

# Longitudinal Automatic landing System - Design for CHARLIE Aircraft by Root-Locus

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**Abstract-** The present paper aims at designing of Automatic Landing System for CHARLIE aircraft based on root locus modern control system. The control method is used to determine the gains of the controllers. To apply the root locus method, the nonlinear model of the flight system with controllers is established and developed. Then the nonlinear model is linearized about certain operating point. A CHARLIE aircraft with flight condition parameters used for proposed flight path control system is described. The block diagram of the proposed control system with required controller gains is established. The transfer functions for open loop and then the closed loop are obtained with the help of automatic control principle. The root locus for open loop is drawn and then K values are found for given damping ratios. Finally the step responses of the closed loop system with Automatic Landing system controller autopilot are drawn. The digital simulation results prove the effectiveness of proposed control system in terms of fast response after applying external disturbances.

**Index Terms-** Aircraft, flight control, pitch Attitude, Root-Locus and PI control.

## I. INTRODUCTION

The rapid advancement of aircraft design from the very limited capabilities of Wright brothers first successfully airplane today's high performance military, commercial, and general aviation aircraft required the development of many technologies, those are aerodynamics, Structures, materials, propulsion, and flight controls. The development of automatic control system has played an important role in the growth of civil and military aviation. Modern aircraft include a variety of automatic control system that aids the flight crew in navigation, flight management and augmenting the stability characteristic of the airplane. For this situation an autopilot is designed that control the pitch of aircraft that can be used by the flight crew to lessen their workload during cruising and help them land their aircraft [1]. The autopilot is an element within the flight control system. It is a pilot relief mechanism that assists in maintaining an attitude, heading, altitude or flying to navigation or landing references [2]. Designing an autopilot requires control system theory background and knowledge of stability derivatives and different altitudes for a given airplane [3]. Lot of works been done to control the autopilot of an aircraft for the purpose of flight stability and yet this researches still remains an open issue in the present and future works [4,5,6,7,8]. The rest of the paper is organized as follows: section 2 provides background information on the Modeling of an Aircraft Equation of Motion,

while section 3 details the Aircraft Control System, section 4 presents ALS Design for CHARLIE aircraft, section 5 shows Step response of CHARLIE aircraft system and finally, section 6 presents the conclusion of this work.

## II. AIRCRAFT EQUATION OF MOTION

### 2.1 Aircraft Equations of Longitudinal Motion

In order to obtain the transfer function of the aircraft, it is first necessary to obtain the equations of motion for the aircraft. The equations of motion are derived by applying Newton's Laws of motion which relate to the summation of the external forces and moments to the linear and angular accelerations of the system or body. Certain assumptions must be made to do this application. By the way, the application is done according to [9].

Furthermore in longitudinal dynamics in order to get the linearized and Laplace transformed equations of motion, stability derivatives have to be also calculated. Then the related force term and moment term are handled, the longitudinal equations of motion for the aircraft are written as;

$$\begin{aligned} \dot{u} &= X_u u + X_w w - g \cos \gamma_0 \theta \\ \dot{w} &= Z_u u + Z_w w + U_0 q - g \sin \gamma_0 \theta + Z_{\delta E} \delta_E \\ \dot{q} &= M_u u + M_w w + M_{\dot{w}} \dot{w} + M_q q + M_{\delta E} \delta_E \\ \dot{\theta} &= q \end{aligned} \quad (1)$$

### 2.2 Transfer functions obtained from Short Period Approximation

The short period approximation consists of assuming that any variations, u, which arise in airspeed as a result of control surface deflection, atmospheric turbulence, or just aircraft motion, are so small that any terms in the equations of motion involving u are negligible. In other words, the approximation assumes that short period transients are of sufficiently short duration that  $U_0$  remain essentially constant, i.e.  $u = 0$ . Thus, the equations of longitudinal motion may now be written as:

$$\begin{aligned} \dot{w} &= Z_w w + U_0 q + Z_{\delta E} \delta_E \\ \dot{q} &= M_w w + M_{\dot{w}} \dot{w} + M_q q + M_{\delta E} \delta_E = (M_w + M_w Z_w) w + (M_q + U_0 M_w) q + (M_{\delta E} + Z_{\delta E} M_w) \delta_E \end{aligned} \quad (2)$$

