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Coupled fixed point theorems for generalized (α, ψ)-contractive type maps

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Abstract: In this paper, we introduce generalized (α, ψ)-contractive maps and prove the existence and uniqueness of coupled fixed points for generalized (α, ψ)-contractive maps in G-metric spaces. Furthermore, we provide examples in support of our results.

Keywords: α-admissible, (α, ψ)-contractive maps, mixed monotone property, coupled fixed point, generalized (α, ψ)-contractive map.

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1 Introduction

Fixed points and fixed point theorems have always a prominent role to find the existence of solutions of problems that arise in theoretical mathematics. In 1922, Banach [2] proved a remarkable results in this direction that each contraction in a complete metric space has a unique fixed point. Later many authors have directed their attention to this concept and have generalized the Banach fixed point theorems in various ways. In 2012, Samet, Vetro and Vetro [7] introduced a new concept namely '(α, ψ)-contractive' mappings and proved the related fixed points of such mappings in metric space setting.

Recently, Mustafa and Sims [4] introduction a new concept namely generalized metric space called G-metric space and characterized Banach fixed point theeorem in the context of G-metric space. For more works on the existence of fixed points and coupled fixed points in G-metric spaces, we refer [4].

In 1987, Guo and Lakshmi Kantham [12] introduced the notion of a coupled fixed points for mixed monotone operators. The concept of a coupled fixed point was reconsidered by Gnana-Bhaskar and Lakshmi Kantham [11] in 2006. They proved

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and discussed the existence and uniqueness of a coupled fixed point of an operator $F : X \times X \to X$ on a partially ordered metric spaces, we refer [11].

Later, Alghamdi and Karapinar [5] introduced the new concept namely (G, β, ψ)-contractive type maps which are generalizations of (α, ψ)-contractive maps, proved existence and uniqueness of fixed points of such contractive maps in G-metric spaces.

2 Preliminaries

Throughout this paper we denote by Ψ the family of nondecreasing functions $\psi : [0,\ \infty) \to [0,\ \infty)$ which satisfies $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$ where ψ^n is the nth iterate of ψ.

Remark 2.1. Any function $\psi \in \Psi$ satisfies $\lim_{n\to\infty} \psi^n(t) = 0$, $\psi(t) < t$ for any $t > 0$ and ψ is continuous at 0.

Definition 2.2. (Samet, Vetro and Vetro [7, Definition 2.1]) Let $(X,\ d)$ be a metric space and $T : X \to X$. We say that T is (α, ψ)-contractive mapping if there exist two functions $\alpha : X \times X \to [0,\ \infty)$ and $\psi \in \Psi$ such that

$\alpha(x,\ y)d(Tx,\ Ty) \leq \psi(d(x,\ y))$ for all $x,\ y \in X$. (2.2.1)

Definition 2.3. (Samet, Vetro and Vetro [7, Definition 2.1]) Let $(X,\ d)$ be a metric space, $T : X \to X$ and $\alpha : X \times X \to [0,\ \infty)$. We say that T is α-admissible if $x,\ y \in X$

$\alpha(x,\ y) \geq 1 \Rightarrow \alpha(Tx,\ Ty) \geq 1$. (2.3.1)

For examples on α-admissible functions, we refer [7] and for more works on α-admissible functions, we refer [[8], [10], [9], [6]].

Theorem 2.4. (Samet, Vetro and Vetro [7, Theorem 2.1]) *Let $(X,\ d)$ be a complete metric space and $T : X \to X$. Suppose that there exist two functions $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that T is (α, ψ)-contractive map.*
Also, assume that

(*i*) *T is α-admissible;*

(*ii*) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$; either*

(*iii*) *T is continuous; (or)*

(*iv*) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.*

Then T has a fixed point. i.e., there exists $u \in X$ such that $Tu = u$.

Definition 2.5. (Karapinar and Samet [8, Definition 2.1]) Let (X, d) be a metric space and $T : X \to X$ be a given mapping. We say that T is a generalized (α, ψ)-contractive mapping if there exist two functions $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that for all $x,\ y \in X$

$\alpha(x, y)d(Tx, Ty) \leq \psi(M(x, y))$, where (2.5.1)

$M(x, y) = \max\{d(x, y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\}$.

Theorem 2.6. (Karapinar and Samet [8, Theorem 2.3]) *Let (X, d) be a complete metric space and $T : X \to X$. Suppose that there exist two functions $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that T is a generalized (α, ψ)-contractive map*
Also, assume that the following conditions are satisfied:

(*i*) *T is α- admissible;*

(*ii*) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$; either*

(*iii*) *T is continuous; (or)*

(*iv*) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,\ x_{n+1}) \geq 1$ for all n and $x_n \to x$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},\ x) \geq 1$ for all k.*

Then there exists $u \in X$ such that $Tu = u$.

Mustafa and Sims [4] introduced the concept of G-metric space and proved fixed point results in complete G-metric spaces. After that, Alghamdi and Karapinar [5] proved some fixed point results in complete G-metric spaces.

Definition 2.7. [11] Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$ the mapping F is said to have the *mixed monotone property* if $F(x, y)$ is monotone non-decreasing in x and monotone non-increasing in y, that is for any $x, y \in X$,
$x_1, x_2 \in X,\ x_1 \preceq x_2 \Rightarrow F(x_1, y) \preceq F(x_2, y)$
$y_1, y_2 \in X,\ y_1 \preceq y_2 \Rightarrow F(x, y_1) \succeq F(x, y_2)$.

Definition 2.8. [11] An element $(x, y) \in X \times X$ is called a *coupled fixed point* of the mapping $F : X \times X \to X$ if $F(x, y) = x$, $F(y, x) = y$.

