# Of Relative Motion And Absolute Motion: Fallaces Sunt Rerum Species

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**Abstract**: In the normal format of circumstances, circumscribed delusional beliefs can follow brain injury. Authors suggest that these involve anomalous perceptual experiences **created by a** deficit to the person's perceptual system, and misinterpretation of these experiences due to biased reasoning. We say notwithstanding the brain injury, indoctrination of beliefs is one that is responsible for disastrous consequences, detrimental ramifications and deleterious implications leading to illusion, as is the case with "Time Delusion" All space and time is within us. Creation essentially is subjective with actor and observer and script writer being the same.Authors use the Capgras delusion (the claim that one or more of one's close relatives has been replaced by an exact replica or impostor) to illustrate this argument. Our account maintains that people voicing this delusion suffer an impairment that leads to faces being perceived as drained of their normal affective significance, and an additional reasoning bias that leads them to put greater weight on forming beliefs that are observationally adequate rather than beliefs that are a conservative extension of their existing stock. We show how this position can integrate issues involved in the philosophy and psychology of belief, and examine the scope for mutually beneficial interaction.*(Delusions and Brain Injury: The Philosophy and Psychology of Belief TONY STONE1,*, ANDREW W. YOUNG2 Article first published online: 4 MAY 2007 DOI: 10.1111/j.1468-0017.1997.tb00077.xMind & Language)* **.**To quoteMarga Reimer*(Only a Philosopher or a Madman: Impractical Delusions in Philosophy and Psychiatry Marga Reimer Philosophy, Psychiatry, & Psychology Volume 17, Number 4, December 2010 pp. 315-328 | 10.1353/ppp.2010.0028)*although it is natural to regard psychiatric delusions as beliefs, there seem to be significant differences between at least some such delusions and ordinary beliefs. These differences include the comparatively *weak influence of psychiatric delusions* on the subject's behavior, emotional life (affect), and "web of beliefs." They also include the notorious resistance of psychiatric delusions (versus ordinary beliefs) to counterevidence. Such differences have led some psychiatrists and philosophers to speculate that psychiatric delusions *may not be* genuine beliefs. Marga Reimertakes issue with such speculation, noting that the particular features of psychiatric delusions that motivate it, characterize the endorsement of philosophical doctrines whose epistemic status as beliefs is rarely questioned. Attention is drawn to a fallacy that has led some theorists to conclude, from the fact that psychiatric delusions are not paradigmatic beliefs, that they are (probably) not genuine beliefs. We call this fallacy the "fallacy of ignoring anomalies. In the following model we give the predicational anteriorities, character constitution, primordial exactitude, modern theoretical study analysis, ontological consonance, apocryphal aneurism, atrophied asseverations and beliefs of "modern man" besieged by tenets of "scientific" endeavour. That there is data that contradicts Einsteinian theories is simply overlooked blatantly and flagrantly, without giving prominence and precedence to the fact that such a contradiction should have. Perception is not reality. Despite knowing the predicament, studies have been made and claims and counter claims are made which is baffling and befuddling. VonNeumann reluctantly agreed that consciousness is to be added for perception to obtain reality. What is that factor is disputable. But one thing is certain, that we are getting it all wrong. The traditional psychiatric view of delusion has come under increasing attack over recent years. As Georgaca has noted, a number of problems with the assumptions in definitions of delusion can be identified when

they are viewed from a social constructionist perspective *(Georgaca 2000, 2004; Harper 1992, 1996; Heise 1988).*The distinction between endorsement and explanationist models is orthogonal to the distinction between one-factor and two-factor versions of empiricism *(see Davies et al. 2001(As quoted in Experience, Belief, and the Interpretive Fold Tim Bayne, Elisabeth Pacherie)*

## INTRODUCTION—VARIABLES USED

**Acknowledgements:** *Classification* of the protagonist or antagonist laws based on the characterstics and pen chance, predilection, proclivity and propensities of the systems under investigation: we acknowledge in unmistakable and unambiguous terms the help of Stanford encyclopedia, Kant's writings, and Deleuze's logic of sense, google search and Wikipedia. Concerted and orchestrated efforts are made to unify the various substantive statements about the necessity of perception, reality, consciousness, Totalism, stratification and other factors of cardinal importance. (Emphasis ours)

The capacity of active self-maintenance and self-generation that underlies regeneration, reproduction, and healing, are ways that organisms become distinctive wholes. The autopoetic system is not entirely autonomous, however. It must be structurally coupled to the World, to the Medium in Which IT exists. According to Katherine Hayles, the "Second Wave" of cybernetics Started with an article Entitled "What the Frog's Eye Tells the Frog's Brain," in Which the authors, including Humberto Maturana, showed that a frog's visual system does not so much represent reality as construct it. As the authors noted, "The frog's eye speaks to the brain in a language already highly organized and interpreted instead of transmitting some more or less accurate copy of the distribution of light upon the receptors." (How We Became Posthuman, p.131, p. 135) (See vision) *Humberto Maturana* developed the epistemological implications of this study by stressing the internal organization and circularity of cognition and of living systems in general. It is this recursivity which characterizes "second order" cybernetics, which entails the observation of observation (in a formal and not psychological frame.) Thus, second-order cybernetic systems are both closed and open. Autopoetic Systems are closed at the Level of Operation or Organization, but are at the Open Level of Structure. For Varela, "An autopoetic Machine Continuously Generates and specifies ITS Own Organization through ITS Operation as a system of Own Production of ITS Components." (Cf. Kant's distinctions between mechanism and organism in mechanism / vitalism) Note, however, that the theory of autopoesis defines the relations of the components without reference to the whole. Fylix Guattari argues for a machinistic autopoesis, "the non-discursive and self-enunciating nexus of the machine." (As a critique of any unifying idea of the signifier - esp. The Lacanian one).Evolutionary biologists describe *the Relationship Between Adaptation and ongoing* Changes in the Environment as the "Red Queen Hypothesis," named After the Queen in Through the Looking Glass Who found she HAD to keep running just to stay in place. According to Lewontin's constructionist view, the world is changing because the organisms are changing. The Red Queen's running only makes the problem worse. (P.58) For Lewontin, IT is a General Principle of Historical development of Biological Systems That the State of Possible Conditions Which make the Coming into Being of the system are abolished by IT. In 1892, Charles Darwin categorized human affects into seven or eight discrete expressions, each with its own facial display: happiness, sadness, fear, anger, disgust, surprise, interest, perhaps shame, and their combination. (The Expression of the Emotions in Man and Animals) According to Darwin, "The same state of mind is expressed throughout the world with remarkable uniformity." He postulated that these innate patterns of feeling and facial display evolved as social signals "understood" by all members to enhance species survival. (Cf ritualization) For Darwin, our expressions of emotion are universal (that is, innate not learned) and they are products of our evolution. They are Also, at Least to some extent, involuntary, and feigned emotions are Rarely Fully convincing. Neither Our expressions NOR Our emotions are Unique to Human Beings; Other Animals have some of the Same emotions, and some of the expressions Shown by Animals ressemble our own. For Darwin,

"He, who admits on general grounds that the structure and habits of all animals have been gradually evolved, will look at the whole subject of expression in a new and interesting light." *(Introduction to First Edition)*

**Maya - The Vedic Phantasmagoria**

From time immemorial certain metaphysical and other questions have bothered the human mind about this universe and the Nature. One of the questions is more like a riddle, “Is the world only an appearance or a true reality?" If it is a reality, is it an outcome of collation of gross atoms or both gross and subtle atoms? Many other questions also bother the human beings. Why did I come to this world and did I have a choice to refuse? Has the law of Karma relating to good and bad actions, thoughts and desires of the earlier births in the past made us to come or some cosmic power lured me to come and then left me alone to face the turbulent waves of the vast ocean of materialism? What made Shakespeare through one of his main characters Hamlet to say," To be or not be, that is the question?" Shakespeare again repeated in his "The Tempest" that the world is a dream. Why did Buddha find this world as full of miseries and sufferings? What made guru Nanak- the founder of Sikh religion largely based on Vedic Dharma to say," Nanak dukhiya sab sansar" (In this world everyone is suffering owing to wide spread miseries, unhappiness and destitution.) Why did Schopenhauer talk in whispers to plants, flowers and shrubs and then bent his ears to get reply to his metaphysical question from these plants? His questions to plants related to “who he was?” and other similar inquiries. After all why these search to know "thy self", not only by Schopenhauer but also earlier by Socrates and still earlier by the ancient Vedic seers, sages and metaphysicists?

Why there are so many theories about the world ranging from extreme materialism of dreaded Charvakas of the ancient India and idealism of Plato and Immanuel Kant apart from pure idealism and absolute monism of Adi Shankaracharya? If for Shankaracharya the ineffable and formless God is the only and absolute Reality, to Kapila Rsi of the Sankhya darshan - one of the six Schools of Indian philosophies, both God and Nature are real. In his ethical metaphysics, agnostic Buddha found only Nature is real. Holy Koran refers to this world as an illusion of comfort with alluring and deceiving appearance. Surah 57 Para 20 says that life of this world is naught but the stuff of illusion. The Presidency of Islamic Researchers, IFTA have translated the same Surah in section 3.para 20 as, “Know yea all that life of this world is but play and a pastime." To many saints of ancient India, Prakrti -the divine Nature is the adequate and efficient cause of the creation of this world along with its animate and inanimate life. However, Bhagavad Gita holds a different view that it is only the Supreme Lord who has assigned the task of creation of the Universe to Prakrti (the Nature) and she performs this divine role under His supervision (B.G.9-10).

In the field of physical sciences the matter of Newton, Tyndall and other scientists of classical mechanics was inert and indestructible with atom as the smallest unit. Now the scientists of particle physics have found some unsuspected vitality in the matter and its millions of atoms. Matter is no longer inert and experiences some kind of “fatigue" normally faced by human beings and animals as noticed in it by Sir Jagdish Chander Bose. There is no proof that he was influenced by Vedic metaphysics which says that the spirit of God pervades in all animate and inanimate life/things. The scientist Abdus Salam of Pakistan and Nobel Prize winner in Physics found symmetry between an event and its mirror image and unification of basic two forces of Nature and called these mystical ideas of beauty and harmony based on the laws of nature. (The Times of India dated 24.11-1996 - Current Topics). Newton’s indestructible smallest atom has now been split into more than one hundred sub atomic particles and each such particle is having unsuspected vitality and some of these have been found moving with tremendous speed continuously. Does it mean that the physical sciences particularly particle physics are moving towards

the Vedic truth that the phenomenal world is an Illusion -Maya or what later Immanuel Kant of Germany described it as phantasmagoria?Does it mean that a human made category like *the smallest unit of matter* is applicable to the Physical Universe?

Shunya Vadis of ancient India found through their metaphysics that the entire universe is built on Shunya- vacuum or void as the innermost part of a gross atom is Shunya and only the spirit of God dwells there. Some of the modern scientists tell us that an atom contains sub-atomic particles as well as waves, wavicles, clouds and events reaching a subtle stage in the innermost part. However, a confirmation of the illusory nature of the world is yet to come from the scientists, in spite of the fact that ether the finest gross main element (maha bhuta) still remains a mystical ghost to them. The innermost part of atom is still finer than ether's invisible particles. Ether is without dimensions and weight, but cause of creation of air as mentioned in Vedas. It also has tremendous energy and serves as a medium for the sound waves to pass and go across the world as well as other nearby planets in fraction of a second. How does this energy come in the mystical ghost described in Vedas as Akash? Does it have subtle particles or only gross or both? Which philosophers should be believed who say that matter consists of four main elements air, water, fire and earth i.e. excluding ether or those who have found five main elements (maha bhuta) including ether? Is it possible to count main elements? Is it possible to only one fundamental element to appear in the form of five main elements?

Besides Vedas, many other scriptures of major religions of the world and some metaphysicists as well as philosophers have found the *world as phenomenon, Maya, Mithya or illusion. Greek philosopher Heraclitus noticed instability in the world of senses that never is, being in the process of becoming***.** The world as it is or thing in itself is quite different from the world as it appears. Plato found that the world is blue print of Reality and this appearance always leads us astray (Protagoras-356.D). Omar Khayyam in the 12th century AD found this world as a magic box lit by the Sun-candle of God. Immanuel Kant while agreeing with Plato that the world of senses is a phenomenon but explained it as phantasmagoria. *Schopenhauer went to the extent of saying that the world exists as "I feel it ", otherwise it remains a dream. Ancient philosophers Pythagoras and Parmenides also found the illusory nature of the world of senses***.**

A large number of individuals in the world had been in search of God, His form and attributes and also where does He dwell? Lately this search is getting more intense owing to wide spread miseries, diseases and evil effect of the prevalent extreme materialism, apart from the vulgarization of culture and religion. This search is also due to the fact that what is presented to our senses does not give satisfactory answer to many of our question, activities and phenomenon, besides the other reasons that majority of the people are getting dissatisfied and even disillusioned with the world of diversity, flux, change and are seeking the Grand Designer (Vishwa karma) who created all this. *Intellectual arguments, empirical knowledge and our sense experience have led to a great confusion in all areas of metaphysics, philosophy, science, economics, politics, religion and even social behaviour and other fields of human activities.*

This phenomenon is observed more in the metaphysics of ancient seers and sages. It may appear rather strange but the fact remains that after studying the same hymns, (mantras and riks) in the Vedas, the ancient metaphysicists propounded six vastly different philosophies (Sad Darshan) but still came to almost similar conclusions in regard to ideal human conduct, social behaviour, one global family and spiritual brotherhood. None of these six major and a few minor Schools of Indian philosophy found any current of materialism isolated from spiritualism flowing in the vast ocean of knowledge contained in the Vedas. *Most of these schools found the concept of Vedic phantasmagoria-Maya as extremely useful for the human beings and also for any society.Vedic metaphysics tells us that unless the concept of Maya is properly understood and so long as its illusory effect stays, the multiplicities of philosophical ideas,*

*concepts and thoughts as well as gods and goddesses can never disappear.* In this confusion caused by Maya, individuals can never agree to any standard guidelines for their behavior and conduct.

The Vedas refer to *this mystical power (Shakti) of the divine Nature as Maya, which forces us to do all these unusual and divergent activities. It is due to the effect of Maya that many of us when we start seeking God, obstacles come from our senses and also from the material phenomenal world around us. Maya is a supernatural power, mysterious will and wondrous skill. Vedas refer to it as Shakti Energy (extremely powerful cosmic energy of God). Rig Veda 6-47-18 and 6-45-16 refer to it as creative Art of God (Maya- Bhi).*

Swami Shivananda, an eminent Vedic scholar who has also studied many other scriptures of Hindus and other religions, has described *Maya as a huge cosmic saw*. Multiplicity, lust, greed, pride, hatred, jealousy, egoism and self-interest are its sharp teeth. Below and between its teeth are love, cooperation, humility, purity and truth. Those in pursuit of money, *matter in any of its form and other mundane activities are very often caught in the teeth of saw and they first start losing the divine instruments like Buddhi (intellect) of their inner world and finally the instruments of their outer world like sense organs and finally their gross bodies as well.* According to Bhagavad Gita, God being merciful and benevolent gives long rope to individuals to develop the divine qualities hidden in them and known to their immortal soul.

There are many things which human senses cannot see or see it differently, like ether, water vapors in the air, ships and aero planes from a distance, rope appearing as snake in dim light etc. A large number of physical and mental illusions, mirages are due *to time and space*. In the desert water and even illusory lake appears during the bright Sun light, in a moving train there is an illusion of trees moving very fast, sea shells appear as silver from a distance during day light. Human senses not only accept these illusions as true and real based on one's knowledge, but also create illusory ghosts, devils and many mythological figures, gods and goddesses. Thus at the cosmic level all ***phenomenon in the world are related to Time and Space***. Time is not any material entity or empirical concept that could be derived from experience, but it does help in appreciating any phenomenon. There has never been a time when you and we have not existed, nor will there be a time when we will cease to exist. As the same person inhabits the body through childhood, youth, and old age, so too at the time of death he attains another body. The wise are not deluded by these changes. (Bhagavad Gita quote) Strictly speaking, Time appears as illusion that human memory creates to a man. Any sequence of observable events creates so called "impenetrable" Time-line. Obviously, any number of events observable *in another sequence* gives different Time-line *for the same number of events*.

Owing to our illusory senses, experience is highly inadequate to know the true concept of Time. Most of the human beings cannot equate properly any duration of time e.g. past one year and future one year. While the past one-year and even a number of years appear to be of short duration, the future one-year is perceived much longer. When you are happy or enjoying some transitory pleasure in a club or living in the company of your beloved or close friend or even passing through a stage of material prosperity, the time appears to be moving very fast. The time slows down during your grief, sickness, sorrow, pain etc., and we get the feeling the time is being elongated. That is subjective aspect of Time.

*This phenomenon of shortening or elongation of similar period of time is perceived by human senses even though all times are part of the same "time", which is eternal and lives in God. Thus any reason or judgment based on the perception of human senses can never be perfect. From this phenomenon one also observes that Time is a-priori and is beyond any sense experience.* It is more of a transcendental reality than an absolute reality. Because of this attribute of time, the world looks both real and unreal at different moments of time. As already brought out, human senses cannot appreciate the various attributes

of time and its role as the fourth dimension to space. Our soul being immortal corresponds to Time that is eternal and gross body to three-dimensional space. Thus our "real self"- the observer is four-dimensional. In spite of this our senses perceive only three dimensions of the gross body. *Thus shadow looks more real than reality owing to vehement effect of Maya.*By applying the mental, unreal constructs of space and time to itself, the universe creates the stupendous illusion that many things exist where, in truth, there is but oneness.

In the new matrix of space and time, the universe seems to be composed of infinity of different things, all located separately in space and time. Some of the things in this universe are just "stuff" - material - but other things are on an altogether higher level. They are minds, like fragments, or sparks, of the Great Mind of the universe. Just as the universe is capable of thought, so are these sparks. They are "souls".

There is another idea of Time. They think that Time itself makes interaction with human brain and creates "sense of Time". In that case Time becomes "observable" like some sort of telepathy. Obviously, that idea makes not any sense because an isolated man in a cell looses sense of Time soon.

Again the Space is only "one". However, for human beings its dimensions are limited to consciousness and the degree of knowledge. To a child space is small; to a scientist it is large and expanding. The spiritualists, metaphysicists and Vedic seers find it still wider rather infinite and for them it includes heaven, number of worlds where six other communities of human beings live like pitries, angels, karma devas, Gandharvas etc., and also Brahma lok or Vaikuntha -the eternal abode of God (Yajur Veda 26-1, 2). Saint Augustine had also somewhat similar description of Space i.e. the city of Earth and the city of God. One is within the vision of human beings through their senses and can be seen with the help of material scientific instruments.

