Solution of Electrical Circuit using Euler's Method

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Abstract- In this paper we shall express about concept that describe a very different and extremely clever way of solving the Initial Value Problem of ordinary differential equation .

This concept is so- called Euler's Method that we shall applied it in physical fields to solve an electrical circuit either to be Series connection or Parallel one .

In the solution first of all we solve the equation of the (I. V . P) by using the Method of Integrating factor to find the exact solution of the problem then after that we use Euler's Method in Series connection or Parallel one , with different values for the variables first of all big values then after that small one and see which of the connection is divergent or convergent .

Index Terms- Ordinary Differential Equation , Numerical Method(Euler's Method), Integrating factor , Initial Value Problem , Electrical Circuit .

I. Introduction

Ordinary Differential Equation

The differential equation is an equation involving one dependent variable and its derivatives with respect to one or more independent variables . The order of a differential equation is the order of the highest derivative present.

An ordinary differential equation is one in which there is only one independent variable, so that all the derivatives occurring in it are ordinary derivatives.[5]

A linear ordinary differential equation of order *n*, in the dependent variable *y* and the independent variable *x* , is an equation that is in, or can be expressed in the form :

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$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0 \quad [1.1]$$

$$\frac{d^4y}{dx^4} + x^2\frac{d^3y}{dx^3} + x^3\frac{dy}{dx} = xe^x \quad [1.2]$$

A non- linear ordinary differential equation is an ordinary differential equation that is not linear.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0 \quad [1.3]$$

The equation is nonlinear because the dependent variable *y* appears to the second degree in the term 6y.

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0 \quad [1.4]$$

The equation owes its nonlinearity to the presence of the term $5\left(\frac{dy}{dx}\right)^3$ which involves the third power of the first derivative.

A linear 1st order O.D.E. in *y(x)* has the form

$$\frac{dy}{dx} + f(x)\,y = r(x). \quad [1.5]$$

Here *f(x)* and *r(x)* are given functions.

The above linear O.D.E. is said to be **homogeneous** if *r(x) = 0*
(that is, *r(x)* is identically zero over the interval of interest). Thus, a homogeneous 1st order linear O.D.E .is of the form

$$\frac{dy}{dx} + f(x)y = 0 \quad [1.6]$$

If the linear O.D.E.

$$\frac{dy}{dx} + f(x)y = r(x). \quad [1.7]$$

is such that the function *r(x)* has non-zero values for at least some values of *x* (within the interval of interest) then the O.D.E. is said to be **nonhomogeneous** The initial value problem for an nth order differential equation

$$F\left(x, y, \frac{dy}{dx}, \ldots\ldots\ldots \frac{d^ny}{dx^n}\right) = 0 \quad [1.8]$$

we mean: Find a solution to the differential equation on an interval *I* that satisfies at x_o the *n* initial conditions

$y(x_o) = y_o$,

$$\frac{dy}{dx}(x_0) = y_1 , \quad [1.9]$$

$$\frac{d^{n-1}y}{dx^{n-1}}(x_0) = y_{n-1} \quad [1.10]$$

where x_o belong to *I* and $y_o, y_1 \ldots, y_{n-1}$ are given constants . [9]

II. NUMERICAL METHOD

If an ordinary differential equation cannot solved exactly, one may resort to numerical methods for finding approximately the required solution .

In this paper I choose the oldest numerical method for approximating the solution of differential equation is **Euler' s Method** that dates from about 1768 It is simple in concept and easy to execute . The simplicity of Euler's Methods makes it a good way to begin to explore the numerical approximation of solutions of relatively simple differential equations[9] . [8] , [10]

III. THE METHOD OF INTEGRATING FACTOR FOR SOLVING : Y´ + P(T)Y = G(T)

1. Put the equation in standard form : $y' + p(t)y = g(t)$.
2. Calculate the integrating factor $\mu(t) = e^{\int p(t)dt}$.
3. Multiply the equation by $\mu(t)$ and write it in the form $[\mu(t)y]' = \mu(t)g(t)$.
4. Integrate this equation to obtain $\mu(t)y = \int \mu(t)g(t)\,dt + c$.
5. Solve for y.

(where p and g are given functions.) .[5] , [8]

IV. INITIAL VALUE PROBLEM (I.V.P)

Definition :. By an initial value problem for an nth order differential equation ,

$$F\left(x, y, \frac{dy}{dx}, \ldots\ldots, \frac{d^ny}{dx^n}\right) = 0 \quad [4.1]$$

We mean: find a solution to the differential equation on an interval I that satisfies at x_o the n initial conditions :

$y(x_o) = y_o$,

$\frac{dy}{dx}(x_0) = y_1$, . [4 .2] .

$\frac{d^{n-1}y}{dx^{n-1}}(x_0) = y_{n-1}$ [4 .3]

where x_o belong to I and y_o , y_1 . . . , y_{n-1} are given constants.[10]

V. ELECTRICAL CIRCUIT

Basic electrical components: the basic components which may be found in an electric circuit include resistor , inductor and capacitor . The resistor is a device which restricts the flow of electric current to within a safe level in the circuit .