Definition 2.9. [11] Let X be a non-empty set and $F : X \times X \to X$ be a mapping. An element $x \in X$ is called a *fixed point* of F if $x = F(x, x)$.

Theorem 2.10. [*11*] *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a mapping having mixed monotone property on X. Assume that there exists $k \in [0, 1)$ such that*
$d(F(x, y), F(u, v)) \leq \frac{k}{2}[d(x, u) + d(y, v)]$
for all $x, y, u, v \in X$ with $x \succeq u$ and $y \preceq v$. Suppose that either

(*i*) *F is continuous (or);*

(*ii*) *X has the following property*

(*a*) *if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \to x$ then $\{x_n\} \leq x$ for all n.*

(*b*) *if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \to x$ then $\{x_n\} \leq x$ for all n.*

Further, if there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then there exist $x, y \in X$ such that $x = F(x, y)$ and $y = F(y, x)$. i.e., F has a coupled fixed point in X.

Lemma 2.11. [6] *Let $F : X \times X \to X$ be a given mapping. Define the mapping $T_F : X \times X \to X \times X$ by $T_F(x, y) = (F(x, y), F(y, x))$ for all $(x, y) \in X \times X$. Then (x,y) is fixed point of T_F if and only if (x, y) is a coupled fixed point of F.*

Lemma 2.12. [6] *Let (X, G) be a G-metric space. A mapping $F : X \times X \to X$ is said to be continuous if for any two G-Convergent sequence $\{x_n\}$ and $\{y_n\}$ converging to x and y, respectively, $\{F(x_n, y_n)\}$ is G-converging to $F(x, y)$.*

Alghamdi and Karapinar [5] proved the following results.

Theorem 2.13. *Let $(X,\ G)$ be a complete G-metric space and let $F : X \times X \to X$ be a given mapping. Suppose there exist $\psi \in \Psi$ and the function $\beta : X^2 \times X^2 \times X^2 \to [0, \infty)$ such that*
$\beta((x, y), (u, v), (u, v))G(F(x, y), F(u, v), F(u, v)) \leq \frac{1}{2}(\psi(G(x, u, u)+G(y, v, v)))$ (2.13.1)
for all $(x, y), (u, v) \in X \times X$. Suppose that

(i) *for all $(x, y), (u, v) \in X \times X$, we have*
$\beta((x, y), (u, v), (u, v)) \geq 1$ *implies*
$\beta((F(x, y), F(y, x)), (F(u, v), F(v, u)), (F(u, v), F(v, u))) \geq 1$

(ii) *there exist $(x_0, y_0, y_0) \in X \times X \times X$ such that*
$\beta((x_0, y_0), (F(x_0, y_0), (F(y_0, x_0)), (F(x_0, y_0), (F(y_0, x_0))) \geq 1$ *and*
$\beta((F(y_0, x_0), (F(x_0, y_0)), (F(y_0, x_0), (F(x_0, y_0)(y_0, x_0))) \geq 1$

(iii) *either F is continuous; (or)*

(iv) *if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that*
$\beta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))$

then F has a coupled fixed point. i.e., there exist $(x^*, y^*) \in X \times X$ such that $F(x^*, y^*) = x^*$ and $F(y^*, x^*) = y^*$.

Definition 2.14. [12] Let X be a nonempty set $F : X \times X \to X$ and $g : X \to X$ then

(i) An element $(x, y) \in X \times X$ is called a *coupled coincidence point* of the mappings F and g if $F(x, y) = g(x)$, $F(y, x) = g(y)$.

(ii) An element $(x, y) \in X \times X$ is called a *common coupled coincidence point*of the mappings F and g if $F(x, y) = g(x) = x$, $F(y, x) = g(y) = y$.

3 Main Result

We introduce the concept of F is generalized (α, ψ)-contractive type mappings as follows:

Definition 3.1. Let (X, G) be a G-metric space. Let $F : X \times X \to X$ be a map if there exist two function $\alpha : X^2 \times X^2 \times X^2 \to [0, \infty)$ and $\psi \in \Psi$ such that
$\alpha((x, y), (u, v)(u, v))G(F(x, y), F(u, v), F(u, v)) \leq \frac{1}{2}\psi(\max\{G(x, u, u) + G(y, v, v),$
$\frac{1}{2}(G(x, F(x, y), F(x, y)) + G(y, F(y, x), F(y, x))) + (G(u, F(u, v), F(u, v))$
$+ G(v, F(v, u), F(v, u))), \frac{1}{2}(G(x, F(u, v), F(u, v)) + G(y, F(v, u), F(v, u)))$
$+ (G(u, F(x, y), F(x, y)) + G(v, F(y, x), F(y, x)))\})$ (3.1.1)
for all $(x, y), (u, v) \in X \times X$,
then we say that F is *generalized* (α, ψ)*-contractive map* in two variables.

