A Computational Model of High Jump Height

S.P.D.S.S.K. Karunarathna[1], T.S. Hithakshika[1]
Department of Mathematical Sciences, Wayamba University of Sri Lanka

Abstract

Elite high jump athletes use fosbury flop to compete their jump. This technique comprises three phases, namely, approach phase, takeoff phase and flight phase. The performance of the athlete is depended on these three phases, and the takeoff phase plays a salient role in the athlete performance. As the takeoff phase makes a tremendous contribution to change the high jump height in comparison to approach and flight phases, we aim to propose a dynamical model to describe the takeoff phase using biomechanical characteristics.

In prior works, the takeoff phase is considered in the two-dimensional space. In this study, the takeoff phase is modeled in the three-dimensional space. Such an approach has not been considered in the literature. As the position of the peak of the jump relative to the bar is determined by the takeoff distance, it is important to know the takeoff distances that give the maximum performance for athletes. However, existing methods have not provided a variable to work out the takeoff distance. *Our propose model provides necessary details to compute takeoff distances for players.*

Using our model, we observe that the takeoff distance is altered for the same athlete with change of *takeoff plane* while other parameters are fixed. In particular, an athlete is considered: body mass is 75 kg, coefficient of air resistant is 0.75 kg/m, takeoff velocity is 5.4 m/s, and takeoff angle (azimuthal angle) is 54°. The takeoff distance is altered between 1.26 m and 1.03 m while the takeoff plane is changed from 30° to 45°. For elite athletes, the takeoff distance may be changed between 0.94 - 1.50 m. Consequently, the performance of athletes can be changed by 12.96 cm. Moreover, our model turns out that the takeoff distance is depended on the takeoff velocity, takeoff angles and coefficient of air resistant.
corresponding author: karunarathnakln@gmail.com

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1 Introduction

Thousands billion dollars were spent on the sports industry around the world [1]. This statistic concludes the competition in the sports industry and the importance of sports. Several researches have been carried out to study the performance through the motion of the athlete [2, 3, 4, 5]. In particular, mathematicians start to model dynamics of sporting events. Exploring the dynamics of sporting events lead to enhance athlete's performance and reduce the risk of injury [1].

Track and field comprises several events such as sprints, middle-distance, long-distance, hurdles, relays, jumps and throws. Among these events, the high jump has been one of the most intensely studied events in track and field [1, 6, 7, 8]. However, the knowledge of it is still imperfect, and there is room for doubts and disagreements. In the literature, regression model and gray prediction model were used to predict the high jump performance. However, as growth rate of the high jump achievements is gradually decline, the predicted results have significantly deviated from the actual situation [9]. The approach phase, the takeoff phase and the flight phase are components of the high jump. An individual model for each of these phases and combination of these models were studied in [1]. However, Cooke [1] completely ignored the air resistance and takeoff angle, in particular, polar angle is ignored. As many of the characteristics of the flight phase are determined by the takeoff phase [7], flight of the athlete is described using takeoff phase together with biomechanical characteristics.

The most important biomechanical parameters are pointed out in [8, 10, 11, 12, 13]. Results of high jumps are determined by rational biomechanical characteristics, namely: speed of running, speed of repulsion, takeoff angle of a sportsman's body mass center, position of a sportsman's body masses center in the phases of repulsion and bar over passing [8]. A mathematical model were proposed by Adashevskiy [8] to analyze the influences of biomechanical characteristics on the high jump height. The takeoff distance, that is, the distance between the takeoff position and the plane of the bar and stands, is important to achieve athlete's best performance because the value of this distance determines maximum height of the jump relative to the bar. However, there is no way to discuss the takeoff distance using mathematical model of Adashevskiy [8]. Moreover, existing works concluded that high jumpers need to be able to judge whether the takeoff point is too close or too far from the bar [14].

Unlike prior work, we propose a mathematical model using biomechanical parameters to describe the three-dimensional motion of the athlete in the air. Using this model , we predetermine the takeoff distances for players. In addition, the actual motion of the athlete is described.

