

Software Development in Intuitionistic Fuzzy Relational Calculus

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June 9, 2016

Abstract

In this study, MATLAB program for computing power of an IF matrix, strength of connectedness and index matrix of an IFG and its line IFG have been developed. This has been formulated using the theoretical aspects which are proved in [4]. These programs have been verified with suitable illustrations. The proposed MATLAB program is simple, efficient and takes less computational time in image segmentation. The author hopes that this paper will benefit other researchers who are working in the field image segmentation, fuzzy graph theory and intuitionistic fuzzy graph theory.

2010 Mathematics Subject Classification: 05C72, 03E72, 03F55, 97R10.

Keywords: maxminsum(A,B), minmaxsum(A,B), power(A,k), StrengthOfConn(A), Indexmatrix(A), Linematrix(A).

I Introduction

MATLAB stands for MATrix LABoratory. It is a high-level programming language and interactive environment for numerical computation, visualization and programming. MATLAB is developed by MathWorks. Cleve Moler is Chairman and Chief Scientist at The MathWorks.

The founders of The MathWorks recognized the need among engineers and scientists for more powerful and productive computation environments beyond those provided by languages such as Fortran and C. In response to that need, the founders combined their expertise in mathematics, engineering, and computer science to develop MATLAB, a high-performance technical computing environment. MATLAB combines comprehensive math and graphics functions with a powerful high-level language” (MathWorks Inc.) and is primarily a tool for matrix computations. It provides vast library of mathematical functions for linear algebra, statistics, Fourier analysis, filtering, optimization, numerical integration and solving ordinary differential equations. MATLAB has a large collection of toolboxes for variety of applications. A toolbox consists of functions that can be used to perform some computations in the toolbox domain. Some examples of MATLAB toolboxes are: signal processing, image processing, neural network, control system, statistics, symbolic mathematics, optimization and system identification.

Peeva K. and Y. Kyosev [8] introduced fuzzy relational calculus toolbox and created library for intuitionistic fuzzy set in fuzzy relational calculus toolbox. But still there is no MATLAB program to find about intuitionistic fuzzy graph using its index matrix and incidence matrix. So, motivated by the work of Peeva, K. and Y. Kyosev[8], we developed a MATLAB program for strength of connectedness, power of an IF matrix and index matrix of line IFG using the same fuzzy relational calculus toolbox. Fuzzy connectedness method has achieved good performance for image segmentation. There are several image segmentation methods based on fuzzy concept among which fuzzy connectedness is a well-known techniques for this purpose. However, in the computation of fuzzy connectedness, these MATLAB programs can also be used to find strength of connectedness for fuzzy graph by omitting the non membership degree of vertices and edges. The theoretical results for writing such programs have been taken from[4][2][1][7], where Atanassov introduced the index matrix representation of intuitionistic fuzzy graphs and discussed its operations in [2][1], R.Parvathi et al.[7] discussed operations on intuitionistic fuzzy graphs using index matrices and M.G.Karunambigai et al. [4] analyzed the properties of the power of an intuitionistic fuzzy graph and given the relationship between the index matrix of an intuitionistic fuzzy graph and power of an intuitionistic fuzzy graph. The present research work is a continuous study of [4].

In this Section I, the basic definitions and theorems which are used for further sections have been presented. A MATLAB program for operations of IF matrices like maxmin sum, minmax sum and power of an IF matrix have been presented with suitable illustration in Section 2. In Section 3, a MATLAB program for strength of connectedness and index matrix of an IFG and its line IFG have been formulated with suitable illustrations.

II Preliminaries:

Here, the basic definitions and theorems of intuitionistic fuzzy graph have been presented.

Definition 2.0.1 [5] *An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$ said to be a minmax IFG if*

- (1) $V = \{v_1, \dots, v_n\}$ such that $\mu_i : V \rightarrow [0, 1]$ and $\nu_i : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of an element $v_i \in V$ respectively and $0 \leq \mu_i + \nu_i \leq 1$, for every $v_i \in V$,
- (2) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \rightarrow [0, 1]$ and $\nu_{ij} : V \times V \rightarrow [0, 1]$ are such that
 $\mu_{ij} \leq \min(\mu_i, \mu_j)$
 $\nu_{ij} \leq \max(\nu_i, \nu_j)$,

denotes the degree of membership and non-membership of an edge $e_{ij} = (v_i, v_j) \in E$ respectively, where, $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, for every $e_{ij} = (v_i, v_j) \in E$. The degree of hesitance(hesitation degree) of the vertex $v_i \in V$ in G is $\Pi_i = 1 - \mu_i - \nu_i$ and the degree of hesitance(hesitation degree) of an edge $e_{ij} = (v_i, v_j) \in E$ in G is $\Pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$.

