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Skewness Corrected Control Charts for Random Queue Length

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Abstract- Classical Shewhart kind control charts are based on normality assumption and ignore the skewness of the plotted statistic in the construction of control charts. Generally the skewness is large enough to be overlooked and in such situation the traditional chart is improper to give satisfactory performance. In this paper, we present a comparative study and present best control chart based on skewness and kurtosis for random queue length for M/M/1 queueing model evaluated on the basis of performance measure false alarm rate.

Index Terms- Control limits, false alarm rate, Haim Shore method, kurtosis correction method, (M/M/1): $(\infty/FCFS)$ queueing system, Shewhart method, skewness correction method, skewness and kurtosis correction method

1. Introduction

Queueing system has the following types of system responses of interest:

(i) Some measure of waiting time that a customer might be forced to bear that is the time a customer spends in the queue and the total time a customer spends in the system (queue plus service).

(ii) The number of customers in the queue and the total number of customers in the system.

(iii) A measure of facility utilization.

As most queueing systems have stochastic elements, these measures are often random variables and their probability distributions or their average values are desired. Depending on the type of system under study, one may be of more interest than the other. The problem is to find out the values of appropriate measures for a given system, which quantifies the phenomenon of waiting in queues that is the average number of customers in queue, average waiting time in queue/system and average facility utilization. Thus average queue length and average waiting time are the main observable characteristics of any queueing system. In this paper, control charts are proposed for the monitoring of random queue length of (M/M/1): $(\infty/FCFS)$ queueing systems. A service system in which customers arrive at random to avail service is known as queueing system and if a server is busy then the customer has to wait, resulting in a queue. The customer waiting in queue is served when a server becomes available according to first come, first serve queue discipline. The characterizing feature of a single server queue is that both the inter-arrival time and the service times are exponentially distributed with parameters λ and μ respectively. Further there is no limit to queue size. Let random variable (r.v.) N denote the number of customers in the system (either served or waiting). In this paper control charts are constructed for N by various methods which can serve as follows:

1. To monitor the stability of the queueing system in terms of N where an out of control signal signify change in any of the parameter that determine N say traffic intensity or in arrival or service rate of the system or rise in variance.
2. The upper control limit of N can be taken as upper patience limit of a customer in queue which can be used further in studying one of the important performance measures of queueing system.

3. In case these control limits are informed beforehand to customers, then abandonment of service by customer because of impatience reduces. Performance of any queueing system is judged to be satisfactory if a customer has to wait for minimum period of time within his/her patience limit in the system to get service, in concise queue length should be small.

All of the cases discussed above rely heavily on the goodness of the control chart limits [8]. Mean, standard deviation, skewness and kurtosis are used in calculating the control limits. It is also assumed that the control charts relate to individual observations. Through numerical analysis of the performance measure, control charts obtained by different methods are compared to find the best performing control chart.

2. Literature Review

Control charts are one of the powerful tools of statistical process control, widely accepted and applied in industry. Basically used to improve productivity, prevent defects and unnecessary process adjustments, also provides information in diagnosis and process capability. Control chart effectiveness is based on control limits to detect whether a process is in control. The traditional charts are based on the assumption that the distribution of the quality characteristic is normal or approximately normal [2]. However, in many situations this normality assumption of process population is not valid when the process distribution is found to be skewed.

If the underlying process distribution is skewed, then methods with which false alarm rates can be controlled to the desired level for an arbitrary skewed distribution are required. A great deal of literature developed various methods to deal with skewness of process distribution. Chan and Cui [2] took the degree of skewness of the underlying process distribution and proposed skewness correction (SC) method for constructing control charts for skewed distribution. Kurtosis correction method was developed to set up control charts for the symmetric and leptokurtic distribution [8]. A skewness and kurtosis correction (SKC) method was proposed to derive control charts with no assumption of the underlying process distribution, that takes into account the degree of skewness and kurtosis of the process distribution and provides the control limits using three standard deviation and adding the known function of skewness and kurtosis [9].Control charts for attributes data were developed, where the monitoring statistic may have a skewed distribution and implemented for the study of queue length of M/M/s system [7].This paper uses control charts developed through heuristic methods with no assumptions on the process distribution for obtaining upper control limit for characteristic i.e. number of customers in the queueing system. Performance of control charts are judged on the basis of false alarm rate.

