Dynamic Influences of Constant and Variable Elastic Foundations on Elastic Beam under Exponentially Varying Magnitude Moving Load

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Abstract- Dynamic effects of elastic foundation on the flexural motions of Bernoulli-Euler beam moving with variable velocity and under the action of an exponentially varying magnitude distributed load is studied in this present paper. The governing differential equations are solved by the method of finite sine transforms and finite difference method to obtain solutions to the fourth order partial differential equations. The axial force, the elastic foundations and mass intensity of the beam are assumed uniform. Numerical results in plotted curves are displayed and found to be in agreement with those found in literature when vital parameters of the governing equation are analyzed.

Index Terms- Distributed load, Elastic foundation, Finite difference method, Prestressed beam, Uniform beam

I. Introduction

Beams resting on elastic subgrade have many practical applications in modern engineering and because of this, they constitute great technical problems in structural designs. The problem of the dynamic vibration of beams an elastic foundation have been studied by many authors and analytical solutions of the governing equations have been proposed [1-4].

Several researchers have developed different methods to find solutions to flexural behavior of uniform and non-uniform structures resting on an elastic foundation and under the action of moving loads [5-8].

Analytical methods are used in the determination of mathematical functions which define solution in closed form while numerical methods solve the fundamental equations that describe the problem in approximate numerical way. The finite difference method is used for numerical computation of different physical problems in a finite number of sufficiency small intervals. Among the authors that used finite difference methods to solve the dynamic problem of deflections of structural element subjected to moving load is Afsar [9]. The paper proposed the displacement potential formulations for the solution of general anisotropic composite structures and solved cantilever beam with uniformly distributed load.

Awodola [10] examined the influence of variable velocity on the vibration of simply-supported Bernoulli-Euler beam under exponentially varying magnitude moving load. He assumed the beam to rest on uniform constant foundation and the moving load assumed to be concentrated force.

Recently, Soltani et al [11] presented free vibration of beams with variable flexural rigidity resting on two parameter elastic foundations. In their study, natural frequency of non-parametric beams is investigated by using finite difference method (FDM).

More recently, Ogunyebi et al [12] presented the flexural motions of Beam on an elastic foundation subjected to moving concentrated force. They employed the method of Garlekin's to reduce the fourth order partial differential equation to second order ordinary differential equations with constant coefficients and obtained analytical solutions that shed light on the vital information about the vibrating structural member when on an elastic subgrade at uniform velocity.

However, in above studies, the more practical situation where the influence of constant and variable elastic foundation on thin beam with constant and variable velocity and under the action of an exponentially varying magnitude moving distributed loads have not been addressed to the author's best knowledge and this is taken into consideration here. Therefore, this present study presents the dynamic influence of constant and variable foundations on the flexural motions of thin beam with constant and variable velocity on exponentially varying magnitude distributed loads.

II. FORMULATION OF THE GOVERNING EQUATION

Consider a simply supported beam resting on elastic foundation, the differential equation governing the transverse motion of the beam under moving distributed loads at constant and variable velocity may be written in the form

$$EI\frac{\partial^4 U(x,t)}{\partial x^4}+m\frac{\partial^2 U(x,t)}{\partial t^2}-N\frac{\partial^2 U(x,t)}{\partial x^2}+2m\omega_n\frac{\partial U(x,t)}{\partial t}+FU(x,t)=q(x,t) \quad (1)$$

where EI is the flexural stiffness of the beam (E the Young modulus and I the moment of inertia of the cross section), m mas intensity, $U(x,t)$ is the vertical response of the beam, N is the prestress, ω_n is the circular frequency of the beam, F is the constant foundation, $q(x,t)$ is the excitation acting on the beam, x is the coordinate axis along the beam and t is the time coordinate.

