has a unique positive solution , which is an equilibrium solution for the system 582

$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0$ 583

$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0$ 584

$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0$ 585

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$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0$ 587

$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0$ 588

$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0$ 589

has a unique positive solution , which is an equilibrium solution for the system 560

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) +$
$(a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Definition and uniqueness of T^*_{37} :- 561

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$, $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

(e) By the same argument, the equations(SOLUTIONAL) admit solutions G_{36}, G_{37} if

$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$

$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi(G^*) = 0$ 562

Finally we obtain the unique solution OF THE SYSTEM

G_{37}^{*} $given\ by\ \varphi((G_{39})^{*})=0$, T_{37}^{*} $given\ by\ f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a_{36}^{'})^{(7)}+(a_{36}^{''})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a_{38}^{'})^{(7)}+(a_{38}^{''})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b_{36}^{'})^{(7)}-(b_{36}^{''})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b_{38}^{'})^{(7)}-(b_{38}^{''})^{(7)}((G_{39})^{*})]}$$ 563

Definition and uniqueness of T_{21}^{*} :- 564

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^{*} for which $f(T_{21}^{*})=0$. With this value , we obtain from the three first equations

$$G_{20}=\frac{(a_{20})^{(3)}G_{21}}{[(a_{20}^{'})^{(3)}+(a_{20}^{''})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}=\frac{(a_{22})^{(3)}G_{21}}{[(a_{22}^{'})^{(3)}+(a_{22}^{''})^{(3)}(T_{21}^{*})]}$$

565

Definition and uniqueness of T_{25}^{*} :- 566

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^{*} for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a_{24}^{'})^{(4)}+(a_{24}^{''})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a_{26}^{'})^{(4)}+(a_{26}^{''})^{(4)}(T_{25}^{*})]}$$

Definition and uniqueness of T_{29}^{*} :- 567

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^{*} for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a_{28}^{'})^{(5)}+(a_{28}^{''})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a_{30}^{'})^{(5)}+(a_{30}^{''})^{(5)}(T_{29}^{*})]}$$

Definition and uniqueness of T_{33}^{*} :- 568

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a_{32}^{'})^{(6)}+(a_{32}^{''})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a_{34}^{'})^{(6)}+(a_{34}^{''})^{(6)}(T_{33}^{*})]}$$

(f) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 569

$$\varphi(G)=(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G)+(b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(G)\right]+(b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0)>0$, $\varphi(\infty)<0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*})=0$

(g) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 570

$\varphi(G_{19}) = (b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$

$\left[(b_{16}^{'})^{(2)}(b_{17}^{''})^{(2)}(G_{19}) + (b_{17}^{'})^{(2)}(b_{16}^{''})^{(2)}(G_{19})\right] + (b_{16}^{''})^{(2)}(G_{19})(b_{17}^{''})^{(2)}(G_{19}) = 0$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 571

(a) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if 572

$\varphi(G_{23}) = (b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$

$\left[(b_{20}^{'})^{(3)}(b_{21}^{''})^{(3)}(G_{23}) + (b_{21}^{'})^{(3)}(b_{20}^{''})^{(3)}(G_{23})\right] + (b_{20}^{''})^{(3)}(G_{23})(b_{21}^{''})^{(3)}(G_{23}) = 0$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

573

(b) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if 574

$\varphi(G_{27}) = (b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$

$\left[(b_{24}^{'})^{(4)}(b_{25}^{''})^{(4)}(G_{27}) + (b_{25}^{'})^{(4)}(b_{24}^{''})^{(4)}(G_{27})\right] + (b_{24}^{''})^{(4)}(G_{27})(b_{25}^{''})^{(4)}(G_{27}) = 0$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

(c) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if 575

$\varphi(G_{31}) = (b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$\left[(b_{28}^{'})^{(5)}(b_{29}^{''})^{(5)}(G_{31}) + (b_{29}^{'})^{(5)}(b_{28}^{''})^{(5)}(G_{31})\right] + (b_{28}^{''})^{(5)}(G_{31})(b_{29}^{''})^{(5)}(G_{31}) = 0$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