3. Probability distribution of the system size N

(M/M/1): (∞/FCFS) is taken for study as it models a large number of queueing problems has Poisson input, exponential service time, single server with first come, first serve queue discipline and infinite capacity. In the construction of control chart for N, characteristics of distribution of r.v. N is needed.

Let r.v. N be the number of customers in the system (both waiting and in service) in steady state. The steady state probability distribution of r.v. N is given by, $p_n = P[N = n] = \rho^n(1-\rho), n = 0,1,2,\ldots,$ where $\rho < 1$ (1)

and $\rho = \frac{\lambda}{\mu}$ is the traffic intensity or utilization rate for single server queue, λ is the mean arrival rate and μ is the mean service rate.The r.v. N follows geometric distribution with parameter $(1-\rho)$. The distribution of the r.v. N depends on two parameters λ and μ only through their ratio. The model clearly suggests that, if control is possible, the parameters should be adjusted to make ρ approach 1, in order to achieve full utilization of the server.

3.1. Distributional properties of random variable N

The distributional properties of r.v. N namely mean, variance, skewness and kurtosis are displayed in the table given below.

Table I: Distributional properties of random variable N

E(N)	V(N)	γ_1	γ_2
$\frac{\rho}{1-\rho}$	$\frac{\rho}{(1-\rho)^2}$	$\frac{1+\rho}{\sqrt{\rho}}$	$(\frac{1}{\rho}+\rho)+4$

From the above table, it can be noted that various characteristics of N depends on traffic intensity ρ.

4. Numerical analysis of characteristics of distribution of r.v N

To study the effect of traffic intensity on the distributional properties of random variable N, a set of increasing values of ρ are selected mean, variance, skewness and kurtosis are computed and displayed in Table II. From this table, it is noted that as the values of traffic intensity (ρ) increases, the mean and variance increases rapidly. But for large values of ρ, the rate of increment in variance as compared to mean is larger, as such the expected value of N does not appear to have much relevance for large values of ρ. Although these quantities can be made arbitrarily large for sufficiently large ρ (<1), the system may therefore take long time to reach steady state equilibrium. It is observed that skewness, and kurtosis of the distribution of r.v. N decreases with increasing values of traffic intensity.

Table II: Mean, Variance, Skewness, and Kurtosis of r.v. N

ρ	E(N)	V(N)	γ_1	γ_2
0.1	0.11111	0.123	3.47851	14.1
0.2	0.25	0.313	2.68328	9.2
0.3	0.42857	0.612	2.37346	7.633333
0.4	0.66667	1.11	2.21359	6.9
0.5	1	2	2.12132	6.5
0.6	1.5	3.75	2.06559	6.266667
0.65	1.85714	5.31	2.04657	6.188462
0.7	2.33333	7.78	2.03189	6.128571
0.75	3	12	2.02073	6.083333
0.8	4	20	2.01246	6.05
0.85	5.66667	37.8	2.00661	6.026471
0.9	9	90	2.00278	6.011111
0.91	10.1111	112	2.00222	6.008901
0.92	11.5	144	2.00174	6.006957
0.93	13.2857	190	2.00132	6.005269
0.94	15.6667	261	2.00096	6.00383
0.95	19	380	2.00066	6.002632
0.96	24	600	2.00042	6.001667
0.97	32.3333	1080	2.00023	6.000928
0.98	49	2450	2.0001	6.000408
0.99	99	9900	2.00003	6.000101
0.991	110.111	12200	2.00002	6.000082
0.992	124	15500	2.00002	6.000065
0.993	141.857	20300	2.00001	6.000049
0.994	165.667	27600	2.00001	6.000036

0.995	199	39800	2.00001	6.000025
0.996	249	62300	2	6.000016
0.997	332.333	111000	2	6.000009
0.998	499	250000	2	6.000004
0.999	999	999000	2	6.000001

5. Construction of control charts for random variable N

Knowing the characteristics of the probability distribution of r.v. N control charts are constructed for N using different methods. These charts will be referred as C-charts. As the probability function of a geometric distribution is a monotonously decreasing function in n and since control limits for queueing system are to be computed, it would be meaningless to define lower control limit (LCL) for this distribution, however, obtaining upper control limit i.e. UCL would obviously make sense. Since, it will help in monitoring improbable high values of N (the monitoring statistic) and further help in controlling abandonment due to impatience. Hence, in all control charts defined for N, only UCL, α_u and ARL would be computed. The criteria used for out of control condition will be whenever a point falls outside the UCL, the system is said to be out of control. In other words, customers are running out of patience and might abandon the system. And hence investigation and corrective action are required to be taken to find and eliminate the causes responsible for this behaviour [6].