The boundary condition of the structure without loss of generality for the simply supported beam whose length L has the form

$$U(0,t)=U(l,t)=0 \quad (2)$$

$$\frac{\partial^2}{\partial x^2}U(0,t)=\frac{\partial^2}{\partial x^2}U(l,t)=0 \quad (3)$$

and the initial condition given as

$$U(x,t)=\frac{\partial^2}{\partial t^2}U(x,t)=0 \quad (4)$$

Adopting [10], the moving distributed load at constant velocity of the prestressed thin beam at constant foundation is given as

$$q(x,t)=qH(x-ct) \quad (5)$$

where $q(t)$ is the variable magnitude of the load.

The Heaviside function $H(x-ct)$ is defined as

$$H(x-ct)=\begin{cases}0, \; for \; x<0\\ 1, \; for \; x>0\end{cases} \quad (6)$$

with the properties

$$\frac{d}{dx}H(x-ct)=\delta(x-ct) \quad (7)$$

$$f(x)H(x-ct)=\begin{cases}0, \; for \; x<ct\\ f(x), \; for \; x\geq ct\end{cases} \quad (8)$$

where $\delta(x-ct)$ represent the Dirac delta function and Heaviside function engineering function used to measure engineering applications which often involved function that are either “off” or “on”.

In this study, an exponentially varying magnitude moving load $q(t)$ is chosen to be of the form

$$q(t)=qe^{2t} \quad (9)$$

so that equation (5) becomes

$$q(x,t)=qe^{2t}H(x-ct) \quad (10)$$

CASE I

In this section, the beam is assumed to be uniform that is the beam properties, Young modulus, the moment of inertia and the mass per unit length of the beam do not vary along the span of the beam. Substituting equation (10) into equation (1) one obtains

$$EI\frac{\partial^4 U(x,t)}{\partial x^4}+m\frac{\partial^2 U(x,t)}{\partial t^2}-N\frac{\partial^2 U(x,t)}{\partial x^2}+2m\omega_n\frac{\partial U(x,t)}{\partial t}+FU(x,t)$$
$$=qe^{2t}H(x-ct) \quad (11)$$

Equation (11) is a fourth order partial differential equation governing the prestressed uniform beam under exponentially varying magnitude distributed moving load with variable coefficients.

III. PROBLEM SOLUTION

The finite Fourier sine transforms is employed to solve the motion equation (11) that governs the problem of thin beam under moving exponentially distributed forces resting on constant elastic foundation at constant and variable velocity. This is given as

$$U(j,t)=\int_0^L U(x,t)\sin\frac{j\pi x}{L}dx \tag{12}$$

with the inverse

$$U(x,t)=\frac{2}{L}\sum_{j=1}^{\infty}U(j,t)\sin\frac{j\pi x}{L}dx \tag{13}$$

substituting equation (12) into equation (11), noting the properties of the Heaviside function and in conjunction with conditions given in (2) and (3), one obtains

$$\ddot{U}(j,t)+G_{j1}\dot{U}(j,t)+G_{j2}U(j,t)=G_{j3}e^{2t}\left[\cos\frac{j\pi x}{L}-\cos j\pi\right] \tag{14}$$

where

$$\left.\begin{aligned} &G_{j1}=\frac{2m\omega_n}{m}, G_{j2}=G_{j2a}-G_{j2b}+G_{j2c} \\ &G_{j2a}=\frac{EI}{m}\left(\frac{m\pi}{L}\right)^4, G_{j2b}=\frac{N}{m}\left(\frac{m\pi}{L}\right)^2, G_{j2c}=\frac{F_c}{m} \\ &G_{j3}=\frac{q}{m} \end{aligned}\right\} \tag{15}$$