Einstein could perhaps think of the phenomenon of illusory nature of senses as it is found to a certain extent in his theory of Relativity. The world as reality or a relative reality largely depends on the concept of Space each individual has. The individuals having limited knowledge of space invariably find the world as absolutely real and others with more knowledge of space find it either relatively real or even mithya - not so real or even illusory. Thus *Vedic Maya is more like metaphysical concept of Time (kala) and Space (dis), not a substance but relative to each individual based on his/her pure or empirical knowledge, which various metaphysicists refer to, as a-priori and a-posteriori knowledge.*

Rig Veda 1-131-1 and 6-47-18 describe the universe as His creative Art and for every form He is the Model. All the riches, matter, clouds, Sun and other objects belong to Him. Shvetashvatra Upanishad 4 - 9, 10 describes Brahma as *the Illusion Maker and Prakrti as Maya***.** Being *Shakti energy (cosmic power) of God, Maya hinders the truth to the human senses, which tend to even misrepresent the reality*. The Vedic metaphysicists find Maya more like human being, as he looks and what he is different. The human senses and etani- outward looking mind hide the true reality of a person.

Maya is the principal concept which manifests, perpetuates and governs the illusion and dream of duality in the phenomenal Universe. The substance emanated from Brahman through which the world of form is manifested. It has a captivating nature, which blinds atman (Self) to the transcendent Truth. It is to be seen through, like an epiphany, in order to achieve Moksha - liberation of the atman from the cycle of samsara. Ahamkara (ego-consciousness) and karma are seen as part of the binding forces of Maya. Maya may be understood as the phenomenal Universe of perceived duality, a lesser reality-lens superimposed on the unity of Brahman. Maya is neither true nor untrue. Since Brahman is the only Truth, Maya cannot be true. Since Maya causes the material world to be seen, it cannot be untrue. Hence, Maya is described as indescribable. Maya has two principle functions - one is to veil Brahman and obscure and conceal it from our consciousness. The other is to present and promulgate the material world and the veil of duality

instead of Brahman. The veil of Maya is pierce able and with diligence and grace, may be permanently rent. Consider an illusion of a rope being confused as a snake in the darkness. Just as this illusion gets destroyed when true knowledge of the rope is perceived, similarly, Maya gets destroyed for a person when they perceive Brahman with transcendental knowledge. Maya is the veritable fabric of duality and she performs this role at the behest of the Supreme Lord. God is not bound by Maya, just as magicians are not illusioned and deluded by their own magic.

Our comment: Note the standard enigmatic phrases - Maya is neither true nor untrue; Maya is indescribable; Maya is the fabric of duality. What do these statements actually mean? We are told that Maya is a substance that emanates from Brahman and that it performs its role at the behest of the Supreme Lord. We are not told why God veils himself with Maya, or why he creates Maya in the first place. Why does he want to hide himself from our consciousness? Why does he make Maya seductive to us? The only way to make sense of it is to conclude that God's business isn't with us at all, but with himself. We are the product of God's struggle with his own nature and his own doubts about himself.

Diverse thinkers, from the ancient Greek philosophers through contemporary quantum cosmology and eternal inflation theory, *have called time an illusion*. For them, the perception of time passing from present moment to present moment is an artefact of our psychology, so that anything real or true is real or true eternally and timelessly. The belief that reality lies in a timeless realm of truth, rather than in the flow of events our perceptions show us, might be supported by scientific argument but equally it reflects a metaphysical prejudice. Contemporary attempts to extend quantum theory to the cosmological, to encompass the whole Universe and not just a sub-system of it, are often couched *in equations which suggest time is emergent from a timeless reality*. But these attempts suffer from problems, both technical and conceptual, that are even more challenging than the usual conundrums of quantum theory. Several advances in the study of quantum gravity have shown that our four-dimensional space-time is only recovered in a version of the theory in which time is real and not emergent.

Rig Veda 7 -33-3 attributes the cosmic illusion Maya in Prakrti to a formless and subtle “being of light” Indra deva, who represents power and strength of the Nature. Owing to his power of creating this illusion, Indra deva transforms himself into many forms like clouds, ether, and thunder etc. “Rupam rupam prati rupo bababhuva”. Thus this illusion is only in Brahmanda that includes subtle Prakrti and its gross manifestation- the ever-expanding universe and cosmos. The same illusion is not in Brahma who is the Only Reality. He is Brahman as bliss. He is Vishnu as all pervasive in Vishwa- the universe. Shankaracharya explains Bhuman is no other than the innermost “self” of a person. Those who realise their "Self " after reaching the stage of Turiya which is more or less akin to Kaivalya of Patanjali’s Yoga Shastra, normally cross this cosmic illusion- though the illusion stays but it does not affect them. It is a stage similar to material world when a person comes to know the illusory nature of water, as a mirage in the desert, the mirage still stays but this does not affect the person. Turiya and kaivalya are the stages when one can communicate with his/her partly omniscient manifested soul. Only God is omniscient and all knowing.

These metaphysical stages pertain to supreme consciousness and the method prescribed by Shankaracharya and Patanjali to reach these stages is quite different. At this stage the external world of senses looks empirically real but not transcendentally. Vedic metaphysicists describe the empirically real world as the world of Forms (Nama rupa) limited by time and space and the other world of a-priori principles having no form, *is beyond time and space*. The absolute truth, perfection and bliss pertain to this other world where all forms (Nama rupa) disappear. It is for this reason that many individuals say that some saints, seers and metaphysicists while live in this phenomenal world but actually they are other worldly. As light and darkness, knowledge and nescience cannot remain together, in the same manner

both the worlds cannot be known together. Only with perfect knowledge Brahma jnan all other lower degrees of knowledge merge and get harmonized. Deepada budadalle kathhale!

*Maya being "shakti energy" of God and made effective through Nature has two distinct qualities. (a) It hides the truth. (b). It also misrepresents the truth.* In both cases senses are under its influence. Being the Creative Art of Brahma, its main purpose is to discipline the senses and harmonise the inner and outer world and finally take a person to a stage where everything merges in One- the supreme Reality**.** During the stages of nescience- material and intellectual knowledge bereft of divine and spiritual knowledge, individuals are normally ignorant of this Shakti energy, which is creative Art of God. When such individual want to know His attributes, Form, His eternal laws and Commandments (Vedic Rta) and become the seeker of God, first of all the cosmic illusion Maya hides Brahma. Then it projects the unreal world of Nama rupa which is only in name and form and poses great hindrance in the initial stages. The sole purpose of Maya is to ensure that all impurities in thoughts, actions, ideas and desires are eliminated before a person becomes the real seeker of God and he/she does not seek Him just for ostentation, social recognition or any other material and mundane gain**.**

This stage in the Vedic metaphysics is described as Moksha (final liberation) when the individual comes under His protection and attains permanent bliss. According to Upanishads the individual becomes Brahma himself as his/ her pure soul merges with *God "aham Brahma asi" - I am Brahma," aham atma Brahma" - my soul is Brahma.* At this stage all forms, names (Nama rupa), multiplicity of gods/ goddesses, mythologies, religious fairy tales, aimless ceremonies and sterile rituals disappear and one finds God and His spirit in all animate and inanimate life/things on this earth. All human beings become spiritual brothers and sisters and love for the entire mankind along with selfless service is the first indication of crossing the huge cosmic Saw of Maya. It is only such persons who can build the global family and universal brother hood, which Vedas describe as Vasudhevan Kutumbhkam (global family) and VishvaBandhutva (universal brotherhood). According to Atma - bodha, God is revealed as One and illuminator of all.

Yajur Veda 40-15 to 17 mentions that the face of truth is covered with a golden disc of matter. Maya creates this golden disc and hides the God who is resplendent Protector and His name is O.M. "OM Khamma Brahma" In Rig-Veda it is described as *"OM Tat Sat"- that supreme reality is OM*. In the Vedas OM is the cosmic Word and described as Shabad Brahma. This was the First Word spoken by God and It created tremendous cosmic energy, which resulted into the formation of subtle primordial matter of purity, activity and passivity and these combined and thus the subtle Prakrti was formed. The primordial matter in the form of sattavic, Rajasic and Tamasic gunas when joined in certain proportion, created gross matter and the universe with five mahatbhuta - main elements i.e. air, water, fire, earth and ether was formed. *This world with primordial subtle atoms of three gunas of purity, activity and passivity thus originated from the cosmic Word. Shabad Brahma OM. IT not only created enormous cosmic energy but also created veil of Maya during the formation of the gross universe.*

Time is not experienced, only the Now is experienced. In this talk, Eckhart brings a sense of clarity around our experience of time. He describes how the unconscious mind is always unhappy in the present moment because it is always looking to the future for something better. He explains how the mind creates a story of me to build up a false sense of self, the ego. The ego always hopes to find what seems to be missing in the present moment by looking towards the next moment which never arrives except as the Now. This unconscious state of being relies on our thinking mind, on our understanding of past and future as important, crucial elements to our existence. Eckhart describes past and future as only thought-forms, concepts created by the mind which are used to understand change. The mind-made sense of self, or ego, is always searching for meaning. At the most basic level, the creation of a self implies that there

must also be the other. Eckhart explains that the more one identifies with thinking, the more the ego is in control. Thus, it becomes more difficult to sense your own aliveness, the shared consciousness of all life. Eckhart describes how the ego struggles to create meaning in an unconscious world by seeking it out through interactions and behaviors that provoke responses. For example, he notes that conflict can be one way for people to feel alive, if only on a negative level. Pain, suffering, and human drama can create a secondary sense of aliveness, which he explains is a substitution for the real sense of aliveness - that which comes from simply feeling the timeless consciousness that you are, which resides beneath all forms, including thought. (*Natalie Schreiber*)

*Brian Greene's "The Illusion of Time, part of the series "The Fabric of the Cosmos" aired Sunday evening, July 22, 2012, on "Nova" on PBS. Here's a summary, followed by a couple of thoughts.*

Mahayana Enlightenment, these two worlds (relative samsara and relative nirvana, or relative form and relative emptiness) merge into a perfect Unity of Absolute Emptiness or Absolute Nirvana, and then there is neither motion nor stillness, or you can say that there is both of them together (as I explained in the first paragraph), or you can say neither of those and give the logically precise answer: silence. The Madhyamika Shastra (XV.3) put it best:

"It cannot be called void or not void,

Or both or neither;

But in order to point it out,

It is called 'the Void.' "

"Time is not what it seems…There may be no distinction between the past, present, and future." Discoveries in quantum physics suggest that *time is entirely different from how we perceive it to be in our daily lives*.

All cultures, including very ancient ones, have found time fascinating. The Maya, for example, calculated time with three different, interrelated calendars; for the sun, the moon, and Venus.

Isa Upanishad, which is the last chapter of Yajur Veda, has a number of passages about the absolute reality of God and the relative reality of the world. Shankaracharya in his Brahma sutras analyses this contradiction between appearance and reality and concludes it is due to apara jnan (lower knowledge) of senses and matter that the world looks real. It is a stepping-stone to para jnan (higher knowledge) of God, soul, spirit and Prakrti. *He explained it in his doctrine of "self evolving Brahma" which many savants relate to Vedic metaphysics of Brahma Parinama vada. As in an individual the "self" is real and the reality of the gross body is a lower truth, the same is true at the cosmic level. Brahma is real and Prakrti is lower truth. A number of Vedic hymns contain a philosophy of absolute and pure* monism-advaita vedanism. In a few words it means *"unity in diversity."* According to Shankaracharya who is advaita Vedantist, owing to lack of knowledge of Maya, when this diversity increases in the universe, it merges back in the subtle Prakrti and finally merges with Brahma when Pralaya or final dissolution occurs. The same re-emerges in the next cycle of Creation (Srishti), which symbolizes Unity. *This cycle of Srishti and Pralaya continues, but it does not affect Brahma who is eternal.*

The cosmic Age when effect of Maya is maximum owing to inadequacy of knowledge due to nescience or predominance of intellectual knowledge (apara jnan), that celestial Period is Kali Yuga. On the other extreme when its effect is minimum and the people follow divine laws of social, moral and physical order, pursuit of money and matter is on the path of Dharma (righteousness) and the predominant guna is

that of purity, truth and honesty known as Sattavic guna, that celestial Age is Sat or Krita Yuga. The other two celestial Periods of Treta and Dwapar Yuga fall in between. Thus Maya is not only extremely powerful for not only concealing the true character of the Reality but also one of the causes of Creation and Dissolution of the Universe as well as for the four celestial Ages. In view of its *vehement effect Sri Rama Krishna calls it Maha Maya- the great Illusion of the divine mother Prakrti.* Unless one severs the shackles of Maha Maya, he/she cannot realise his/her real "self" and also the God. The saint of Bhakti cult Kabir says, "Unless you leave not Maya and you continue enjoying the illusion of money, power, false prestige, Maya leaves you not. Only with devotion to God, He alone will loosen us from this yoke." It is Maya that creates infatuation and makes you feel that children belong to you and you give love to children as loan to be returned in your old age. Only when the vehement effect of Maya starts disappearing that you give love to your *children as a selfless donation and not a loan*. Very often we forget that children do not belong to us, they only come through us. Bhagavad Gita clearly says that during the entire process of procreation God is present. He is thus the procreating Father of all of us.

Vedic metaphysics clearly mentions that all material things belong to God and these are given to us for minimum use as need based and not greed based. One of the greatest delusion which Maya creates that you start believing due to your egoism that *you are the doer, even though all actions are performed due to modes of Prakrti with primordial matter in the form of three gunas (B.G. 3-27).* It is Maya that is the cause of insatiable desire and thus deludes the human beings (B.G.3-39, 40). It is through its cosmic illusion we are chained to Destiny. To overcome the vehement effect of Maya the Vedic rsis and munnies had advised that we should obtain *the higher knowledge of Nirguna Brahma who is ineffable and formless*. Also one should seek a preceptor who is an illumined soul capable of removing inner darkness. In Vedas, the term used for preceptor is Guru. This word consists of Gu and Ru. Gu means darkness both inner and outside and Ru is to dispel. Thus Guru is dispeller of all kinds of darkness of his / her students/disciples. The Vedic guru is the one who knows all ten physical sciences, mathematics, social sciences, military science and complete spiritual and divine knowledge. It is for the Hindus now to judge which of the present day guru or swami meets the Vedic qualifications of a preceptor. All others can be considered as spreading false spiritual and divine knowledge, leading to more and more vehement effect of Maya.

"This is not unchanging, yet it is not moving. It has never been void; there is no question of inside or outside, no separation of absolute and relative. Realize that this is your own original face: even if it appears as ordinary or holy, even if it divides into objective and subjective experiences, all comes and goes completely within it, all arises and vanishes herein. It is like the water of the ocean making waves; though they rise again and again, never is any water added. It is also like waves dying away; though they die out and vanish, not a drop is lost." (*Zen Master Keizan, Transmission of Light*)

Maya is a part of lower (material) nature of God, which includes five main elements, mind, reason and ego (B.G. 7-4, 5). Bhagavad Gita also says that one has to pass through this lower nature of Brahma i.e. subtle Prakrti and gross universe, through discipline, sense control, moderation in thoughts, desires and living to reach God's higher divine nature. *The whole creation is deluded by objects evolved from three modes of Prakrti consisting of three gunas with subtle atoms and under their effect particularly* Tamasic and Rajasic gunas individuals fail to recognise God (B.G. 7-13). Lord Krishna in Bhagavad Gita uses the words Gun maiyi mmam Maya (wondrous veil of Mine). This Mine (Mmam) word is meant to be God as in the Bhagavad Gita lord Krishna is described as incarnation of God. However, in the Vedas there is no incarnation of God as a human being or any other material form. Thus Bhagavad Gita to this extent deviates from Vedic metaphysic. However, in many other aspects it teaches the Vedic metaphysical knowledge. Lord Himself says in Bhagavad Gita that it is difficult to cross Maya except through divine knowledge (B.G.7-14**).** *Maya is the divine potency of God (Brahma's Yog Maya) and it gives a delusion*

*of pair of opposites (B.G.7-27).*

Svetasvatra Upanishad IV-9, 10 clearly mentions that *Prakrti is Maya, mighty Lord is Mayin - the illusion maker and the whole world with beings is part of Him.* There is also detailed description of Maya in Yog Vashishta, Atma-bodha-3, Karika, Svetasvatra and Mundaka Upanishads. All these scriptures clearly mention that at the intellectual level and also for the ignorant people it remains a riddle and cannot be explained as the concept of inner world is lacking in them. For such persons the vehement effect of Maya starts diminishing and they also find disengagement of spirit from matter. All gods and goddesses, Isvara (personal God/gods) with attributes and form, start merging in One God. If Vedic metaphysics about Maya is accepted as true then its vehement effect about multiplicity of gods (33 millions) amongst Hindus is the maximum.

"The life of a sentient being is a long dream. Existence only appears to be real. When one finally awakens, or attains Buddha hood, existence is seen for what it is--a sequence of illusions. Until that time, people will remain obsessed by the body, mind, and external phenomena, not realizing that they are illusory. You will live in a dream, thinking that it is reality. . . .

"Sentient beings mistakenly view their moment-to-moment illusory existence as a continuous, connected lifetime. Because they are unaware that their life is unreal, they do not attempt to wake up." (*Ch'an Master Sheng-yen, Complete Enlightenment, pp. 108-109*)

**How the Cosmic Illusion Works**

Maya being the material cause of the universe, it works through its veiling power, which creates difference between the seer and the scene. It works vehemently in all the subtle three guna but more vehemently in Tamasic guna of stupor, inertia, passivity, hypocrisy and deceit and works least vehemently when sattavic guna of truth, purity and transparency is predominant. In the case of predominance of sattavic guna it helps the individuals even to work with the poor and destitute without any self-interest. Working for the poor with self-interest of just keeping you occupied or for social recognition and material gain, is due to predominance of Tamasic guna.

However, Vedic metaphysics makes it clear that the effect *of Maya completely disappears and it stops working in a shuniya sattava stage, when a person goes beyond guna. Those who lead a life of moderation in action, thoughts and desires can only achieve this stage and finally move towards need* based living by reducing material needs as much as possible. Gandhi ji had already found out by following need-based living that the mother Earth has plenty for all of us to meet our needs but not enough for our greed. *When the vehement effect of Maya stops working, one comes to know about the absolute supreme Reality.*

Once you know His true nature, one finds Him pervading everywhere in animate and inanimate life/things. These individuals can dare to tear the veil off the face of Nature and have at any risk a glimpse of the beyond (*Swami Vivekananda*). Nature then no longer appears as real but one finds it a shadow or sport (Lila) of the God and a phenomenon only. Vedic phantasmagoria and material things returning to Him akin to Pralya and Srishti. On Creation or Srishti all material things come from Him and on Pralaya or Dissolution return to Him. Vedas do emphasise that life of *human beings in this world is naught but the stuff of illusion*.