5.1. Control chart C_1: Shewhart method

The conventional Shewhart type control charts are used to determine whether a process is in a state of statistical control to bring out-of-control process into in-control and to monitor the process to confirm that it stays in control [5]. The limits are constructed by assuming that the underlying distribution is normal or approximately normal. Here, control chart can be used for on-line system surveillance and to improve the system.

Control limits for $\mathbf{C_1}$-chart for r.v. N when parameters are known [6]:

$$\left.\begin{aligned} UCL &= E(N) + L * \sqrt{V(N)} \\ CL &= E(N) \\ LCL &= E(N) - L * \sqrt{V(N)} \end{aligned}\right\} \qquad (2)$$

where E(N) and V(N) are displayed in Table I and L(=3) is the “distance” of the upper control limit from the centre line, expressed in standard deviation units.

Control limits are used in determination of a performance measure of queueing system, hence a need for better performing charts.

5.2. Control chart C_2 : Skewness correction (SC) method

Skewness correction method corrects the Shewhart chart according to the degree of skewness of the distribution of r.v. N. It provides asymmetric control limits using ±3 standard deviations plus the same known function of the degree of skewness estimated from subgroups, and with no assumptions on the distribution.

The skewness correction C_2 chart is based on the following control limits, when parameters are known [2]:

$$\left.\begin{aligned} UCL &= E(N) + (3 + c_4^*)\sqrt{V(N)} \\ CL &= E(N) \\ LCL &= E(N) + (-3 + c_4^*)\sqrt{V(N)} \end{aligned}\right\} \qquad (3)$$

where c_4^* denote skewness correction and is given by $c_4^* = \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2}$ and E(N) , V(N) and $\gamma_1(N)$ are displayed in Table I. When $\gamma_1 = 0$, the SC chart reduces to the Shewhart chart. However, if the distribution is positively skewed, then the distance of the UCL from the CL is larger than that of the LCL from the CL. If the distribution is negatively skewed, then the distances are reversed, but the bandwidth of the control chart is always six.

5.3. Control chart C_3: Haim Shore method

Sometimes the skewness is too large to be ignored and may be crucial to the proper functioning. Ignoring skewness affects the performance of the control chart and results in raised Type I risk. These findings have instigated in the development of various control charts that takes into consideration the non-normality of the monitoring statistic. Control charts for attributes data were developed, where the monitoring statistic may have a skewed distribution and implemented it for the study of queue length of M/M/s system[7]. Unlike the traditional chart which uses only standard normal quantiles, the control chart for attributes may use logistic quantiles. Since this may help for monitoring statistic with exceptionally high skewness and high kurtosis.

The control limits for N, obtained by Haim Shore method which involves the first three moments of the r.v. N are:

$For\ \gamma_1(N) \geq 0.5$

$$UCL = E(N) + 3.642\sqrt{V(N)} + 0.9146\sqrt{V(N)}\,\gamma_1 - \frac{1}{2}$$

$$= \frac{\rho}{1-\rho} + 3.642\frac{\sqrt{\rho}}{1-\rho} + 0.9146\frac{1+\rho}{1-\rho} - \frac{1}{2} \quad (4)$$

$$CL = E(N) = \frac{\rho}{1-\rho} \quad (5)$$

$$LCL = E(N) - 3.642\sqrt{V(N)} + (1.40)(0.9146)\sqrt{V(N)}\,\gamma_1 + \frac{1}{2}$$

$$= \frac{\rho}{1-\rho} - 3.642\frac{\sqrt{\rho}}{1-\rho} + (1.40)(0.9146)\frac{1+\rho}{1-\rho} + \frac{1}{2} \quad (6)$$

where E(N),V(N) and γ_1 are displayed in Table I. It is observed that skewness depends only on the traffic intensity and from Table II the skewness of the distribution of N decreases as ρ increases. Also, the minimum value of skewness is nearly 2 which are observed for high values of ρ. Therefore control limits given by expressions (4), (5) and (6) are used, as the distribution of N is highly skewed and for high values of skewness the probability limits, based on normal quantiles, may not be appropriate due to the long tails associated with highly skewed distributions or distributions that do not converge to the normal. Hence, control limits based on the logistic quantile are used which are proved to be more efficient, as it has longer tails relative to normal variable.