(d) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if 578

579

$\varphi(G_{35}) = (b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

580

$\left[(b_{32}^{'})^{(6)}(b_{33}^{''})^{(6)}(G_{35}) + (b_{33}^{'})^{(6)}(b_{32}^{''})^{(6)}(G_{35})\right] + (b_{32}^{''})^{(6)}(G_{35})(b_{33}^{''})^{(6)}(G_{35}) = 0$

581

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G^{*}) = 0$

Finally we obtain the unique solution of 89 to 94 582

G_{14}^{*} *given by* $\varphi(G^{*}) = 0$, T_{14}^{*} *given by* $f(T_{14}^{*}) = 0$ and

$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]}$, $G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$

$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G^*)]}$, $T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

G_{17}^* $given\ by\ \varphi((G_{19})^*) = 0$, T_{17}^* $given\ by\ f(T_{17}^*) = 0$ and 584

$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}^*)]}$, $G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}^*)]}$ 585

$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b_{16}')^{(2)} - (b_{16}'')^{(2)}((G_{19})^*)]}$, $T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b_{18}')^{(2)} - (b_{18}'')^{(2)}((G_{19})^*)]}$ 586

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution 588

G_{21}^* $given\ by\ \varphi((G_{23})^*) = 0$, T_{21}^* $given\ by\ f(T_{21}^*) = 0$ and

$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]}$, $G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$

$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}{}^*)]}$, $T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}{}^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 589

G_{25}^* $given\ by\ \varphi(G_{27}) = 0$, T_{25}^* $given\ by\ f(T_{25}^*) = 0$ and

$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}^*)]}$, $G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}^*)]}$

$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b_{24}')^{(4)} - (b_{24}'')^{(4)}((G_{27})^*)]}$, $T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b_{26}')^{(4)} - (b_{26}'')^{(4)}((G_{27})^*)]}$ 590

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 591

G_{29}^* $given\ by\ \varphi((G_{31})^*) = 0$, T_{29}^* $given\ by\ f(T_{29}^*) = 0$ and

$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}^*)]}$, $G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}^*)]}$

$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]}$, $T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]}$ 592

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 593

G_{33}^* $given\ by\ \varphi((G_{35})^*) = 0$, T_{33}^* $given\ by\ f(T_{33}^*) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}((G_{35})^{*})]}$ 594

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 595

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(1)}$ *and* $(b_i^{''})^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^{*} + \mathbb{G}_i$, $T_i = T_i^{*} + \mathbb{T}_i$ 596

$\frac{\partial (a_{14}^{''})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)}$, $\frac{\partial (b_i^{''})^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14}$ 598

$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14}$ 599

$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14}$ 600

$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right)$ 601

$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}^{'})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right)$ 602

$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right)$ 603

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(2)}$ *and* $(b_i^{''})^{(2)}$ 604
Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote 605

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^{*} + \mathbb{G}_i$, $T_i = T_i^{*} + \mathbb{T}_i$ 606

$\frac{\partial (a_{17}^{''})^{(2)}}{\partial T_{17}}(T_{17}^{*}) = (q_{17})^{(2)}$, $\frac{\partial (b_i^{''})^{(2)}}{\partial G_j}((G_{19})^{*}) = s_{ij}$ 607

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^{*}\mathbb{T}_{17}$ 609

$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^{*}\mathbb{T}_{17}$ 610

$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^{*}\mathbb{T}_{17}$ 611

$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^{*}\mathbb{G}_j\right)$ 612

$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}^{'})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^{*}\mathbb{G}_j\right)$ 613

$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^{*}\mathbb{G}_j\right)$ 614

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}$ and $(b_i^{''})^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 615