Definition 2.0.2 [5] *Let $G = (V, E)$ be an intuitionistic fuzzy graph. A walk is a sequence of vertices and edges, where the endpoints of each edge are the preceding and following vertices in the sequence, such that either one of the following conditions is satisfied.*

- 1) $\mu_{ij} > 0$ & $\nu_{ij} = 0$ for some i & j .
 - 2) $\mu_{ij} > 0$ & $\nu_{ij} > 0$ for some i & j .
- If a walk begins at v_i and ends at v_j then it is an $v_i - v_j$ walk. A walk is closed if it begins and ends at the same vertex.*

Definition 2.0.3 [5] *Let $G = (V, E)$ be an intuitionistic fuzzy graph. A path P in an intuitionistic fuzzy graph G is a sequence of distinct vertices v_1, v_2, \dots, v_n such that either one of the following conditions is satisfied.*

- 1) $\mu_{ij} > 0$ & $\nu_{ij} = 0$ for some i & j .
- 2) $\mu_{ij} > 0$ & $\nu_{ij} > 0$ for some i & j .

Definition 2.0.4 [5] *Let $G = (V, E)$ be an intuitionistic fuzzy graph. The length of a path $P = v_1v_2\dots v_{n+1}$ ($n > 0$) in G is n .*

Definition 2.0.5 [5] *Let $G = (V, E)$ be an intuitionistic fuzzy graph. The length of a path $P = v_1v_2\dots v_{n+1}$ ($n > 0$) in G is n .*

Definition 2.0.6 [5] An intuitionistic fuzzy graph $G = (V, E)$ is connected if any two vertices are joined by a path.

Definition 2.0.7 [5] Let $G = (V, E)$ be an intuitionistic fuzzy graph. The μ - strength of a path $P = v_1v_2\dots v_n$ in an intuitionistic fuzzy graph G is denoted by $S_{\mu(G)}(P)$ and is defined as $\min\{\mu_{ij}\}$, for all $i, j = 1, 2, \dots, n$

Definition 2.0.8 [5] Let $G = (V, E)$ be an intuitionistic fuzzy graph. The ν - strength of a path $P = v_1v_2\dots v_n$ in an intuitionistic fuzzy graph G is denoted by $S_{\nu(G)}(P)$ and is defined as $\max\{\nu_{ij}\}$, for all $i, j = 1, 2, \dots, n$

Definition 2.0.9 [5] If $v_i, v_j \in V \subseteq G$, the μ - strength of connectedness between the vertices v_i and v_j in G is $CONN_{\mu(G)}(v_i, v_j) = \max\{S_{\mu(G)}(P)\}$ and ν - strength of connectedness between the vertices v_i and v_j in G is $CONN_{\nu(G)}(v_i, v_j) = \min\{S_{\nu(G)}(P)\}$ for all possible paths between v_i and v_j .

Definition 2.0.10 [5] An intuitionistic fuzzy graph, $G = (V, E)$ is said to be a strong intuitionistic fuzzy graph if

$$\mu_{ij} = \min(\mu_i, \mu_j) \text{ and } \nu_{ij} = \max(\nu_i, \nu_j), \forall e_{ij} \in E.$$

Definition 2.0.11 [6] An intuitionistic fuzzy matrix(IFM) is a matrix of order $m \times n$ and is defined as $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$, where $a_{\mu_{ij}} \in [0, 1]$, $a_{\nu_{ij}} \in [0, 1]$ such that $0 \leq a_{\mu_{ij}} + a_{\nu_{ij}} \leq 1$, $1 \leq i \leq m$ and $1 \leq j \leq n$. It can also be represented in the matrix form,

$$A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n} = \begin{pmatrix} \langle a_{\mu_{11}}, a_{\nu_{11}} \rangle & \langle a_{\mu_{12}}, a_{\nu_{12}} \rangle & \dots & \langle a_{\mu_{1n}}, a_{\nu_{1n}} \rangle \\ \langle a_{\mu_{21}}, a_{\nu_{21}} \rangle & \langle a_{\mu_{22}}, a_{\nu_{22}} \rangle & \dots & \langle a_{\mu_{2n}}, a_{\nu_{2n}} \rangle \\ \dots & \dots & \dots & \dots \\ \langle a_{\mu_{m1}}, a_{\nu_{m1}} \rangle & \langle a_{\mu_{m2}}, a_{\nu_{m2}} \rangle & \dots & \langle a_{\mu_{mn}}, a_{\nu_{mn}} \rangle \end{pmatrix}$$

Definition 2.0.12 The number of rows and columns that IF matrix has called its dimension or its order. That is, the dimension or order of IF matrix with m rows and n columns is $m \times n$. The individual items in an IF matrix are called its elements or entries.

Definition 2.0.13 [6] Let $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$ be a intuitionistic fuzzy matrix. The transpose of the matrix A is denoted by A^T and is defined as $A^T = \{ \langle a_{\mu_{ji}}, a_{\nu_{ji}} \rangle \}_{n \times m}$.