5.4. Control chart C_4 : Kurtosis correction (KC) method

Tadikamalla and Popescu [8] developed a KC method to construct control chart when the process distribution is symmetrical, but is leptokurtic, it has the advantage of providing better control limits and are easier to use and make no assumption on the functional form of underlying distribution. This method shifts the control limits to both sides by the same amount which is function of kurtosis, which is estimated by sample kurtosis. It is observed that the use of the α-quantile of the normal distribution is not appropriate, if the process distribution is skewed.

The control limits based on KC method for C_4 control chart if the process distribution and parameters are known for r.v. N [8],

$$\left.\begin{aligned} UCL &= E(N) + \left(3 + \frac{\gamma_2(N)}{1+0.33\gamma_2}\right) * \sqrt{V(N)} \\ CL &= E(N) \\ LCL &= E(N) - \left(3 + \frac{\gamma_2(N)}{1+0.33\gamma_2(N)}\right) * \sqrt{V(N)} \end{aligned}\right\}$$

(7)

where E(N),V(N) and γ_2 are displayed in Table I. When kurtosis of underlying distribution is zero, the control chart reduces to Shewhart chart.

5.5. Control chart C_5 : Skewness and kurtosis correction (SKC) method

Wang (2009)[9] developed a SKC method to construct control chart with no assumptions on the process distribution, both the degree of skewness and kurtosis of the process distribution is taken into account. It provides the control limits using 3 standard deviation with addition of the known function of skewness and kurtosis. If kurtosis of the process distribution is zero then SKC method reduces to SC method and to Shewhart if both skewness and kurtosis of the distribution are found to be zero.

The control limits based on SKC method if the process distribution and parameters are known for r.v. N ,

$$\left.\begin{aligned} UCL &= E(N) + \left(3 + \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2} + \frac{\frac{5}{4}\gamma_2}{1+3|\gamma_2|}\right) * \sqrt{V(N)} \\ CL &= E(N) \\ LCL &= E(N) - \left(3 + \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2} + \frac{\frac{5}{4}\gamma_2}{1+3|\gamma_2|}\right) * \sqrt{V(N)} \end{aligned}\right\} \quad (8)$$

where E(N),V(N), γ_1 and γ_2 are as displayed in Table I.

6. Performance measures

In this section performance measures of control chart are studied.

6.1. False alarm rate for control chart

Let α_u denote the probability of type I error generated in the upper tail of the control chart. The Type-I error rate is defined as the probability of signalling an out of control even though the process is actually in-control. It is defined as, $\alpha_u = P[N > UCL] = \rho^{UCL}$ where UCL is as obtained for control charts based on different methods. This implies that the probability of exceeding some limit on the number of customers in the system is a geometrically decreasing function of that number and decays very rapidly.

6.2. Average run length

Let ARL_{Ci}, i=1,…,5 denote the average run length of control chart C_i. ARL is the average number of points that must be plotted before a point indicates an out of control condition. If the system observations are uncorrelated, then for any control chart; the average run length is given by [6],

$$ARL_{C_i} = \frac{1}{\alpha_u}$$, i=1,…,5 where α_u is the false alarm rate . (9)

7. Performance comparison of different control charts

The control limits of control chart based on different methods depend on traffic intensity ρ. To study the effect of ρ on control limits and performance measures, same set of increasing values of ρ are selected and CL, UCL,FAR and ARL were computed and comparisons were carried out.