2 Prior Work

Fosbury flop; modern high jump technique, with curved approach is the most common method used by current elite athlete. The high jump consists three individual stages. In the first stage, the athlete runs along a linear trajectory. In the second stage, the athlete runs along a curve instead of running along a straight line. The third stage of the high jump is the actual jump. An individual model for each of these stages and combination of these models were investigated in [1]. The actual jump is modeled as a projectile motion and the equation of the athlete's trajectory is [1]:

$$y = y_0 + v_i t + \frac{gt^2}{2}, \tag{1}$$

where y_0 is the initial distance of the athlete's center of mass from the ground, v_i is the initial velocity, g is the standard gravity, t is time. In this work, angles at repulsion and air resistance are completely ignored. According to authors knowledge, the motion of the athlete can be described using newton's second law. In this case, we assume athlete as a particle. The motion of the athlete then can be written as

$$\ddot{y} = -g. \tag{2}$$

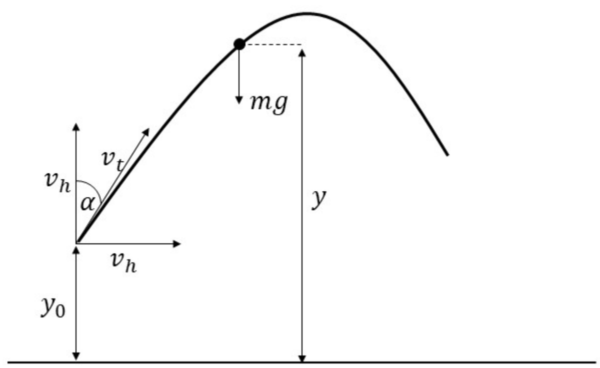

Figure 1: The motion of athlete is modeled as a projectile motion [1]. The athlete's trajectory is shown.

By integrating twice, we obtain

$$y = y_0 + v_t \cos\alpha - \frac{gt^2}{2}, \tag{3}$$

where v_t is takeoff velocity (initial velocity), α is the takeoff angle, and y_0 is the initial distance of the athlete's center of mass from the ground.

However, takeoff speed and angle of sportsman's masses center at repulsion are the main biomechanical characteristics of high jumps [8]. Adashevskiy et al. [8] proposed a mathematical model to determine the influences of biomechanical characters on the height of jump: speed and corner of flight of center of mass during pushing away, positions of center of mass body of sportsman in the phases of pushing away and transition through a slat, forces of resistance of air environment, influences of moment of inertia of body. The motion of the athlete was expressed as follows [8]:

$$m\ddot{x} = -kv^2\cos\alpha; \quad m\ddot{y} = -mg - kv^2\sin\alpha; \quad J\ddot{\phi} = -k\dot{\phi}^2, \tag{4}$$

where α is angle between current projections of body masses center speed and speed vector, and

$$\cos\alpha = \frac{\dot{x}}{v}; \;\; \sin\alpha = \frac{\dot{y}}{v}; \;\; v = \sqrt{\dot{x}^2 + \dot{y}^2}. \tag{5}$$

Here $J-$ moment of inertia, $\ddot{\phi}-$ corresponds to angle acceleration of the body. The takeoff distance is crucial because it determines the position of the peak of the jump relative to the bar [6]. However, as Adashevskiy et al. [8] consider athlete's body moves in a one of anatomical plane, there is a limitation for finding the takeoff distances.

In contrast to all of the above mention methods, we develop a mathematical model to describe the flight of the athlete in the air. Propose model describes the three-dimensional motion of the athlete in the air. Our model turns out that the salient of the takeoff distance and takeoff plane.

3 Our Contribution

A dynamic model of the high jump height is developed. The model describes the three-dimensional motion of the athlete in the air. In this paper, we assume that the athlete is a particle, that is, the center of gravity is the particle. The force of gravity and air resistance are accounted in the model.

$$\ddot{x} = F_1(m, v, \theta, \phi, k) \tag{6}$$
$$\ddot{y} = F_2(m, v, \theta, \phi, k) \tag{7}$$
$$\ddot{z} = F_3(m, v, \theta, \phi, k) \tag{8}$$

where m is body mass, v is absolute speed of body, θ, ϕ are takeoff angles of sportsman's masses center at repulsion, that is, polar angle θ, and azimuthal angle ϕ, and k is the air resistant coefficient. Estimation of air resistant forces for objects is explained in [8].