Definition 2.0.14 [4] Let $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$ and $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{m \times n}$ be two intuitionistic fuzzy matrices. Then two IF matrices A and B are equal to each other, if they have the same dimensions $m \times n$ and the same elements $a_{\mu_{ij}} = b_{\mu_{ij}}, a_{\nu_{ij}} = b_{\nu_{ij}}$ for $i = 1, \dots, m$ and $j = 1, \dots, n$. It is denoted by $A = B$

Definition 2.0.15 [4] Let $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$ and $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{m \times n}$ be two intuitionistic fuzzy matrices. Then the two types of sum of A and B are defined as

1) max min sum of IF matrices : $A +_{\max \min} B = \{ \langle c_{\mu_{ij}}, c_{\nu_{ij}} \rangle \}_{m \times n} = [\langle \max(a_{\mu_{ij}}, b_{\mu_{ij}}), \min(a_{\nu_{ij}}, b_{\nu_{ij}}) \rangle, 1 \leq i \leq m, 1 \leq j \leq n$.

2) min max sum of IF matrices : $A +_{\min \max} B = \{ \langle c_{\mu_{ij}}, c_{\nu_{ij}} \rangle \}_{m \times n} = [\langle \min(a_{\mu_{ij}}, b_{\mu_{ij}}), \max(a_{\nu_{ij}}, b_{\nu_{ij}}) \rangle, 1 \leq i \leq m, 1 \leq j \leq n$.

Notation 2.0.1 Throughout this paper, we denote " $+_{\max \min}$ " as "+".

Definition 2.0.16 [4] Let $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$ and $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{n \times p}$ be two intuitionistic fuzzy matrices. Then the two types of product of A and B are defined as

1) max min product of IF matrices : $A \bullet_{\max \min} B = \{ \langle c_{\mu_{ij}}, c_{\nu_{ij}} \rangle \}_{m \times p} = [\langle \max(\min(a_{\mu_{ij}}, b_{\mu_{jk}})), \min(\max(a_{\nu_{ij}}, b_{\nu_{jk}})) \rangle, 1 \leq i \leq m, 1 \leq j \leq p, 1 \leq k \leq n$ and

2) min max product of IF matrices: $A \bullet_{\min \max} B = \{ \langle c_{\mu_{ij}}, c_{\nu_{ij}} \rangle \}_{m \times p} = [\langle \min(\max(a_{\mu_{ij}}, b_{\mu_{jk}})), \max(\min(a_{\nu_{ij}}, b_{\nu_{jk}})) \rangle, 1 \leq i \leq m, 1 \leq j \leq p, 1 \leq k \leq n$.

Definition 2.0.17 [4] Let $A = \{ \langle a_{\mu_{ij}}, a_{\nu_{ij}} \rangle \}_{m \times n}$ be intuitionistic fuzzy matrix and k is a positive integer. Then the k^{th} power of an intuitionistic fuzzy matrix is denoted by A^k and is defined as max min product of k - copies of an intuitionistic fuzzy matrix A .

Definition 2.0.18 [2] Let $G = (V, E)$ be an intuitionistic fuzzy graph. The index matrix representation of intuitionistic fuzzy graph (IMIFG) is of the form $[V, E \subset V \times V]$ where $V = \{v_1, v_2, \dots, v_n\}$ and

$$E = \{ \langle \mu_{ij}, \nu_{ij} \rangle \}_{m \times n} = \begin{matrix} & v_1 & v_2 & \dots & v_n \\ \begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{matrix} & \left(\begin{array}{cccc} \langle \mu_{11}, \nu_{11} \rangle & \langle \mu_{12}, \nu_{12} \rangle & \dots & \langle \mu_{1n}, \nu_{1n} \rangle \\ \langle \mu_{21}, \nu_{21} \rangle & \langle \mu_{22}, \nu_{22} \rangle & \dots & \langle \mu_{2n}, \nu_{2n} \rangle \\ \dots & & & \\ \langle \mu_{n1}, \nu_{n1} \rangle & \langle \mu_{n2}, \nu_{n2} \rangle & \dots & \langle \mu_{nn}, \nu_{nn} \rangle \end{array} \right) \end{matrix}$$

where $\langle \mu_{ij}, \nu_{ij} \rangle \in [0, 1] \times [0, 1] (1 \leq i, j \leq n)$, the edge between two vertices v_i and v_j is indexed by $\langle \mu_{ij}, \nu_{ij} \rangle$.

Note 2.0.1 Index matrix representation of any intuitionistic fuzzy graph is an intuitionistic fuzzy matrix.