Example 3.2. Let $X = \{0, 1, 3\}$. We define $G : X^3 \to \mathbb{R}_+$ by
$G(x, y, z) = d(x, y)+d(y, z)+d(z, x)$. Let $A = \{(3, 0), (0, 3)\}$, $B = \{(0, 0), (1, 1), (3, 3), (3, 1), (1, 3)\}$ and $C = \{(1, 0), (0, 1)\}$. We define mapping $F : X \to X$ by $F(x, y) = \begin{cases} 1 & if\ (x, y) \in A \\ 0 & if\ (x, y) \in B \\ 3 & if\ (x, y) \in C. \end{cases}$
We define $\alpha : X^3 \to [0, +\infty)$ and $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ by
$\alpha((x, y)(u, v), (u, v)) = \begin{cases} \frac{6}{5} & if\ (x, y) \in A,\ (u, v) \in B \\ 0 & otherwise \end{cases}$ and $\psi(t) = \frac{11t}{12}$ for all $t > 0$.
Now, we verify the inequality (3.1.1) as follows:
<u>Case (i)</u>: $(x, y) = (3, 0)$ and $(u, v) = (0, 0)$
In this case, $F(3, 0) = F(0, 3) = 1$, $F(0, 0) = 0$, $\alpha((3, 0), (0, 0), (0, 0)) = \frac{6}{5}$ and $M((3, 0), (0, 0)) = 6$
$\alpha((x, y), (u, v), (u, v))G((x, y), (u, v), (u, v)) = \alpha((3, 0), (0, 0), (0, 0))G((3, 0), (0, 0), (0, 0))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3, 0), (0, 0)))) = \frac{1}{2}(\psi(M((x, y), (u, v))))$.
<u>Case (ii)</u>: $(x, y) = (3, 0)$ and $(u, v) = (1, 1)$
In this case, $F(3, 0) = F(0, 3) = 1$, $F(1, 1) = 0$, $\alpha((3, 0), (1, 1), (1, 1)) = \frac{6}{5}$ and $M((3, 0), (1, 1)) = 6$
$\alpha((x, y), (u, v), (u, v))G((x, y), (u, v), (u, v)) = \alpha((3, 0), (1, 1), (1, 1))G((3, 0), (1, 1), (1, 1))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3, 0), (1, 1)))) = \frac{1}{2}(\psi(M((x, y), (u, v))))$.
<u>Case (iii)</u>: $(x, y) = (3, 0)$ and $(u, v) = (3, 3)$
In this case, $F(3, 0) = F(0, 3) = 1$, $F(3, 3) = 0$, $\alpha((3, 0), (3, 3), (3, 3)) = \frac{6}{5}$ and $M((3, 0), (3, 3)) = 10$
$\alpha((x, y), (u, v), (u, v))G((x, y), (u, v), (u, v)) = \alpha((3, 0), (3, 3), (3, 3))G((3, 0), (3, 3), (3, 3))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3, 0), (3, 3)))) = \frac{1}{2}(\psi(M((x, y), (u, v))))$.
<u>Case (iv)</u>: $(x, y) = (3, 0)$ and $(u, v) = (3, 1)$
In this case, $F(3, 0) = F(0, 3) = 1$, $F(3, 1) = F(1, 3) = 0$, $\alpha((3, 0), (3, 1), (3, 1)) = \frac{6}{5}$ and $M((3, 0), (3, 1)) = 6$
$\alpha((x, y), (u, v), (u, v))G((x, y), (u, v), (u, v)) = \alpha((3, 0), (3, 1), (3, 1))G((3, 0), (3, 1), (3, 1))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3, 0), (3, 1)))) = \frac{1}{2}(\psi(M((x, y), (u, v))))$.
<u>Case (v)</u>: $(x, y) = (3, 0)$ and $(u, v) = (1, 3)$
In this case, $F(3, 0) = F(0, 3) = 1$, $F(3, 1) = F(1, 3) = 0$, $\alpha((3, 0), (1, 3), (1, 3)) = \frac{6}{5}$

and $M((3,0),(1,3)) = 10$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((3,0),(1,3),(1,3))G((3,0),(1,3),(1,3))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3,0),(1,3)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Case (vi): $(x,y) = (0,3)$ and $(u,v) = (0,0)$
In this case, $F(0,3) = F(3,0) = 1$, $F(0,0) = 0$, $\alpha((0,3),(0,0),(0,0)) = \frac{6}{5}$ and $M((0,3),(0,0)) = 6$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((0,3),(0,0),(0,0))G((0,3),(0,0),(0,0))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((3,0),(0,0)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Case (vii): $(x,y) = (0,3)$ and $(u,v) = (1,1)$
In this case, $F(0,3) = F(3,0) = 1$, $F(1,1) = 0$, $\alpha((0,3),(1,1),(1,1)) = \frac{6}{5}$ and $M((0,3),(1,1)) = 6$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((0,3),(1,1),(1,1))G((0,3),(1,1),(1,1))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((0,3),(1,1)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Case $(viii)$: $(x,y) = (0,3)$ and $(u,v) = (3,3)$
In this case, $F(0,3) = F(3,0) = 1$, $F(3,3) = 0$, $\alpha((0,3),(3,3),(3,3)) = \frac{6}{5}$ and $M((0,3),(3,3)) = 9$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((0,3),(3,3),(3,3))G((0,3),(3,3),(3,3))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((0,3),(3,3)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Case (ix): $(x,y) = (0,3)$ and $(u,v) = (3,1)$
In this case, $F(3,0) = F(0,3) = 1$, $F(3,1) = F(1,3) = 0$, $\alpha((0,3),(3,1),(3,1)) = \frac{6}{5}$ and $M((0,3),(3,1)) = 10$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((0,3),(3,1),(3,1))G((0,3),(3,1),(3,1))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((0,3),(3,1)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Case (x): $(x,y) = (0,3)$ and $(u,v) = (1,3)$
In this case, $F(3,0) = F(0,3) = 1$, $F(3,1) = F(1,3) = 0$, $\alpha((0,3),(1,3),(1,3)) = \frac{6}{5}$ and $M((0,3),(1,3)) = 10$
$\alpha((x,y),(u,v),(u,v))G((x,y),(u,v),(u,v)) = \alpha((0,3),(1,3),(1,3))G((0,3),(1,3),(1,3))$
$= \frac{12}{5} \leq \frac{1}{2}(\psi(M((0,3),(1,3)))) = \frac{1}{2}(\psi(M((x,y),(u,v))))$.
Therefore, the inequality (3.1.1) satisfies. Hence F is *generalized* (α,ψ)*-contractive map* in two variables.