For those who are still involved and deeply got entangled in this material world under the influence of Maya with the predominance of Rajasic and Tamasic guna, Vedic metaphysics help them to get out of its vehement effect in stages. The first step is to know and understand Prakrti and her attributes. It is just as

if an infant first knows and understands his/her mother. In the Vedas the supreme Mother is subtle but extremely powerful Prakrti - the divine Nature. The gross earth is only benign Mother, which helps us selflessly in our material need-based requirements and while moving very fast around herself and the unmoved mover of the universe (suriya) - the Sun, on an axle that does not rust. Also the mother earth does not kick or give a jerk, while moving so fast.

*The supreme Mother Prakrti is subtle and effable.* While matter in this world only provides material and intellectual knowledge, Prakrti helps the human beings in acquiring the knowledge of Spirit, cosmic laws of social and moral order, selfless service, spiritual brother hood etc. The knowledge provided by the infinite supreme Mother also known as Maha Maya, is spiritual knowledge (Vijnan) as distinct from divine knowledge (jnan) of God, soul, *time and space* and the vast inner world of the human beings. In Vedic metaphysics both *Time and Space* being eternal live in God. *Thus Vedas have three kind of knowledge i.e. Jnan, Vijnan and Ajnan and their complete understanding is Brahma jnan.* Material or intellectual knowledge in isolation is ignorance or ajnan/avidya. All three kinds of knowledge in isolation can at best lit small candles to remove petty darkness in very small areas but only Brhama jnan- the perfect knowledge of this trinity can remove the total darkness of the entire mankind. This Trinity of knowledge is like the Sun. When it rises the darkness disappears in the entire world. *It is for this reason that savitar deva (the Sun) is described in the Vedas as in-charge of perfect knowledge. The mother hymn "Gayatri Mantra" is prayer to savitra deva to provide us knowledge in all our three stages- while being awake, during sleep and in our dreams.*

Only when a person thoroughly understands the attributes and functioning of Prakrti through material and spiritual knowledge, one can move towards para jnan that is higher knowledge. For those persons who are moving towards higher knowledge after acquiring material and spiritual knowledge, Maya starts disappearing for them and they find 33 beings of light merging with One God. Prakrti herself then merges with God and He is seen everywhere, as the veil of Maya is torn and all visions Beyond get clear. Rig-Veda 3-55-1 describes it as Mahad Aksharam- the Great Eternal in whom every material and subtle things finally merge. Since all 33 devas finally merge in Him, Vedic seers and wise men also named Him "Brahma Deva." *In the darkness of māyā, I mistook the rope for the snake, but that is over, and now I dwell in the eternal home of the Lord. (Sggs 332).*

Even though William Shakespeare is not considered as the scientist of the soul, *still he also found this phenomenal world as "dream" and "empty shell." In the Tempest IV- 1-151, he has written, "our life is rounded with a sleep, is a baseless fabric of the vision and all, which it inherits, shall dissolve."*He further writes, " in this stage world looks empty shell with clouds capped towers, gorgeous palaces, the solemn temples, the great globe itself and will get dissolved." *Thus during Maya's working stage, truth is found in the garb of untruth and virtue in sin and evil. There is a famous fable depicting effect of Maya*. There were two sisters named Truth and Untruth. They went together for a swim. After their bath by mistake the sister Truth put on the clothes of her sister. Untruth not finding her clothes put on the clothes of her sister Truth. Since then Truth is freely moving in the garb of untruth and vice versa. This effect of Maya is also seen when sin is described as virtue.

*Time, if we can intuitively grasp such an identity, is a delusion: the difference and inseparability of one moment belonging to its apparent past from another belonging to its apparent present is sufficient to disintegrate it. ( Jorge Luis Borges, A New Refutation of Time)*

Advaita Vedantists like Shankaracharya had found that during its working stage Maya creates Nama Rupa (name and form). We make jugs, plates, statues and many other things out of clay. While clay

could be "real" to senses, jugs, plates are only real in appearance i.e., in name and form (Nama rupa). Again clay itself is made of other subtle and gross atoms and particles (anu, kanu and tan-matra). Thus clay is also real only in name and form and not absolutely real. The same is applicable to human beings that are real in name and form as billions of living cells atoms of various elements, primordial matter of three Gunas, inner and outer instruments make the human being. Thus absolutely Reality has to be found beyond name and form. Maya is real like clay with lower knowledge of matter (avidya) and with higher knowledge it no longer remains absolutely real and finally disappears when Reality is found. Shankaracharya thus concludes that Maya belongs to lower level of Reality and not absolutely Real. According to Vedic metaphysics, the working of Maya stops for any individual when seer and seen are unified. This state of super consciousness by the Vedic metaphysicists is described as Turiya.

**How to attain Turiya**

Many Vedic metaphysicists have referred to five Koshas (sheaths) or coverings in human body, which have to be crossed before reaching the stage of super consciousness. The first is annomaye kosha or food sheath. You cross this sheath when you eat to live and not live to eat and follow the principle of moderation in your food habits. You neither over eat nor starve yourself.

"If you are attached to your thinking, then everything has name and form. This is the world of opposites. But name and form are always changing, changing, changing. Because of this, everything is impermanent. Everything is like a dream, is like dew, is like a bubble or a flash of lightning. Nothing stays but is always in a process of change. Rather than being some constant, fixed reality, this whole universe constantly appears and disappears. But there is a way to experience the true nature of this constantly changing universe. Simply do not become attached to any outside world. Don't become attached to names and forms. If you keep that point, then your mind is not moving. You attain that names and forms are fundamentally empty. This whole universe is completely empty. You are completely empty. Nothing ever comes or goes. Nothing ever appears or disappears. When you keep this mind, you soon attain your true self." (*Zen Master Seung Sahn, The Compass of Zen, pp. 127-129*)

"It is like a cinema-show. There is the light on the screen and the shadows flitting across it impress the audience as the enactment of some piece. If in the same play an audience also is shown on the screen as part of the performance, the seer and the seen will then both be on the screen. Apply it to yourself. You are the screen, the Self has created the ego, the ego has its accretions of thoughts which are displayed as the world, the trees and the plants. . . . In reality, all these are nothing but the Self. If you see the Self, the same will be found to be all, everywhere and always. Nothing but the Self exists." (*Sri Ramana Maharishi*)

Reverting back to the subject of Vedic sheaths, only after crossing the first Annomaye Kosha of food, one can move towards crossing the second Pranomaye Kosha relating to vital breath. When the taste of food moves from the tongue to the mind, one should start living in the natural surroundings with trees, plants and plenty of fresh air around. If some breathe control exercises are done along with your normal day to day work and also keeping the gross body free from toxic and poisonous gases, this would help in crossing the second obstacle in the form of sheath in the body. Having crossed these two coverings mostly in the gross body and partly in the subtle body (Pranaomaye Kosha), the third sheath to be crossed is Manomaye Kosha pertaining to human mind. Human senses and turbulent etani-outward looking mind would create a lot of hurdles to the person desirous of crossing these Kosha. However, for attaining the stage of Turiya all these sheaths are required to be crossed.

After crossing this Kosha, only one formless, ineffable God (Nirguna Brahma) appears and the true nature of Prakrti, meaning of spiritual brotherhood and one global family are understood. The fifth and

last sheath to be crossed is anandamaya kosha, when aim is to seek permanent Bliss. Unfortunately the wall of intellectual reasoning, pragmatism and material knowledge created by Maya along with human senses, senses organs, mind and five Koshas make it difficult for a person to become his/her own self and reach a stage of super consciousness (Turiya). When one crosses the last sheath, one finds that his/her real self is always at peace only the senses create disturbances. Thus the controlled senses and the attribute of permanent peace of the real self take an individual to *self- realization, bliss and super consciousness*.

Shankaracharya had observed that reason, character, personality of the individual, guna, bhuta and maha-bhuta, karma (good and bad deeds), and physical sciences all belong to realm of senses or Maya. This is the world of multiplicity owing to effect of guna, so number of personal gods (saguna Brahma) appears as real. He refers to them as Iswaras. These personal gods/Gods can be one or many for different individuals but they all belong to the realm of Maya and are compatible with Supreme Brahma. These Iswaras are the purveyor of rewards, punishment and arbiter of one's karma. The good and bad actions as karma also form part of Maya and also successive births take place in the realm of Maya. For escape you have to release yourself from Iswaras and get absorbed in Brahma.

**The Illusory Effects of Maya**

*Maya is an imaginary reality. The human beings see the same phenomenon; study the same subject and listen to same discourse but tend to interpret differently. Very often, many of them get even dogmatic and sometimes, they are prepared to quarrel.* When the effect of Maya gets vehement, it leads to communal riots, demolition of temples, mosques and churches of the same God. They tend to justify with material and intellectual arguments their evil action. The permanent truths contained in all the scriptures of various religions are ignored and a few non-permanent differences are highlighted for such actions of vandalism. The higher knowledge of Vedas guides the human beings to avoid contrary truths. In cases like communal riots, quarrel over petty issues etc.; *Maya creates a golden disc and hides the inner world of divine instruments, one's real self and even Brahma and makes you feel that the outer material world is the real and the individuals belonging to other religions and faith are your enemies*.

Though Maya affects all the human beings, but its vehemence is the maximum when Tamasic guna becomes predominant in an individual. It creates false illusion of optimism or even excessive pessimism resulting in blind faith and unscientific outlook. Owing to impure effect of three gunas in Prakrti, one can see the effect of Maya in the pair of opposites in the divine Nature like day and night, summer and winter, hot and cold etc. In the human beings this effect is *found as pleasure and pain, good and bad conduct evil and virtue etc.* These pairs of opposites caused by Maya are one the main reasons of multiplicity of gods, religions, communities, thoughts and ideas. When it acts vehemently on individuals, it makes them outwardly restrain their organs of senses and actions but mentally dwell on objects of senses like, filthy lucre, matter, sex, power, status etc. They become men of deluded intellect and hypocrites (*B.G. III-6*).

Maya hides from such individuals of deluded intellect, the higher knowledge of a-priori principles known to the manifested soul and transcendental research done by rsis and munnies of yore. Their eleven servants i.e. five senses, five sense organs and outward looking mind become masters and hide the inner world from them and make them feel that the outer world of phenomenon is the only real world. Subjects like laws of God, Vedic Varna ashram institution and metaphysical concepts like soul and spirit, Turiya and inner instruments like buddhi (intellect) become a subject of laughing matter with them. They would never believe in life after death, transmigration of soul, effect of karma - good or bad deeds of the human beings as the real cause of rebirth. Like the Charvakas who were the dreaded materialists of the ancient India, these people with predominance of Tamasic guna considered Vedic metaphysics as creation of

cunning and mischievous Rsis and Munies of yore and have no relevance to any age. This absolute realism of the material world is the cause of quarrel amongst nations, disorder in society, extremely divergent ideas and views on economic, political, social, religious and other matters amongst human beings.

Under the vehement effect of Maya, many individuals cannot make proper distinction between right and wrong, good or bad, virtue and evil, pleasure and bliss. To know the proper distinction of these concepts of pairs of opposites had been the subjects of research by Vedic seers, sages and metaphysicists. Even the medieval and modern philosophers in all parts of the world had been contemplating to find out the true significance of these pairs of opposites. Plato's concern was about the absolute nature of right and wrong and after metaphysical research gave his findings in his theory of Forms and Ideas and included the same in his three treatises Republic, Phaedo and Gorgias. It is this doctrine of Maya or Plato's world of phenomenon, which helps to know in this regard and also to attain determinate intellect. Bhagavad Gita clearly says that without this determinate intellect, it is not possible to know right cessation of duties. Thus the knowledge of absolute nature of right and wrong and also knowing right cessation of duties is a part of higher knowledge.

It is due to Maya there is flux in Prakrti. Lord Buddha independently observed it and his ethical metaphysics is largely based on flux in Nature. He advised detachment to matter and all material things to avoid the effect of Flux. Only those who consider the passing state as permanent, they invariably cling to it desperately and blindly run after all material things. Even they tend to resist the laws of change and flux, which are beyond resistance. It is these laws of change and flux, which provide permanence to non-permanent things and characters. Being agnostic Buddha did not feel necessary to bring God in his ethical philosophy. He linked illusion in Nature and universe to flux. With right knowledge of ashata marga (eight- fold path), Panch sheel and Dhamma this effect can be considerably reduced and when you reach the stage of Nirvana, the effect of Flux disappears.

The permanency of various mythological stories, fables and characters is well known in many religions. Many mythological characters are now more real than many other real characters. The survival of mythological heroes of Hindus like five Pandavas, Sharvan Kumar, Eklavya and many others along with gods and goddesses, for the last a few thousand years is nothing but providing permanency to imaginary and non permanent characters. It is that what Shankaracharya calls it lower knowledge or apar jnan. Ramayana, Mahabharata, Panch tantra and Puranas along with some other major and minor Hindu scriptures contain details of the cosmic sport (Lila). Vedic Maya is a highly metaphysical concept and the cosmic sport is a part of it.

It is not only in Hindu religion that mythological gods and other heroes have become more real than reality and their idols, statues of stone, marble, and bronze are worshipped, this phenomenon is also noticed in other religions in the world. The eminent English philosopher and a Nobel Prize winner *Bertrand Russell*, in his book "Why I am not a Christian?' had even doubted whether Christ was ever born. However, he did not get into any controversy over this aspect. Obviously he was not a fanatic or fundamentalist and at best was an agnostic. His findings were more metaphysical than historical or religious. *It is all the effect of phantasmagoria in the Nature*. So long as the person is under the influence of this illusion, *multiplicity of mythological gods and goddesses cannot be avoided*. This effect is multiplied many fold with the predominance of Tamasic guna and many Hindus now believe that there are 330 millions gods and goddesses who should be worshipped.

Thus when a man start getting out of the vehement effect of Maya by constantly reducing your Tamasic guna and acquiring more of subtle particles of sattavic guna the man tend to become extremely useful for the mankind. All his/her thoughts, ideas and actions move in the direction of enlightened liberalism and

the man find living God everywhere. The inanimate objects, things like ocean, sea, earth, mountains etc., appear as having life in them, where jelly fish to powerful whales, lowest and humblest creatures to the strongest animals and powerful human beings can live in harmony. Living sea and the earth appear more as manifestation of God ever keen to help the animate life to live there full life by providing all varieties of food and conditions for their development and for mankind even to reach their perfection in all areas of their interest. Only One nameless God appears.

Owing to ignorance and not knowing this concept of Maya or remaining under its influence due to predominance of Tamasic and Rajasic gunas is misery, getting out of it is enlightenment, peace and contentment. Those who get out of it never perish even though their gross bodies may only die. Their thoughts, ideas and philosophy guide the mankind in all ages. It is the effect of Maya that all events in your life become passing appearance due to Flux in Prakrti caused by this cosmic illusion**.** Our gross body's structure, mind, thoughts remain always in flux. Due to effect of Maya, the human bodies as well as thoughts vary considerably during childhood, youth, and middle an old age. For some persons changes are faster and are perceptible owing to its more vehement effect. But the soul which is one's real self, continues with the individual to guide him/her right path, action and thoughts from the birth to death and even in the next birth. Its power of guidance and inner light go on decreasing when the subtle particles of bad and evil actions, thoughts and desires of the human beings accumulate over it and reduce its illumination. Such persons get into utter inner darkness and feel highly insecure. Those persons who are transparent, truthful, enlightened by their soul lead a life of complete freedom without much fears of losing the gross body. They are always convinced that their real "self" never dies and is immortal. As Bhagavad Gita says, for such individuals losing gross body is more like change of clothes. Thus Maya along with the predominance of kind of primordial matter bring complete change in human personality.

It is because of this phenomenon of flux and Maya along with their perceptible effects that one finds this world both real and unreal based on the stage of one's knowledge of supreme Reality, Prakrti and matter. Since the ancient seers and metaphysicists of the post Vedic period also had higher knowledge of varying degree, so from the study of same Vedas different metaphysical philosophies emerged. There were six major and a few minor philosophies. These are popularly known as sad darshana. Sad is six and Darshan is a "world view." All these philosophies describe the worldview somewhat differently while agreeing on certain common Vedic truths. Nyaya Darshan and Mimamsas School found philosophy of realism in the Vedas and other schools found in these Shrutis absolute monism, pure idealism or dualism and even qualified monism and subjective idealism. One minor school of Shuniya Vadis found the spirit of God all pervasive along with life and vitality in all inanimate things. In other parts of the world based on degree of knowledge, experience, observations etc. and the effect of Maya and flux, hundreds of different philosophies have been propounded, from idealism to pragmatism and scientific rationalism, materialism to mysticism and many other extremes. It is for this reason that Vedas advise human beings to tear the veil of Maya and Flux in Prakrti.

The concept of Yugas or four Celestial periods is not only in the Vedas but also found in other scriptures and philosophy of certain lovers of wisdom in all parts of the world. The scriptures of Sikh religion refer to these four celestial periods, Buddhist and Jain's metaphysics also confirm about this concept. Greek philosophers and some historians refer to four such periods, though Hasoid- an eminent Greek philosopher refers to five such periods' i.e. golden, silver, brass, heroic and iron Ages. Plato found only four such cosmic Ages i.e. without the heroic Age of Hasoid. Vedas, Upanishads, Purans and other Hindu scriptures only refer to four Ages- Krita or Sat Yuga, Treta, Dwapar and Kali Yuga. However, one cult, which migrated to India from the areas now form part of Pakistan and having mostly Hindu followers, refers to five Yugas by adding Sangam Yuga, which is the transition between Kali and Krita Yugas. While Vedas refer to the present Age as Kali Yuga akin to Iron Age of Greeks, this cult describes

the present celestial period as Sangam Yuga that is shortly to merge with Sat Yuga or Golden Age after the cosmic dissolution (Pralaya). This word "shortly" is *being shifted from decade to decade to rope in more followers and keep them united and integrated to the cult by creating* fear complex. Only the true and faithful followers and devotees of that cult will survive and enter that Golden Age. While in the Vedas and Bhagavad Gita the total duration of these four celestial periods is over 4 million years, for this cult it is only 5000 years for one celestial cycle of five Yugas. Thus Maya creates such a great cosmic illusion that untruth is made to appear as truth and vice versa.

