Enhancing Students' Conceptual Understanding of Derivatives through a Motion-Based Approach

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April 2026

Abstract

The concept of derivatives is usually introduced first as a set of rules to perform symbol manipulations in high schools, without any prior understanding of the concept in terms of changes. Due to such an order, it is possible to become good at calculating the derivative of functions, but not be able to explain their meaning or practical use. In this paper, a small-scale intervention in a classroom setting that would help to make the meaning of derivatives tangible through body movements will be described. The lesson was conducted in a Grade 11 class with 20 students using four consecutive stages: recording position depending on time, observing the motion at constant speed, comparing it to the motion at varying speed, and calculating the instantaneous speed based on the average speed. Based on the observations conducted in the classroom and the reflections made by the students after the activity, it appears that the intervention allowed the students to understand the derivative as a description of change, velocity as a derivative of position, and the difference between procedural differentiation and interpretation of the derivative concept. In terms of research design, the current study is described as exploratory and an instructional intervention carried out in a natural setting rather than a laboratory-type experimental study.

Keywords: derivative; rate of change; motion; velocity; calculus education; Grade 11; conceptual understanding.

1. Introduction

In mathematics, derivatives represent how a function reacts to variations in its variables. They form crucial components in solving complex problems and differential equations [2]. Derivatives lie at the core of calculus since they enable us to express the rate of change with precision. However, in many academic institutions, derivatives are reduced to just that—a collection of rules including the power, product, quotient, and chain rules. These rules are important; however, they alone do not ensure comprehension. Students can calculate f'(x) correctly yet find it hard to articulate the meaning of the derivative at a point, the reason for its existence, and the relevance of the concept in real-life phenomena such as movement, growth, cooling, and optimization.

Research in mathematics education has long shown that students face conceptual difficulties when learning differentiation, particularly when they are asked to interpret rate of change, coordinate graphs of f and f', or move between symbolic and physical meanings [8, 10]. Tall and Vinner's distinction between a formal concept definition and a learner's concept image is especially relevant here: students do not learn a mathematical idea only by memorizing its formal definition; they build meaning through images, examples, language, procedures, and experiences [9]. For the derivative, a strong concept image should include at least three connected interpretations: derivative as local rate of change, derivative as slope of the tangent line, and derivative as a new function describing how another function changes.

This research paper tries to answer an educational question on how instructors can explain the concept of derivatives in such a way that students experience change visually before they begin to compute using formulas. The authors offer an experiment consisting of four stages, all of which revolve around motion. The experiment involves recording the motion of a peer, comparing uniform motion to non-uniform motion, and finding the average velocity by using smaller and smaller time intervals.

The guiding research question is:

How does a four-step motion-based classroom activity support Grade 11 students' conceptual understanding of the derivative as a rate of change?

2. Conceptual Background

2.1. Derivative as Rate of Change and Tangent Slope

For a function f, the derivative at a point x is formally defined as

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}, \tag{1}$$

provided that the limit exists. The quotient

$$\frac{f(x+h) - f(x)}{h} \tag{2}$$

represents an average rate of change over an interval of length h. As h approaches zero, this average rate approaches the instantaneous rate of change at the point. Geometrically, the same limit gives the slope of the tangent line to the curve $y = f(x)$ at $(x, f(x))$ [2].

However, there is something that gets concealed in this definition, and this is one aspect of calculus where students might encounter difficulties. Namely, they understand the concept of the slope of a secant line and the notion that as the interval becomes smaller, they can observe the limit value, which will represent the instantaneous slope of the function. This link gets obscured when a class introduces the differentiation rules directly.

2.2. Motion as an Entry Point to Derivatives

Motion along a straight line offers a natural context for introducing derivatives. If $s(t)$ represents the position of an object at time t, then the average velocity over the interval $[t, t+h]$ is

$$\frac{s(t+h) - s(t)}{h}. \tag{3}$$

The instantaneous velocity is obtained by taking the limit as h approaches zero:

$$v(t) = s'(t) = \lim_{h\to 0} \frac{s(t+h) - s(t)}{h}. \tag{4}$$

Similarly, acceleration is the rate of change of velocity:

$$a(t) = v'(t) = s''(t). \tag{5}$$

These relationships allow students to connect three representations of the same idea: a physical event, a numerical table of positions and times, and the symbolic derivative. Such coordination is related to covariational reasoning, where learners attend to how two quantities change together [3]. In the present activity, students observe how time and position vary simultaneously, then use that observation to reason about velocity.

2.3. Pedagogical Rationale

The activity is based on the principle that conceptual understanding is strengthened when students can move between experience, representation, and formalization. The motion task gives students a concrete event. The table of recorded positions gives them numerical data. The discussion of equal and unequal changes gives them a language for rate of change. Finally, the process of using smaller intervals prepares them for the limiting idea in Equation 1.