Definition 2.0.19 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph where $V = \{v_1, v_2, \dots, v_n\}$. The incidence matrix of an intuitionistic fuzzy graph G is $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{n \times m}$, where n and m represents the number of vertices and number of edges of G respectively, whose entries of B are as follows:

$$B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{n \times m} = \begin{cases} \langle \mu(e_j), \nu(e_j) \rangle, & \text{if an edge } e_j \text{ is incident on the vertex } v_i \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

It can also be represented in the matrix form,

$$B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{n \times m} = \begin{matrix} & e_1 & e_2 & \dots & e_n \\ \begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{matrix} & \left(\begin{array}{cccc} \langle \mu(e_1), \nu(e_1) \rangle & \langle \mu(e_2), \nu(e_2) \rangle & \dots & \langle \mu(e_n), \nu(e_n) \rangle \\ \langle \mu(e_1), \nu(e_1) \rangle & \langle \mu(e_2), \nu(e_2) \rangle & \dots & \langle \mu(e_n), \nu(e_n) \rangle \\ \dots & \dots & \dots & \dots \\ \langle \mu(e_1), \nu(e_1) \rangle & \langle \mu(e_2), \nu(e_2) \rangle & \dots & \langle \mu(e_n), \nu(e_n) \rangle \end{array} \right) \end{matrix}$$

where $\langle \mu(e_j), \nu(e_j) \rangle \in [0, 1] \times [0, 1]$.

Definition 2.0.20 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph, where $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{e_1, e_2, \dots, e_k\}$. Then the line intuitionistic fuzzy graph is denoted by $G_L = (V_L, E_L)$, where the vertices of G_L are in one-one correspondence with the edges of G and there exist an edge between the vertices of G_L if and only if the corresponding edges of G are adjacent. The membership and non-membership value of V_L and E_L are defined as follows:

$$\begin{aligned} \mu_L(v_i) &= \mu(e_i) \text{ and } \nu_L(v_i) = \nu(e_i), \forall e_i \in E. \\ \mu_L(v_i, v_j) &= \begin{cases} \min(\mu_L(v_i), \mu_L(v_j)), & \text{if } e_i \text{ and } e_j \text{ are adjacent in } G \\ (0, 1), & \text{otherwise} \end{cases} \\ \nu_L(v_i, v_j) &= \begin{cases} \max(\nu_L(v_i), \nu_L(v_j)), & \text{if } e_i \text{ and } e_j \text{ are adjacent in } G \\ (0, 1), & \text{otherwise} \end{cases} \end{aligned}$$

Definition 2.0.21 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph with the underlying crisp graph $G^* = (V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$. Then the power of an intuitionistic fuzzy graph G is denoted by, $G^k = (V^k, E^k)$, where $V^k = V$ and the vertices v_i and v_j are adjacent in G^k if and only if $d_{G^*}(v_i, v_j) \leq k$ (Refer Definition 1.4). The membership and non-membership

values of the edges of G^k are defined as follows:

$$(\mu^k(v_i, v_j), \nu^k(v_i, v_j)) = \begin{cases} (\min(\mu_i, \mu_j), \max(\nu_i, \nu_j)), & \text{if } d_{G^*}(v_i, v_j) \leq k \\ (0, 1), & \text{otherwise} \end{cases}$$

Theorem 2.0.1 [4] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two strong IFGs. Then $A(G_1 \cup G_2) = A(G_1) + A(G_2)$ if and only if $V_1 = V_2$.

Theorem 2.0.2 [4] Let $G = (V, E)$ be an IFG and let $A = \{ \langle \mu_{ij}, \nu_{ij} \rangle \}$ be the index matrix of G . Then for each positive integer k , the

$$(i, j)^{\text{th}} \text{ entry of } A^k = \text{strength of connectedness of } v_i - v_j \text{ walks of length } k. \quad (1)$$

Theorem 2.0.3 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph, where $V = \{v_1, v_2, \dots, v_n\}$ be the vertices of G . Let $A = \{ \langle \mu_{ij}, \nu_{ij} \rangle \}$ be the index matrix of G . Then $(CONN_{\mu(G)}(v_i, v_j), CONN_{\nu(G)}(v_i, v_j)) = (i, j)^{\text{th}}$ entry of $A + A^2 + \dots + A^{n-1}$, $\forall v_i \neq v_j \in V$ and

Theorem 2.0.4 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph. Let $A = \{ \langle \mu_{ij}, \nu_{ij} \rangle \}_{m \times m}$ and $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{m \times n}$ be the index matrix and incidence matrix of G respectively. Then the entries of $B \bullet_{\max - \min} B^T$ are

$$\{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{m \times m} = \begin{cases} (\mu_{ij}, \nu_{ij}), & \text{if } i \neq j \\ (\max(\mu_{ik}), \min(\nu_{ik})), & \text{if } i = j, \forall v_k \in N_G(v_i) \end{cases}$$