The effect of Maya varies considerably in each celestial period, owing to different proportion of gunas. Each of the successive periods commencing from Krita, the positive energy generated goes on becoming less and negative energy in the society increases till in *the iron or Dark Age (Kali Yuga), negative energy reaches its peak. Since successively evil and corrupt activities go on increasing, it leads to progressive* decline in the divine guidance and people start giving more importance to nescience and material and intellectual knowledge which steadily gets isolated from divine and spiritual knowledge. Vedas refer to this as Avidya or ignorance. Thus, Avidya replaces Vedic Vidya and this material education produces intellectual, industrial and other social, economic and political giants but moral infants. There is continuous degradation in each successive celestial period. Vedic Kali Yuga or Plato's Iron Age is the period when both state and society become Tamo Pradhan and Maya spreads its vast net, making you feel that all matter, world, Nature are all real and soul, spirit, God, buddhi (intellect) appear as unreal.

It is through the knowledge and complete understanding of Maya and by increasing subtle sattavic guna; a man can cross the rabbles of senses. The science of Axiology is highly linked with Maya and it tends to synthesise sensory and super sensory aspects of values by creating Trivarga (trinity of values)- sensate, ideational and idealistic. Without proper understanding of the doctrine of Maya, sensate values spread in society, particularly when Tamasic gunas predominate along *with material and intellectual knowledge* bereft of divine and spiritual knowledge. The other two values start spreading when most of the persons in any society understand the Vedic concept of Maya and move from avidya towards Vidya. The predominance of idealistic and ideational values finally leads to the creation of a perfect society**.** In such a society, people get an absolute value system, what Vedic metaphysics describes, Satyam, Shivam, and Sundaram. It simply means value system based on truth (satyam), goodness (shivam) and divine beauty (sundaram). Many dualists even describe Brahma the Vedic formless and ineffable God as Satyam, Shivam, and Sundram. It would thus be seen that understanding of the concept of Maya helps in acquiring trinity of values and also to achieve an ideal state and society.

**Absence of the Knowledge of Maya**

The great cosmic Saw Maya when creates its most vehement effect on human senses, mind and gunas, it is not possible *to know completely about God, soul, spirit and all other divine and spiritual truths and concepts like Vedic Moksha*. Without its knowledge no one can understand that we are all spiritual brothers and sisters. This noble concept in Vedas is described as viswa bandhutva (universal brotherhood) and viswa Kutumbhkam (global family) and the presence of His spirit in all animate and inanimate life/things. Without the proper knowledge and understanding of Maya, people grieve that others do not recognise their merits and when you understand this phantasmagoria of cosmic illusion, a man grieve at his/her own incapacity (*Analects 14-32*). Also as Psalm 144- 3,4 describes that one should know that man is like a breath and his days are like passing shadow. Even if a person does not know this Vedic doctrine of cosmic Saw with its sharp teeth, with sattavic gunas predominance of transparency, purity and truthfulness, its effect starts becoming less and less.

These persons with predominance of Sattvic gunas know that human body is impure, bad smelling and replete with various types of stench which trickle here and there. The same feeling is not there with

persons having predominance of Tamasic gunas. One possessed of such a gross body thinks highly of him and even despises others. It is all due to lack of one's lack of knowledge of insight and divinity of the inner world (Sutta Nipata 205-6). It is the absence of knowledge of Maya and with Tamasic gunas, the danger of phantasmagoria is the maximum and fallacies like myths start emerging and even become more real than the reality itself. The deep rooted human needs of many people are satisfied through these myths, just like small children who feel more satisfied with fairy tales than the stories based on hard facts of life including factual historical events explained through small stories.

For individuals with predominance of Tamasic and Rajasic gunas, these myths, rituals, ceremonies are beyond practical and scientific reasoning. For others with sattavic gunas and firm belief in immanent and inherent principles or what Immanuel Kant describes as *a-priori principles*, they avoid material and intellectual arguments and reasoning about myths and they normally have complete faith and love for One God. Because of wide spread material and intellectual knowledge amongst Hindus, the majority of them have virtually no idea of Vedic Maya and some of them who have heard this word Maya distorted it as "Chhal, Kapat" (deception, cheating). Tricksters and magicians are some time referred as Mayavi persons, wealthy lady is Maya Devi and wealthy person who worship all deities as Mammon or Kuber is Maya Das. Those who understand this concept of Maya in all its divine and spiritual aspects describe Maya Das as the servant of Mayin - the great illusion maker who is formless and ineffable One God.

**MAYIN and MAYA (The Illusion Maker and the Cosmic Illusion)**

According to Shankaracharya if a manis atheist and look at the world as real, then he/she very often come to the conclusion that God is the Creator and He is Saguna Brahma with form and attributes assigned by human beings. In the Vedas Sagun Brahma is personal God also described as sagun Isvara. Nirguna Brahman is formless and ineffable impersonal Universal God. Thus for the individuals who consider this world as real, normally have blind faith in personal God, gods and goddesses and they do not consider Him as Mayin or Illusion Maker. It is for this reason that the devotees of major and minor deities normally do not *know much about Maya except vaguely and they find both subtle Nature and its gross manifestation as Universe and personal God/gods as real. On the contrary those who know and understand the Vedic cosmic illusion Maya do not consider* Nirguna Brahma as the Creator. Their belief in lower and higher deities is normally not there but has firm scientific faith in the One Impersonal God. The metaphysics contained in the Upanishadic part of Vedas, relate mostly to impersonal God and His knowledge is Brahma Jnan. Knowing Him one knows the entire knowledge of spirit and matter. Thus the Reality of the Universe is relative from absolutely real to illusory depending to what kind and degree of knowledge one possesses.

The metaphysicists of Isa Upanishad clearly tell us "That Reality" is only Brahma and He is beyond attributes, categories, modes and characteristics being *Nirguna Brahma*. To describe Him even the words recoil. However, Prakrti - the divine Nature, consists of three subtle gunas in certain proportion to maintain its equilibrium and has limited attributes of selfless service, energy, power, harmony, heat, electricity, etc., but excluding Time and Space and the divine attributes of soul. Time and Space are eternal and do not face Srishti and Pralaya and remain uniform in all the four celestial periods of Krita, Treta, Dwapar and Kali Yugas. Prakrti has three gunas which provide all characteristics to Her including formation of subtle and gross atoms, particles (Anu, kanu, tanmatras etc.), while *Nirguna Brahma is beyond attributes as gunas do not affect Him. The cosmic illusion as Maya is also due to gunas and has limited attributes that can be known through Vedic knowledge and by going beyond gunas; one crosses Maya and reaches Nirguna Brahma. Isa Upanishad further says, "Renounce all vain appearance, become transparent and truthful and covet not other man's wealth."*

Aristotle held a metaphysical view that God is not a Creator but an Unmoved Mover. This metaphysical

finding of Aristotle was without linking the world as phantasmagoria or phenomenal world as his renowned teacher Plato held. Thus, the influence of Plato particularly his Forms and Ideas went deep in Aristotle that helped him in arriving at metaphysical discovery almost similar to Vedic metaphysics of the Advaita Vedantists. It would thus appear that permanent truths are the same whether one arrives at based on the Vedic doctrine of Maya, gunas or Forms and Ideas of Plato along with the deductive approach to philosophy or just Aristotelian inductive methods of philosophy. Only those means, concepts and doctrines are good which lead to discovery of *permanent truths*.

The ancient seers and compilers of Upanishads have found God as Mayin - the great Illusion Maker who Himself is not affected and remains Whole. "Whole is that, whole too is this, from the whole, whole cometh, take whole from the whole, whole remains." Thus Maya**,** *Time, Space, Prakrti and Universe's creation and dissolution and re-creation after each cycle of four celestial periods (One Kalpa) do not affect Him. This Advaita Vedantist philosophy of the Upanishads leaves no scope for the divine Nature* and Universe to be real. Universe is at best a shadow of God that appears as gross to us, though in reality it is subtlest of the subtle, being the manifestation of God and human senses invariably find it real. By casting this cosmic shadow HE remains the same. To the absolute monists like Shankaracharya, Prakrti and gross universe are mere reflection of the Supreme Reality, a shadow cast by it on the empirical plane in the Space. This shadow or reflection or His illusion is Vedic phantasmagoria and Plato's world of phenomenon and creator of this Illusory Maya is Mayin. This two tier Reality that is different to the human manifested soul (Jivatma) and senses (indriya) is found in many Hindu scriptures. When absolute Reality is understood all gods, goddesses, personal *God a Saguna Brahma or Saguna Iswara merge with One God. All kinds of mythologies and fairy tales of the organised religions in the* world look as unreal.

Later scriptures relegated metaphysical concepts, doctrines and subjects like Prakrti, Maya, distinction between soul and spirit, Braham Jnan, Vidya, material knowledge as ignorance or nescience and many others. Those scriptures tried to bring down the great importance of Vedic metaphysics through mythology, proxy worship, giving more importance to intellectual and material knowledge in the form of certain *fairy tales* and even combining soul and spirit into one. Thus for most of the Hindus atma, Jivatma and jiva are the same and these three terms are inter changeable.

The distortion and laxity in the interpretation of various Vedic metaphysical concepts made the effect of Maya more vehement. This created thousands of metaphysical views and led to mushroom growth of fake god men, false gurus and tricksters who ignored altogether Vedic knowledge and spread their ignorance as spiritual and divine knowledge. Thousands of Hindu gurus, cults and swamis are spreading their own limited knowledge by stressing vague concept of love, harmony, tolerance, devotion to God and gurus, etc., and creating their own techniques of worship, meditation, raj yoga, saral yoga, sahaj yoga, prem yoga and even Yogas of physical exercises. Magic, mythology, blind faith, unscientific outlook, superstitions, hallucination and spreading material knowledge that their followers are the children of the gurus, founders of the cults etc.. Many of them openly declare that they are the incarnation of God/god /goddess in this world and have been sent to spread their knowledge, (which is worse than ignorance) to the entire mankind. Thus, they resort to all kinds of tricks, methods and techniques to spread themselves in all parts of the world and then proudly advertise that they have thousands of branches in India and abroad.

The philosophy contained in Upanishads describes the material world as maya matram - a mere false show, like what a magician creates things and they again disappear but both their appearance and disappearance does not affect the magician. Thus Sristi and Pralaya -creation and dissolution of the universe do not affect the pure state of Brahma. On attaining Vidya or Brahama Jnan, Atma (soul) and Paramarta (supreme soul) become One. Human soul merges with impersonal God. Advaita Vedantists

like Shankaracharya find Unity in the Vedic Atma, Parmatma and Brahma. Those individuals who have not attained Brahma Jnan, they find diversity in these divine concepts and find soul, spirit, *God as Nirguna Brahma and saguna Brahma not a unity but different and dissimilar and it leads to worship of many deities and seek blessings from each of them***.**

Thus based on degree of Vidya or Avidya individuals find unity in diversity and also diversity in unity. Material knowledge and intellectual arguments will justify both. It is for this reason that Vedic metaphysics advises the need for knowledge based on immanent and a-priori principles. The Indian Machiavellian philosopher of 3rd century BC Kautaliya also known as Chanakya, took advantage of different schools of Indian philosophy and also found wide spread ignorance.

## EXISTENTIALISM

The roots of existentialism began with Kierkegaard in the first half of the 19th century. He was critical of Hegel's philosophical system which analysed being (or existence) in an abstract and impersonal way. Kierkegaard was concerned with the individuals' subjective experience of what it is to exist as a human being. For Kierkegaard the individual constantly has to choose what he/she is to become without recourse to the findings of science and philosophy. As we saw in the philosophy of god section Kierkegaard thought that the individual could chose to have a religious faith in the face of an absurd world

### Edmund Husserl

The German phenomenologist Edmund Husserl (1859-1938) was influential in the development of methods that were later used by the existentialists. A phenomenologist is interested in things as they *appear to consciousness*, rather than things-in-themselves (*See Kant*). This emphasis on the individuals subjective consciousness was continued in the 20th century as existentialism developed.

### Martin Heidegger

Martin Heidegger (1889-1976) studied under Husserl. Heidegger was interested in the "*question of being*". He thought that *western philosophy had been over obsessed with the problem of knowledge*. For Heidegger the individual as being-in-the-world was characterized by action and anxiety: knowing the world is not our primary way of being in the world. In his later works, Heidegger became more interested in the history of concepts in language. He regarded his investigations as an attempt to disclose or uncover the concealed *nature of being*. His most fundamental question was: why should there be being at all, when there could be nothing? Although Heidegger claimed he was not an "existentialist", his influence on Sartre and the existentialist movement is undeniable.

### Jean Paul Sartre

Jean Paul Sartre (1905-80) is, perhaps, the best-known existentialist. He was a gifted playwright and novelist who was offered the Nobel Prize for literature in 1964, but refused it. Sartre thought that there was no fixed human nature or essence and so the individual has to choose his/her being. This choice brings with it responsibility. Those who do not choose, but base their lives on pre-arranged moral and philosophical systems are said to be acting in bad faith.

### Albert Camus

Albert Camus (1913-1960) was a journalist, novelist and philosopher. He thought that life is essentially absurd: the modern world is full of injustice; millions work in repetitive exploitative jobs. Camus

thought that we should rebel against these absurdities by refusing to participate in them.Existentialism in the 20th century reflects the loss of certainties in the post-modern world. If there are no clear philosophical answers to *the question of existence*, then each individual has to design their own life as a project. The choice and responsibility of that project falls entirely on them.

**Kierkegaard: The solitary wanderer**

Fate was to be the 'exception', the lonely wanderer.Kierkegaard thought that philosophers who claimed that philosophy could show us the ultimate nature of spirit *were deluded*. Hegel claimed *tohave overcome* paradox, but Kierkegaard *was not convinced*. Existence, for Kierkegaard, was paradoxical. The individual must find his/her spiritual path, not through the comfortable dogmatic rituals of the established church or the pseudo-clarity of Hegelian dialectics, but through action, action that is conscious of religious conviction.Kierkegaard held that religious faith was central to an authentic existence. His Christian existentialism has continued to be influential. Theologians have had to face the horrific absurdities of the 20th century and religious (or theistic) existentialism shows how the individual can, with faith be authentic in an uncertain world.

**Sartre: 'Existence precedes Essence'**

Jean Paul Sartre was one of the leaders of the French post war left wing intellectual movement, co-founding with Maurice Merleau-Ponty (1908-61) the journal Les Temps Moderns. His experiences as a resistance fighter shaped his philosophy that was influenced by the ideas of Husserl and Heidegger.Unlike Kierkegaard, Sartre was an atheist. As God does not exist, there are no 'essences'. By essence, Sartre is talking about a pre-defined human nature. What Sartre meant by the phrase 'existence precedes essence' is this: If there is no cosmic designer, then there is no design or essence of human nature. Human existence or being differs from the being of objects in that human being is self-conscious. This self-consciousness also gives the human subject the opportunity to define itself. The individual creates his/her self by making self-directed choices.

As human existence is self-conscious without being pre-defined, we, as autonomous beings are "condemned to be free": compelled to make future directed choices. These choices induce anxiety and uncertainty in to our psyches. *If we, as individuals, simply follow custom or social expectations in order to escape this angst, we have escaped the responsibility of making our own choices, of creating our own essence*. We have acted in bad faith.To act authentically we *must* take responsibility for our future. ***We cannot choose what gender, class, or country we were born into, but we can choose what we make of them. We are free to create our own interpretation of ourselves in relation to the world, to create a project of possibilities, of authentic actions as the expression of freedom.****(Wikipedia, author's blogs and other collective essays)*

After this long preamble we a going to the mathematical section. We hope that all readers take the following section with open mind and relationship with recent publication titled 'Key Aspects of Relative Motion in Historical Retrospective and Absolute Motion in Scientific Perspective' (by Allan Zade) - published at: "International Journal of Scientific and Research Publications (IJSRP), Volume 3, Issue 6, June 2013 Edition".

## NOTATION

## Module One

The Earth **is the** motionless center of the Universe

$G_{13}$ : Category one of **motionless** center of the Universe. Generally speaking in any investigating system, it is possible to find a centre and that is the centre we are talking about here.

$G_{14}$ : Category two of motionless center of the Universe

$G_{15}$ : Category three of motionless center of the Universe

$T_{13}$ : Category one ofEarth. This does not mean there are three earths. There are many investigating systems, perceived, observable, unobserved, factual, and paradigmatic. Characterstics of the investigating systems form the bastion, stylobate and sentinel in the classification doxa.

$T_{14}$ : Category two ofEarth

$T_{15}$ : Category three ofEarth

## Module Two

The Earth is an typical planet of the Solar system moving **by** its trajectory around the Sun

$G_{16}$ : Category one of its trajectory around the Sun. As said, we can construe the trajectories in the manner Sir Allan Zade has explained. It is not necessarily earth and Sun.

$G_{17}$: Category two of its trajectory around the Sun

$G_{18}$: Category three of its trajectory around the Sun

$T_{16}$: Category one of Earth is an typical planet of the Solar system moving

$T_{17}$ : Category two of Earth is an typical planet of the Solar system moving

$T_{18}$ : Category three of Earth is an typical planet of the Solar system moving

## Module three

basic principle of the method **eliminates** uniqueness of a property of an object associated with the property by the human definition and makes the connection of the universal property with the same property of the object by a new definition

$G_{20}$ : Category one of uniqueness of a property of an object associated with the property by the human definition and makes the connection of the universal property with the same property of the object by a new definition

$G_{21}$ : Category two of uniqueness of a property of an object associated with the property by the human definition and makes the connection of the universal property with the same property of the object by a new definition

$G_{22}$ : Category three of uniqueness of a property of an object associated with the property by the human definition and makes the connection of the universal property with the same property of the object by a new definition

$T_{20}$ : Category one of basic principle of the method

$T_{21}$: Category two of basic principle of the method

$T_{22}$ : Category three of basic principle of the method

## Module four

Time **is (=) a** logical link in human mind to any physical process that has observable duration. So we shall retain this as a frame of reference

$G_{24}$ : Category one of logical link in human mind to any physical process that has observable duration. So we shall retain this as a frame of reference

$G_{25}$ : Category two of logical link in human mind to any physical process that has observable duration. So we shall retain this as a frame of reference

$G_{26}$ : Category three of logical link in human mind to any physical process that has observable duration. So we shall retain this as a frame of reference

$T_{24}$ : Category one ofTime. Any subdivision is allowed. Note that the model makes allowance for the passage from one category to another. Only thing that should be remembered is the predicational anteriority could be finalized based on the characterstics of the investigating system involved. This holds good for the entire monograph. We shall not repeat the same again.