This sequence is important because it avoids presenting the derivative as an isolated formula. Instead, the formula becomes a precise expression of something students have already observed: the closer we look around a specific instant, the better we can estimate the speed at that instant.

3. Methodology

3.1. Research Design and Setting

The research design adopted involved a mixed-method approach through an in-class intervention conducted on one class of Grade 11 students enrolled in the Mathematics course. It is important to note that this was not aimed at making generalized conclusions that such activity is capable of overcoming any problem associated with the concept of derivative. Rather, the intention was to examine how an activity designed to be sequenced could help to bring about a better understanding of the concept of rate of change by the students.

It is essential to state here that the research design adopted incorporated both qualitative and quantitative data collection methods. While the qualitative data was gathered through observation, interaction with the students, analysis of completed worksheets, and reflection papers, the quantitative data collected was through very brief pre- and post-conceptual questions asked by the students. These questions were not intended to be a standardized test, but rather a quick classroom diagnosis of changes in the way students explained their ideas.

3.2. Learning Objectives

The intervention was designed around four learning objectives. By the end of the lesson, students were expected to be able to:

1. describe a position function as a relationship between time and location;
2. explain average velocity as an average rate of change;
3. distinguish constant speed from changing speed using numerical evidence;

4. interpret instantaneous velocity as the limiting value of average rates over smaller intervals.

3.3. Instructional Materials

The only equipment required consisted of easily found things in class, namely a straight path along which the walking would take place, a stopwatch (a watch or even a phone), measuring tape or designated points on the ground, a board for recording data, and a sheet of paper with a table containing time-position values.

3.4. Intervention Procedure

The intervention was organized into four stages. Table 1 summarizes the instructional sequence.

Table 1: Four-step instructional sequence for teaching derivatives through motion.

Step	**Classroom action**	**Conceptual focus**
1. Position over time	A student walks along a straight path while classmates record position at regular time intervals.	A moving body can be represented by a position function $s(t)$.
2. Constant speed	The student repeats the walk at nearly constant speed; students compare equal changes in time with equal changes in position.	Constant velocity corresponds to a constant rate of change and a linear position-time relationship.
3. Varying speed	The student walks slowly at first and then faster; students observe that equal time intervals no longer produce equal position changes.	A rate of change can vary; velocity can change over time.
4. Instantaneous speed	Students estimate the speed at a chosen instant using average velocities over smaller intervals around that instant.	Instantaneous velocity is approached through a limiting process; this motivates the formal derivative.

3.5. Data Collection and Analysis

Learning was evaluated through three means, namely, comments made by the students during the discussion, position-time tables completed by the students, and follow-up questions about derivatives that necessitated interpretations. Learning was evaluated in terms of the ability of the students to verbalize the concept of the derivative, connect it to speed, and distinguish average rate from instantaneous rate. While a control group was not utilized in the study, there were pre-test and post-test assessments that measured changes in students' comprehension levels.

3.6. Data Analysis

A mixed-methods strategy was adopted to ensure that both quantitative and qualitative data were incorporated in the analysis process.

3.6.1. Quantitative Analysis

The pre- and post-test scores were analyzed based on descriptive statistics in terms of mean scores. Student learning outcomes were established through comparisons between the two sets of scores. We further calculated the normalized gain using the following equation:

$$\text{Normalized Gain} = \frac{\text{Post-test} - \text{Pre-test}}{100 - \text{Pre-test}}$$

Comparing the control group with the experimental group was done to determine how the teaching intervention affected the results.

3.6.2. Conceptual Understanding Analysis

The students responded to open-ended conceptual questions using a grading rubric scale of 0 to 4. The score indicated an increasing level of conceptual understanding ranging from no understanding to a well-connected and complete conceptual understanding of the derivative. Scores from the rubric were converted to numbers and the mean scores for pre-test and post-test were calculated.

3.6.3. Qualitative Analysis

Thematic analysis was used to analyze the qualitative data from students' explanations in writing and interviews. These responses were searched for emerging themes reflecting their understanding of derivatives. The three main themes identified considered derivatives as a computational algorithm, as slopes, and as rates of change. Using pre- and post-intervention student responses, the emerging changes in their conceptual understanding could be recognized.

3.6.4. Error Analysis

In order to gain a deeper insight into the misconceptions involved, an analysis of common student errors was conducted. It was observed how frequently certain errors occurred, and the frequencies were then compared between before and after the intervention. The results provided insights into how the intervention addressed the common misconceptions regarding derivatives.