Theorem 2.0.5 [4] Let $G = (V, E)$ be an intuitionistic fuzzy graph and $G_L = (V_L, E_L)$ be a line intuitionistic fuzzy graph. Let $A = \{ \langle \mu_{ij}, \nu_{ij} \rangle \}$ and $B = \{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}$ be the index matrix and incidence matrix of G respectively. Then the entries of $B^T \bullet_{\max - \min} B$ are

$$\{ \langle b_{\mu_{ij}}, b_{\nu_{ij}} \rangle \}_{n \times n} \begin{cases} (\mu_L(v_i, v_j), \nu_L(v_i, v_j)), & \text{if } i \neq j \\ (\max(\mu_{ik}), \min(\nu_{ik})), & \text{if } i = j, \forall v_k \in N_G(v_i) \end{cases}$$

where $(\mu_L(v_i, v_j), \nu_L(v_i, v_j))$ is the membership and non membership value of an edge $e_{ij} \in E_L$.

III MATLAB programming for operation of IF Matrices

In this section, MATLAB program has been developed to find

- 1) max-min sum and min-max sum of IF Matrices
- 2) power of an IF matrix

Also, an example that executes the function maxminsum, minmaxsum and powerifm has been presented.

3.1 MATLAB programming for max-min sum and min-max sum of IF Matrices

The MATLAB program maxminsum (A, B) generates the max-min sum of two IF matrices A and B .

Parameters Description:

A – intuitionistic fuzzy matrix of order $m \times n$,

B – intuitionistic fuzzy matrix of order $m \times n$,

The source code of the MATLAB program maxminsum (A, B) is presented below. Line starting with a percentage sign are interpreted as comment lines by MATLAB and are ignored.

```
1 function maxminsum = maxminsum(A,B)
2 %The following command calls im function to construct new intuitionistic ...
   fuzzy matrix with degree of membership min(A.m,B.m) and degree of ...
   non-membership max(A.n,B.n)
3 maxminsum = im(max(A.m,B.m), min(A.n,B.n));
```

The MATLAB program minmaxsum (A, B) generates the min-max sum of two IF matrices A and B .

Parameters Description:

A – intuitionistic fuzzy matrix of order $m \times n$,

B – intuitionistic fuzzy matrix of order $m \times n$,

The source code of the MATLAB program minmaxsum (A, B) is presented below.

```
1 function C = minmaxsum(A,B)
2 %The following command calls im function to construct new intuitionistic ...
   fuzzy matrix C such that each entry in C has two values– degree of ...
   membership min(A.m,B.m) and degree of non-membership max(A.n,B.n)
```

$$C = \text{im}(\min(A.m, B.m), \max(A.n, B.n));$$

Problem:

The following example shows how to evaluate the max-min sum and min-max sum of the IF matrices using the above function. Let

$$A = \begin{pmatrix} \langle 0, 1 \rangle & \langle .5, .3 \rangle & \langle .5, .2 \rangle \\ \langle .3, .3 \rangle & \langle 0, 1 \rangle & \langle .1, .4 \rangle \\ \langle .3, .1 \rangle & \langle .1, .5 \rangle & \langle 0, 1 \rangle \\ \langle .1, .1 \rangle & \langle .2, .5 \rangle & \langle .1, .7 \rangle \end{pmatrix}$$

and

$$B = \begin{pmatrix} \langle 0, 1 \rangle & \langle .3, .3 \rangle & \langle .2, .1 \rangle \\ \langle .3, .5 \rangle & \langle 0, 1 \rangle & \langle .4, .6 \rangle \\ \langle .1, .1 \rangle & \langle .5, .4 \rangle & \langle 0, 1 \rangle \\ \langle .1, .6 \rangle & \langle .5, .1 \rangle & \langle .7, .3 \rangle \end{pmatrix}$$

Find $A +_{\max - \min} B$ and $A +_{\min - \max} B$ using MATLAB.