$T_{25}$ : Category two ofTime

$T_{26}$ : Category three ofTime

## Module five

Time **does not** exist (and never existed) as a physical property of the Universe

$G_{28}$ : Category one of existence (and never existed) as a physical property of the Universe

$G_{29}$ : Category two of existence (and never existed) as a physical property of the Universe

$G_{30}$ : Category three of existence (and never existed) as a physical property of the Universe

$T_{28}$ : Category one of Time

$T_{29}$ : Category two ofTime

$T_{30}$ : Category three of Time

## Module six

Anything that **does not** exist as a part of physical reality of the Universe **cannot be** expanded, contracted, dilated, bent or altered any other way

$G_{32}$ : Category one of expansion, contraction, dilation, alteration, warping, woofing, wefting, or clef ting of such non existent “anything”

$G_{33}$ : Category two of expansion, contraction, dilation, alteration, warping, woofing, wefting, or clef ting of such nonexistent “anything”

$G_{34}$ : Category three of expansion, contraction, dilation, alteration, warping, woofing, wefting, or clef ting

of such nonexistent "anything"

$T_{32}$ : Category one ofanything that **does not** exist as a part of physical reality of the Universe

$T_{33}$ : Category two ofanything that **does not** exist as a part of physical reality of the Universe

$T_{34}$ : Category three ofanything that **does not** exist as a part of physical reality of the Universe

## Module seven

An objects involved in a relativistic experiment **restores(eb)** any of its real property as soon as it comes back to the initial reference frame despite any number of reference frames it changes during the relativistic experiment

$G_{36}$ : Category one of objects involved in a relativistic experiment

$G_{37}$ : Category two of objects involved in a relativistic experiment

$G_{38}$: Category three of objects involved in a relativistic experiment

$T_{36}$ : Category one of any of its real property as soon as it comes back to the initial reference frame despite any number of reference frames it changes during the relativistic experiment

$T_{37}$ : Category two of any of its real property as soon as it comes back to the initial reference frame despite any number of reference frames it changes during the relativistic experiment

$T_{38}$ : Category three of any of its real property as soon as it comes back to the initial reference frame despite any number of reference frames it changes during the relativistic experiment

## Module eight

two previously synchronized clocks **should (eb) have** the same indications after any relativistic experiment and stay in synchronization after any relative motion at any speed after any duration of the experiment.

$G_{40}$ : Category one oftwo previously synchronized clocks

$G_{41}$ : Category two oftwo previously synchronized clocks

$G_{42}$ : Category three oftwo previously synchronized clocks

$T_{40}$ : Category one ofsame indications after any relativistic experiment and stay in synchronization after any relative motion at any speed after any duration of the experiment

$T_{41}$ : Category two ofsame indications after any relativistic experiment and stay in synchronization after any relative motion at any speed after any duration of the experiment

$T_{42}$ : Category three ofsame indications after any relativistic experiment and stay in synchronization after any relative motion at any speed after any duration of the experiment

## Module Nine

data of the Hafele–Keating experiment remain **in conflict with** theoretical framework of Relativity

The Einstein field equations (EFE) may be written in the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $R$ the scalar curvature, $g_{\mu\nu}$ the metric tensor, $\Lambda$ is the cosmological constant, $G$ is Newton's gravitational constant, $c$ the speed of lightin vacuum, and $T_{\mu\nu}$ the stress–energy tensor

$G_{44}$ : Category one of theoretical framework of Relativity (Please see the above equation. Data of Hafele–Keating experimentcontradicts each and every term in the equation $R_{\mu\nu}$ is the Ricci curvature tensor, $R$ the scalar curvature, $g_{\mu\nu}$ the metric tensor, $\Lambda$ is the cosmological constant, $G$ is Newton's gravitational constant, $c$ the speed of lightin vacuum, and $T_{\mu\nu}$ the stress–energy tensor. In essence it contradicts the entire equation to which we can apply the model

$G_{45}$ : Category two of theoretical framework of Relativity

$G_{46}$ : Category three of theoretical framework of Relativity

$T_{44}$ : Category one ofdata of the Hafele–Keating experiment. There are various systems which satisfy Hafele–Keating experimental axiomatic predications and postulation alcovishness. Classification, we repeat is based on that like say in Newton's Gravitational field. One can always do that with any two objects. Constancy of the gravitational field should not act as an encumbrance with the "Total gravity" and the conduction of experiments with various astronomical objects. Sir Allan Zade has reinforced this comment in more ways than one in his papers on "Time Delusion"

$T_{45}$ : Category two of data of the Hafele–Keating experiment

$T_{46}$ : Category three of data of the Hafele–Keating experiment

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)} (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ ,
$(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)} (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$,
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$,
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)} (a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{2}$
$, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,

$$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$$
$$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$$

are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a_{20}^{''})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b_{20}^{''})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a_{24}^{''})^{(4)}(T_{25},t) =$**First augmentation factor**

$-(b_{24}^{''})^{(4)}\big((G_{27}),t\big) =$**First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a_{28}^{''})^{(5)}(T_{29},t) =$**First augmentation factor**

$-(b_{28}^{''})^{(5)}\big((G_{31}),t\big) =$**First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a_{32}^{''})^{(6)}(T_{33},t)$ =**First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a_{36}^{'})^{(7)} + (a_{36}^{''})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a_{37}^{'})^{(7)} + (a_{37}^{''})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b_{37}^{'})^{(7)} - (b_{37}^{''})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$

$+(a_{36}^{''})^{(7)}(T_{37},t)$ =**First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a_{40}^{'})^{(8)} + (a_{40}^{''})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a_{41}^{'})^{(8)} + (a_{41}^{''})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a_{42}^{'})^{(8)} + (a_{42}^{''})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b_{40}^{'})^{(8)} - (b_{40}^{''})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b_{41}^{'})^{(8)} - (b_{41}^{''})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b_{42}^{'})^{(8)} - (b_{42}^{''})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a_{44}^{'})^{(9)} + (a_{44}^{''})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a_{45}^{'})^{(9)} + (a_{45}^{''})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a_{46}^{'})^{(9)} + (a_{46}^{''})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b_{44}^{'})^{(9)} - (b_{44}^{''})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b_{45}^{'})^{(9)} - (b_{45}^{''})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \qquad 54$$

$+(a''_{44})^{(9)}(T_{45},t)$ =**First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big)$ =**First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \qquad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \qquad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \qquad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \qquad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \qquad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{c} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{matrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{matrix}\right] T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{matrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{matrix}\right] T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{matrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{matrix}\right] T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category

1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{matrix} (a_{20}^{'})^{(3)}\boxed{+(a_{20}^{''})^{(3)}(T_{21},t)}\boxed{+(a_{16}^{''})^{(2,2,2)}(T_{17},t)}\boxed{+(a_{13}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{24}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{28}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{32}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{36}^{''})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a_{40}^{''})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a_{44}^{''})^{(9,9,9)}(T_{45},t)} \end{matrix}\right] G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{matrix} (a_{21}^{'})^{(3)}\boxed{+(a_{21}^{''})^{(3)}(T_{21},t)}\boxed{+(a_{17}^{''})^{(2,2,2)}(T_{17},t)}\boxed{+(a_{14}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{25}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{29}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{33}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{37}^{''})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a_{41}^{''})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a_{45}^{''})^{(9,9,9)}(T_{45},t)} \end{matrix}\right] G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{matrix} (a_{22}^{'})^{(3)}\boxed{+(a_{22}^{''})^{(3)}(T_{21},t)}\boxed{+(a_{18}^{''})^{(2,2,2)}(T_{17},t)}\boxed{+(a_{15}^{''})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{26}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{30}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{34}^{''})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{38}^{''})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a_{42}^{''})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a_{46}^{''})^{(9,9,9)}(T_{45},t)} \end{matrix}\right] G_{22} \quad 69$$

$\boxed{+(a_{20}^{''})^{(3)}(T_{21},t)}$, $\boxed{+(a_{21}^{''})^{(3)}(T_{21},t)}$, $\boxed{+(a_{22}^{''})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}^{''})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}^{''})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{24}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}^{''})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{28}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{32}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{36}^{''})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}^{''})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}^{''})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{40}^{''})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}^{''})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}^{''})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{44}^{''})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}^{''})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{46}^{''})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{matrix} (b_{20}^{'})^{(3)}\boxed{-(b_{20}^{''})^{(3)}(G_{23},t)}\boxed{-(b_{16}^{''})^{(2,2,2)}(G_{19},t)}\boxed{-(b_{13}^{''})^{(1,1,1,)}(G,t)} \\ \boxed{-(b_{24}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{36}^{''})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b_{40}^{''})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b_{44}^{''})^{(9,9,9)}(G_{47},t)} \end{matrix}\right] T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{matrix} (b_{21}^{'})^{(3)}\boxed{-(b_{21}^{''})^{(3)}(G_{23},t)}\boxed{-(b_{17}^{''})^{(2,2,2)}(G_{19},t)}\boxed{-(b_{14}^{''})^{(1,1,1,)}(G,t)} \\ \boxed{-(b_{25}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{37}^{''})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b_{41}^{''})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b_{45}^{''})^{(9,9,9)}(G_{47},t)} \end{matrix}\right] T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}^{'})^{(3)} \boxed{-(b_{22}^{''})^{(3)}(G_{23},t)} \boxed{-(b_{18}^{''})^{(2,2,2)}(G_{19},t)} \boxed{-(b_{15}^{''})^{(1,1,1,)}(G,t)} \\ \boxed{-(b_{26}^{''})^{(4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b_{30}^{''})^{(5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b_{34}^{''})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{38}^{''})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b_{42}^{''})^{(8,8,8,8)}(G_{43},t)} \boxed{-(b_{46}^{''})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \qquad 72$$

$\boxed{-(b_{20}^{''})^{(3)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{16}^{''})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{13}^{''})^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}^{''})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}^{''})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}^{''})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}^{''})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}^{''})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}^{''})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}^{''})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}^{''})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}^{''})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}^{''})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a_{24}^{'})^{(4)} \boxed{+(a_{24}^{''})^{(4)}(T_{25},t)} \boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)} \boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a_{16}^{''})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a_{20}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}^{''})^{(7,7,7,7,7)}(T_{37},t)} \boxed{+(a_{40}^{''})^{(8,8,8,8,8)}(T_{41},t)} \boxed{+(a_{44}^{''})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \qquad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a_{25}^{'})^{(4)} \boxed{+(a_{25}^{''})^{(4)}(T_{25},t)} \boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)} \boxed{+(a_{33}^{''})^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a_{17}^{''})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a_{21}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}^{''})^{(7,7,7,7,7)}(T_{37},t)} \boxed{+(a_{41}^{''})^{(8,8,8,8,8)}(T_{41},t)} \boxed{+(a_{45}^{''})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \qquad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a_{26}^{'})^{(4)} \boxed{+(a_{26}^{''})^{(4)}(T_{25},t)} \boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)} \boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a_{18}^{''})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a_{22}^{''})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}^{''})^{(7,7,7,7,7)}(T_{37},t)} \boxed{+(a_{42}^{''})^{(8,8,8,8,8)}(T_{41},t)} \boxed{+(a_{46}^{''})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \qquad 75$$

$\boxed{(a_{24}^{''})^{(4)}(T_{25},t)}$, $\boxed{(a_{25}^{''})^{(4)}(T_{25},t)}$, $\boxed{(a_{26}^{''})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}^{''})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}^{''})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}^{''})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}^{''})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}^{''})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}^{''})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}^{''})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}^{''})^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficien*

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$,
$\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *an*
$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$,
$\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3
$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$,
$\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3
$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3
$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \qquad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \qquad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \qquad 78$$

*Where* $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$
, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$ *are fourth detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$,
$\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$ *are fifth detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$ *are sixth detrition coefficients for*
$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$
, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$ *are seventh detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
*are eighth detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for

category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} \boxed{-(b''_{28})^{(5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} \boxed{-(b''_{29})^{(5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} \boxed{-(b''_{30})^{(5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \qquad 84$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for ca*

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$ , $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$ , $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$ , $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$ , $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \qquad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$ , $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$ , $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$ , $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$ , $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$,

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$

Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \qquad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \qquad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition

coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - \begin{bmatrix} (a'_{36})^{(7)} \boxed{+(a''_{36})^{(7)}(T_{37},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)} \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 91$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - \begin{bmatrix} (a'_{37})^{(7)} \boxed{+(a''_{37})^{(7)}(T_{37},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 92$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - \begin{bmatrix} (a'_{38})^{(7)} \boxed{+(a''_{38})^{(7)}(T_{37},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 93$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - \begin{bmatrix} (b'_{36})^{(7)} \boxed{-(b''_{36})^{(7)}(G_{39},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 94$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}^{'})^{(7)} & \boxed{-(b_{37}^{''})^{(7)}(G_{39},t)} & \boxed{-(b_{17}^{''})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}^{''})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b_{25}^{''})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}^{''})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}^{''})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ & \boxed{-(b_{14}^{''})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}^{''})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}^{''})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}^{'})^{(7)} & \boxed{-(b_{38}^{''})^{(7)}(G_{39},t)} & \boxed{-(b_{18}^{''})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}^{''})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b_{26}^{''})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}^{''})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}^{''})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ & \boxed{-(b_{15}^{''})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}^{''})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}^{''})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}^{''})^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}^{''})^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}^{''})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}^{''})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}^{''})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}^{''})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}^{''})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}^{''})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}^{''})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}^{''})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}^{''})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}^{''})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}^{''})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}^{''})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}^{''})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}^{''})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}^{''})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}^{''})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}^{'})^{(8)} & \boxed{+(a_{40}^{''})^{(8)}(T_{41},t)} & \boxed{+(a_{16}^{''})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}^{''})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a_{24}^{''})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}^{''})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}^{''})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ & \boxed{+(a_{13}^{''})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}^{''})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}^{''})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \qquad 95$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}^{'})^{(8)} & \boxed{+(a_{41}^{''})^{(8)}(T_{41},t)} & \boxed{+(a_{17}^{''})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}^{''})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a_{25}^{''})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}^{''})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}^{''})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ & \boxed{+(a_{13}^{''})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}^{''})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}^{''})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)} G_{41} - \left[ \begin{array}{l} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)} T_{41} - \left[ \begin{array}{l} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)} T_{40} - \left[ \begin{array}{l} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)} T_{41} - \left[ \begin{array}{l} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)} & \boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13} \qquad 96$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)} & \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)} & \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b_{28}^{''})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}^{''})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}^{''})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{32}^{''})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}^{''})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}^{''})^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{15}^{''})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}^{''})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{37}^{''})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}^{''})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}^{''})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{42}^{''})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}^{''})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}^{''})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3
Where we suppose

$$(a_i)^{(1)}, (a_i^{'})^{(1)}, (a_i^{''})^{(1)}, (b_i)^{(1)}, (b_i^{'})^{(1)}, (b_i^{''})^{(1)} > 0, i, j = 13,14,15 \qquad 97$$

The functions $(a_i^{''})^{(1)}, (b_i^{''})^{(1)}$ are positive continuousincreasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i^{''})^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i^{''})^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i^{'})^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$\lim_{T_2\to\infty} (a_i^{''})^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty} (b_i^{''})^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i^{''})^{(1)}(T_{14}^{'},t) - (a_i^{''})^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}^{'}|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i^{''})^{(1)}(G^{'},t) - (b_i^{''})^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G^{'}||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(1)}(T_{14}^{'},t)$ and $(a_i^{''})^{(1)}(T_{14},t)$ . $(T_{14}^{'},t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$ . It is to be noted that $(a_i^{''})^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i^{''})^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ are positive constants

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}} < 1$$

**Definition of**$(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\widehat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$and the constants$(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, i, j = 16,17,18$$

The functions$(a_i'')^{(2)}, (b_i'')^{(2)}$are positive continuousincreasing and bounded.

**Definition of**$(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of**$(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\widehat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\widehat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ .$(T_{17}', t)$and$(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\widehat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$,are positive constants 109

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$which together with $(\widehat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$and the constants$(a_i)^{(2)}, (a_i^{'})^{(2)}, (b_i)^{(2)}, (b_i^{'})^{(2)}, (p_i)^{(2)},\ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\widehat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i^{'})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i^{'})^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i^{'})^{(3)}, (a_i^{''})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (b_i^{''})^{(3)} > 0, i, j = 20,21,22 \quad 112$$

The functions$(a_i^{''})^{(3)}, (b_i^{''})^{(3)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i^{''})^{(3)}(T_{21}, t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i^{''})^{(3)}(G_{23}, t) \le (r_i)^{(3)} \le (b_i^{'})^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \to \infty} (a_i^{''})^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty} (b_i^{''})^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of**$(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i^{''})^{(3)}(T_{21}^{'}, t) - (a_i^{''})^{(3)}(T_{21}, t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}^{'}|e^{-(\widehat{M}_{20})^{(3)}t}$$

$$|(b_i^{''})^{(3)}(G_{23}^{'}, t) - (b_i^{''})^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}^{'}||e^{-(\widehat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(3)}(T_{21}^{'}, t)$ and$(a_i^{''})^{(3)}(T_{21}, t)$ .$(T_{21}^{'}, t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\widehat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i^{''})^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{20})^{(3)} = 1$ then the function $(a_i^{''})^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would

be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$,are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constantsThere exists two constants$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$which together with$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$and the constants$(a_i)^{(3)}, (a_i^{'})^{(3)}, (b_i)^{(3)}, (b_i^{'})^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i^{'})^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i^{'})^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i^{'})^{(4)}, (a_i^{''})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (b_i^{''})^{(4)} > 0, i, j = 24,25,26 \quad 117$$

The functions$(a_i^{''})^{(4)}, (b_i^{''})^{(4)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i^{''})^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i^{''})^{(4)}\big((G_{27}), t\big) \le (r_i)^{(4)} \le (b_i^{'})^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$\lim_{T_2 \to \infty} (a_i^{''})^{(4)}(T_{25}, t) = (p_i)^{(4)} \quad 118$$

$$\lim_{G \to \infty} (b_i^{''})^{(4)}\big((G_{27}), t\big) = (r_i)^{(4)}$$

**Definition of**$(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i^{''})^{(4)}(T_{25}^{'}, t) - (a_i^{''})^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i^{''})^{(4)}((G_{27})^{'}, t) - (b_i^{''})^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'}, t)$ and$(a_i^{''})^{(4)}(T_{25}, t)$ .$(T_{25}^{'}, t)$and$(T_{25}, t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i^{''})^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would

be absolutely continuous.