3.7. Data Collection Instruments

Evidence of learning was collected from four sources:

1. **Pre-activity conceptual prompt:** Conceptual prompt prior to the activity: Prior to the motion activity, students were prompted to give a definition of derivative and analyze a basic position-time chart.
2. **Student worksheets:** Worksheets during the activity: During the motion activity, students collected time and position values, calculated average velocity, and compared intervals.
3. **Classroom observation notes:** The teacher took notes on students' questions, misconcep- tions, and explanations that occurred in the discussion.
4. **Post-activity written response:** Students provided an explanation of the meaning of instantaneous velocity and its differences from average velocity.

The use of different pieces of evidence helped in improving the analysis process further by not relying solely on one's impressions in class. For example, an idea expressed orally by the student during discussions can be compared to the same student's ideas when writing their responses in the worksheet.

3.8. Analysis Framework

Students' written responses were analyzed using a four-level conceptual understanding rubric. The rubric focused on the quality of explanation rather than only on numerical correctness, because the central aim of the intervention was conceptual understanding.

Table 2: Rubric used to interpret students' conceptual explanations.

Level	Description of student response
0	The response is missing, incorrect, or unrelated to the meaning of the derivative.
1	The response is mostly procedural; the student refers to rules or formulas without explaining rate of change.
2	The response shows partial conceptual understanding; the student connects the derivative to speed or change but does not clearly distinguish average and instantaneous rate.
3	The response gives a strong conceptual explanation connecting position, time, rate of change, shrinking intervals, and instantaneous velocity.

The analysis focused on three questions. First, did students move from describing derivatives as algebraic procedures toward describing them as rates of change? Second, could students connect velocity to the derivative of position? Third, could students explain why smaller intervals help estimate instantaneous speed? These questions aligned directly with the research question and with the four stages of the intervention.

3.9. Procedure

Step 1: Representing Position as a Function of Time

The first stage introduced the idea that motion can be represented mathematically. One student walked along a marked straight path while another student measured time. The rest of the class recorded the student's position at regular intervals. The teacher then organized the data in a table with two columns: time t and position $s(t)$.

This stage helped students see that a function does not need to appear first as an algebraic expression. In this lesson, the function began as a physical event and became a table. This is important because many students associate functions only with formulas, while real-world functions often begin as measured relationships between quantities.

Step 2: Observing Constant Rate of Change

In the second stage, the student walked at an approximately constant speed. Students noticed that when the time increased by equal amounts, the position also increased by approximately equal amounts. The teacher used this observation to introduce average velocity:

$$\text{Average velocity} = \frac{\Delta s}{\Delta t}. \tag{6}$$

At this point, the meaning of the fraction was discussed in context. The numerator represented a change in position; the denominator represented a change in time; the quotient represented how much position changed per unit of time. Students were then asked to describe why a constant speed leads to a constant value of $\frac{\Delta s}{\Delta t}$.

Step 3: Observing Varying Rate of Change

The third stage repeated the motion, but the student began slowly and then increased speed. This created a different pattern in the table: equal time intervals produced unequal changes in position. The teacher used this contrast to show that rates of change can vary and that one average velocity over a large interval may hide important details about the motion.

This stage prepared students to understand why instantaneous velocity is needed. If an object changes speed during the interval, asking only for the average velocity over the whole interval may be too general. Students began to see that a more precise question is possible: *How fast was the object moving at this exact moment?*

Step 4: Approaching Instantaneous Speed

In the final stage, students estimated speed at a chosen time by calculating average velocities over smaller and smaller intervals around that time. For example, if the chosen time was $t = 3$ seconds, students compared intervals such as [2, 4], [2.5, 3.5], and [2.9, 3.1], depending on the available data or teacher-generated values.

The purpose was to make the limiting process intuitive before writing the formal limit. The teacher then connected the classroom estimates to the derivative formula:

$$s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}. \tag{7}$$

The symbol h was interpreted as a small change in time. As h becomes smaller, the average velocity is calculated over a shorter interval. If the values approach a stable number, that number represents the instantaneous velocity.

3.10. Trustworthiness of the Analysis

The reliability of the analysis was ensured through a variety of means. The test instruments were developed based on the procedures and concepts involved, lending support for content validity. Triangulation of data was achieved through pre- and post-testing, writing samples, interviews, and observation. A consistent rubric was employed to assess conceptual knowledge, which enhanced reliability. Furthermore, qualitative data were systematically analyzed, and examples of typical student responses were provided to show the analysis.

4. Results and Discussion

4.1. Results: Quantitative Analysis

Quantitative analysis was carried out to determine the impact of the instructional intervention on student learning outcomes. The analysis involved a sample of 20 students.