Solution:

```
%Enter the degree of membership of A in the variable a.m
>>a.m = [0 .5 .5 ;.3 0 .1 ;.3 .1 0 ; .1 .2 .1];
%Enter the degree of non-membership of A in the variable a.n
>>a.n = [1 .3 .2;.3 1 .4 ; .1 .5 1;.1 .5 .7 ];
>>A = im(a.m,a.n)
%This command returns an IF matrix A with degree of membership a.m and degree of
% non- membership a.n

<0 1> <.5 .3> <.5 .2>
```

```
<.3 .3> <0 1> <.1 .4>  
<.3 .1> <.1 .5> <0 1>  
<.1 .1> <.2 .5> <.1 .7>
```

```
%Enter the degree of membership of B in the variable b.m
```

```
>>b.m = [0 .3 .2;.3 0 .4 ; .1 .5 0;.1 .5 .7 ];
```

```
%Enter the degree of non-membership of B in the variable b.n
```

```
>>b.n = [1 .3 .1;.5 1 .6; .1 .4 1;.6 .1 .3];
```

```
>>B = im(b.m,b.n)
```

```
%This command returns an IF matrix B with degree of membership b.m and degree of  
% non- membership b.n
```

```
<0 1> <.3 .3> <.2 .1>  
<.3 .5> <0 1> <.4 .6>  
<.1 .1> <.5 .4> <0 1>  
<.1 .6> <.5 .1> <.7 .3>
```

```
>>maxminsum(A,B);
```

```
% This command returns the max-min sum of IF matrices A and B
```

```
ans =
```

```
<0 1> <.5 .3> <.5 .1>  
<.3 .5> <0 1> <.4 .4>  
<.3 .1> <.5 .4> <0 1>  
<.1 .1> <.5 .1> <.7 .3>
```

```
>>minmaxsum(A,B)
```

```
% This command returns the min-max sum of IF matrices A and B
```

```
ans =
```

```
<0 1> <.3 .3> <.2 .2>  
<.3 .5> <0 1> <.1 .6>  
<.1 .1> <.1 .5> <0 1>  
<.1 .6> <.2 .5> <.1 .7>
```

3.2 MATLAB Programming for Power of IF Matrix

The MATLAB program `powerifm(A, k)` generates the k^{th} power of an intuitionistic fuzzy matrix A .

Parameters Description:

A – intuitionistic fuzzy matrix of order $m \times m$,

k – power of an IF matrix A

The source code of the MATLAB program `powerifm(A, k)` is presented below.

```

1 function powerifm = powerifm(A,k)
2 temp = A;
3 for i =2 :k
4 powerifm = SC_comp(A,temp);
5 temp = power;
6 end
    
```

Problem:

The following example shows how to evaluate the power of the IF matrix using the above function. Let

$$A = \begin{pmatrix}
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle .1, .4 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle .1, .4 \rangle & \langle 0, 1 \rangle & \langle .1, .6 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle .1, .6 \rangle & \langle 0, 1 \rangle & \langle .3, .5 \rangle \\
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle .3, .5 \rangle & \langle 0, 1 \rangle
 \end{pmatrix}$$

Find A^3 using MATLAB.

Solution:

`%Enter the degree of membership of A in the variable a.m`

```
>>a = [0 .6 0 0 0;.6 0 .1 0 0;0 .1 0 .1 0];
>>a1 = [0 0 .1 0 .3; 0 0 0 .3 0];
A.m =vertcat(a,a1);
%Enter the degree of non- membership of A in the variable a.n
>>b = [1 .3 1 1 1;.3 1 .4 1 1; 1 .4 1 .6 1];
b1 = [ 1 1 .6 1 .5; 1 1 1 .5 1];
A.n=vertcat(b,b1);
>>A = im(A.m,A.n)
%This command returns an IF matrix A with degree of membership a.m and degree of
% non- membership a.n
A =

<0,1> <.6,.3> <0,1> <0,1> <0,1>
<.6,.3> <0,1> <.1,.4> <0,1> <0,1>
<0,1> <.1,.4> <0,1> <.1,.6> <0,1>
<0,1> <0,1> <.1,.6> <0,1> <.3,5>
<0,1> <0,1> <0,1> <.3,.5> <0,1>

>>powerifm(A,3)

This command returns the result of the IF matrix A^3

ans =

<0,1> <.6,.3> <0,1> <.1,6> <0,1>
<.6,.3> <0,1> <.1,.4> <0,1> <.1,.6>
<0,1> <.1,.4> <0,1> <.1,.6> <0,1>
<.1,.6> <0,1> <.1,.6> <0,1> <.3,5>
<0,1> <.1,.6> <0,1> <.3,.5> <0,1>
```

IV Programming for Computing Strength of Connectedness of any Pair of Vertices in an IFG and to Compute the Index Matrix of an IFG and its Line IFG

In this section, a MATLAB program has been developed to find

- 1) Strength of connectedness between any pair of vertices in an IFG
- 2) Index matrix of an intuitionistic fuzzy graph and its line IFG.

4.1 MATLAB programming Strength of connectedness between vertices

The MATLAB program `StrengthOfConn(A).m` returns the strength of connectedness between vertices of an IFG G , where A is the index matrix of the intuitionistic fuzzy graph G .