**Definition of** $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$,are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$which together with$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$and the constants$(a_i)^{(4)}, (a_i^{'})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}, i = 24,25,26$,satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i^{'})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i^{'})^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, i, j = 28,29,30 \quad 122$$

The functions$(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(5)},\ (r_i)^{(5)}$:

$$(a_i^{''})^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i^{''})^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i^{'})^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$\lim_{T_2 \to \infty} (a_i^{''})^{(5)}(T_{29}, t) = (p_i)^{(5)} \quad 123$$

$$\lim_{G \to \infty} (b_i^{''})^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of**$(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i^{''})^{(5)}(T_{29}^{'}, t) - (a_i^{''})^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}^{'}|e^{-(\widehat{M}_{28})^{(5)}t}$$

$$|(b_i^{''})^{(5)}((G_{31})^{'}, t) - (b_i^{''})^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\widehat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'}, t)$ and$(a_i^{''})^{(5)}(T_{29}, t)$ .$(T_{29}^{'}, t)$and$(T_{29}, t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\widehat{M}_{28})^{(5)}\big]$. It is to

be noted that $(a_i^{''})^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{28})^{(5)}=1$ then the function $(a_i^{''})^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{28})^{(5)},(\hat{k}_{28})^{(5)}$ : 125

$(\widehat{M}_{28})^{(5)},(\hat{k}_{28})^{(5)}$,are positive constants

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}<1$$

**Definition of** $(\hat{P}_{28})^{(5)},(\hat{Q}_{28})^{(5)}$ : 126

There exists two constants$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$which together with$(\widehat{M}_{28})^{(5)},(\hat{k}_{28})^{(5)},(\hat{A}_{28})^{(5)}$ *and* $(\hat{B}_{28})^{(5)}$and the constants$(a_i)^{(5)},(a_i^{'})^{(5)},(b_i)^{(5)},(b_i^{'})^{(5)},(p_i)^{(5)},\ (r_i)^{(5)}, i=28,29,30$,satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[\,(a_i)^{(5)}+(a_i^{'})^{(5)}+\ (\hat{A}_{28})^{(5)}+\ (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}]<1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[\ (b_i)^{(5)}+(b_i^{'})^{(5)}+\ (\hat{B}_{28})^{(5)}+\ (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}]<1$$

Where we suppose

$$(a_i)^{(6)},(a_i^{'})^{(6)},(a_i^{''})^{(6)},(b_i)^{(6)},(b_i^{'})^{(6)},(b_i^{''})^{(6)}>0, i,j=32,33,34 \qquad 127$$

The functions$(a_i^{''})^{(6)},(b_i^{''})^{(6)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(6)},\ (r_i)^{(6)}$:

$$(a_i^{''})^{(6)}(T_{33},t)\le(p_i)^{(6)}\le(\hat{A}_{32})^{(6)}$$

$$(b_i^{''})^{(6)}((G_{35}),t)\le\ (r_i)^{(6)}\le(b_i^{'})^{(6)}\le(\hat{B}_{32})^{(6)}$$

$$\lim_{T_2\to\infty}(a_i^{''})^{(6)}(T_{33},t)=(p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i^{''})^{(6)}\big((G_{35}),t\big)=\ (r_i)^{(6)}$$

**Definition of**$(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)},(p_i)^{(6)},\ (r_i)^{(6)}}$ are positive constantsand $\boxed{i=32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(6)}(T_{33}^{'},t)-(a_i^{''})^{(6)}(T_{33},t)|\le(\hat{k}_{32})^{(6)}|T_{33}-\ T_{33}^{'}|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i^{''})^{(6)}((G_{35})^{'},t)-(b_i^{''})^{(6)}\big((G_{35}),t\big)|<(\hat{k}_{32})^{(6)}||(G_{35})-(G_{35})^{'}||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'},t)$ and$(a_i^{''})^{(6)}(T_{33},t)$ .$(T_{33}^{'},t)$and$(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)},(\widehat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)}=1$ then the function $(a_i^{''})^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$,are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ ,\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}<1$$

**Definition of** $(\hat{P}_{32})^{(6)},(\hat{Q}_{32})^{(6)}$ : 130

There exists two constants$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$which together with$(\widehat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)},(\hat{A}_{32})^{(6)}$*and* $(\hat{B}_{32})^{(6)}$and the constants$(a_i)^{(6)},(a_i^{'})^{(6)},(b_i)^{(6)},(b_i^{'})^{(6)},(p_i)^{(6)},\ (r_i)^{(6)},i=32,33,34,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left[(a_i)^{(6)}+(a_i^{'})^{(6)}+\ (\hat{A}_{32})^{(6)}+\ (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right]<1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left[\ (b_i)^{(6)}+(b_i^{'})^{(6)}+\ (\hat{B}_{32})^{(6)}+\ (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right]<1$$

Where we suppose

(A) $(a_i)^{(7)},(a_i^{'})^{(7)},(a_i^{''})^{(7)},(b_i)^{(7)},(b_i^{'})^{(7)},(b_i^{''})^{(7)}>0,i,j=36,37,38$ 131

(B) The functions$(a_i^{''})^{(7)},(b_i^{''})^{(7)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(7)},\ (r_i)^{(7)}$:

$$(a_i^{''})^{(7)}(T_{37},t)\le(p_i)^{(7)}\le(\hat{A}_{36})^{(7)}$$

$$(b_i^{''})^{(7)}(G_{39},t)\le\ (r_i)^{(7)}\le(b_i^{'})^{(7)}\le(\hat{B}_{36})^{(7)}$$

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*(C)* $\lim_{T_2\to\infty}(a_i^{''})^{(7)}\,(T_{37},t)=(p_i)^{(7)}$

*(D)*

$$\lim_{G\to\infty}(b_i^{''})^{(7)}\left((G_{39}),t\right)=\ (r_i)^{(7)}$$

**Definition of**$(\hat{A}_{36})^{(7)},(\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)},(\hat{B}_{36})^{(7)},(p_i)^{(7)},\ (r_i)^{(7)}}$ are positive constants and $\boxed{i=36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i^{''})^{(7)}(T_{37}^{'},t)-(a_i^{''})^{(7)}(T_{37},t)|\leq(\hat{k}_{36})^{(7)}|T_{37}-T_{37}^{'}|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i^{''})^{(7)}((G_{39})^{'},t)-(b_i^{''})^{(7)}\big((G_{39}),t\big)|<(\hat{k}_{36})^{(7)}||(G_{39})-(G_{39})^{'}||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(7)}(T_{37}^{'},t)$ and$(a_i^{''})^{(7)}(T_{37},t)$ .$(T_{37}^{'},t)$and$(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)},(\hat{M}_{36})^{(7)}\big]$ . It is to be noted that $(a_i^{''})^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)}=1$ then the function $(a_i^{''})^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)},(\hat{k}_{36})^{(7)}$ : 134

(E) $(\hat{M}_{36})^{(7)},(\hat{k}_{36})^{(7)}$,are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}},\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}<1$$

**Definition of** $(\hat{P}_{36})^{(7)},(\hat{Q}_{36})^{(7)}$ : 135

(F) There exists two constants$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$which together with$(\hat{M}_{36})^{(7)},(\hat{k}_{36})^{(7)},(\hat{A}_{36})^{(7)}$*and* $(\hat{B}_{36})^{(7)}$and the constants$(a_i)^{(7)},(a_i^{'})^{(7)},(b_i)^{(7)},(b_i^{'})^{(7)},(p_i)^{(7)},\ (r_i)^{(7)},i=36,37,38$,satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)}+(a_i^{'})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}]<1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)}+(b_i^{'})^{(7)}+(\hat{B}_{36})^{(7)}+(\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}]<1$$

Where we suppose

$$(a_i)^{(8)},(a_i^{'})^{(8)},(a_i^{''})^{(8)},(b_i)^{(8)},(b_i^{'})^{(8)},(b_i^{''})^{(8)}>0,i,j=40,41,42 \quad 136$$

The functions$(a_i^{''})^{(8)},(b_i^{''})^{(8)}$are positive continuousincreasing and bounded

**Definition of**$(p_i)^{(8)},\ (r_i)^{(8)}$: 137

$$(a_i^{''})^{(8)}(T_{41},t)\leq(p_i)^{(8)}\leq(\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i^{''})^{(8)}((G_{43}),t)\leq\ (r_i)^{(8)}\leq(b_i^{'})^{(8)}\leq(\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i^{''})^{(8)}(T_{41},t)=(p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i^{''})^{(8)}\left((G_{43}),t\right) = (r_i)^{(8)} \qquad 141$$

**Definition of**$(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(8)}(T_{41}^{'},t) - (a_i^{''})^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}^{'}|e^{-(\hat{M}_{40})^{(8)}t} \qquad 142$$

$$|(b_i^{''})^{(8)}((G_{43})^{'},t) - (b_i^{''})^{(8)}\left((G_{43}),t\right)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})^{'}||e^{-(\hat{M}_{40})^{(8)}t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(8)}(T_{41}^{'},t)$and $(a_i^{''})^{(8)}(T_{41},t)$ .$(T_{41}^{'},t)$and$(T_{41},t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$ . It is to be noted that $(a_i^{''})^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i^{''})^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$,are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$which together with$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$and the constants$(a_i)^{(8)}, (a_i^{'})^{(8)}, (b_i)^{(8)}, (b_i^{'})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i^{'})^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i^{'})^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i^{'})^{(9)}, (a_i^{''})^{(9)}, (b_i)^{(9)}, (b_i^{'})^{(9)}, (b_i^{''})^{(9)} > 0, i,j = 44,45,46 \qquad 146\text{A}$$

The functions$(a_i^{''})^{(9)}, (b_i^{''})^{(9)}$are positive continuousincreasing and bounded.

**Definition of**$(p_i)^{(9)}, (r_i)^{(9)}$:

$$(a_i^{''})^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i^{''})^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i^{'})^{(9)} \le (\hat{B}_{44})^{(9)}$$

$\lim_{T_2\to\infty} (a_i^{''})^{(9)}\,(T_{45},t) = (p_i)^{(9)}$

$$\lim_{G\to\infty} (b_i^{''})^{(9)}\,(G_{47},t) = \ (r_i)^{(9)}$$

**Definition of**$(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)},\ \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i^{''})^{(9)}(T_{45}^{'},t) - (a_i^{''})^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}^{'}|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i^{''})^{(9)}((G_{47})^{'},t) - (b_i^{''})^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})^{'}||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(9)}(T_{45}^{'},t)$ and$(a_i^{''})^{(9)}(T_{45},t)$ .$(T_{45}^{'},t)$and$(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$ . It is to be noted that $(a_i^{''})^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i^{''})^{(9)}(T_{45},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$,are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants$(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$which together with$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)}$*and* $(\hat{B}_{44})^{(9)}$and the constants$(a_i)^{(9)}, (a_i^{'})^{(9)}, (b_i)^{(9)}, (b_i^{'})^{(9)}, (p_i)^{(9)},\ \ (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i^{'})^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i^{'})^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$G_i(t) \le \ \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \ (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)$ , $T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)$ , $T_i(0)$ :

$$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153
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**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \,,\; T_i(0) = T_i^0 \,,\; G_i^0 \leq (\hat{P}_{13})^{(1)} \,, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a_{13}^{'})^{(1)} + a_{13}^{''})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a_{14}^{'})^{(1)} + (a_{14}^{''})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{14}\left(s_{(13)}\right)\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{15}\left(s_{(13)}\right)\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a_{16}^{'})^{(2)} + a_{16}^{''})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a_{20}^{'})^{(3)} + a_{20}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \,,\; T_i(0) = T_i^0 \,,\; G_i^0 \le (\,\hat{P}_{24}\,)^{(4)} \,, T_i^0 \le (\,\hat{Q}_{24}\,)^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\,\hat{P}_{24}\,)^{(4)} e^{(\,\hat{M}_{24}\,)^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\,\hat{Q}_{24}\,)^{(4)} e^{(\,\hat{M}_{24}\,)^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}^{'})^{(4)} + a_{24}^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\ \hat{P}_{28}\ )^{(5)} \ , T_i^0 \leq (\ \hat{Q}_{28}\ )^{(5)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\ \hat{P}_{28}\ )^{(5)} e^{(\ \hat{M}_{28}\ )^{(5)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\ \hat{Q}_{28}\ )^{(5)} e^{(\ \hat{M}_{28}\ )^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}\left(s_{(28)}\right) - \left( (a_{28}^{'})^{(5)} + a_{28}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{28}\left(s_{(28)}\right) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}\left(s_{(28)}\right) - \left( (a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{29}\left(s_{(28)}\right) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}\left(s_{(28)}\right) - \left( (a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)} \left(T_{29}\left(s_{(28)}\right), s_{(28)}\right) \right) G_{30}\left(s_{(28)}\right) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}\left(s_{(28)}\right) - \left( (b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)} \left(G_{31}\left(s_{(28)}\right), s_{(28)}\right) \right) T_{28}\left(s_{(28)}\right) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}\left(s_{(28)}\right) - \left( (b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)} \left(G_{31}\left(s_{(28)}\right), s_{(28)}\right) \right) T_{29}\left(s_{(28)}\right) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}\left(s_{(28)}\right) - \left( (b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)} \left(G_{31}\left(s_{(28)}\right), s_{(28)}\right) \right) T_{30}\left(s_{(28)}\right) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\ \hat{P}_{32}\ )^{(6)} \ , T_i^0 \leq (\ \hat{Q}_{32}\ )^{(6)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\ \hat{P}_{32}\ )^{(6)} e^{(\ \hat{M}_{32}\ )^{(6)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\ \hat{Q}_{32}\ )^{(6)} e^{(\ \hat{M}_{32}\ )^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}\left(s_{(32)}\right) - \left( (a_{32}^{'})^{(6)} + a_{32}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{32}\left(s_{(32)}\right) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}\left(s_{(32)}\right) - \left( (a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{33}\left(s_{(32)}\right) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}\left(s_{(32)}\right) - \left( (a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)} \left(T_{33}\left(s_{(32)}\right), s_{(32)}\right) \right) G_{34}\left(s_{(32)}\right) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)} \left( G_{35}(s_{(32)}), s_{(32)} \right) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)} \left( G_{35}(s_{(32)}), s_{(32)} \right) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)} \left( G_{35}(s_{(32)}), s_{(32)} \right) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{36})^{(7)} \text{ , } T_i^0 \leq (\hat{Q}_{36})^{(7)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^{0} + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a_{36}^{'})^{(7)} + a_{36}^{''})^{(7)} \left( T_{37}(s_{(36)}), s_{(36)} \right) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^{0} + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a_{37}^{'})^{(7)} + (a_{37}^{''})^{(7)} \left( T_{37}(s_{(36)}), s_{(36)} \right) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^{0} + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)} \left( T_{37}(s_{(36)}), s_{(36)} \right) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^{0} + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)} \left( G_{39}(s_{(36)}), s_{(36)} \right) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^{0} + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b_{37}^{'})^{(7)} - (b_{37}^{''})^{(7)} \left( G_{39}(s_{(36)}), s_{(36)} \right) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)} \left( G_{39}(s_{(36)}), s_{(36)} \right) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{40})^{(8)} \text{ , } T_i^0 \leq (\hat{Q}_{40})^{(8)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a_{40}^{'})^{(8)} + a_{40}^{''})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a_{41}^{'})^{(8)} + (a_{41}^{''})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a_{42}^{'})^{(8)} + (a_{42}^{''})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b_{40}^{'})^{(8)} - (b_{40}^{''})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b_{41}^{'})^{(8)} - (b_{41}^{''})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b_{42}^{'})^{(8)} - (b_{42}^{''})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{44})^{(9)} \text{ , } T_i^0 \leq (\hat{Q}_{44})^{(9)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a_{44}^{'})^{(9)} + a_{44}^{''})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[ (a_{45})^{(9)} G_{44}(s_{(44)}) - \left( (a_{45}^{'})^{(9)} + (a_{45}^{''})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[ (a_{46})^{(9)} G_{45}(s_{(44)}) - \left( (a_{46}^{'})^{(9)} + (a_{46}^{''})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^{0} + \int_{0}^{t} \left[ (b_{44})^{(9)} T_{45}\left(s_{(44)}\right) - \left( (b_{44}^{'})^{(9)} - (b_{44}^{''})^{(9)} \left( G_{47}\left(s_{(44)}\right), s_{(44)} \right) \right) T_{44}\left(s_{(44)}\right) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^{0} + \int_{0}^{t} \left[ (b_{45})^{(9)} T_{44}\left(s_{(44)}\right) - \left( (b_{45}^{'})^{(9)} - (b_{45}^{''})^{(9)} \left( G_{47}\left(s_{(44)}\right), s_{(44)} \right) \right) T_{45}\left(s_{(44)}\right) \right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^{0} + \int_{0}^{t} \left[ (b_{46})^{(9)} T_{45}\left(s_{(44)}\right) - \left( (b_{46}^{'})^{(9)} - (b_{46}^{''})^{(9)} \left( G_{47}\left(s_{(44)}\right), s_{(44)} \right) \right) T_{46}\left(s_{(44)}\right) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^{0} + \int_{0}^{t} \left[ (a_{13})^{(1)} \left( G_{14}^{0} + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^{0} + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left( e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^{0}) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^{0} \right) e^{\left( -\frac{(\hat{P}_{13})^{(1)} + G_{14}^{0}}{G_{14}^{0}} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_{i}^{0})$is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}$ , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^{0} + \int_{0}^{t} \left[ (a_{16})^{(2)} \left( G_{17}^{0} + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} \qquad 169$$

$$= \left(1 + (a_{16})^{(2)} t\right) G_{17}^{0} + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left( e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^{0}) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[ \left( (\hat{P}_{16})^{(2)} + G_{17}^{0} \right) e^{\left( -\frac{(\hat{P}_{16})^{(2)} + G_{17}^{0}}{G_{17}^{0}} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}$ , $G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^{0} + \int_{0}^{t} \left[ (a_{20})^{(3)} \left( G_{21}^{0} + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)}+G_{21}^{0}\right)e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}$ , $G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{P}_{24})^{(4)}+G_{25}^{0}\right)e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)}+G_{29}^{0}\right)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)}+G_{33}^{0}\right)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(a) The operator $\mathcal{A}^{(7)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)}+G_{37}^{0}\right)e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$maps the space of functions satisfying Equations into itself .Indeed it is obvious that 180

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}=$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)}+G_{41}^{0}\right)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 8
Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \qquad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \qquad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator$\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of**$\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_{0}^{t}\{(a_{13}^{'})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a_{13}^{''})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a_{13}^{''})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}^{''})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a_{13}^{'})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right) \quad 186$$

And analogous inequalities for$G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{13}^{''})^{(1)}$ and $(b_{13}^{''})^{(1)}$depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_{+}$.