Here, we observe that the inequality (2.13.1) fails to hold.
For, by choosing $(x,y) = (3,0)$ and $(u,v) = (3,1)$ we have
$\alpha((3,0),(3,1),(3,1))G((3,0),(3,1),(3,1)) = \frac{12}{5} \nleq \frac{1}{2}(\psi(G(3,3,3) + G(0,1,1)))$
so it is a generalization.

Theorem 3.3. *Let $(X,\ G)$ be a complete G-metric space. Let $F : X \times X \to X$ be generalized (α,ψ)-contractive map in two variables satisfying the following conditions.*

(*i*) *for all $(x,y),(u,v) \in X \times X$, we have*
$\alpha((x,y),(u,v),(u,v)) \geq 1$ *implies*
$\alpha((F(x,y),F(y,x)),(F(u,v),F(v,u)),(F(u,v),F(v,u))) \geq 1$

(*ii*) *there exist $(x_0,y_0,y_0) \in X \times X \times X$ such that*
$\alpha((x_0,y_0),(F(x_0,y_0),(F(y_0,x_0)),(F(x_0,y_0),(F(y_0,x_0))) \geq 1$ *and*
$\alpha((F(y_0,x_0),(F(x_0,y_0)),(F(y_0,x_0),(F(x_0,y_0)(y_0,x_0))) \geq 1$

(*iii*) *F is continuous.*

then F has a coupled fixed point. i.e., there exist $(x^*, y^*) \in X \times X$ such that $F(x^*, y^*) = x^*$ and $F(y^*, x^*) = y^*$.

Proof. Let (Y, δ) be a complete G-metric space with $Y = X \times X$ and $\delta((x,y),(u,v),(s,t)) = G(x,u,s) + G(y,v,t)$ for all $(x,y),(u,v),(s,t) \in Y$.
By using (3.3.1) and (G4) we get

$$\alpha((x,y),(u,v),(u,v))G(F(x,y),F(u,v),F(u,v)) \leq \tfrac{1}{2}\psi(\max\{G(x,u,u)+G(y,v,v),$$
$$\tfrac{1}{2}(G(x,F(x,y),F(x,y))+G(y,F(y,x),F(y,x)))+(G(u,F(u,v),F(u,v))$$
$$+G(v,F(v,u),F(v,u))),\tfrac{1}{2}(G(x,F(u,v),F(u,v))+G(y,F(v,u),F(v,u)))$$
$$+(G(u,F(x,y),F(x,y))+G(v,F(y,x),F(y,x)))\})$$
$$\alpha((x,y),(u,v),(u,v))G(F(x,y),F(u,v),F(u,v)) \leq \tfrac{1}{2}\psi(\max\{\delta((x,y),(u,v),(u,v)),$$
$$\tfrac{1}{2}(\delta((x,y),F(x,y),F(y,x))+\delta((u,v),F(u,v),F(v,u))),$$
$$\tfrac{1}{2}(\delta((x,y),F(u,v),F(v,u))+\delta((u,v),F(x,y),F(y,x)))\}) \tag{3.3.1}$$

Similarly

$$\alpha((v,u),(v,u),(y,x))G(F(v,u),F(v,u),F(y,x)) \leq \tfrac{1}{2}\psi(max\{G(v,v,y)+G(u,u,x),$$
$$\tfrac{1}{2}(G(F(y,x),F(y,x),y)+G(F(x,y),F(x,y),x))+(G(F(v,u),F(v,u),v)$$
$$+G(F(u,v),F(u,v),u)),\tfrac{1}{2}(G(F(y,x),F(y,x),v)+G(F(x,y),F(x,y),u))+$$
$$(G(F(v,u),F(v,u),y)+G(F(u,v),F(u,v),x))\})$$
$$\alpha((v,u),(v,u),(y,x))G(F(v,u),F(v,u),F(y,x)) \leq \tfrac{1}{2}\psi(\max\{\delta((v,u),(v,u),(y,x)),$$
$$\tfrac{1}{2}(\delta((y,x),F(y,x),F(x,y))+\delta((v,u),F(v,u),F(u,v))),$$
$$\tfrac{1}{2}(\delta((y,x),F(v,u),F(u,v))+\delta((v,u),F(y,x),F(x,y)))\}) \tag{3.3.2}$$

Adding the inequalities (3.3.1) and (3.3.2) we get

$$\alpha((x,y),(u,v),(u,v))G(F(x,y),F(u,v),F(u,v))+$$
$$\alpha((v,u),(v,u),(y,x))G(F(v,u),F(v,u),F(y,x)) \leq \psi(\max\{\delta((x,y),(u,v),(u,v)),$$
$$\tfrac{1}{2}[\delta((x,y),(F(x,y),F(y,x)),(F(x,y),F(y,x)))+\delta((u,v),(F(u,v),F(v,u)),(F(u,v),F(v,u))],$$
$$\tfrac{1}{2}[\delta((x,y),(F(u,v),F(v,u)),(F(u,v),F(v,u)))$$
$$+\delta((u,v),(F(x,y),F(y,x)),(F(x,y),F(y,x))]\}) \tag{3.3.3}$$

Choose $\beta : Y \times Y \times Y \to [0,\infty)$ is given by

$$\beta((x,y),(u,v),(u,v)) = min\{\alpha((x,y),(u,v),(u,v)),\alpha((v,u),(v,u),(y,x))\}$$

