Analysis of Curved Plate Elements using Open Source Software

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Abstract- Analysis of curved plate elements requires a high computational effort to obtain a reliable solution for a buckling load for design purposes. Available programs are expensive to acquire and they need thorough knowledge for effective use. There is therefore need to code cheaper and accessible programs in line with using sustainable methods to better the livelihood of mankind. To address this issue a theory is formulated based on the Euler-Bernoulli beam model. This model is applicable to thin elements which include plate and membrane elements.

This paper illustrates a finite element theory to calculate the master stiffness of a curved plate. The master stiffness takes into account the stiffness, the geometry and the loading of the element. The determinant of this matrix is established from which the buckling load which is unknown in the matrix is evaluated by the principal of bifurcation.

The curved element is divided into 2,3,6,9 and 12 elements; this demonstrates the computational effort to a reliable solution. As expected, that as you divide the curve into smaller constituent elements, the solution of the buckling load is tedious as more mathematical operations are involved hence the need to program the operations.

Numerical analysis is carried out by abstracting the procedural development of the theory and programming it to run in a visual basic platform. The results obtained are giving a good agreement with results obtained with classical plate equations. This program is proposed to increase computational efficiency in the analysis of curved plates at a sustainable cost. It can also be used to establish the relationship between buckling load and curvature of plates.

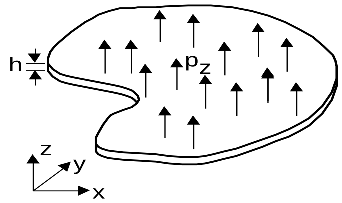
Index Terms- finite element method, analysis, curved plates, program

I. INTRODUCTION

A plate is a planar body whose thickness is small compared with its other dimensions. Curved plate structures are frequently used in; aerospace vehicles, domes, roof structures and pressure vessels. A plate structure may be as simple as the web of a stiffener or as complex as an integrally stiffened plate supported by heavy frames and rings.

Thin plates are characterized by a structure that is bounded by upper and lower surface planes separated by a distance h (figure 1). The x-y coordinate axes are located on the neutral plane of the plate and the z-axis is normal to the x-y plane.

For this paper it was assumed that h is a constant and those material properties are homogeneous through the thickness. Consequently, the location of the x-y axes(figure 2) lie at the mid-surface plane (z=0) with the upper and lower surfaces corresponding to z=h/2 and z=-h/2, respectively.



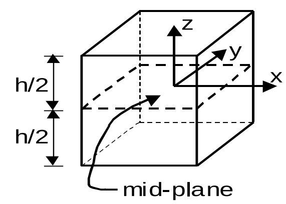


Figure 1 Structure of thin plate **Figure 2 Location of thin plate axes**

In the behavior of plate structures under in plane compression, a critical point exists where an infinitesimal increase in load can cause the plate surface to buckle; the load at this critical point defines the buckling strength of the plate.

Any further increase in load beyond the load at the initiation of buckling increases the buckling deformations until collapse occurs. Thus, the load at collapse defines the post buckling or crippling strength of the plate. The behavior of plate structures in this regard differs markedly from the behavior of columns and many other thin curved shell structures for which the buckling load corresponds closely to the collapse load.

Buckling of a plate structure can cause an unacceptable degradation. It can trigger general buckling of larger structures because of a redistribution of stresses; it can also affect the response by the structure to excessive displacement or fatigue which may be a cause of failure.

A lot of research has been done in this area of research which has been referenced.Available programs give criteria to do analysis of curved elements,but they are expensive to acquire and use. So, there is need to have an accessible criterion at an affordable cost.

1.1 Objectives

1) To develop an analytical program of curved plates.
2) To develop a numerical method for analysis of curved plates
3) To write a finite element analysis program to analyze curved plate elements using the method developed.

1.2 Scope for the work

An analytical formulation of the curved-plate non-linear equilibrium equations will be made. The analytical formulation will be implemented into a computer based program.

A convergence study using a segmented-plate approach will be performed for a simple example problem to obtain baseline results for use in future comparisons. Results will be compared with results from classical plate equations.