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$

$\frac{\partial (a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^{*}) = (q_{21})^{(3)} \quad , \frac{\partial (b_i^{''})^{(3)}}{\partial G_j}\left((G_{23})^{*}\right) = s_{ij}$

616

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21}$ 618

$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21}$ 619

$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}^{'})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21}$ 6120

$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^{*}\mathbb{G}_j\right)$ 621

$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}^{'})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right)$ 622

$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right)$ 623

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 624

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 625

$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$

$\frac{\partial (a_{25}^{''})^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial (b_i^{''})^{(4)}}{\partial G_j}\left((G_{27})^{*}\right) = s_{ij}$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25}$ 627

$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25}$ 628

$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}^{'})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25}$ 629

$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_{j}\right)$ 630

$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}^{'})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_{j}\right)$ 631

$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_{j}\right)$ 632

633

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}$ and $(b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$ **:-** 634

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{29}^{''})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i^{''})^{(5)}}{\partial G_j}\left((G_{31})^* \right) = s_{ij}$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29}$ 636

$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}^{'})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^{*}\mathbb{T}_{29}$ 637

$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}^{'})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^{*}\mathbb{T}_{29}$ 638

$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^{*}\mathbb{G}_{j}\right)$ 639

$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}^{'})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^{*}\mathbb{G}_{j}\right)$ 640

$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}^{'})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^{*}\mathbb{G}_{j}\right)$ 641

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}$ and $(b_i^{''})^{(6)}$ 642
Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$ **:-** 643

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{33}^{''})^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i^{''})^{(6)}}{\partial G_j}\left((G_{35})^* \right) = s_{ij}$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}^{'})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33}$ 645

$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}^{'})^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33}$ 646

$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}^{'})^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33}$ 647

$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}^{'})^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^{*}\mathbb{G}_{j}\big)$ 648

$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}^{'})^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^{*}\mathbb{G}_{j}\big)$ 649

$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}^{'})^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^{*}\mathbb{G}_{j}\big)$ 650

Obviously, these values represent an equilibrium solution of 79,20,36,22,23, 651

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(7)}$ and $(b_i^{''})^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 652

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

653

$$\frac{\partial(a_{37}^{''})^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i^{''})^{(7)}}{\partial G_j}\big(\,(G_{39})^{**}\,\big) = s_{ij}$$

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain 654

655

$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}^{'})^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37}$ 656

$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}^{'})^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37}$ 657

$\frac{d\mathbb{G}_{38}}{dt} = -\big((a_{38}^{'})^{(7)} + (p_{38})^{(7)}\big)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37}$ 658

$\frac{d\mathbb{T}_{36}}{dt} = -\big((b_{36}^{'})^{(7)} - (r_{36})^{(7)}\big)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\big(s_{(36)(j)}T_{36}^{*}\mathbb{G}_{j}\big)$ 659

$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}^{'})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right)$ 660

$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}^{'})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right)$ 661

2. 662

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(14)}T_{14}^{*}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^{*} + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^{*}\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(13)}T_{13}^{*}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(15)}T_{13}^{*}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^{*} + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^{*} + (b_{17})^{(2)}s_{(16),(17)}T_{17}^{*}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^{*} + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^{*}\right)$$

$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T_{17}^{*} + (b_{17})^{(2)}s_{(16),(16)}T_{16}^{*}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b_{16}^{'})^{(2)} + (b_{17}^{'})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right) (\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right)(q_{18})^{(2)}G_{18}$$

$$+\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^{*} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^{*}\right)$$

$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}T_{17}^{*} + (b_{17})^{(2)}s_{(16),(18)}T_{16}^{*}\right)\} = 0$$