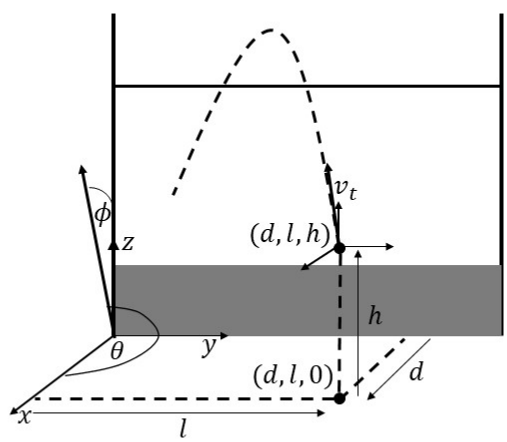

Figure 2: The motion of the athlete in three-dimensional space is shown. The velocity v_t is the takeoff velocity, and θ, ϕ are takeoff angles, that is, polar and azimuthal angles. The polar angle describes the *takeoff plane.*

The following system of second-order differential equations describes the motion of the athlete:

$$\begin{aligned} m\ddot{x} &= -kv^2 \cos\theta \sin\phi \\ m\ddot{y} &= -kv^2 \sin\theta \sin\phi \\ m\ddot{z} &= -mg - kv^2 \cos\phi, \end{aligned} \tag{9}$$

where $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$, $\cos\theta = \frac{\dot{x}}{v}$, $\sin\theta = \frac{\dot{y}}{v}$, $\cos\phi = \frac{\dot{z}}{v}$. It follows

$$\begin{aligned} \ddot{x} &= -\frac{k}{m}\dot{x}\ \sqrt{\dot{x}^2 + \dot{y}^2} \\ m\ddot{y} &= -\frac{k}{m}\dot{y}\ \sqrt{\dot{x}^2 + \dot{y}^2} \\ m\ddot{z} &= -g - \frac{k}{m}\dot{z}\ \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}, \end{aligned}$$

Using a change of variables, we obtain the following system of first-order differential equations:

$$\begin{aligned} \dot{X} &= -\frac{k}{m}X\ \sqrt{X^2 + Y^2} \\ \dot{Y} &= -\frac{k}{m}Y\ \sqrt{X^2 + Y^2} \\ \dot{Z} &= -g - \frac{k}{m}Z\ \sqrt{X^2 + Y^2 + Y^2}, \end{aligned}$$

It can be written in the matrix form as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{X} \\ \dot{y} \\ \dot{Y} \\ \dot{z} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} X \\ -\frac{k}{m}X\ \sqrt{X^2 + Y^2} \\ Y \\ -\frac{k}{m}Y\ \sqrt{X^2 + Y^2} \\ Z \\ -g - \frac{k}{m}Z\ \sqrt{X^2 + Y^2 + Y^2} \end{bmatrix}. \tag{10}$$

The takeoff position of athlete is $(d, l, 0)$, and h is the initial distance of the athlete's center of mass from the ground. The initial position of the the athlete's center of mass then is (d, l, h). The takeoff velocity of athlete is v_t. The initial conditions then are given as

$$x(0) = d, y(0) = l, z(0) = h,$$

and

$$\dot{x}(0) = v_t \cos(\theta) \sin(\phi), \dot{y}(0) = v_t \sin(\theta) \sin(\phi), \dot{z}(0) = v_t \cos(\phi).$$

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4 Results and Discussion

The effect of air resistant on the high jump height is examined using Cooke [1], Adashevskiy et al. [8] and our approaches. To do so, we did a gedanken experiment. In this case, we consider a particular high jump athlete with body mass 75 kg and height of center of mass 0.92 m. We assume athlete's takeoff velocity and takeoff angles are 5.8 m/s and 54°, respectively. The standard gravity is considered as 9.8 kg/s^2. Table 1 shows high jump height with different k values (air resistant).

Table 1: Estimated high jump heights for a particular athlete using Cooke approach, Adashevskiy approach and our approach are shown. Athlete's weight and height of center of mass are 75 kg and 0.92 m, respectively. Moreover, we consider takeoff velocity 5.8 m/s and takeoff angle 54°. The standard gravity is 9.8 kg/s^2.

k Values	Cooke Approach (m)	Adashevskiy Approach	Our Approach (m)
0.15	2.0432	2.0392	2.0391
0.30	2.0432	2.0355	2.0355
0.45	2.0432	2.0319	2.0319
0.60	2.0432	2.0283	2.0283
0.75	2.0432	2.0248	2.0247
0.90	2.0432	2.0212	2.0211

As we expected, our approach and Adashevskiy approach are coincided (the numerical error between our approach and Adashevskiy approach is 10^{-4} m). According to the Table 1, we may observe that Cooke [1] approach is overestimating the maximum jump height, and the jump heights are not changing with different k values since Cooke approach is ignoring the air resistant k.