1.3 Methods of analysis

Finite element methods are now widely used to solve structural, fluid, and multi-physics problems numerically[1].The Euler –Bernoulli beam model applies since only thin elements are considered(shear deformations are neglected)[2].

Two methods of analysis of curved elements exist: the Eigenvalue Buckling Analysis and Nonlinear buckling Analysis.

1.3.1 The Eigenvalue Buckling Analysis

The Eigenvalue analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This is analogous to the classical plate equation approach to elastic buckling analysis[3].However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength.

1.3.2 Nonlinear Buckling Analysis

This method takes account of imperfections and nonlinearities of real-world structures. In this method the load is increased until the solution fails to converge, indicating that the structure cannot support the applied load (or that numerical difficulties prevent solution)[4]. If the structure does not lose its ability to support additional load when it buckles, a nonlinear analysis can be used to track post-buckling behavior.

II. Basic Element Shapes

For the discretization of problems involving curved geometries, finite elements with curved sides are useful. The ability to model curved boundaries has been made possible by the addition of midsized nodes. Finite elements with straight sides are known as linear elements, whereas those with curved sides are called higher order elements[5].

2.1Size of Elements

The size of elements influences the convergence of the solution directly. If the size of the elements is small, the final solution is expected to be more accurate.

2.2 Number of Elements

The number of elements is related to the accuracy required and the number of degrees of freedom involved [5]. Although an increase in the number of elements generally means more accurate results, for any given problem, there will be a certain number of elements beyond which the accuracy cannot be improved by any significant amount shown graphically in figure 3[5].

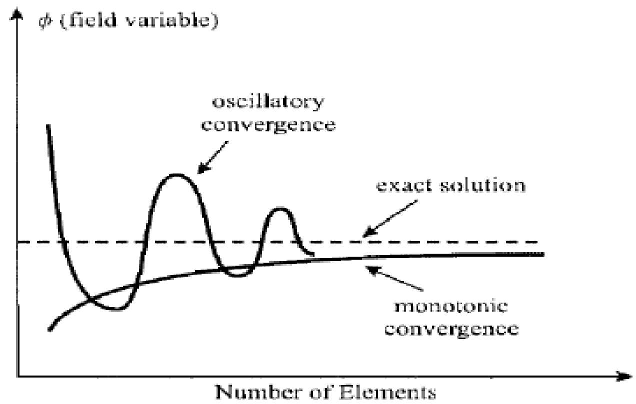


Fig 3 Relationship between the number of elements and accuracy[5]

III. Program Development

Computer programming languages are built around two approaches;

(1) Procedural programming and

(2) Object oriented programming.

In procedural programming, the program is prepared by a series of steps or routines that follow the data provided. The main drawback of the procedural programming languages is that they are not structured and the flow of the program largely depends on conditional statements that induce more chances of errors. These languages are good for small programs and are difficult to maintain when they become larger.

The object oriented programming languages are built on the concept of abstraction. Large complex procedures are subdivided into small procedures by abstraction, encapsulation and inheritance. Each of these sub procedures represents different objects with their own separate identity[6,7]. The program developed is object oriented and follows the following steps used in the formulation of the theory;

1. Identify the principal theory

$\{Q^1\} = [[K] + [G] + [L]] \{q^1\} = \{0\}$[8]

2. Divide the curved plate into appropriate elements and calculate the arc length (L) and the internal angle (ψ) and angle (α) of each element relative to the x axis.

It is noted that the unknown vector does not involve the rotation angle; the essential boundary condition can be imposed with the penalty function method [9,10].