In Tables III and IV, FAR and corresponding ARL obtained by using different control charts are displayed respectively. From these tables, it is observed that as ρ increases FAR (α_u) decreases consequently ARL increases for the control charts based on different methods.

i. When ρ=0.1 and 0.2 , $\gamma_1 = 3.48$ and 2.68 and $\gamma_2 = 14.1$ and 9.2 respectively, it is observed that,

$\alpha_{C_4} < \alpha_{C_3} < \alpha_{C_5} < \alpha_{C_2} < \alpha_{C_1}$.

Consequently,

$ARL_{C_4} > ARL_3 > ARL_{C_5} > ARL_{C_2} > ARL_{C_1}$.

As UCL of control chart based on KC method is found to be highest of all methods, so corresponding FAR is lowest and consequently ARL is highest. Whereas, UCL obtained by Shewhart method is lowest, so FAR is found to be highest and resultantly ARL is lowest of all the methods. The argument is as the underlying distribution is highly skewed and leptokurtic, the FAR is large as skewness is high and also because of the discrepancy between the variability pattern of the asymmetric distribution and the normality assumed in constructing the control chart.

ii. For $\rho \geq 0.3, \gamma_1 = 2.37$ to 2 and $\gamma_2 = 7.63$ to 6,it is observed that,

$\alpha_{C_3} < \alpha_{C_4} < \alpha_{C_5} < \alpha_{C_2} < \alpha_{C_1}$.

Consequently,

$ARL_{C_3} > ARL_4 > ARL_{C_5} > ARL_{C_2} > ARL_{C_1}$.

FAR obtained from control chart based on Haim Shore method is lowest consequently the ARL is highest of all methods and lowest for Shewhart method. Hence control chart based on Haim Shore method is best of all, second best is control chart based on KC method and third best under this situation is control chart based on SKC method.

iii. From Table II, it is observed that for skewness around 2, traffic intensity is nearer to 1.From figure 2 it is observed that as skewness decreases FAR decreases whereas ARL increases(observe figure 4) .On the other hand from figure 1 as traffic intensity(ρ) increases FAR decreases whereas ARL increases(observe figure 3). On the basis of FAR, control charts C_3, C_4 and C_5 show comparable performance.

Table III: Comparison of False Alarm Rate of control charts based on various methods

ρ	γ_1	$(\alpha_u)_{C_1}$	$(\alpha_u)_{C_2}$	$(\alpha_u)_{C_3}$	$(\alpha_u)_{C_4}$	$(\alpha_u)_{C_5}$
0.1	3.47851	0.06836	0.02282	0.009803	0.00909	0.018728
0.2	2.68328	0.04498	0.01203	0.006205	0.00579	0.00968
0.3	2.37346	0.03536	0.0087	0.004562	0.00458	0.006945
0.4	2.21359	0.02994	0.0071	0.003604	0.00392	0.005636
0.5	2.12132	0.02641	0.00614	0.002972	0.00348	0.004862
0.6	2.06559	0.0239	0.0055	0.002523	0.00317	0.004345
0.65	2.04657	0.02289	0.00525	0.002344	0.00304	0.004145
0.7	2.03189	0.02201	0.00503	0.002187	0.00293	0.003973
0.75	2.02073	0.02122	0.00484	0.002049	0.00283	0.003822
0.8	2.01246	0.02052	0.00467	0.001926	0.00274	0.00369
0.85	2.00661	0.01989	0.00453	0.001816	0.00265	0.003572
0.9	2.00278	0.01932	0.00439	0.001718	0.00258	0.003466
0.91	2.00222	0.01921	0.00437	0.001699	0.00256	0.003447
0.92	2.00174	0.0191	0.00434	0.001681	0.00255	0.003427

0.93	2.00132	0.019	0.00432	0.001663	0.00254	0.003408
0.94	2.00096	0.01889	0.0043	0.001646	0.00252	0.00339
0.95	2.00066	0.01879	0.00427	0.001628	0.00251	0.003371
0.96	2.00042	0.0187	0.00425	0.001612	0.0025	0.003353
0.97	2.00023	0.0186	0.00423	0.001595	0.00248	0.003336
0.98	2.0001	0.0185	0.00421	0.001579	0.00247	0.003319
0.99	2.00003	0.01841	0.00418	0.001563	0.00246	0.003302
0.991	2.00002	0.0184	0.00418	0.001561	0.00246	0.0033
0.992	2.00002	0.01839	0.00418	0.00156	0.00246	0.003298
0.993	2.00001	0.01838	0.00418	0.001558	0.00245	0.003297
0.994	2.00001	0.01837	0.00418	0.001557	0.00245	0.003295
0.995	2.00001	0.01836	0.00417	0.001555	0.00245	0.003293
0.996	2	0.01835	0.00417	0.001554	0.00245	0.003292
0.997	2	0.01834	0.00417	0.001552	0.00245	0.00329
0.998	2	0.01833	0.00417	0.00155	0.00245	0.003289
0.999	2	0.01833	0.00417	0.001549	0.00245	0.003287