Defined $T : Y \to Y$ by $T(x,y) = (F(x,y),F(y,x))$ Since T is continuous and $G-\beta-\psi$ contractive mapping of equation (3.3.1)

$$\beta((x,y),(u,v),(u,v))\delta((F(x,y),(y,x)),(F(u,v),F(v,u)),(F(u,v),F(v,u)))$$
$$\leq \psi(\max\{\delta((x,y),(u,v),(u,v)),\tfrac{1}{2}[\delta((x,y),(F(x,y),F(y,x)),(F(x,y),F(y,x)))$$
$$+\delta((u,v),(F(u,v),F(v,u)),(F(u,v),F(v,u))],\tfrac{1}{2}[\delta((x,y),(F(u,v),F(v,u)),(F(u,v),F(v,u)))$$
$$+\delta((u,v),(F(x,y),F(y,x)),(F(x,y),F(y,x))]\})$$

That is

$$\beta((x,y),(u,v),(u,v))\delta(T(x,y),T(u,v),T(u,v)) \leq (\max\{\delta((x,y),(u,v),(u,v)),$$
$$\tfrac{1}{2}[\delta((x,y),(F(x,y),F(y,x)),(F(x,y),F(y,x)))+\delta((u,v),(F(u,v),F(v,u)),(F(u,v),F(v,u))],$$
$$\tfrac{1}{2}[\delta((x,y),(F(u,v),F(v,u)),(F(u,v),F(v,u)))+\delta((u,v),(F(x,y),F(y,x)),(F(x,y),F(y,x))]\})$$

Therefore T is a generalized (β,ψ) contractive map.
We now show that T is β-admissible.
Let $(x,y),(u,v) \in Y$ be such that

$$\beta((x,y),(u,v),(u,v)) = \min\{\alpha((x,y),(u,v),(u,v)),\alpha((v,u),(v,u),(y,x))\}$$
$$\geq 1 \tag{3.3.4}$$

Since $\alpha((x,y),(u,v),(u,v)) \geq 1$ and $\alpha((v,u),(v,u),(y,x)) \geq 1$
Hence by (i),

$$\begin{aligned}\beta(T(x,y),T(u,v),T(u,v)) &= \beta((F(x,y),(y,x)),(F(u,v),F(v,u)),(F(u,v),F(v,u)) \\ &= \min\{\alpha((F(x,y),(y,x)),(F(u,v),(v,u)),(F(u,v),F(v,u)), \\ &\quad \alpha((F(v,u),(u,v)),(F(v,u),(u,v)),(F(y,x),F(x,y)))\} \geq 1\end{aligned}$$

By using (3.3.4) and (i). Hence T is $\beta-$admissble.
Now by (ii), there exists $(x,y) \in X \times X$ such that

$$\begin{aligned}\beta((x_0,y_0),(T(x_0,y_0),T(x_0,y_0)) &= \beta((x_0,y_0),(F(x_0,y_0),F(y_0,x_0) \\ &= \min\{\alpha((x_0,y_0),(F(x_0,y_0),F(y_0,x_0)),(F(x_0,y_0),F(y_0,x_0))), \\ &\quad \alpha((x_0,y_0),(F(x_0,y_0),F(y_0,x_0)),(F(x_0,y_0),F(y_0,x_0)))\} \geq 1 \text{ by (ii)}\end{aligned}$$

We show that T is continuous.
Let $(a,b) \in X \times X$. We have $T(x,y) = (F(x,y),F(y,x))$ for all $x,y \in X$.
Therefore

$$\begin{aligned}\lim_{(x,y)\to(a,b)} T(x,y) &= \lim_{(x,y)\to(a,b)} (F(x,y),F(y,x)) \\ &= (\lim_{(x,y)\to(a,b)} (F(x,y), \lim_{(x,y)\to(a,b)} (F(y,x)) \\ &= (F(a,b),F(b,a)) = T(a,b)\end{aligned}$$

and hence T is continuous. Hence T has a fixed point (x^*,y^*) in Y. Therefore by Lemma 2.11 and (x^*,y^*) is a coupled fixed point of F. □

Theorem 3.4. *Let $(X,\ G)$ be a complete G-metric space. Let $F : X \times X \to X$ be generalized (α,ψ) contractive map in two variables satisfying the following conditions.*

- (i) *for all $(x,y),(u,v) \in X \times X$, we have*
  $\alpha((x,y),(u,v),(u,v)) \geq 1$ *implies*
  $\alpha((F(x,y),F(y,x)),(F(u,v),F(v,u)),(F(u,v),F(v,u))) \geq 1$
- (ii) *there exist $(x_0,y_0,y_0) \in X \times X \times X$ such that*
  $\alpha((x_0,y_0),(F(x_0,y_0),(F(y_0,x_0)),(F(x_0,y_0),(F(y_0,x_0))) \geq 1$ *and*
  $\alpha((F(y_0,x_0),(F(x_0,y_0)),(F(y_0,x_0),(F(x_0,y_0)(y_0,x_0))) \geq 1$
- (iii) *if $\{x_n\}$ and $\{x_n\}$ are sequence of X such that*
  $\alpha((x_n,y_n),(x_{n+1},y_{n+1}),(x_{n+1},y_{n+1})) \geq 1$
  and $\alpha((y_{n+1},x_{n+1}),(y_{n+1},x_{n+1}),(y_n,x_n)) \geq 1$
  $\{x_n\}$ and $\{y_n\}$ are convergent to x and y respectively. then
  $\alpha((x_n,y_n),(x,y),(x,y)) \geq 1$ *and* $\alpha((y,x),(y,x),(y_n,x_n)) \geq 1$
  then F has a coupled fixed point. i.e there exist $(x^*,y^*) \in X \times X$ such that
  $F(x^*,y^*) = x^*$ *and* $F(y^*,x^*) = y^*$.