3. Enter the following specific member properties (variables).

a) Area in m^2

b) Moment of inertia I in m^4

c) Length L in m

d) Young's modulus E in N/m^2

e) (ψ) and (α) in degrees for each element

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4. For each element calculate:

a) The stiffness matrix K from the matrix below

$$
[K] = \begin{array}{cccccccc}
 & u_1 & u_2 & w_1 & w_2 & \theta_1 & \theta_2 & \\
\left[\begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\right. & \frac{AE}{L} & -\frac{AE}{L} & 0 & 0 & 0 & 0 & Fx^1 \\
 & -\frac{AE}{L} & \frac{AE}{L} & 0 & 0 & 0 & 0 & Fx^2 \\
 & 0 & 0 & \frac{12EI}{L3} & -\frac{12EI}{L3} & -\frac{6EI}{L2} & -\frac{6EI}{L2} & Fz^1 \\
 & 0 & 0 & -\frac{12EI}{-L3} & \frac{12EI}{L3} & \frac{6E}{L2} & \frac{6EI}{L2} & Fz^2 \\
 & 0 & 0 & -\frac{6EI}{L2} & \frac{6EI}{L2} & \frac{4EI}{L} & \frac{2EI}{L} & M_1 \\
 & 0 & 0 & -\frac{6EI}{L2} & \frac{6EI}{L2} & \frac{2EI}{L} & \frac{4EI}{L} & M_2
\end{array}
$$

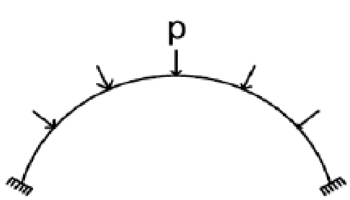
a) The geometric matrix from the matrix below

$$
[G] = \begin{array}{cc}
u_1^1 \quad\quad u_2^1 & w_1^1 \quad w_2^1 \quad \theta_1^1 \quad \theta_2^1 \\
\left(\begin{array}{c|c}
3EA[u^0]\left[\gamma_k \gamma_j \gamma_1\right] & EA[w^0]\left[\phi_k \phi_j \gamma_1\right] \\
3EA[u^0]\left[\gamma_k \gamma_j \gamma_2\right] & EA[w^0]\left[\phi_k \phi_j \gamma_2\right] \\
EA[w^0]\left[\phi_1 \phi_k \gamma_j\right] & EA[u^0]\left[\phi_1 \phi_j \gamma_k\right] \\
EA[w^0]\left[\phi_2 \phi_k \gamma_j\right] & EA[u^0]\left[\phi_2 \phi_j \gamma_k\right] \\
EA[w^0]\left[\phi_3 \phi_k \gamma_j\right] & EA[u^0]\left[\phi_3 \phi_j \gamma_k\right] \\
EA[w^0]\left[\phi_3 \phi_k \gamma_j\right] & EA[u^0]\left[\phi_2 \phi_j \gamma_k\right]
\end{array}\right)
\end{array}
$$

b) Calculate the load matrix L selecting a specific load case

$[L] = [L_P] + [L_F]$

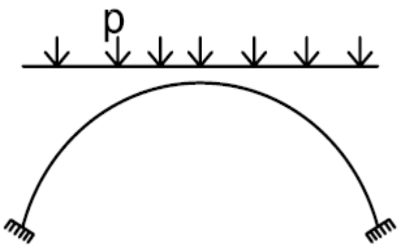
Load case i-Loads remain normal to the element



$$
[L_P] = P \begin{pmatrix}
0 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{L}{12} & \frac{L}{12} \\
0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{L}{12} & -\frac{L}{12} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

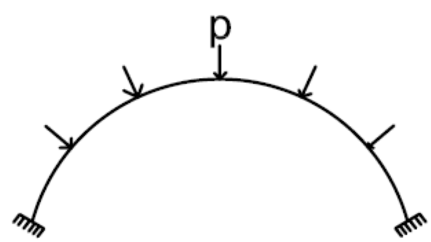
$$
[L_F] = P \begin{pmatrix}
0 & 0 & 0 & 0 & -F_{Z1} & 0 \\
0 & 0 & 0 & 0 & 0 & -F_{Z2} \\
0 & 0 & 0 & 0 & -F_{X1} & 0 \\
0 & 0 & 0 & 0 & 0 & -F_{X2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Load case ii-Loads remain parallel to the original direction



$$L_P = L_F = [0]$$

Load case iii-Loads remain directed towards a fixed point



$$
[L_P] = P \begin{pmatrix}
-\frac{L}{3R} & -\frac{L}{6R} & 0 & 0 & 0 & 0 \\
-\frac{L}{6R} & -\frac{L}{3R} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