If the state vector for short period motion is now defined as:

$$X \triangleq \begin{bmatrix} w \\ q \end{bmatrix} \quad (3)$$

And the control vector,  $u$ , is taken as the elevator deflection,  $\delta_E$ , then eqs (2) may be written as a state equation:

$$\dot{x} = Ax + Bu$$

Where:

$$A = \begin{bmatrix} Z_w & U_0 \\ (M_w + Z_w) & (M_q + U_0 M_w) \end{bmatrix}$$

$$B = \begin{bmatrix} Z_{\delta E} \\ (M_{\delta E} + Z_{\delta E} M_w) \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} (s - Z_w) & -U_0 \\ (M_w + M_w Z_w) & (s - [M_q + U_0 M_w]) \end{bmatrix}$$

$$\Delta_{sp}(s) = \det[sI - A] = s^2 - [Z_w + M_q + M_w U_0]s + [Z_w M_q - U_0 M_w] \quad (4)$$

It is easy to show that:

$$\frac{w(s)}{\delta_E(s)} = \frac{(U_0 M_{\delta E} - M_q Z_{\delta E}) \{1 + \frac{Z_{\delta E}}{(U_0 M_{\delta E} - M_q Z_{\delta E})} s\}}{\Delta_{sp}(s)} = \frac{K_w(1+sT_1)}{\Delta_{sp}(s)}$$

Where:

$$K_w = (U_0 M_{\delta E} - M_q Z_{\delta E})$$

$$T_1 = \frac{Z_{\delta E}}{K_w}$$

Also:

$$\frac{q(s)}{\delta_E(s)} = \frac{(Z_{\delta E} M_w - M_{\delta E} Z_w) \{1 + \frac{M_{\delta E} + Z_{\delta E} M_w}{(Z_{\delta E} M_w - M_{\delta E} Z_w)} s\}}{\Delta_{sp}(s)} = \frac{K_q(1+sT_2)}{\Delta_{sp}(s)}$$

Where:

$$K_q = (Z_{\delta E} M_w - M_{\delta E} Z_w)$$

$$T_2 = \frac{(M_{\delta E} + Z_{\delta E} M_w)}{K_q} \quad (5)$$

### 2.3 Flight Path Angle

There is a useful kinematic relationship which can be found by means of the short period approximation: to change the flight path angle,  $\gamma$ , of an aircraft it is customary to command a change in the pitch attitude,  $\theta$ , of the aircraft. Since

$$\gamma = \theta - \alpha \quad (6)$$

$$\frac{Y(s)}{\delta_E(s)} = 1 - \frac{\alpha(s)}{\theta(s)} \cdot \frac{\delta_E(s)}{\delta_E(s)} \quad (7)$$

$$\frac{Y(s)}{\delta_E(s)} = \frac{-Z_w}{(s - Z_w)} \quad (8)$$

## III. AIRCRAFT CONTROL SYSTEM

In figure 1 a block diagram of typical aircraft system is shown [10]

The angle  $\gamma_0$  represents the required glide path angle according to assigned landing path.

The error signal is  $\epsilon$  is the difference between  $\gamma_0$  and  $\gamma$ .

$\delta_E$  is the command signal to the aircraft elevator.

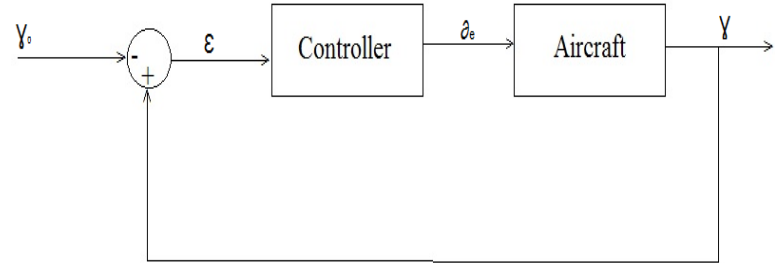


Figure 3.1 Block diagram of aircraft control system.

### 3.1 CHARLIE Aircraft Mathematical Model:

CHARLIE Aircraft: is a very large, four – engine passenger jet aircraft [9].

Table 1 Longitudinal Motion Stability derivative

$Z_w$	$Z_{\delta E}$	$M_w$	$M_{\dot{w}}$	$M_q$	$M_{\delta E}$
-0.512	-1.96	-0.006	-0.0008	-0.357	-0.378