Equation (14) is a second order ordinary differential equation. To solve equation (14), use is made of finite difference method. And to this end, the derivative of displacement of the prestressed structure resting on constant elastic foundation is given as

$$\dot{U}(j,t)=\frac{U_{j+1}-U_{j-1}}{2h} \tag{16}$$

and

$$\ddot{U}(j,t)=\frac{U_{j+1}-2U_j+U_{j-1}}{h^2} \tag{17}$$

where h is the mesh size.
Substituting equations (16) and (17) into equation (14), and after rearrangements, one obtains

$$U_{j+1}=\frac{1}{(2+hG_{j1})}\left[H_m^*+U_{j-1}(G_{j1}-2)+U_j(4-2h^2G_{j2})\right] \tag{18}$$

where $H_m^*=G_{j3}e^{2t}\left[\cos\frac{j\pi x}{L}-\cos j\pi\right]$

and when equation (13) is considered in equation (**1**8), one obtains

$$U(x,t)=\frac{2}{L}\sum_{j=1}^{\infty}\left\{\frac{1}{(2+hG_{j1})}\left[H_m^*+U_{j-1}(G_{j1}-2)+U_j(4-2h^2G_{j2})\right]\right\}\times\frac{\sin j\pi x}{L} \tag{19}$$

which represents the response amplitude of the prestressed simply supported beam on variable exponentially moving distributed load on constant elastic foundation.

CASE II

In this section, a more practical situation where an elastic beam is resting on variable elastic foundation and traversed by moving distributed load on variable velocity is considered.
Thus, the thin beam under moving exponentially distributed forces resting on variable elastic foundation at variable velocity is given as

$$=qe^{2t}H\left[x-(x_n+\alpha\sin\omega_n t)\right] \tag{20}$$

where $q(x,t)$ is the variable magnitude of the load $\sin\omega_n$ is the distance function which makes the velocity of the moving load a variable, x_n and α are constant [12].

The variable elastic foundation of the prismatic beam is given as

$$F(x,t) = F_v(4x - 3x^2 + x^3) \tag{21}$$

where F_v is the variable foundation of the structure. To this end, the governing equation (1) is re-rewritten as

$$EI\frac{\partial^4 U(x,t)}{\partial x^4} + m\frac{\partial^2 U(x,t)}{\partial t^2} - N\frac{\partial^2 U(x,t)}{\partial x^2} + 2m\omega_n\frac{\partial U(x,t)}{\partial t} + F_v(4x-3x^2+x^3)U(x,t)$$
$$= qe^{2t}H\left[x - (x_n + \alpha\sin\omega_n t)\right] \tag{22}$$

where all other parameters are as previously defined.
Equation (22) is similar to equation (11), and using similar approach as before, the solution to equation (22) is

$$V_{j+1} = \frac{1}{(2+hS_{m0})}\left[B_m^* + V_{j-1}(S_{m0}-2) + V_j(4-2h^2S_{m1})\right] \tag{23}$$

where $B_m^* = S_{m2}e^{2t}\cos\left[j\pi - (x_n + \alpha\sin\omega_n t)\right] - \cos j\pi$
and in view of equation (13), equation (23) becomes

$$V(x,t) = \frac{2}{L}\sum_{j=1}^{\infty}\left\{\frac{1}{(2+hS_{m0})}\left[B_m^* + V_{j-1}(S_{m0}-2) + V_j(4-2h^2S_{m1})\right]\right\} \times \frac{\sin j\pi x}{L} \tag{24}$$

which represents the response amplitude of the prestressed simply supported beam on variable exponentially moving distributed load on variable elastic foundation at variable velocity.

IV. NUMERICAL RESULTS AND DISCUSSION

This section presents the numerical results of the theoretical solution of the simply supported beam with variable velocity moving on exponentially varying magnitude distributed loads.
An elastic beam of length $12.192m$ has been considered. The value of the axial force varies between 0 and $30000000N$,while the values of the foundation moduli varies between 0 and $900000N/m^3$.
Furthermore, the constant bending stiffness is $6.068\times10^6 m^3/s^2$ and values h, x_o, α, β of are taken to be $0.4m, 5.0, 0.2$ and 8.0 respectively.
For various values of the axial force and foundation moduli, figures 1 to 5 show the deflection profile of the beam under the action of exponentially varying magnitude distributed load moving with variable velocity.
Figure1 and figure 2 display the deflection of axial force and foundation modulli on the prestressed simply supported beam with constant velocity respectively. The graphs show that the response amplitude decreases as the value of the axial force and foundation moduli increases. The influences of the variable velocity on the transverse displacement of the simply supported uniform beam displayed in figure3 and figure 4 respectively show that an increase in the value of the axial force and foundation moduli decrease the deflection of the beam under the action of exponentially varying magnitude distributed load moving with variable velocity.