Table IV: Comparison of Average Run Length (ARL) of control charts based on various methods

ρ	ARL_{c_1}	ARL_{c_2}	ARL_{c_3}	ARL_{c_4}	ARL_{c_5}
0.1	14.6286	43.8232	102.006	110.051	53.3969
0.2	22.2306	83.1528	161.152	172.8334	103.311
0.3	28.2805	114.895	219.201	218.2565	143.981
0.4	33.3957	140.916	277.49	255.2174	177.417
0.5	37.861	162.907	336.477	287.1177	205.673
0.6	41.8406	181.975	396.363	315.5815	230.139
0.65	43.6828	190.646	426.676	328.8233	241.246
0.7	45.4403	198.833	457.246	341.5136	251.72
0.75	47.1213	206.588	488.078	353.714	261.628
0.8	48.7331	213.955	519.175	365.4752	271.027
0.85	50.2817	220.97	550.541	376.8394	279.965
0.9	51.7724	227.666	582.176	387.8421	288.483
0.91	52.064	228.97	588.535	390.0021	290.139
0.92	52.3535	230.261	594.906	392.149	291.781
0.93	52.641	231.542	601.287	394.2831	293.407
0.94	52.9264	232.811	607.679	396.4046	295.019
0.95	53.2099	234.069	614.082	398.5138	296.616
0.96	53.4913	235.317	620.495	400.6107	298.199
0.97	53.7709	236.554	626.92	402.6957	299.768
0.98	54.0485	237.78	633.355	404.7689	301.324
0.99	54.3242	238.996	639.802	406.8304	302.866
0.991	54.3517	239.117	640.447	407.0359	303.019

0.992	54.3792	239.238	641.092	407.2413	303.172
0.993	54.4066	239.359	641.738	407.4466	303.326
0.994	54.434	239.48	642.383	407.6518	303.479
0.995	54.4614	239.601	643.029	407.8568	303.632
0.996	54.4888	239.721	643.675	408.0618	303.784
0.997	54.5162	239.842	644.321	408.2666	303.937
0.998	54.5435	239.962	644.967	408.4713	304.09
0.999	54.5708	240.082	645.613	408.676	304.242