a) Rearrange all the matrices in the following order

$\underline{K}$, $\underline{G}$, and $\underline{L}_F$

$$[\underline{K}^1] = \begin{array}{c} \begin{array}{cccccc} U_1 & w_1 & \theta_1 & u_2 & w_2 & \theta_1 \end{array} \\ \left(\begin{array}{c|c} K^1_{11} & K^1_{12} \\ \hline K^1_{21} & K^1_{22} \end{array}\right) \end{array} \begin{array}{c} F_{X1} \\ F_{Z1} \\ M_1 \\ F_{X2} \\ F_{Z2} \\ M_2 \end{array}$$

5. Matrix transformation

Rotate all element matrices to the global coordinate system by the operation

$K^g = T^T K^{(e)} T$

Where

K^g = Global stiffness matrix

T^T = transpose of T matrix

$K^{(e)}$ = Local stiffness matrix

T= ɤφ where

$$\text{ɤ} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\varphi = \begin{pmatrix} \cos\psi & \sin\psi & 0 & 0 & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\psi & \sin\psi & 0 \\ 0 & 0 & 0 & -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Calculate the master stiffness from the summation of all the matrices of the system

Eg $K_1+K_2+K_3=k$, $G_1+G_2+G_3=G$, $L1+L_2+L_3 = L$ For three elements

6. Calculate the determinant of the resulting 6x6 matrix from the sum below

$|\bar{K} + p\,\bar{K}_0| = 0$ (Bifurcation theory)

Where

$\rho[\bar{K}_0] = [\bar{G}] + [\bar{L}_\rho] + [\bar{L}_F],\ [\bar{L}_F] = 0$

From the determinant the solution for p is the value for the buckling load.

IV. Sample Problem

The figure 4 is composed of a thin membrane forming a circular arch with uniform pressure. Determine the buckling pressure for three possible load cases.

Input data.

1) Area (A)= 4.05 x 10^{-4} m^2
2) Moment of inertia (I) = 1.31×10^{-6} m^4
3) Young's Modulus (E) =6.9×10^{10} N/m^2
4) Radius (R) =2.54m

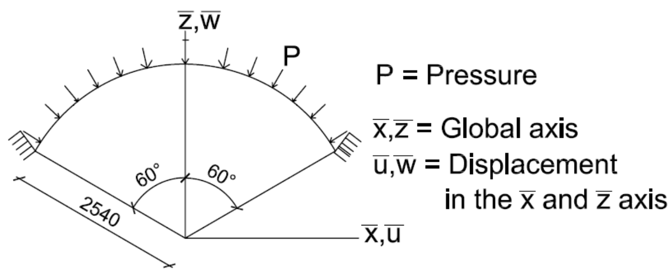


Figure 4 Uniform circular arch under uniform pressure p

4.1 DISCUSSION OF RESULTS

i) Summary of Results

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Table 1 Relationship of number of elements and resistance to buckling pressure

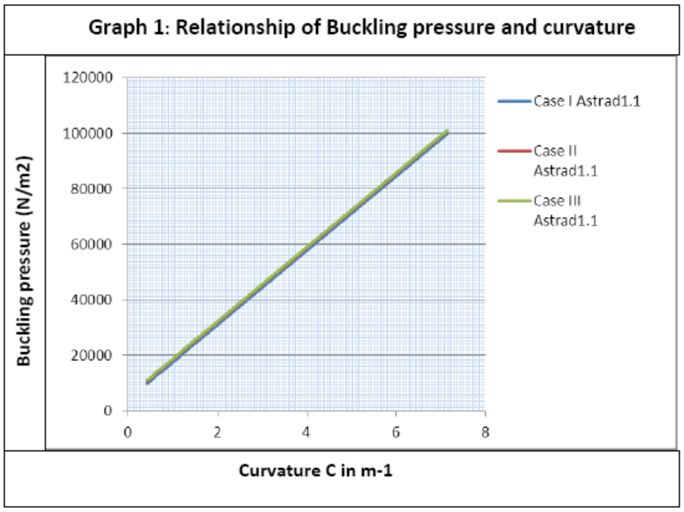
Number of elements	Buckling pressure (N/m^2)		
	Case I	Case II	Case III
	Astrad1.1	Astrad1.1	Astrad1.1
2	16580.425	16580.84	16580.85
3	9952.05	11299.77	11859.48
6	10012.20	10806.076	11964.198
9	9962.183	10760.107	11300.183
12	9962.706	10801.956	11089.702
Exact solution	9959.463	10673.981	11113.549