Parameters Description:

A – intuitionistic fuzzy matrix of order $m \times m$,

The source code of the MATLAB program `powerifm(A, k)` is presented below.

```
1 function StrengthOfConn = StrengthOfConn(A)
2 k= length(A.m( :,1));
3 for i = 2:k-1
4 B= A;
5 Ai = SC_comp(A,B);
6 sum = maxminsum(A,Ai);
7 StrengthOfConn = sum;
8 A = sum;
9 end
10 for i = 1:k
11 for j = 1:k
12 if i==j
13 X(i,j) == 0;
14 Y(i,j)==1;
15 else
16 X(i,j) == 1;
17 Y(i,j)==0;
```

```

18 end
19 end
20 end
21 Z = im(X,Y);
22 StrengthOfConn =minmaxsum(StrengthOfConn,Z);
    
```

Problem:

The following example shows how to evaluate the Strength of connectedness of the IF matrix using the above function. Let $G = (V, E)$ be an intuitionistic fuzzy graph and A be its index intuitionistic fuzzy graph. Let

$$A = \begin{pmatrix}
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle .1, .4 \rangle & \langle .6, .3 \rangle & \langle .6, .3 \rangle \\
 \langle 0, 1 \rangle & \langle .1, .4 \rangle & \langle 0, 1 \rangle & \langle .1, .6 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle .1, .6 \rangle & \langle 0, 1 \rangle & \langle .3, .5 \rangle \\
 \langle 0, 1 \rangle & \langle .6, .3 \rangle & \langle 0, 1 \rangle & \langle .3, .5 \rangle & \langle 0, 1 \rangle
 \end{pmatrix}$$

Find Strength of connectedness of G using MATLAB.

Solution:

```

%Enter the degree of membership of A in the variable a.m
>>a = [0 .6 0 0 0;.6 0 .1 .6 .6;0 .1 0 .1 0];
>>a1 = [0 .6 .1 0 .3; 0 .6 0 .3 0];
>>A.m = vertcat(a,a1);
%Enter the degree of membership of A in the variable a.n
>>b = [1 .3 1 1 1;.3 1 .4 .3 .3; 1 .4 1 .6 1];
>> b1 = [1 .3 .6 1 .5; 1 .3 1 .5 1];
>>A.n = vertcat(b,b1);
>>A = im(A.m,A.n)
%This command returns an IF matrix A with degree of membership a.m and degree of
% non- membership a.n
A =
    
```

```
<0,1> <.6,.3> <0,1> <0,1> <0,1>
<.6,.3> <0,1> <.1,.4> <.6,.3> <.6,.3>
<0,1> <.1,.4> <0,1> <.1,.6> <0,1>
<0,1> <.6,.3> <.1,.6> <0,1> <.3,5>
<0,1> <.6,.3> <0,1> <.3,.5> <0,1>
```

```
>>StrengthOfConn (A)
```

This command returns the strength of connectedness of G
 ans =

```
<0,1> <.6,.3> <.1,.4> <.6,.3> <.6,.3>
<.6,.3> <0,1> <.1,.4> <.6,.3> <.6,.3>
<.1,.4> <.1,.4> <0,1> <.1,.4> <.1,.4>
<.6,.3> <.6,.3> <.1,.4> <0,1> <.6,.3>
<.6,.3> <.6,.3> <.1,.4> <.6,.3> <0,1>
```

4.2 MATLAB Programming for Index matrix of intuitionistic fuzzy graph

The MATLAB program Indexmatrix(A) generates the index matrix of intuitionistic fuzzy graph G, where A is the incidence matrix of intuitionistic fuzzy graph G.

Parameters Description:

A– intuitionistic fuzzy matrix of order $m \times n$,

The source code of the MATLAB program Indexmatrix(A) is presented below.

```
1 function Indexmatrix = Indexmatrix(A)
2 B = transpose(A);
3 C = SC_comp(A,B);
4 k= length(A.m(:,1));
5 for i = 1:k
6 for j = 1:k
7 if i==j
8 X(i,j) == 0;
9 Y(i,j)==1;
```



```

10 else
11 X(i,j) == 1;
12 Y(i,j)==0;
13 end
14 end
15 end
16 Z = im(X,Y);
17 Indexmatrix =minmaxsum(C,Z);
    
```

Problem:

The following example shows how to evaluate the index matrix of intuitionistic fuzzy graph using the above function.

Let $G = (V, E)$ be an intuitionistic fuzzy graph and A be its incidence matrix of intuitionistic fuzzy graph. Let

$$A = \begin{pmatrix}
 \langle 1, .7 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .4, .5 \rangle & \langle .4, .4 \rangle \\
 \langle .1, .7 \rangle & \langle .2, .6 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle .2, .6 \rangle & \langle .3, .6 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .3, .6 \rangle & \langle .4, .4 \rangle & \langle 0, 1 \rangle & \langle .4, .4 \rangle \\
 \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .4, .4 \rangle & \langle .4, .5 \rangle & \langle 0, 1 \rangle
 \end{pmatrix}$$

Find Index matrix of G using MATLAB.