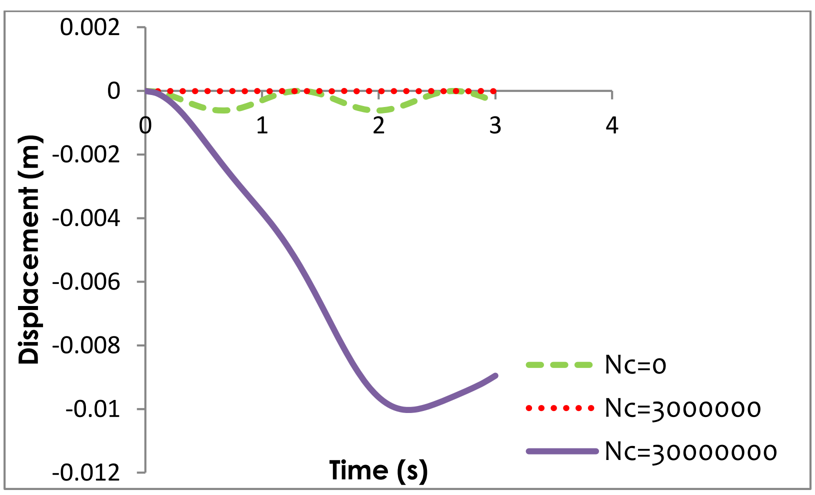

Figure 1: Deflection of simply supported beam at different values of axial force Nc and fixed value of foundation moduli moving with constant velocity.

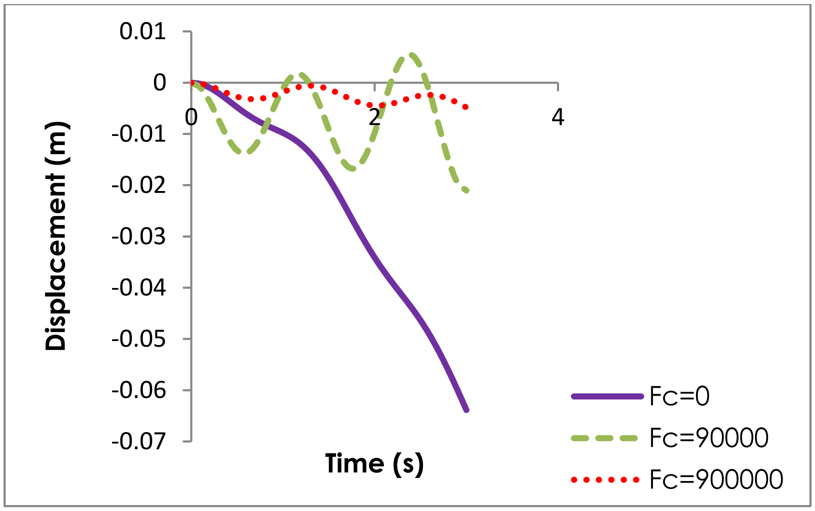

Figure 2: Deflection of simply supported beam at different values of foundation moduli Fc and fixed value of axial force moving with constant velocity.