ii) Effect of varying plate properties

a) CurvatureTable 2

b) :Relationship of Buckling pressure and Curvature

Curvature C in m^{-1}	Buckling pressure (N/m^2)		
	Case I	Case II	Case III
	Astrad1.1	Astrad1.1	Astrad1.1
0.4082	9962.706	10801.96	11089.7
0.4274	10108.31	10941.42	11233.42
0.5155	11284.06	12117.15	12409.16
0.6494	13070.34	13903.44	14195.44
0.7463	14362.83	15195.93	15487.93
1.0638	18600.95	19434.05	19726.05
1.3514	22436.5	23269.6	23561.7
1.8519	29114.52	29947.62	30239.62
2.9412	43649.38	44482.48	44774.48
7.1429	99716.68	100549.8	100841.8



c) Cross sectional Area

i) Table 3 : Area = 3.05 x 10^{-4}m^{2}

Number of elements	Buckling pressure (N/m^2) Case I Astrad1.1	Case II Astrad1.1	Case III Astrad1.1
2	15035.443	15035.886	15035.886
3	8348.574	9695.646	10255.803
6	8408.561	9202.913	10360.651
9	8358.275	9157.455	9696.275
12	8362.302	9195.403	9487.40

ii) Table 4:Area = 2.05 x 10^{-4}m^{2}

Number of elements	Buckling pressure (N/m^2) Case I Astrad1.1	Case II Astrad1.1	Case III Astrad1.1
2	13738.393	13738.836	13738.836
3	7051.524	8398.596	8958.953
6	7111.511	7905.596	9063.511
9	7061.225	7860.405	8399.225
12	7065.252	7898.353	8190.353

4.2 DISCUSSION OF THE RESULTS

i) Nature of results

As the number of elements increases, the results have an oscillatory convergence to the exact solution.

More divisions results into output of higher accuracy but requires more computational effort as there are more calculations. The program will increase the efficiency of the final result as the computations have been programmed.

ii) Loading cases

The results show that a curved plate resists a higher load when it is directed towards the center of the arc. The loading cases vary as to the use of the plate element e.g. as a water structure or roof element. The program can be useful in a quick analysis considering the particular load case, given the load cases programmed are those frequently encountered.

ii) Relationship between load and curvature.

From the results, load resistance of a curved plate is directly proportional to curvature.

4.3 CONCLUSION AND RECOMMENDATIONS

The research had 3 specific objectives which have been achieved

a) Analytical program

An analysis criterion based on Euler Bernoulli theory was developed. This is applicable only to thin elements which includes thin plates and membranes.

b) Numerical method

The six step procedure of analysis to arrive at a buckling load forms a summary of the numerical method.

c) Finite element program

The six steps above were programmed to run on a visual basic platform. This program is referred as Astrad 1.1.This code was named with a future intention of redeveloping it to include analysis of thicker elements. The program is less costly and requires less effort to use.

In order to access the efficiency and accuracy of the program, an example is analyzed whose results is tabulated in tables 1,2,3 and 4.The analysis shows a good agreement with classical plate equations. The program

It can be seen that a curved element resists more loads directed towards its center than other loading cases. Curvature and plate thickness proportionately influence plate resistance to load. Use of the program is useful for the study of curved plate elements because it manipulates the given plate and loading parameters to give output.

The program should be developed further to carter for more attributes like modification of thin plates into a composite element and with stiffeners.

Further research into the inclusion of the Timoshenko theory into the program to carter for thicker elements should also be done.

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