If insteadof proving the existence of the solution on$\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(1)}$ and $(b_i^{''})^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0e^{\left[-\int_0^t\{(a_i^{'})^{(1)}-(a_i^{''})^{(1)}\left(T_{14}\left(s_{(13)}\right),s_{(13)}\right)\}ds_{(13)}\right]}\geq 0$$

$T_i\ (t)\geq T_i^0e^{\left(-(b_i^{'})^{(1)}t\right)}>0$ for $\mathrm{t}>0$

**Definition of**$\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}^{'})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}^{'})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}^{'})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(1)}(G(t),t)) = (b_{14}^{'})^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i^{''})^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}^{''})^{(1)}(G(t),t) = (b_{15}^{'})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of**$\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a_{16}^{'})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a_{16}^{''})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a_{16}^{''})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}^{''})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\begin{aligned}\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|&e^{-(\widehat{M}_{16})^{(2)}t}\\&\leq\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a_{16}^{'})^{(2)}+(\widehat{A}_{16})^{(2)}\right.\\&\left.+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)\end{aligned}$$

And analogous inequalities for$G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:**The fact that we supposed $(a_{16}^{''})^{(2)}$ and $(b_{16}^{''})^{(2)}$depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If insteadof proving the existence of the solution on$\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(2)}$ and $(b_i^{''})^{(2)}, i=16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i\ (t)=0$ and $T_i\ (t)=0$ 200

it results

$$G_i\ (t)\geq G_i^0e^{\left[-\int_0^t\{(a_i^{'})^{(2)}-(a_i^{''})^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$$

$T_i\ (t)\geq T_i^0e^{\left(-(b_i^{'})^{(2)}t\right)}>0$ for $t>0$

**Definition of**$\left((\widehat{M}_{16})^{(2)}\right)_1,\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$ : 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}^{'})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}^{'})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}^{'})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(2)}((G_{19})(t), t)) = (b_{17}^{'})^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i^{''})^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}^{''})^{(2)}\left((G_{19})(t), t\right) = (b_{18}^{'})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator$\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(\left((G_{23})^{(1)},(T_{23})^{(1)}\right),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**<u>Definition of</u>**$\widetilde{G_{23}},\widetilde{T_{23}}:\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right)=\mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a_{20}^{'})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a_{20}^{''})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a_{20}^{''})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a_{20}^{''})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a_{20}^{'})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$ 214

And analogous inequalities for$G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**<u>Remark 11:</u>** The fact that we supposed $(a_{20}^{''})^{(3)}$ and $(b_{20}^{''})^{(3)}$depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If insteadof proving the existence of the solution on$\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(3)}$ and $(b_i^{''})^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**<u>Remark 12:</u>** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i^{'})^{(3)}-(a_i^{''})^{(3)}\left(T_{21}\left(s_{(20)}\right),s_{(20)}\right)\}ds_{(20)}\right]}\geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(3)}t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}$ *and* $G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}^{'})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}^{'})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}^{'})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(3)}\left((G_{23})(t), t\right) = (b_{21}^{'})^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i^{''})^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}^{''})^{(3)}\left((G_{23})(t), t\right) = (b_{22}^{'})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\widehat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\widehat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{24})^{(4)}\right] \leq (\widehat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(\left((G_{27})^{(1)},(T_{27})^{(1)}\right),\left((G_{27})^{(2)},(T_{27})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a_{24}^{'})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a_{24}^{''})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a_{24}^{''})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\begin{aligned}&\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\\&\leq\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a_{24}^{'})^{(4)}+(\widehat{A}_{24})^{(4)}\right.\\&\left.+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)}\right)\right)\end{aligned}$$ 226

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a_{24}^{''})^{(4)}$ and $(b_{24}^{''})^{(4)}$ depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If insteadof proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}, i=24,25,26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0$ *and* $T_i\,(t)=0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(4)} - (a_i^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(4)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}$ *and* $G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}^{'})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}^{'})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}^{'})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(4)}\,((G_{27})(t), t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t), t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \qquad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \qquad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(\left((G_{31})^{(1)},(T_{31})^{(1)}\right),\left((G_{31})^{(2)},(T_{31})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})},\widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a_{28}^{'})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a_{28}^{''})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a_{28}^{''})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}^{''})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\begin{aligned}\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t} \qquad 237\\ \leq\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}^{'})^{(5)}+(\widehat{A}_{28})^{(5)}\right.\\ \left.+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(\left((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)}\right)\right)\end{aligned}$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a_{28}^{''})^{(5)}$ and $(b_{28}^{''})^{(5)}$ depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 238

$(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$.

If insteadof proving the existence of the solution on$\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(5)}$ and $(b_i^{''})^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\ (t) = 0\ and\ T_i\ (t) = 0$ 239

it results

$$G_i\ (t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(5)} - (a_i^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$T_i\ (t) \geq T_i^0 e^{\left(-(b_i^{'})^{(5)}t\right)} > 0$ for t > 0

**Definition of**$\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:**if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}^{'})^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}^{'})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}^{'})^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for$G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:**If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(5)}\ ((G_{31})(t), t)) = (b_{29}^{'})^{(5)}$ then $T_{29}\ \to \infty$. 242

**Definition of**$(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i^{''})^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\ (t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29}\ \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$it results

$T_{29}\ \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}^{''})^{(5)}\left((G_{31})(t), t\right) = (b_{30}^{'})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator$\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}\right), \left((G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**<u>Definition of</u>**$(\widetilde{G_{35}}), (\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}), (\widetilde{T_{35}})\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{32})^{(6)}\left|G_{33}^{(1)} - G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)}\left|G_{32}^{(1)} - G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}} +$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}t} \leq \frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) \quad 248$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If insteadof proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})$ $(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(6)} - (a''_i)^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}$ *and* $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a'_{34})^{(6)}$$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\,((G_{35})(t), t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}^{''})^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}^{'})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator$\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(\left((G_{39})^{(1)}, (T_{39})^{(1)}\right), \left((G_{39})^{(2)}, (T_{39})^{(2)}\right)\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**<u>Definition of</u>**$\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$
$$\int_0^t \{(a_{36}^{'})^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$
$$G_{36}^{(2)}\left|(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}^{''})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)}t} \leq \frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right) d\left(\left((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)}\right)\right) \quad 259$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If insteadof proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})$ (*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{(-(b'_i)^{(7)}t)} > 0$ for $t > 0$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(7)}((G_{39})(t),t)) = (b_{37}^{'})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1-e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t),t\right) = (b_{38}^{'})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \tag{267}$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \tag{268}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right) = \sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\} \tag{269}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right) = \mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\, ds_{(40)}+$$
$$\int_{0}^{t}\{(a_{40}^{'})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{-(\widehat{M}_{40})^{(8)} s_{(40)}}+$$
$$(a_{40}^{''})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a_{40}^{''})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)-(a_{40}^{''})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right| \; e^{-(\widehat{M}_{40})^{(8)} s_{(40)}} e^{(\widehat{M}_{40})^{(8)} s_{(40)}}\} ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right| e^{-(\widehat{M}_{40})^{(8)} t} \leq \frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a_{40}^{'})^{(8)}+(\widehat{A}_{40})^{(8)}\right.$$
$$\left.+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right) d\left(\left((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)}\right)\right)$$ 273

And analogous inequalities for$G_i$ $and$ $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a_{40}^{''})^{(8)}$ and $(b_{40}^{''})^{(8)}$depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ $and$ $(\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)} t}$ respectively of $\mathbb{R}_{+}$. 274

If insteadof proving the existence of the solution on$\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(8)}$ and $(b_i^{''})^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0$ $and$ $T_i\,(t)=0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i^{'})^{(8)}-(a_i^{''})^{(8)}(T_{41}(s_{(40)}),s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(8)} t\right)} > 0$ for t > 0

**Definition of**$\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:**if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}^{'})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}^{'})^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}^{'})^{(8)}$$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(8)}((G_{43})(t),t)) = (b_{41}^{'})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i^{''})^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b_{42}^{''})^{(8)}\left((G_{43})(t),t(t),t\right) = (b_{42}^{'})^{(8)}$

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It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(\left((G_{47})^{(1)},(T_{47})^{(1)}\right),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})}\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{44})^{(9)} \left|G_{45}^{(1)} - G_{45}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}}\, ds_{(44)} +$$
$$\int_0^t \{(a_{44}^{'})^{(9)} \left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$(a_{44}^{''})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} +$$
$$G_{44}^{(2)} \left|(a_{44}^{''})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a_{44}^{''})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right| \; e^{-(\widehat{M}_{44})^{(9)} s_{(44)}} e^{(\widehat{M}_{44})^{(9)} s_{(44)}} \} ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{44})^{(9)} t}$$
$$\leq \frac{1}{(\widehat{M}_{44})^{(9)}} \left((a_{44})^{(9)} + (a_{44}^{'})^{(9)} + (\widehat{A}_{44})^{(9)}\right.$$
$$\left. + (\widehat{P}_{44})^{(9)} (\widehat{k}_{44})^{(9)}\right) d\left(\left((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a_{44}^{''})^{(9)}$ and $(b_{44}^{''})^{(9)}$ depending also ontcan be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ *and* $(\widehat{Q}_{44})^{(9)} e^{(\widehat{M}_{44})^{(9)} t}$ respectively of $\mathbb{R}_+$.

If insteadof proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(9)}$ and $(b_i^{''})^{(9)}, i = 44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})$ $(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(9)} - (a_i^{''})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(9)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}^{'})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)} \left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}^{'})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}^{'})^{(9)}$$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for$G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:**If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(9)}\left((G_{47})(t),t\right) = (b_{45}^{'})^{(9)}$ then $T_{45} \to \infty$.

**Definition of**$(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i^{''})^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}^{''})^{(9)}\left((G_{47})(t),t\right) = (b_{46}^{'})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of**$(\sigma_1)^{(1)}$ , $(\sigma_2)^{(1)}$ , $(\tau_1)^{(1)}$ , $(\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}$ , $(\sigma_2)^{(1)}$ , $(\tau_1)^{(1)}$ , $(\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} - (a_{13}^{''})^{(1)}(T_{14}\,,t) + (a_{14}^{''})^{(1)}(T_{14}\,,t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G,t) - (b_{14}^{''})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$$

**Definition of**$(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**$(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**$(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \qquad if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$$

$$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \qquad if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$$

and analogously 284

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \qquad if\ (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$where$(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t)\right. \tag{286}$$

$$\left. \le \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a_{15}')^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a_{15}')^{(1)}t}\right] + G_{15}^0 e^{-(a_{15}')^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \tag{287}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \tag{288}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b_{15}')^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b_{15}')^{(1)}t}\right] + T_{15}^0 e^{-(b_{15}')^{(1)}t} \le T_{15}(t) \le \tag{289}$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ :

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ :

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ :

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :-

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \qquad \boldsymbol{if}\,(\nu_0)^{(2)} < (\nu_1)^{(2)}$$

$$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\,(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)},$$

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}}$

$$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$$

and analogously

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \qquad \boldsymbol{if}(u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \qquad \boldsymbol{if}(\bar{u}_1)^{(2)} < (u_0)^{(2)} \qquad 309$$

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \qquad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t)\right. \qquad 312$$

$$\left. \le \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a_{18}')^{(2)})}[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \qquad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \qquad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18}')^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \le T_{18}(t) \le \qquad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**<u>Definition of</u>**$(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)} \qquad 318$$

$$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$$

**<u>Behavior of the solutions</u>** 319

**<u>Theorem 3:</u>** If we denote and define

**<u>Definition of</u>**$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \leq -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}, \qquad \boldsymbol{if}\,(\nu_0)^{(3)} < (\nu_1)^{(3)}$$

$$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}, \boldsymbol{if}\,(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)},$$

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$$

and analogously 322

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \qquad \boldsymbol{if}\,(u_0)^{(3)} < (u_1)^{(3)}$$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\,(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \qquad \boldsymbol{if}\,(\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \qquad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t}-e^{-(S_2)^{(3)}t}\right]+G_{22}^{0}e^{-(S_2)^{(3)}t}\leq G_{22}(t)\right. \quad 324$$

$$\left.\leq\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}((S_1)^{(3)}-(a_{22}^{'})^{(3)})}[e^{(S_1)^{(3)}t}-e^{-(a_{22}^{'})^{(3)}t}]+G_{22}^{0}e^{-(a_{22}^{'})^{(3)}t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\leq T_{20}(t)\leq T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\leq T_{20}(t)\leq\frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b_{22}^{'})^{(3)})}\left[e^{(R_1)^{(3)}t}-e^{-(b_{22}^{'})^{(3)}t}\right]+T_{22}^{0}e^{-(b_{22}^{'})^{(3)}t}\leq T_{22}(t)\leq \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 328

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a_{20}^{'})^{(3)}$

$$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$$

$$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b_{20}^{'})^{(3)}$$

$$(R_2)^{(3)}=(b_{22}^{'})^{(3)}-(r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)}\leq-(a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}-(a_{24}^{''})^{(4)}(T_{25},t)+(a_{25}^{''})^{(4)}(T_{25},t)\leq-(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)}\leq-(b_{24}^{'})^{(4)}+(b_{25}^{'})^{(4)}-(b_{24}^{''})^{(4)}\big((G_{27}),t\big)-(b_{25}^{''})^{(4)}\big((G_{27}),t\big)\leq-(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)},(\nu_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},\nu^{(4)},u^{(4)}$ : 329

By $(\nu_1)^{(4)}>0$ , $(\nu_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0$ , $(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2+(\sigma_1)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},,(\bar{\nu}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)}>0$ , $(\bar{\nu}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0$ , $(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2+(\sigma_2)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \qquad \boldsymbol{if} (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if} (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}, \qquad \boldsymbol{if} (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$$

and analogously

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$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \qquad \boldsymbol{if} (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if} (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \qquad 333$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \right. \qquad 334$$

$$\left. \le \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a_{26}')^{(4)})} \left[ e^{(S_1)^{(4)} t} - e^{-(a_{26}')^{(4)} t} \right] + G_{26}^0 e^{-(a_{26}')^{(4)} t} \right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \qquad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b_{26}')^{(4)})} \left[ e^{(R_1)^{(4)} t} - e^{-(b_{26}')^{(4)} t} \right] + T_{26}^0 e^{-(b_{26}')^{(4)} t} \le T_{26}(t) \le \qquad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a_{24}')^{(4)}$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \qquad \boldsymbol{if}\,(\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\,(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \qquad \boldsymbol{if}\,(u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\,(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\,(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \leq G_{30}(t)\right. \quad 344$$

$$\left. \leq \frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a_{30}^{'})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a_{30}^{'})^{(5)}t}\right]+G_{30}^{0}e^{-(a_{30}^{'})^{(5)}t}\right)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b_{30}^{'})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b_{30}^{'})^{(5)}t}\right]+T_{30}^{0}e^{-(b_{30}^{'})^{(5)}t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**<u>Definition of</u>**$(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a_{28}^{'})^{(5)}$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b_{28}^{'})^{(5)}$$

$$(R_2)^{(5)} = (b_{30}^{'})^{(5)} - (r_{30})^{(5)}$$

**<u>Behavior of the solutions of equation</u>** 349

**<u>Theorem 2:</u>** If we denote and define

**<u>Definition of</u>**$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a_{32}^{'})^{(6)} + (a_{33}^{'})^{(6)} - (a_{32}^{''})^{(6)}(T_{33}, t) + (a_{33}^{''})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b_{32}^{'})^{(6)} + (b_{33}^{'})^{(6)} - (b_{32}^{''})^{(6)}\big((G_{35}), t\big) - (b_{33}^{''})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

**<u>Definition of</u>**$(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \qquad \boldsymbol{if}\,(\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\,(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \qquad \boldsymbol{if}\,(u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\,(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\,(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \qquad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t)\right. \qquad 355$$

$$\left. \le \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a_{34}^{'})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a_{34}^{'})^{(6)}t}\right] + G_{34}^0 e^{-(a_{34}^{'})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{\left((R_1)^{(6)} + (r_{32})^{(6)}\right)t}} \qquad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{\left((R_1)^{(6)} + (r_{32})^{(6)}\right)t} \qquad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}^{'})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b_{34}^{'})^{(6)}t}\right]+T_{34}^{0}e^{-(b_{34}^{'})^{(6)}t}\leq T_{34}(t)\leq$$ 358

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a_{32}^{'})^{(6)}$

$$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$$

$$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b_{32}^{'})^{(6)}$$

$$(R_2)^{(6)}=(b_{34}^{'})^{(6)}-(r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\leq-(a_{36}^{'})^{(7)}+(a_{37}^{'})^{(7)}-(a_{36}^{''})^{(7)}(T_{37},t)+(a_{37}^{''})^{(7)}(T_{37},t)\leq-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\leq-(b_{36}^{'})^{(7)}+(b_{37}^{'})^{(7)}-(b_{36}^{''})^{(7)}\big((G_{39}),t\big)-(b_{37}^{''})^{(7)}\big((G_{39}),t\big)\leq-(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)},(\nu_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},\nu^{(7)},u^{(7)}$ : 361

By $(\nu_1)^{(7)}>0,(\nu_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_1)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\left(u^{(7)}\right)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},,(\bar{\nu}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)}>0,(\bar{\nu}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\left(\nu^{(7)}\right)^2+(\sigma_2)^{(7)}\nu^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\left(u^{(7)}\right)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)}$ by

$$(m_2)^{(7)}=(\nu_0)^{(7)},(m_1)^{(7)}=(\nu_1)^{(7)},\qquad \boldsymbol{if}\,(\nu_0)^{(7)}<(\nu_1)^{(7)}$$

$$(m_2)^{(7)}=(\nu_1)^{(7)},(m_1)^{(7)}=(\bar{\nu}_1)^{(7)},\boldsymbol{if}\,(\nu_1)^{(7)}<(\nu_0)^{(7)}<(\bar{\nu}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \qquad \boldsymbol{if} (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \qquad \boldsymbol{if} (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if} (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \qquad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t)\right. \qquad 366$$

$$\left. \le \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a_{38}')^{(7)})} [e^{(S_1)^{(7)} t} - e^{-(a_{38}')^{(7)} t}] + G_{38}^0 e^{-(a_{38}')^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \qquad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \qquad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b_{38}')^{(7)})} \left[e^{(R_1)^{(7)} t} - e^{-(b_{38}')^{(7)} t}\right] + T_{38}^0 e^{-(b_{38}')^{(7)} t} \le T_{38}(t) \le \qquad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**<u>Definition of</u>** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}')^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \qquad \boldsymbol{if}\,(\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\,(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \qquad \boldsymbol{if}\,(u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\,(\bar{u}_1)^{(8)} < (u_0)^{(8)}$where$(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \qquad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t)\right. \qquad 377$$

$$\left. \le \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a_{42}')^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a_{42}')^{(8)} t}\right] + G_{42}^0 e^{-(a_{42}')^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b_{42}')^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b_{42}')^{(8)} t}\right] + T_{42}^0 e^{-(b_{42}')^{(8)} t} \le T_{42}(t) \le \qquad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**<u>Definition of</u>**$(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a_{40}')^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b_{40}')^{(8)}$$

$$(R_2)^{(8)} = (b_{42}')^{(8)} - (r_{42})^{(8)}$$

**<u>Behavior of the solutions of equation 37 to 92</u>** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \qquad \boldsymbol{if}\,(\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\,(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \qquad \boldsymbol{if}\,(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \qquad \boldsymbol{if}\,(u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\,(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\,(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left( \frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})} \left[ e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \leq G_{46}(t) \leq \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a_{46}')^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a_{46}')^{(9)}t}] + G_{46}^0 e^{-(a_{46}')^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \leq T_{44}(t) \leq T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b_{46}')^{(9)})} \left[ e^{(R_1)^{(9)}t} - e^{-(b_{46}')^{(9)}t} \right] + T_{46}^0 e^{-(b_{46}')^{(9)}t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[ e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t} \right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a_{44}')^{(9)}$

$$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$$

$$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b_{44}')^{(9)}$$

$$(R_2)^{(9)} = (b_{46}')^{(9)} - (r_{46})^{(9)}$$

**Proof :** From global equations we obtain 383

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a_{13}')^{(1)} - (a_{14}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t) \right) - (a_{14}'')^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