Solution:

```

>>a=[.1 0 0 0 .4 .4;.1 .2 0 0 0 0;0 .2 .3 0 0 0];
>>a1 = [0 0 .3 .4 0 .4; 0 0 0 .4 .4 0];
A.m =vertcat(a,a1);
>>b=[.7 1 1 1 .5 .5;.7 .6 1 1 1 1;1 .6 .6 1 1 1];
>>b1 = [1 1 .6 .4 1 .4; 1 1 1 .4 .5 1];
>>A.n = vertcat(b,b1);
>>A = im(A.m,A.n)
    
```

A =

```
<.1, .7> <0,1> <0,1> <0,1> <.4, .5> <.4 .4>
<.1, .7> <.2, .6> <0,1> <0,1> <0,1> <0, 1>
<0,1> <.2, .6> <.3, .6> <0,1> <0,1> <0, 1>
<0,1> <0,1> <.3, .6> <.4, .4> <0,1> <.4, .4>
<0,1> <0,1> <0,1> <.4, .4> <.4 .5> <0,1>
>>Indexmatrix(A)
```

This command returns the index matrix of intuitionistic fuzzy graph
 ans =

```
<0,1> <.1, .7> <0,1> <.4, .4> <.4, .5>
<.1, .7> <0,1> <.2, .6> <0,1> <0,1>
<0,1> <.2, .6> <0,1> <.3, .6> <0, 1>
<.4, .4> <0,1> <.3, .6> <0,1> <.4, .4>
<.4, .5> <0,1> <0,1> <.4, .4> <0,1>
```

4.3 MATLAB Programming for Index matrix of Line intuitionistic fuzzy graph

The MATLAB program `Linematrix(A)` generates the index matrix of intuitionistic fuzzy graph L , where A is the incidence matrix of intuitionistic fuzzy graph G and L is the line intuitionistic fuzzy graph of G .

Parameters Description:

A – intuitionistic fuzzy matrix of order $m \times n$,

The source code of the MATLAB program `Linematrix(A)` is presented below.

```
1 function Linematrix = Linematrix(A)
2 B = transpose(A);
3 C = SC_comp(B,A);
4 k= length(B.m(:,1));
5 for i = 1:k
6 for j = 1:k
7 if i==j
8 X(i,j) == 0;
```

```

9 Y(i,j)==1;
10 else
11 X(i,j) == 1;
12 Y(i,j)==0;
13 end
14 end
15 end
16 Z = im(X,Y);
17 Linematrix =minmaxsum(C,Z);
    
```

Problem:

The following example shows how to evaluate the index matrix of line intuitionistic fuzzy graph using the above function.

Let $G = (V, E)$ be an intuitionistic fuzzy graph and A be its index intuitionistic fuzzy graph. Let

$$A = \begin{pmatrix}
 \langle .1, .7 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .4, .5 \rangle & \langle .4, .4 \rangle \\
 \langle .1, .7 \rangle & \langle .2, .6 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle .2, .6 \rangle & \langle .3, .6 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
 \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .3, .6 \rangle & \langle .4, .4 \rangle & \langle 0, 1 \rangle & \langle .4, .4 \rangle \\
 \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle .4, .4 \rangle & \langle .4, .5 \rangle & \langle 0, 1 \rangle
 \end{pmatrix}$$

Find $L_G(A)$ using MATLAB.

Solution:

```

>>a=[.1 0 0 0 .4 .4;.1 .2 0 0 0 0;0 .2 .3 0 0 0];
>>a1 = [0 0 .3 .4 0 .4; 0 0 0 .4 .4 0];
>>A.m = vertcat(a,a1);
>>b=[.7 1 1 1 .5 .4;.7 .6 1 1 1 1;1 .6 .6 1 1 1];
>>b1 = [1 1 .6 .4 1 .4; 1 1 1 .4 .5 1];
>>A.n= vertcat(b,b1);
    
```

```
>>A = im(a.m,a.n)
```

```
A =
```

```
<.1,.7> <0,1> <0,1> <0,1> <.4,.5> <.4 .5>
```

```
<.1,.7> <.2,.6> <0,1> <0,1> <0,1> <0 1>
```

```
<0,1> <.2,.6> <.3,.6> <0,1> <0,1> <0 1>
```

```
<0,1> <0,1> <.3,.6> <.4,.4> <0,1> <.4 .4>
```

```
<0,1> <0,1> <0,1> <.4,.4> <.4 .5> <0,1>
```

```
>>Linematrix(A)
```

This command returns the index matrix of line intuitionistic fuzzy graph

```
ans =
```

```
<0 ,1> <.1,.7> <0,1> <0,1> <.1,.7> <.1,.7>
```

```
<.1,.7> <0 ,1> <.2,.6> <0,1> <0,1> <0 ,1>
```

```
<0,1> <.2,.6> <0 ,1> <.3,.6> <0,1> <.3,.6>
```

```
<0,1> <0,1> <.3,.6> <0 ,1> <.4 .5> <.4,.4>
```

```
<.1,.7> <0,1> <0,1> <.4,.5> <0 ,1> <.4,.5>
```

```
<.1,.7> <0,1> <.3,.6> <.4,.4> <.4,.5> <0 ,1>
```

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