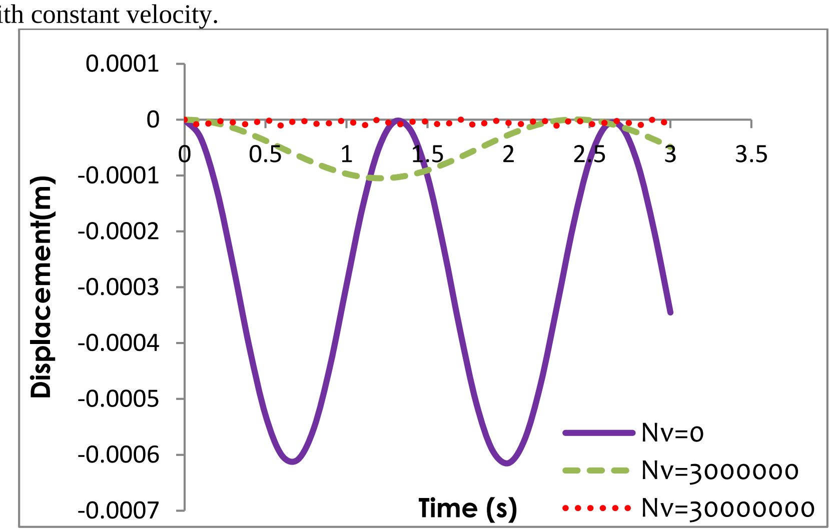

Figure 3: Deflection of simply supported beam at different values of axial force Nv and fixed value of foundation moduli moving with variable velocity.

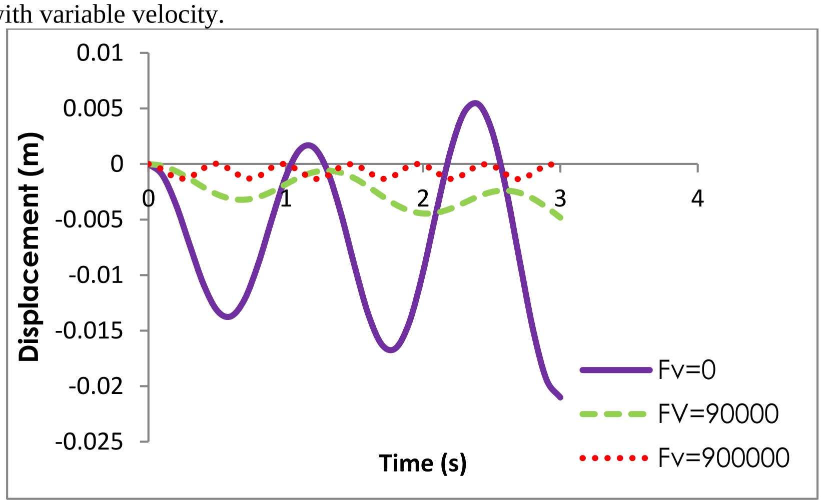

Figure 4: Deflection of simply supported beam at different values of foundation moduli Fv and fixed value of axial force moving with variable velocity.

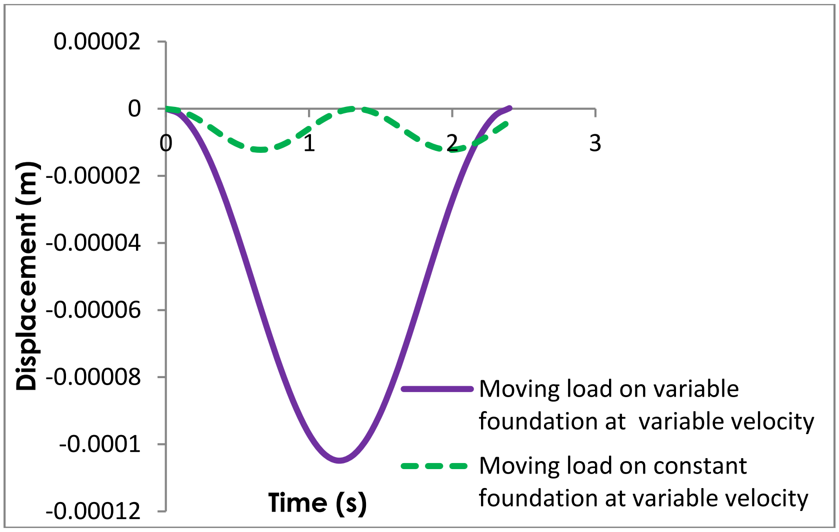

Figure 5: Comparison of deflection of simply supported uniform beam under the action of exponentially varying magnitude distributed load due to constant and variable velocity.