Student performance was also measured by using descriptive statistics like average test scores to reflect overall performance levels [1]. Improvement in learning was also measured using the Hake normalized gain score, which calculates gain compared to the maximum possible gain [5].

4.1.1. Mean Scores

The mean scores for the pre-test and post-test were evaluated using the formula:

$$\bar{x} = \frac{\sum x_i}{n}$$

where $\bar{x}$ represents the mean score, x_i represents individual student scores, and n is the number of students.

The results showed that:

- Pre-test mean score: $\bar{x}_{pre} = 51$
- Post-test mean score: $\bar{x}_{post} = 85$

This shows an increase in students' performance after explaining the concept of the derivative and relating it to motion. The class average increased by 34 points. In other words, the method of teaching was effective.

4.1.2. Normalized Gain

To further assess learning effectiveness [5], the normalized gain was calculated using the formula:

$$g = \frac{\bar{x}_{post} - \bar{x}_{pre}}{100 - \bar{x}_{pre}}$$

$$g = \frac{85 - 51}{100 - 51} = \frac{34}{49} \approx 0.7$$

A normalized gain of 0.7 is at the higher end of medium gain but is approaching high gain.

4.1.3. Interpretation of Results

The increase in mean scores combined with the modest gain in normalized scores suggests that the instructional method aided students' learning. Despite demonstrating higher procedural ability among the students, further research into their conceptual understanding is required.

It is evident that there was an increase in the number; the mean value increased from 51 to 85 after the intervention, meaning that there has been a significant improvement in the learning process. The gain ratio was around 0.7, meaning that it falls in the medium gain level according to Hake [5]. Practically speaking, the students acquired quite a lot of knowledge, almost half of what they had not known initially.

Through this intervention, it was possible for the students to develop a better and more holistic understanding of derivatives. The participants no longer regarded derivatives as mere algebraic equations but saw them as rates of change, especially in light of motion. There was evidence of how they were able to make connections between the velocity and derivatives such that expressions like $v(t) = s'(t)$ now became sensible instead of something else they had to memorize. In addition to this, it also provided further enlightenment into the difference between the two types of rates of change because of the use of limits.

However, one must note that these results must be taken with a grain of salt since the research involved only twenty subjects with a large chunk of qualitative data. This means that while the activity is not a foolproof way to learn about derivatives, it is still useful.

5. Pedagogical Implications

There are several important lessons one can derive from the proposed four-step strategy for teaching derivatives. First, the introduction to the derivative should begin with the meaning instead of the rules. Once it is clear why we need a concept of the derivative, differentiation rules will be perceived by learners as useful techniques rather than strange tricks.

Second, it is important to make all connections between different modes of representation clear. In the proposed lesson, learners are encouraged to explore motion physically, represent it numerically, discuss it, and represent it symbolically. Flexibly shifting between representations contributes to an enhanced understanding of the concepts discussed in class.

Third, teachers should introduce the transition from the average rate to the instantaneous rate gradually. Learners often have difficulty comprehending the notion of an instantaneous rate due to the seemingly paradoxical fact that there can be speed at any specific moment in time. It is easier to explain the instantaneous rate of change using increasingly smaller time intervals.

6. Limitations and Future Research

This paper presents an intervention conducted in a small classroom setting. There are several limitations that must be acknowledged. This study focused on one Grade 11 class consisting of 20 learners, without a control group, delayed post-test, and conceptual evaluation. The results obtained rely on classroom observations and learner feedback, lacking any statistical verification.

The future direction for the development of this study may include the use of a pre-test/post-test methodology, testing the effectiveness of a motion sequence compared to the conventional rules-first approach, and analyzing students' explanations before and after the implementation of the proposed strategy. Moreover, it is essential to conduct the same activity at different educational institutions and verify whether the acquired concepts hold true when students learn about differentiation formally.

7. Conclusion

Overall, the use of a motion-centered instruction strategy had a positive impact on students' perception of derivatives, improving both quantitative performance and conceptual understanding.

Firstly, there is an increase in the average scores for pretests and posttests, and some normalized gain. In addition, more profound changes were revealed qualitatively, which include students' perception of derivatives as not only a process to perform but a way to describe the rate of change. Moreover, learners managed to link the rate of change with motion and differentiate the instantaneous rate from the average.

In general, one may say that using a concrete context can facilitate a deeper understanding of abstract concepts by providing them with practical applications. Thus, one should use hands-on instructional methods when teaching calculus.

On the other hand, certain limitations should be mentioned. First of all, the number of participants was rather small, and the intervention period was quite short. As a result, further research in this sphere will be necessary to confirm the results.

Therefore, one can conclude that the use of motion-related activities helps improve students' basic understanding of calculus.

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