Proof. In above Theorem 3.3 we prove (i) and (ii) conditions
Now to prove condition (iii) by using equation (ii)
let $(x_0,y_0,y_0) \in X \times X \times X$ such that

$$\alpha((x_0,y_0),(F(x_0,y_0),(F(y_0,x_0)),(F(x_0,y_0),(F(y_0,x_0))) \geq 1 \tag{3.4.1}$$

and

$$\alpha((F(y_0,x_0),(F(x_0,y_0)),(F(y_0,x_0),(F(x_0,y_0)(y_0,x_0))) \geq 1. \tag{3.4.2}$$

We define the sequence $(\{x_n\},\{y_n\}) \in X \times X$ by
$x_{n+1} = F(x_n,y_n)$ and $y_{n+1} = F(y_n,x_n)$
Now $x_n = x_{n+1}$ and $y_n = y_{n+1}$ for some n then (x_n,y_n) is a coupled fixed point of X.

Now $x_n \neq x_{n+1}$ and $y_n \neq y_{n+1}$ then by equation (3.4.1) we have
$\alpha((x_0, y_0), (x_1, y_1), (x_1, y_1)) \geq 1$ and
$\alpha((F(x_0, y_0), F(y_0, x_0)), (F(x_1, y_1), F(y_1, x_1)), (F(x_1, y_1), F(y_1, x_1))) \geq 1$
i.e., $\alpha((x_1, y_1), (x_2, y_2), (x_2, y_2)) \geq 1$ continuing this process, we get
$\alpha((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})) \geq 1$
Now $G(F(x_{n-1}, y_{n-1}), F(x_n, y_n), F(x_n, y_n)) \leq$
$\alpha((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n))G(F(x_{n-1}, y_{n-1}), F(x_n, y_n), F(x_n, y_n))$
$\leq \frac{\psi}{2}(\max\{G(x_{n-1}, x_n, x_n) + G(y_{n-1}, y_n, y_n),$
$\frac{1}{2}[G(x_{n-1}, F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1})) + G(y_{n-1}, F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}))$
$+ G(x_n, F(x_n, y_n), F(x_n, y_n)) + G(y_n, F(y_n, x_n), F(y_n, x_n))],$
$\frac{1}{2}[G(x_{n-1}, F(x_n, y_n), F(x_n, y_n)) + G(y_{n-1}, F(y_n, x_n), F(y_n, x_n))$
$+ G(x_n, F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1})) + G(y_n, F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}))]\})$
$\leq \frac{\psi}{2}(\max\{G(x_{n-1}, x_n, x_n) + G(y_{n-1}, y_n, y_n),$
$\frac{1}{2}[G(x_{n-1}, x_n, x_n) + G(y_{n-1}, y_n, y_n) + G(x_n, x_{n+1}, x_{n+1}) + G(y_n, y_{n+1}, y_{n+1})],$
$\frac{1}{2}[G(x_{n-1}, x_{n+1}, x_{n+1}) + G(y_{n-1}, y_{n+1}, y_{n+1}) + G(x_n, x_n, x_n)) + G(y_n, y_n, y_n)])\})$
$\leq \frac{\psi}{2}(\max\{\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)),$
$\frac{1}{2}[\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)) + \delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))],$
$\frac{1}{2}[\delta((x_{n-1}, y_{n-1}), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})) + 0]\})$ (3.4.3)
Similarly, $G(F(y_n, x_n), F(y_n, x_n), F(y_{n-1}, x_{n-1})) \leq \frac{\psi}{2}(\max\{\delta((y_n, x_n), (y_n, x_n), (y_{n-1}, x_{n-1})),$
$\frac{1}{2}[\delta((y_n, x_n), (y_n, x_n), (y_{n-1}, x_{n-1})) + \delta((y_n, x_n), (y_n, x_n), (y_{n-1}, x_{n-1}))],$
$\frac{1}{2}[\delta((y_{n+1}, x_{n+1}), (y_{n+1}, x_{n+1}), (y_{n-1}, x_{n-1})) + 0]\})$ (3.4.4)
On using the notation of β given in the proof of Theorem 3.3, we have
$\beta((x_n, y_n), (x_n, y_n), (x_{n-1}, y_{n-1})) = \min\{\alpha((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)),$
$\alpha((y_n, x_n), (y_n, x_n), (y_{n-1}, x_{n-1}))\} \geq 1.$ (3.4.5)
by using (3.4.3),(3.4.4) and (3.4.5), we have
$G(F(x_{n-1}, y_{n-1}), F(x_n, y_n), F(x_n, y_n)) + G(F(y_n, x_n), F(y_n, x_n), F(y_{n-1}, x_{n-1}))$
$\leq \beta((x_n, y_n), (x_n, y_n), (x_{n-1}, y_{n-1}))G(F(x_{n-1}, y_{n-1}), F(x_n, y_n), F(x_n, y_n))$
$+ G(F(y_n, x_n), F(y_n, x_n), F(y_{n-1}, x_{n-1}))$
$\leq \alpha((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n))G(F(x_{n-1}, y_{n-1}), F(x_n, y_n), F(x_n, y_n))$
$+ \alpha((y_n, x_n), (y_n, x_n), (y_{n-1}, x_{n-1}))G(F(y_n, x_n), F(y_n, x_n), F(y_{n-1}, x_{n-1}))$
$\leq \psi(\max\{\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)), \frac{1}{2}[\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n))$
$+\delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))], \frac{1}{2}[\delta((x_{n-1}, y_{n-1}), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))\})$
$\leq \psi(\max\{\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)), \delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})),$
$\frac{1}{2}[\delta((x_{n-1}, y_{n-1}), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))\})$
$\leq \psi(\max\{\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)), \delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))$
It follows that $\delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})) = G(x_n, x_{n+1}, x_{n+1}) + G(y_{n+1}, y_{n+1}, y_n)$
$\leq \psi(max\{\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n)), \delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1}))\}).$
Since $\psi(t) < t$ for $t > 0$, it follows that