**Definition of** $\nu^{(1)}$ :- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left( (a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} \right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left( (a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

$$\text{it follows } (\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**<u>Definition of</u>** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**<u>Definition of</u>** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a_{13}^{''})^{(1)} = (a_{14}^{''})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for

the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of**$\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of**$(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce$(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)})\, t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)})\, t]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}^{''})^{(2)} = (a_{17}^{''})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}^{''})^{(2)} = (b_{17}^{''})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}^{'})^{(3)} - (a_{21}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}, t)\right) - (a_{21}^{''})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of**$\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**<u>Definition of</u>**$(\bar{\nu}_1)^{(3)}$ :-

From which we deduce$(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**<u>Definition of</u>** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**<u>Definition of</u>** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a_{20}^{''})^{(3)} = (a_{21}^{''})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} =$

$(\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of**$\nu^{(4)}$ :- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of**$(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right) t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right) t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce$(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right) t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right) t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right) t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $v^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain
**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)} G_{25}(t)$**this also defines** $(v_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$,**and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t) v^{(5)} - (a_{29})^{(5)} v^{(5)}$$

**Definition of**$v^{(5)}$ :- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)} v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)} v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of**$(\bar{v}_1)^{(5)}, (v_0)^{(5)}$ :-

For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right) t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right) t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{5+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)}-(\nu_0)^{(5)}}{(\nu_0)^{(5)}-(\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}^{''})^{(5)} = (a_{29}^{''})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$**this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}^{''})^{(5)} = (b_{29}^{''})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$,**and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}^{'})^{(6)} - (a_{33}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}, t)\right) - (a_{33}^{''})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)\leq\frac{d\nu^{(6)}}{dt}\leq-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)},(\nu_0)^{(6)}$ :-

For $0<\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}<(\nu_1)^{(6)}<(\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t)\geq\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}\quad,\quad\boxed{(C)^{(6)}=\frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)}\leq\nu^{(6)}(t)\leq(\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t)\leq\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(6)}=\frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)}\leq\nu^{(6)}(t)\leq(\bar{\nu}_1)^{(6)}$

If $0<(\nu_1)^{(6)}<(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}<(\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)}\leq\frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}\leq\nu^{(6)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\leq(\bar{\nu}_1)^{(6)}$$

If $0<(\nu_1)^{(6)}\leq(\bar{\nu}_1)^{(6)}\leq\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)}\leq\nu^{(6)}(t)\leq\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}\leq(\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$(m_2)^{(6)}\leq\nu^{(6)}(t)\leq(m_1)^{(6)}$, $\boxed{\nu^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$(\mu_2)^{(6)}\leq u^{(6)}(t)\leq(\mu_1)^{(6)}$, $\boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$**this also defines** $(\nu_0)^{(6)}$ **for the special case** .
Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$,**and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of**$\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of**$(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce$(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{\left[-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})\, t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})\, t\right]}} \leq (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \leq (\bar{v}_1)^{(7)} \leq \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(v_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{\left[-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})\, t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})\, t\right]}} \leq (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $v^{(7)}(t)$ :-

$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}$, $\boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{36}^{''})^{(7)} = (a_{37}^{''})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(v_1)^{(7)} = (\bar{v}_1)^{(7)}$ if in addition $(v_0)^{(7)} = (v_1)^{(7)}$ then $v^{(7)}(t) = (v_0)^{(7)}$ and as a consequence $G_{36}(t) = (v_0)^{(7)} G_{37}(t)$**this also defines** $(v_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}^{''})^{(7)} = (b_{37}^{''})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(v_1)^{(7)}$ and $(\bar{v}_1)^{(7)}$,**and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}^{'})^{(8)} - (a_{41}^{'})^{(8)} + (a_{40}^{''})^{(8)}(T_{41}, t)\right) - (a_{41}^{''})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right) t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right) t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right) t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right) t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right) t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}$, $\boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$**this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$,**and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of**$\nu^{(9)}$ :- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \le \frac{d\nu^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of**$(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce$(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right) t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right) t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain
**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a_{44}^{''})^{(9)} = (a_{45}^{''})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$**this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b_{44}^{''})^{(9)} = (b_{45}^{''})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$,**and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a_i^{''})^{(1)} and\ (b_i^{''})^{(1)}$ are independent on $t$ , and the conditions with the notations

$$(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a_{14}^{'})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$$(b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13}^{'})^{(1)}(r_{14})^{(1)} - (b_{14}^{'})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(2)} and\ (b_i^{''})^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \qquad 427$$

$$(a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a_{17}^{'})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \qquad 428$$

$(b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$$(b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16}^{'})^{(2)}(r_{17})^{(2)} - (b_{17}^{'})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \qquad 430$$

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(3)} and\ (b_i^{''})^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$$(a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a_{21}^{'})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$(b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$$(b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20}^{'})^{(3)}(r_{21})^{(3)} - (b_{21}^{'})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i^{''})^{(4)} and\ (b_i^{''})^{(4)}$ are independent on $t$ , and the conditions with the notations

$$(a_{24}^{'})^{(4)}(a_{25}^{'})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a_{24}^{'})^{(4)}(a_{25}^{'})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a_{25}^{'})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$$(b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b_{24}^{'})^{(4)}(r_{25})^{(4)} - (b_{25}^{'})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(5)} and\ (b_i^{''})^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a_{28}^{'})^{(5)}(a_{29}^{'})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a_{29}^{'})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$$(b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b_{28}^{'})^{(5)}(r_{29})^{(5)} - (b_{29}^{'})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i^{''})^{(6)} and\ (b_i^{''})^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a_{32}^{'})^{(6)}(a_{33}^{'})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a_{33}^{'})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$$(b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32}^{'})^{(6)}(r_{33})^{(6)} - (b_{33}^{'})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(7)} and\ (b_i^{''})^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$$(a_{36}^{'})^{(7)}(a_{37}^{'})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$$

$$(a_{36}^{'})^{(7)}(a_{37}^{'})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a_{37}^{'})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$$

$(b_{36}^{'})^{(7)}(b_{37}^{'})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$$(b_{36}^{'})^{(7)}(b_{37}^{'})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b_{36}^{'})^{(7)}(r_{37})^{(7)} - (b_{37}^{'})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(8)} and\ (b_i^{''})^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$$(a_{40}^{'})^{(8)}(a_{41}^{'})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$$

$$(a_{40}^{'})^{(8)}(a_{41}^{'})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a_{41}^{'})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$$

$(b_{40}^{'})^{(8)}(b_{41}^{'})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$$(b_{40}^{'})^{(8)}(b_{41}^{'})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b_{40}^{'})^{(8)}(r_{41})^{(8)} - (b_{41}^{'})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i^{''})^{(9)} and\ (b_i^{''})^{(9)}$ are independent on $t$ , and the conditions (with the notations 45,46,27,28) 436 A

$$(a_{44}^{'})^{(9)}(a_{45}^{'})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$$

$$(a_{44}^{'})^{(9)}(a_{45}^{'})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a_{45}^{'})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$$

$(b_{44}^{'})^{(9)}(b_{45}^{'})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$$(b_{44}^{'})^{(9)}(b_{45}^{'})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b_{44}^{'})^{(9)}(r_{45})^{(9)} - (b_{45}^{'})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a_{13}^{'})^{(1)} + (a_{13}^{''})^{(1)}(T_{14})\right]G_{13} = 0 \qquad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a_{14}^{'})^{(1)} + (a_{14}^{''})^{(1)}(T_{14})\right]G_{14} = 0 \qquad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}(T_{14})\right]G_{15} = 0 \qquad 439$$

$$(b_{13})^{(1)}T_{14} - [(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G)\ ]T_{13} = 0 \qquad 440$$

$$(b_{14})^{(1)}T_{13} - [(b_{14}^{'})^{(1)} - (b_{14}^{''})^{(1)}(G)\ ]T_{14} = 0 \qquad 441$$

$$(b_{15})^{(1)}T_{14} - [(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G)\,]T_{15} = 0 \qquad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17})\right]G_{16} = 0 \qquad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17})\right]G_{17} = 0 \qquad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17})\right]G_{18} = 0 \qquad 445$$

$$(b_{16})^{(2)}T_{17} - [(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}(G_{19})\,]T_{16} = 0 \qquad 446$$

$$(b_{17})^{(2)}T_{16} - [(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}(G_{19})\,]T_{17} = 0 \qquad 447$$

$$(b_{18})^{(2)}T_{17} - [(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}(G_{19})\,]T_{18} = 0 \qquad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})\right]G_{20} = 0 \qquad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})\right]G_{21} = 0 \qquad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})\right]G_{22} = 0 \qquad 451$$

$$(b_{20})^{(3)}T_{21} - [(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23})\,]T_{20} = 0 \qquad 452$$

$$(b_{21})^{(3)}T_{20} - [(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23})\,]T_{21} = 0 \qquad 453$$

$$(b_{22})^{(3)}T_{21} - [(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23})\,]T_{22} = 0 \qquad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25})\right]G_{24} = 0 \qquad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25})\right]G_{25} = 0 \qquad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25})\right]G_{26} = 0 \qquad 457$$

$$(b_{24})^{(4)}T_{25} - [(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \qquad 458$$

$$(b_{25})^{(4)}T_{24} - [(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \qquad 459$$

$$(b_{26})^{(4)}T_{25} - [(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \qquad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29})\right]G_{28} = 0 \qquad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29})\right]G_{29} = 0 \qquad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \qquad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \qquad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \qquad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \qquad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(a) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T_{21}^*$ for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}^*)]}$$

**Definition and uniqueness of** $\Gamma_{25}^*$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T_{25}^*$ for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25}^*)]}$$

**Definition and uniqueness of** $\Gamma_{29}^*$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^*$ for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29}^*)]}$$

**Definition and uniqueness of** $\Gamma_{33}^*$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^*$ for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}^*)]}$$

**Definition and uniqueness of** $\Gamma_{37}^*$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^*$ for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a_{36}^{'})^{(7)} + (a_{36}^{''})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)}(T_{37}^*)]}$$

**Definition and uniqueness of** $\Gamma_{41}^*$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^*$ for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a_{40}^{'})^{(8)} + (a_{40}^{''})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a_{42}^{'})^{(8)} + (a_{42}^{''})^{(8)}(T_{41}^*)]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a_{44}^{'})^{(9)} + (a_{44}^{''})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a_{46}^{'})^{(9)} + (a_{46}^{''})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G) + (b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(G)\right] + (b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b_{16}^{'})^{(2)}(b_{17}^{''})^{(2)}(G_{19}) + (b_{17}^{'})^{(2)}(b_{16}^{''})^{(2)}(G_{19})\right] + (b_{16}^{''})^{(2)}(G_{19})(b_{17}^{''})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b_{20}^{'})^{(3)}(b_{21}^{''})^{(3)}(G_{23}) + (b_{21}^{'})^{(3)}(b_{20}^{''})^{(3)}(G_{23})\right] + (b_{20}^{''})^{(3)}(G_{23})(b_{21}^{''})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b_{24}^{'})^{(4)}(b_{25}^{''})^{(4)}(G_{27}) + (b_{25}^{'})^{(4)}(b_{24}^{''})^{(4)}(G_{27})\right] + (b_{24}^{''})^{(4)}(G_{27})(b_{25}^{''})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b_{28}^{'})^{(5)}(b_{29}^{''})^{(5)}(G_{31}) + (b_{29}^{'})^{(5)}(b_{28}^{''})^{(5)}(G_{31})\right] + (b_{28}^{''})^{(5)}(G_{31})(b_{29}^{''})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b_{32}^{'})^{(6)}(b_{33}^{''})^{(6)}(G_{35}) + (b_{33}^{'})^{(6)}(b_{32}^{''})^{(6)}(G_{35})\right] + (b_{32}^{''})^{(6)}(G_{35})(b_{33}^{''})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b_{36}^{'})^{(7)}(b_{37}^{'})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b_{36}^{'})^{(7)}(b_{37}^{''})^{(7)}(G_{39}) + (b_{37}^{'})^{(7)}(b_{36}^{''})^{(7)}(G_{39})\right] + (b_{36}^{''})^{(7)}(G_{39})(b_{37}^{''})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b_{40}^{'})^{(8)}(b_{41}^{'})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b_{40}^{'})^{(8)}(b_{41}^{''})^{(8)}(G_{43}) + (b_{41}^{'})^{(8)}(b_{40}^{''})^{(8)}(G_{43})\right] + (b_{40}^{''})^{(8)}(G_{43})(b_{41}^{''})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi({G_{43}}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b_{44}^{'})^{(9)}(b_{45}^{'})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b_{44}^{'})^{(9)}(b_{45}^{''})^{(9)}(G_{47}) + (b_{45}^{'})^{(9)}(b_{44}^{''})^{(9)}(G_{47})\right] + (b_{44}^{''})^{(9)}(G_{47})(b_{45}^{''})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a_{13}^{'})^{(1)} + (a_{13}^{''})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}(T_{14}^{*})]}$$

$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)} (G^{*})]}$ , $T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)} (G^{*})]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)} (T_{17}^{*})]}$ , $G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)} (T_{17}^{*})]}$ 513

$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)} ((G_{19})^{*})]}$ , $T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)} ((G_{19})^{*})]}$ 514

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)} (T_{21}^{*})]}$ , $G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)} (T_{21}^{*})]}$

$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)} (G_{23}{}^{*})]}$ , $T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)} (G_{23}{}^{*})]}$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)} (T_{25}^{*})]}$ , $G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)} (T_{25}^{*})]}$

$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)} ((G_{27})^{*})]}$ , $T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)} ((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)} (T_{29}^{*})]}$ , $G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)} (T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)} ((G_{31})^{*})]}$ , $T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)} ((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^*$ given by $\varphi((G_{35})^*) = 0$ , $T_{33}^*$ given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)} (T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)} (T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)} ((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)} ((G_{35})^*)]}$$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a_{36}^{'})^{(7)} + (a_{36}^{''})^{(7)} (T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)} (T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)} ((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)} ((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a_{40}^{'})^{(8)} + (a_{40}^{''})^{(8)} (T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a_{42}^{'})^{(8)} + (a_{42}^{''})^{(8)} (T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b_{40}^{'})^{(8)} - (b_{40}^{''})^{(8)} ((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b_{42}^{'})^{(8)} - (b_{42}^{''})^{(8)} ((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a_{44}^{'})^{(9)} + (a_{44}^{''})^{(9)} (T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a_{46}^{'})^{(9)} + (a_{46}^{''})^{(9)} (T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b_{44}^{'})^{(9)} - (b_{44}^{''})^{(9)} ((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b_{46}^{'})^{(9)} - (b_{46}^{''})^{(9)} ((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(1)}$ *and* $(b_i^{''})^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{14}^{''})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \;\; , \; \frac{\partial (b_i^{''})^{(1)}}{\partial G_j}(G^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}^{'})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(2)}$ and $(b_i^{''})^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$ 532

$\frac{\partial (a_{17}^{''})^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \;\; , \; \frac{\partial (b_i^{''})^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij}$ 533

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}\mathrm{T}_{16}^{*}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}^{'})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}_{17}^{*}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}_{18}^{*}\mathbb{G}_j\right) \quad 539$$

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**Theorem 4:**If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}$ *and* $(b_i^{''})^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}^{''})^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial (b_i^{''})^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}^{'})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}^{'})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}^{'})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}^{'})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^{*}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}^{'})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}^{'})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:**If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(4)}$ *and* $(b_i^{''})^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{25}^{''})^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \;\; , \; \frac{\partial (b_i^{''})^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}^{'})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}^{'})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}^{'})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}^{'})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}^{'})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}$ *and* $(b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{29}^{''})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \;\; , \; \frac{\partial (b_i^{''})^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}^{'})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}^{'})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^{*}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^{*}\mathbb{G}_{j}\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}^{'})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^{*}\mathbb{G}_{j}\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}^{'})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^{*}\mathbb{G}_{j}\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:**If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}$ *and* $(b_i^{''})^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**$\mathbb{G}_i, \mathbb{T}_i$ :- 564

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{33}^{''})^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i^{''})^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}^{'})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}^{'})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}^{'})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}^{'})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_{j}\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}^{'})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_{j}\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}^{'})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_{j}\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(7)}$ *and* $(b_i^{''})^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{37}^{''})^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \;\; , \frac{\partial (b_i^{''})^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}^{'})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}^{'})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}^{'})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}^{'})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}^{'})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}^{'})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(8)}$ *and* $(b_i^{''})^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{41}^{''})^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \;\; , \frac{\partial (b_i^{''})^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}^{'})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}^{'})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}^{'})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}^{'})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_{j}\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}^{'})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_{j}\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}^{'})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^{*}\mathbb{G}_{j}\right) \quad 586$$

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**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(9)}$ *and* $(b_i^{''})^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a_{45}^{''})^{(9)}}{\partial T_{45}}(T_{45}^{*}) = (q_{45})^{(9)} \quad , \frac{\partial (b_i^{''})^{(9)}}{\partial G_j}((G_{47})^{*}) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}^{'})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^{*}\mathbb{T}_{45} \quad 586\text{B}$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}^{'})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^{*}\mathbb{T}_{45} \quad 586\text{C}$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}^{'})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^{*}\mathbb{T}_{45} \quad 586\text{D}$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}^{'})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^{*}\mathbb{G}_{j}\right) \quad 586\text{E}$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\text{F}$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\text{G}$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)}+(b'_{15})^{(1)}-(r_{15})^{(1)}\right)\{\left((\lambda)^{(1)}+(a'_{15})^{(1)}+(p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14}+(b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14}+(b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

+

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

+

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$



$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

+

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

+

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$+$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$+$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$+$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

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