Finally, figure 5 compares the deflection profiles of the constant and variable foundation for a simply supported beam for fixed values of N and K, the response amplitude of a variable foundation is greater than that of a constant foundation problem.

V. CONCLUSION

The flexural behavior of the uniform beam resting on constant and variable elastic foundation under the action of exponentially varying magnitude distributed load moving with constant and variable velocity is examined in this paper. The deflection profile provides a better understanding of the simply supported beam and in addition the possibility of the resonance condition. From the numerical results, it can be clearly see that the presence of elastic foundation and axial force have influences on the dynamic deflection of the beam and this is more noticeable in the case of variable foundation problem at variable velocity. This result is a source of basis for a dependable engineering design for transport and construction engineers.

REFERENCES

[1] James M. L., Smith G. M., J. C Wolford, and P. W Whaley, *Vibration of Mechanical and Structural systems*. Harper Collins College Publishers, 1994.

[2] R.W. Clough and J. Penzien, *Dynamics of Structures.* McGraw-Hill, 1993.

[3] M. Eisenberger, D. Z. Yankelevsky, and J. Clastonic (1986), "Stability of beams on elastic foundation". *Computers & Structures*, Vol. 24, No. 1, pp. 135-139.

[4] F. Miyahara, and J. G. Ergatoudis, (1976), "Matrix analysis of structure-foundation interaction", *Journal of the Structural Division*, Vol. 102, No. 1, pp. 135-139.

[5] M. R. Shadnam, M. Mofid and J. E. Akin (2001), "On the Dynamic response of rectangular plate with moving mass", *Thin-walled Structures,* Vol. 39, pp. 797-806.

[6] C. Omer and Y. Aitung, (2006), "Large deflection static analysis of rectangular plates on two parameter elastic foundations", *International Journal of science and Technology,* Vol.1, No.1, pp. 43-50.

[7] E. Saving, (2001), "Dynamic amplification factor and response spectrum for the evaluation of vibration of beams under successive moving loads", *Sound and Vibratio,* Vol. 248 No. 2, 267-288.

[8] J. Wu, (2006), "Vibration analysis of a portal frame under the action of a moving distributed mass using moving mass element", *Int. Jour. for Numerical Methods in Engineering,* Vol. 62, pp. 2028 – 2052.

[9] A. M. Afsar, S. K. Deb Nath, S. R. Ahmed, and J. L. Song, (2008) "Displacement potential based finite difference solution to elastic field in a cantilever beam of orthotropic composite," *Mechanics of Advanced Materials and Structures*, Vol. 15, No. 5, pp. 386–399.

[10] T. O. Awodola, (2007), "Variable velocity influence on the vibration of Simply supported Bernoulli-Euler beam under exponentially varying magnitude moving load", *Journal of Mathematics and Statistics*, Vol.3, No.4, 228-232.

[11] M. Soltani, A. Sistani, and B. Asgarian, (2016), "Free vibration analysis of beams with flexural rigidity resting on one or two parameter elastic foundations using finite difference method" *The 2016 Structures Congress (Structures16)*. Jeju Island, Korea, August 28 - September 1.

[12] S. N. Ogunyebi, A. Adedowole, T. O. Ogunlade and A.A Oyedele, (2020), "Flexural motions of Beams on an elastic foundation subjected to moving concentrated force", International Journal of Engineering and Advanced technology, Vol. 9, No. 3, 2773-3779.

[13] B. Omolofe and S. N Ogunyebi, (2016), "Dynamic characteristics of a rotating Timoshenko Beam subjected to a variable magnitude load travelling at varying speed, *J. KSIAM.* Vol. 20, No.1, 17–35.

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