$$\begin{aligned}\delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})) &\leq \psi(\delta((x_{n-1}, y_{n-1}), (x_n, y_n), (x_n, y_n))) \\ &\leq \psi^2(\delta((x_{n-2}, y_{n-2}), (x_{n-1}, y_{n-1}), (x_{n-1}, y_{n-1}))) \\ &\;\;\vdots \\ &\leq \psi^n(\delta((x_0, y_0), (x_1, y_1), (x_1, y_1))).\end{aligned}$$

Therefore,$\delta((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})) \to 0$ as $n \to \infty$
so that$G(x_n, x_{n+1}, x_{n+1}) \to 0$ and $G(y_{n+1}, y_{n+1}, y_n) \to 0$ as $n \to \infty$.

Let $\epsilon > 0$ be given, Since $\Sigma_{n=1}^{+\infty}\psi^n(\delta((x_0,y_0),(x_1,y_1),(x_1,y_1))) < \infty$
there exist $N \in Z^+$ such that
$\Sigma_{p=n}^{+\infty}\psi^p(\delta((x_0,y_0),(x_1,y_1),(x_1,y_1))) < \epsilon$ for all $n \geq N(\epsilon)$.
Now we show that $\{x_n\}$ is a Cauchy sequence in X.
Let $m, n \in Z^+$ with $m > n \geq N$.

$$\begin{aligned} G(x_n,x_m,x_m) &= G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+1},x_{n+2}) + ... + G(x_{m-1},x_{m-1},x_m) \\ &\leq \psi^n(\delta((x_0,y_0),(x_1,y_1),(x_1,y_1))) + ... + \leq \psi^{m-1}(\delta((x_0,y_0),(x_1,y_1),(x_1,y_1))) \\ &\leq \Sigma_{p=n}^{+\infty}\psi^p G(x_0,x_1,x_1) < \epsilon \end{aligned}$$

i.e., $G(x_n,x_{n+k},x_{n+k}) < \epsilon$ for all $n+k, n \geq N$
Hence $\{x_n\}$ is a Cauchy sequence in X.
Since X is complete, there exist $x \in X$ such that
$\lim_{n\to\infty} x_n = x$ Similarly we can prove that $\{y_n\}$ is a Cauchy sequence in X.
Since X is complete, there exist $y \in X$ such that $\lim_{n\to\infty} y_n = y$
Thus, $\{x_n\}$ and $\{y_n\}$are sequence in X such that
$\alpha((x_n,y_n),(x_{n+1},y_{n+1}),(x_{n+1},y_{n+1})) \geq 1$ and
$\alpha((y_{n+1},x_{n+1}),(y_{n+1},x_{n+1}),(y_n,x_n)) \geq 1$
$\{x_n\} \to x \in X$ and $\{y_n\} \to y \in X$ as $n \to \infty$.
By (iii) we get, $\alpha((x_n,y_n),(x,y),(x,y)) \geq 1$ and $\alpha((y,x),(y,x),(y_n,x_n)) \geq 1$ for all n. Therefore T and β satisfy all the hypothesis of Theorem 3.4. Hence T has a fixed point and F has a coupled fixed point. □

Condition (H): For all $(x,y),(u,v) \in X \times X$ there exists $(z_1,z_2) \in X \times X$ such that
$\alpha((x,y),(z_1,z_2),(z_1,z2)) \geq 1, \alpha((z_2,z_1),(z_2,z_1),(y,x)) \geq 1$ and
$\alpha((u,v),(z_1,z_2),(z_1,z_2)) \geq 1, \alpha((z_2,z_1),(z_2,z_1),(v,u)) \geq 1$

Theorem 3.5. *In addition to the hypotheses of Theorem* 3.4. *if condition* (H) *holds, then uniqueness of coupled fixed point of* F.

Proof. We show that T and β of Theorem 3.3 satisfy the hypotheses
Let $x,y,u,v \in X$, Then by using (H), we get
$\beta((x,y),(z_1,z_2),(z_1,z2)) = \min\{\alpha((x,y),(z_1,z_2),(z_1,z2)), \alpha((z_2,z_1),(z_2,z_1),(y,x))\}$
Similarly
$\beta((u,v),(z_1,z_2),(z_1,z2)) = \min\{\alpha((u,v),(z_1,z_2),(z_1,z_2)), \alpha((z_2,z_1),(z_2,z_1),(v,u))\}$
Hence T and β satisfy the hypotheses of Theorem 3.3. T has a unique fixed point and consequently by Lemma 2.11 and F has a unique coupled fixed point. □

The following is an example in support of Theorem 3.3.

Example 3.6. Let (X,G) be a G-metric space, where X = [0, 1] and
$G(x,y,z) = |x-y| + |y-z| + |z-x|$ for all $x,y,z \in X$.
We define $F : X \times X \to X$ by $F(x,y) = \frac{1}{4}xy$ for all $x,y \in X$.
We define $\alpha : X^2 \times X^2 \times X^2 \to X$ be such that

$$\alpha((x,y),(u,v),(u,v)) = \begin{cases} 1 & if\ x \geq u, y \leq v \\ \\ 0 & otherwise. \end{cases}$$

Since $|xy - uv| \leq |x-u| + |y-v|$ holds for all $x,y,u,v \in X$.