Figure 3 shows the motions of the athlete in the two-dimensional space with different takeoff velocities while other parameters are fixed. The athlete's trajectories are obtained using our approach. Note that, the takeoff distances are altered with takeoff velocities. Moreover, the takeoff distance is varied with the takeoff plane. However, Adashevskiy approach does not provide a variable and a parameter to compute the takeoff distance. Using our propose method, one can compute the takeoff distance that corresponds to the takeoff velocity and the takeoff plane. To illustrate, we consider a particular athlete: mass of the athlete is $75kg$, the takeoff velocity is $5.4ms^{-1}$, the take-off angle (azimuthal) is 54°, and the air resistant is 0.75. The takeoff distances with the different takeoff planes are illustrated in Table 2. Figure 4 illustrates the motion of the same athlete in the three-dimensional space.

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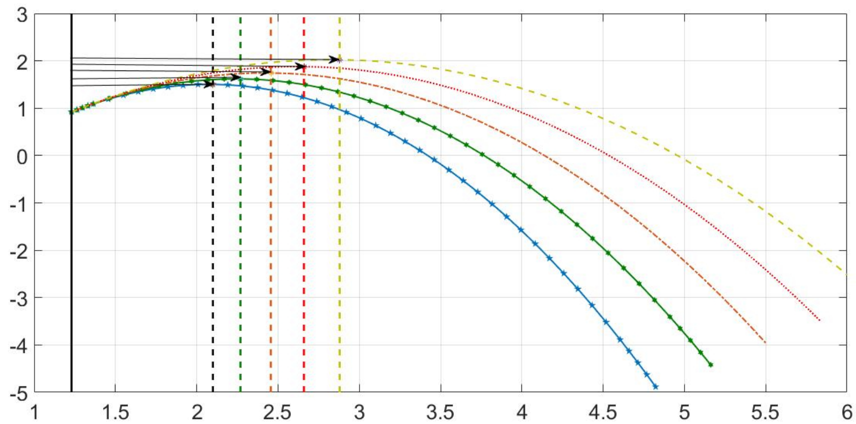

Figure 3: The motions of the athlete for different takeoff velocities are shown. Our approach is used to obtain the motions of the athlete.

Table 2: Change of the takeoff distance with the takeoff plane is shown.

Takeoff Plane	Takeoff Distance (m)
30	1.2583
35	1.1912
40	1.1139
45	1.0282

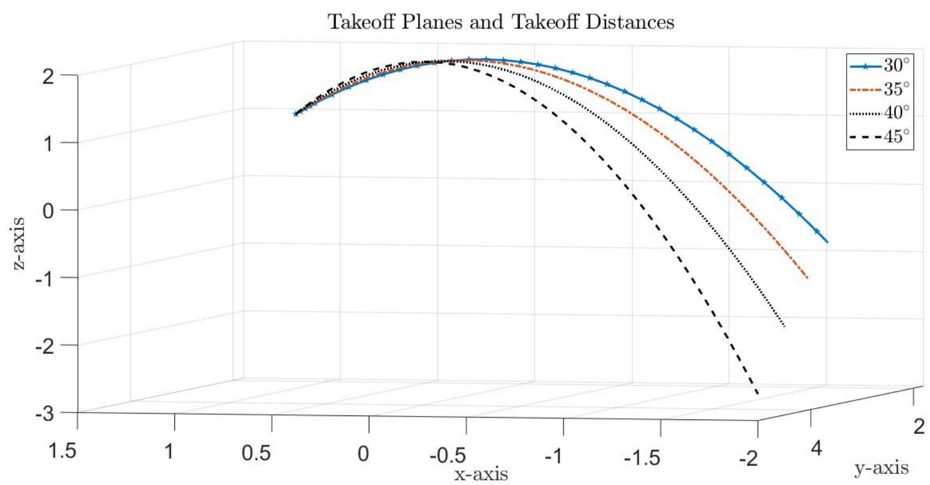

Figure 4: Three-dimensional motions of the athlete with the different takeoff planes are shown.

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