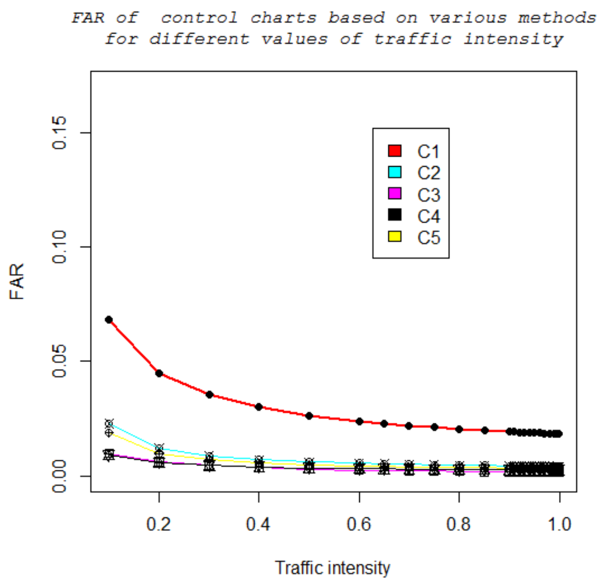

Figure 1

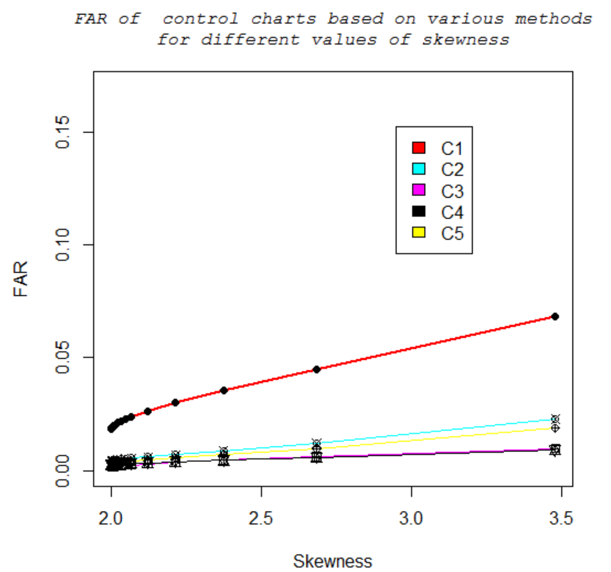

Figure 2

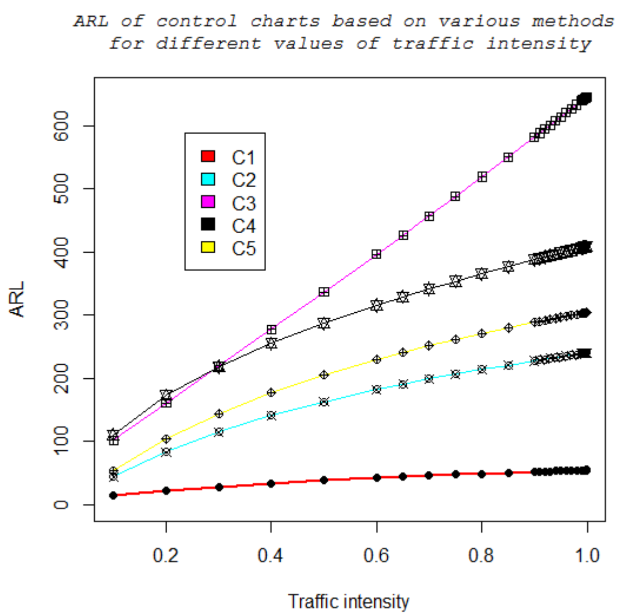

Figure 3

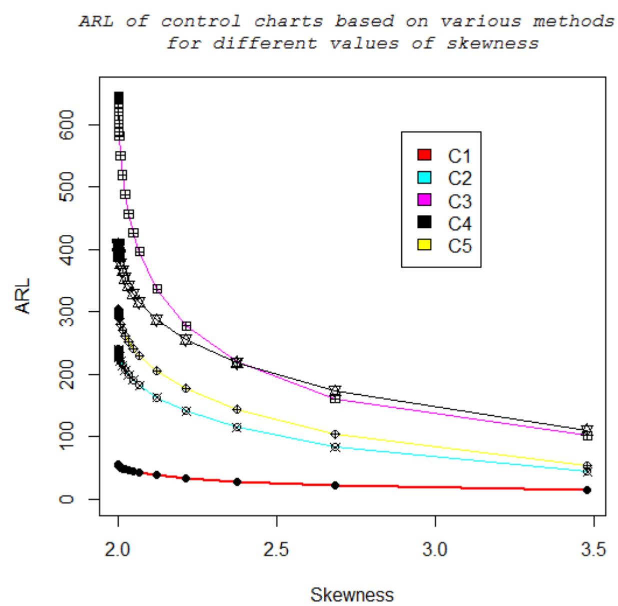

Figure 4

8. Conclusion

The paper proposes performance wise for ρ=0.1 and 0.2 use of control chart C4 based on KC method, since it outperforms rest methods. Further for ρ>0.3 the use of control chart C3 based on Haim Shore method for obtaining UCL for random queue length (N). Since C3 out performs charts based on all other methods as it considers in its construction of control limits skewness of underlying distribution of r.v. N. This control chart can be suggested for intimating system management for taking precautionary measure. Hence control chart C3 is recommended for skewed population and KC method C4 as next best of all charts. For a process in control, the ARL is preferred to be large because an observation plotting outside the control limits represents a false alarm. R software was used in computation.

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