Suppose $x \geq u, y \leq v$ then

$$\begin{aligned}\alpha((x,y),(u,v)(u,v))G(F(x,y),F(u,v),F(u,v)) &= 1.G(\tfrac{xy}{4},\tfrac{uv}{4},\tfrac{uv}{4})\\ &= 2|\tfrac{xy}{4}-\tfrac{uv}{4}|\\ &\leq \tfrac{1}{2}(|x-u|+|y-v|)\\ &= \tfrac{1}{2}\psi(\max\{G(x,u,u)+G(y,v,v),\end{aligned}$$

$\frac{1}{2}(G(x,F(x,y),F(x,y))+G(y,F(y,x),F(y,x)))+(G(u,F(u,v),F(u,v))$
$+G(v,F(v,u),F(v,u))),\frac{1}{2}(G(x,F(u,v),F(u,v))+G(y,F(v,u),F(v,u)))$
$+(G(u,F(x,y),F(x,y))+G(v,F(y,x),F(y,x)))\})$

Thus for (3.1) holds for $\psi(t)=\frac{t}{2}$. Hence F is generalized (α,ψ)-contractive map in two variable. and F is continuous. we choose $x_0=0, y_0=0$ then $F(x_0,y_0)=0=F(y_0,x_0)$
$\alpha((x_0,y_0),(F(x_0,y_0),(F(y_0,x_0)),(F(x_0,y_0),(F(y_0,x_0)))=1$ and
$\alpha((F(y_0,x_0),(F(x_0,y_0)),(F(y_0,x_0),(F(x_0,y_0)(y_0,x_0)))=1$
Hence, hypotheses of Theorem 3.3 satisfy. Then there exist coupled fixed point in F. In this case (1,0) and (0,1) are the coupled fixed point of F.

This example is not necessary for this Theorem 3.4 and it has unique, it is sufficient condition for (H) condition.

Example 3.7. Let (X,G) be a G-metric space, where $X=\mathbb{R}$ and
$G(x,y,z)=|x-y|+|y-z|+|z-x|$ for all $x,y,z\in X$.

We define $F: X\times X\to X$ by $F(x,y)=\begin{cases}\frac{|x^2-y^2|}{8} & if\ x,y\in[0,1]\\ 1 & otherwise.\end{cases}$

We define $\alpha: X^2\times X^2\times X^2\to X$ be such that

$$\alpha((x,y),(u,v),(u,v))=\begin{cases}1 & if\ x,y,u,v\in[0,1]\\ 0 & otherwise.\end{cases}$$

Clearly F is not continuous at (1,1) and F is generalized (α,ψ)-contractive map (i.e., F satisfy equation(3.1)), with $\psi(t)=\frac{t}{2}$ for all $t>0$. In fact, for all $x,y,u,v\in[0,1]$ then

$$\begin{aligned}\alpha((x,y),(u,v)(u,v))G(F(x,y),F(u,v),F(u,v)) &= 2|F(x,y)-F(u,v)|\\ &= 2|\tfrac{|x^2-y^2|}{8}-\tfrac{|u^2-v^2|}{8}|\\ &= \tfrac{1}{4}||x^2-y^2|-|u^2-v^2||\\ &\leq \tfrac{1}{4}|x^2-y^2-u^2+v^2|\\ &\leq \tfrac{1}{4}|(x+u)(x-u)+(v+y)(v-y)|\\ &\leq \tfrac{1}{2}||x-u|+|y-v||\\ &= \tfrac{1}{2}\psi(\max\{G(x,u,u)+G(y,v,v),\end{aligned}$$

$\frac{1}{2}(G(x,F(x,y),F(x,y))+G(y,F(y,x),F(y,x)))+(G(u,F(u,v),F(u,v))$
$+G(v,F(v,u),F(v,u))),\frac{1}{2}(G(x,F(u,v),F(u,v))+G(y,F(v,u),F(v,u)))$
$+(G(u,F(x,y),F(x,y))+G(v,F(y,x),F(y,x)))\})$

Suppose at least one of the $x,y,u,v\in[0,1]$ then we have that $\alpha((x,y),(u,v)(u,v))=0$ and hence both side tends to zero. so that (3.4) holds for all $(x,y),(u,v)\in X\times X$ clearly (i) hold, we choose $x_0=\frac{1}{5}$ and $y_0=\frac{4}{5}$ it hold (ii).
Let $\{x_n\}$ and $\{x_n\}$ are sequence of X such that
$\alpha((x_n,y_n),(x_{n+1},y_{n+1}),(x_{n+1},y_{n+1}))\geq 1\Rightarrow\{x_n\},\{y_n\}$ are sequence in [0,1]
similarly $\alpha((y_{n+1},x_{n+1}),(y_{n+1},x_{n+1}),(y_n,x_n))\geq 1\Rightarrow\{y_n\},\{x_n\}$ are sequence in [0,1]

Let $\{x_n\} \to x$ and $\{y_n\} \to y$. Since [0,1] is closed we have $x, y \in [0,1]$
Therefore $\alpha((x_n, y_n), (x, y), (x, y)) \geq 1$ and $\alpha((y, x), (y, x), (y_n, x_n)) \geq 1$
so that (iii) holds. Therefore F, α and ψ satisfy all the hypotheses of Theorem 3.4
and (0,0) is a coupled fixed point of F.

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