+

$$\left((\lambda)^{(3)} + (b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\{\left((\lambda)^{(3)} + (a_{22}^{'})^{(3)} + \left(p_{22}\right)^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)} + (a_{20}^{'})^{(3)} + \left(p_{20}\right)^{(3)}\right)\left(q_{21}\right)^{(3)}G_{21}^{*} + (a_{21})^{(3)}\left(q_{20}\right)^{(3)}G_{20}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(21)}T_{21}^{*}\right)$$

$$+\left(\left((\lambda)^{(3)} + (a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G_{20}^{*} + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^{*}\right)$$

$$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(20)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(20)}T_{20}^{*}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2 + \left((a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + \left(p_{20}\right)^{(3)} + \left(p_{21}\right)^{(3)}\right) (\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2 + \left((b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right) (\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2 + \left((a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right) (\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)} + (a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^{*} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)$$

$$\left(\left((\lambda)^{(3)} + (b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(22)}T_{21}^{*} + (b_{21})^{(3)}s_{(20),(22)}T_{20}^{*}\right)\} = 0$$

+

$$\left((\lambda)^{(4)} + (b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\{\left((\lambda)^{(4)} + (a_{26}^{'})^{(4)} + \left(p_{26}\right)^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)} + (a_{24}^{'})^{(4)} + \left(p_{24}\right)^{(4)}\right)\left(q_{25}\right)^{(4)}G_{25}^{*} + (a_{25})^{(4)}\left(q_{24}\right)^{(4)}G_{24}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(4)} + (b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(25)}T_{25}^{*} + (b_{25})^{(4)}s_{(24),(25)}T_{25}^{*}\right)$$

$$+\left(\left((\lambda)^{(4)} + (a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)(q_{24})^{(4)}G_{24}^{*} + (a_{24})^{(4)}(q_{25})^{(4)}G_{25}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}+(b_{24}^{'})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b_{24}^{'})^{(4)}+(b_{25}^{'})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a_{24}^{'})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^{*}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)$$

$$\left(\left((\lambda)^{(4)}+(b_{24}^{'})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(26)}T_{24}^{*}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b_{30}^{'})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a_{30}^{'})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G_{29}^{*}+(a_{29})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(29)}T_{29}^{*}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a_{29}^{'})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G_{28}^{*}+(a_{28})^{(5)}(q_{29})^{(5)}G_{29}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(28)}T_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b_{28}^{'})^{(5)}+(b_{29}^{'})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^{*}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(30)}T_{28}^{*}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b_{34}^{'})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a_{34}^{'})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a_{33}^{'})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G_{32}^{*}+(a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b_{32}^{'})^{(6)}+(b_{33}^{'})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$$

$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b_{38}^{'})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a_{38}^{'})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a_{36}^{'})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(37)}T_{37}^{*}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a_{37}^{'})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G_{36}^{*}+(a_{36})^{(7)}(q_{37})^{(7)}G_{37}^{*}\right)$$

$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(36)}T_{36}^{*}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a_{36}^{'})^{(7)}+(a_{37}^{'})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b_{36}^{'})^{(7)}+(b_{37}^{'})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a_{36}^{'})^{(7)}+(a_{37}^{'})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a_{36}^{'})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(a_{38})^{(7)}(\mathrm{q}_{36})^{(7)}\mathrm{G}_{36}^{*}\right)$$

$$\left(\left((\lambda)^{(7)}+(\mathrm{b}_{36}^{'})^{(7)}-(\mathrm{r}_{36})^{(7)}\right)\mathrm{s}_{(37),(38)}\mathrm{T}_{37}^{*}+(\mathrm{b}_{37})^{(7)}\mathrm{s}_{(36),(38)}\mathrm{T}_{36}^{*}\right)\}=0$$

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(20)^ [2] Cockcroft-Walton experiment

(21)^ a b c Conversions used: 1956 International (Steam) Table (IT) values where one calorie ≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers' conversion value of one gram TNT

≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a C_p of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average C_p of 25.8, 5.134 moles of metal, and 132 $J.K^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

(24)^ [3] Article on Earth rotation energy. Divided by c^2.

(25)^ **a b** Earth's gravitational self-energy is 4.6×10^{-10} that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.

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