From eq. (5)

$$K_q = -0.181776$$

$$T_2 = 2.07085$$

$$\Delta_{sp}(s) = S^2 + 0.909 S + 0.484$$

$$\frac{\theta(s)}{\delta_E(s)} = \frac{-0.376(S + 0.484)}{S(S^2 + 0.909 S + 0.484)}$$

$$\frac{Y(s)}{\delta_E(s)} = \frac{0.512}{(s + 0.512)}$$

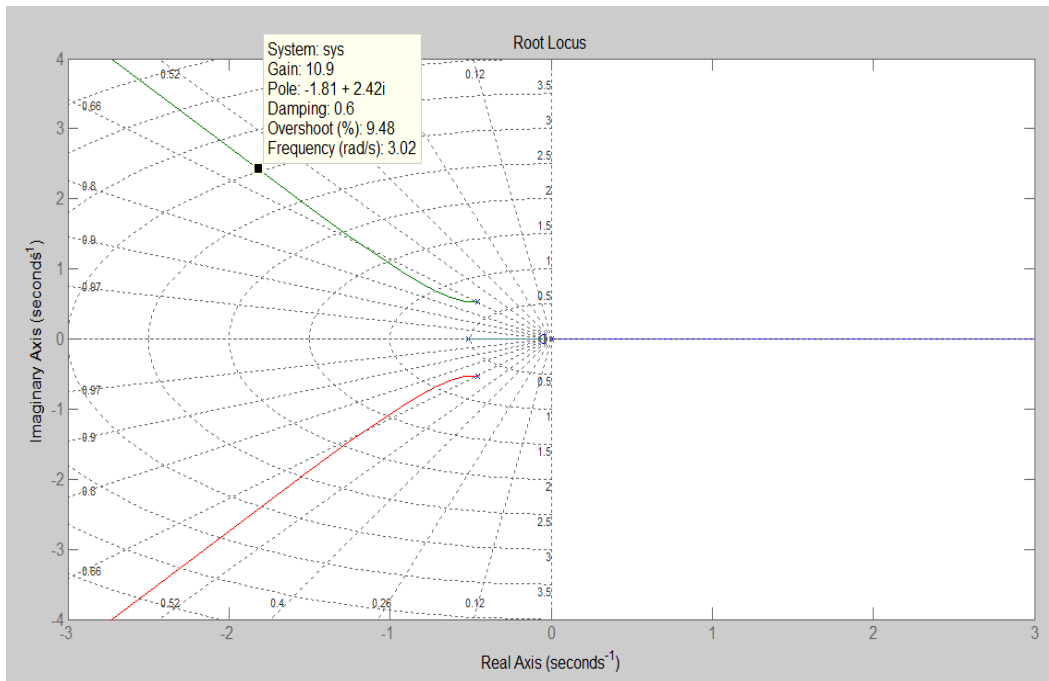
$$\frac{Y(s)}{\delta_E(s)} = \frac{-0.193(S + 0.484)}{S(s + 0.512)(S^2 + 0.909 S + 0.484)}$$

## IV. ALS DESIGN FOR CHARLIE AIRCRAFT

### 4.1 Root Locus for Aircraft

The open loop transfer function CHARLIE aircraft is founded as:

$$\frac{Y(s)}{\delta_E(s)} = \frac{-0.193(S + 0.484)}{S(s + 0.512)(S^2 + 0.909 S + 0.484)}$$



**Figure 4.1** root locus diagram for system at  $\xi = 0.6$

Calculation for controller at  $\xi = 0.6, 0.65$  and  $0.7$  are shown in table 1:

**Table 2** Calculation for the point at  $\xi = 0.6, 0.65$  and  $0.7$

$\xi$	K	$W_n$ rad/sec	$W_d = W_n (\sqrt{1 - \xi^2})$ rad/sec	Overshoot %
0.6	10.9	3	2.4	9.5%
0.65	2.25	1.92	1.46	6.81%
0.7	0.442	1.26	0.9	4.58%

**V. STEP RESPONSE OF CHARLIE AIRCRAFT SYSTEM**

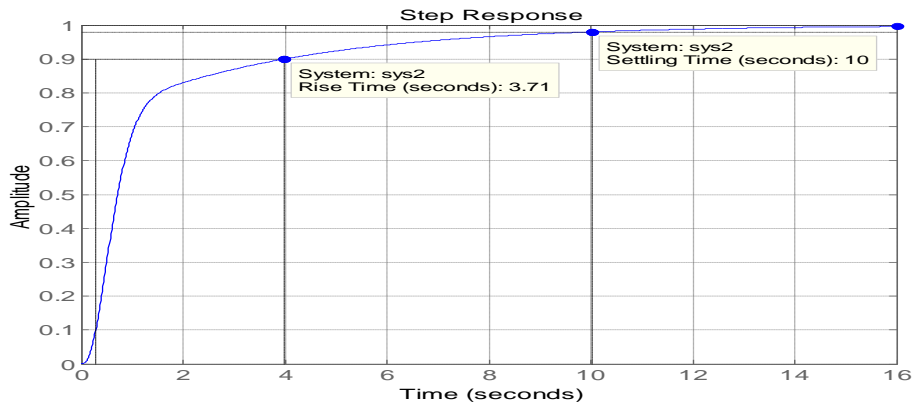
After the root locus for open loop of aircraft is drawn and then K (controller) values are found for given damping ratios, the step responses of the closed loop system with the controller gain are drawn corresponding to K values and rising time, settling time and overshoot are shown in table 3:

**Table 3** summary of performance characteristics for CHARLIE aircraft

K	Response characteristic
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	Rising Time (Tr)	Settling Time (Ts)	Percent Overshoot (% OS)
10.9	849s	2.91e+003s	9.5
2.25	0.548s	3.7s	6.81
0.442	11.1s	21.3s	4.58

By referring to table 3, the result clearly demonstrate that  $K = 2.25$  controller has the fastest response with settling time of 3.71 second and rising time of 0.548 second. These controllers are able to give a good response with percent overshoot 6.81%.

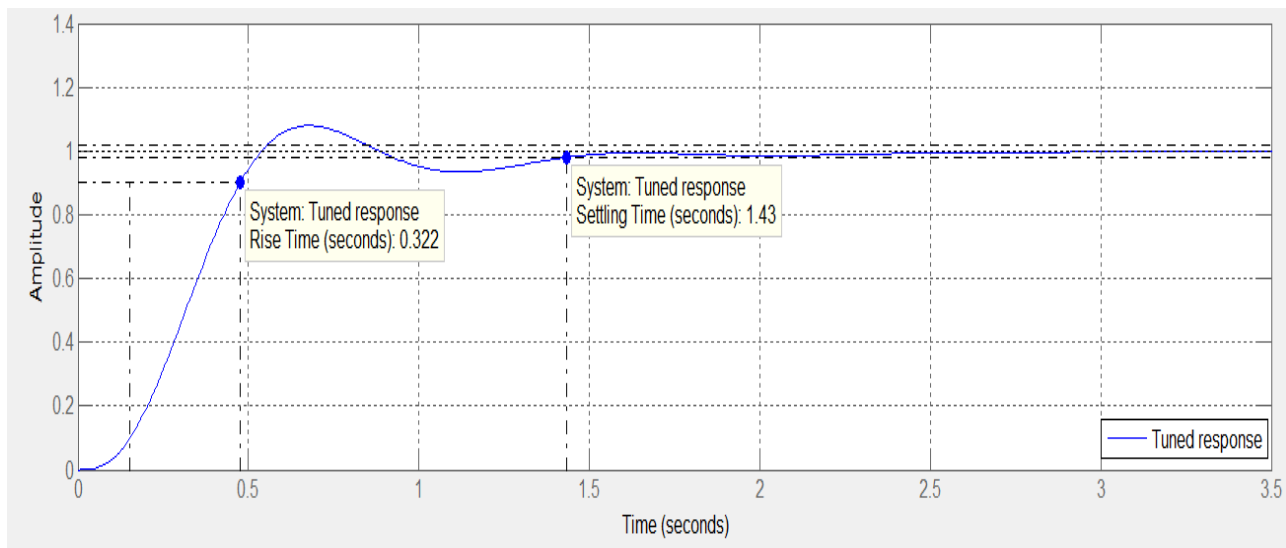


**Figure 5.1 Step response of the closed loop system with  $K=2.25$**

Now, if we replace  $K$  with PI controller. The control performances of PI controller are shown in table 4. A PI type controller can be effective for the considered aircraft. The primary objective of a control system design problem is to determine the controller gain so as to meet the required control performance, from the results in table 3 the last row give a good response with percent overshoot 7.97%. As shown in figure (5.2).

**Table 4 summary of performance characteristics for ALS with PI controller.**

$K$	$K_i$	$K_p$	Rising Time ( $T_r$ )	Settling Time ( $T_s$ )	Percent Overshoot (% OS)
10.9	0.83449	3.5427	0.881 s	17.4 s	8.55
2.25	2.7044	10.8763	0.571 s	11.7 s	9.13
0.442	1.7558	28.5089	0.322 s	1.43 s	7.97



**Figure 5.2 Step response of the closed loop system with  $K= 0.442$ ,  $K_i= 1.7558$  and  $K_p= 28.5089$**

**VI. CONCLUSION**

The transfer functions for open loop and then the closed loop are obtained with the help of automatic control principle. The root locus for open loop is drawn with the help of MATLAB and then the appropriate  $K$  values are determined. Finally the step responses of closed loop Automatic Landing System over wide range of controller gains are drawn. The results validate the powerful of the proposed control system to stabilize the autopilot system.

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