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The inductor is essentially a coil of wire (usually copper) which stores energy in a magnetic field . The capacitor stores energy in an electric field in between two oppositely charged plates [9]

Series-Parallel circuit

With simple series circuits, all components are connected end-to-end to form only one path for electrons to flow through the circuit:

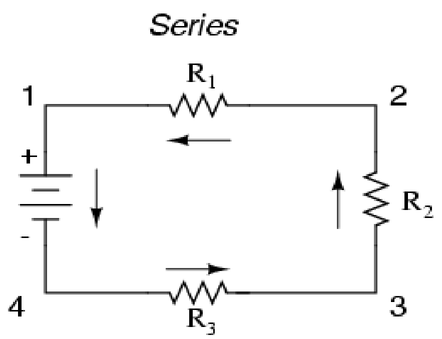

Figure : 1

With simple parallel circuits, all components are connected between the same two sets of electrically common points, creating multiple paths for electrons to flow from one end of the battery to the other:

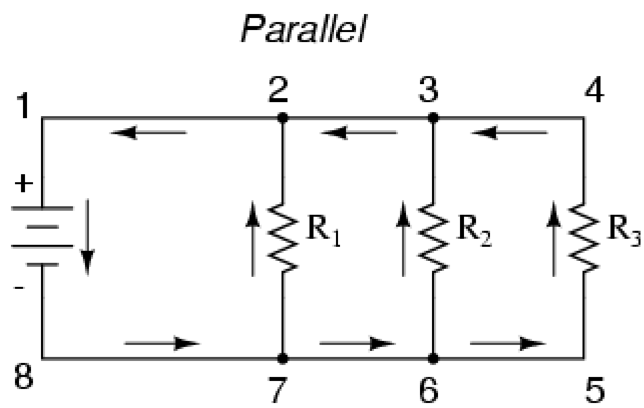

Figure : 2

With each of these two basic circuit configurations, we have specific sets of rules describing voltage, current, and resistance relationships.

Series Circuits:

Voltage drops add to equal total voltage.

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All components share the same (equal) current. Resistances add to equal total resistance.
Parallel Circuits:
All components share the same (equal) voltage. Branch currents add to equal total current.
Resistances diminish to equal total resistance.

VI. Applications for Some Problems

Series circuit - High Values

 Use Euler's method with step size 0.1 to construct a table of approximate values for the solution of the initial-value problem with simple electric circuit
contains from : resistance 12 Ω , inductance 4 H.A battery gives a constant voltage of 60 V.

$$L \frac{dI}{dt} + RI = E(t) \quad [6.1]$$

which is a first-order differential equation that models the current *I* at time *t*.

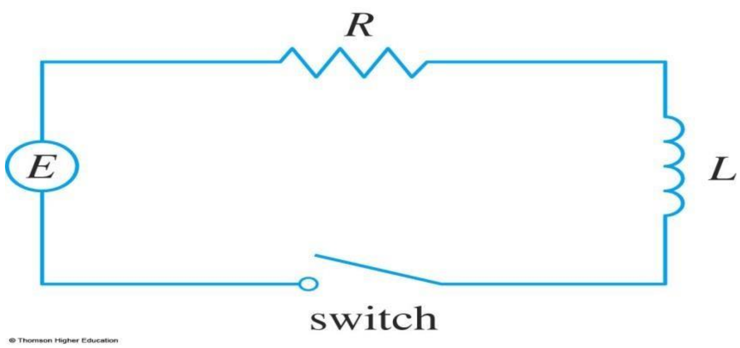

Figure : 3

If the switch is closed when $t = 0$ so the current starts with $I(0) = 0$ Estimate the current in the circuit half a second after the switch is closed.

Solution :

To use Euler's method, note that : $4 \frac{dI}{dt} + 12I = 60$ [6.1]

So $\frac{dI}{dt} = 15 - 3I$ [6.2]

The equation can be written in the form :

$\frac{dI}{dt} + 3I = 15$ [6.3]

Firstly

The exact solution to the initial value problem by using the method of integrating factors is :

Put : $\mu(t) = e^{3t}$

Multiplying the original equation by the integrating factor to obtain

$e^{3t}\frac{di}{dt} + e^{3t}(3I) = 15e^{3t}$ [6.4]

So $(e^{3t}I)' = 15e^{3t}$ [6.5]

 Integrating both sides we have :

$e^{3t}I = \int 15e^{3t}dt$ [6.6]

$e^{3t}I = 5e^{3t} + C$ [6.7]

So $I = 5 + Ce^{-3t}$ [6.8]

To find the value of C we must to use the condition above :

$0 = 5 + C \Rightarrow C = -5$ [6.9]

Now the solution of the equation is :

$I(t) = 5 - 5e^{-3t}$ [6.10]

Secondly

To use Euler's method, note that $f(t, I) = 15 - 3I$ [6.11]

$I_1 = I_0 + hf(t_0, I_0)$

The first step is :

$I_1 = 0 + 0.1(15 - 3(0)) = 0 + 1.5 = 1.5$ [6.12]

Then let us to move so as get the second point :

$I_2 = I_1 + hf(t_1, I_1)$
$= 1.5 + 0.1(15 - 3(1.5)) = 1.5 + 1.05 = 2.55$ [6.13]
Now the third point should be :
$I_3 = I_2 + hf(t_2, I_2)$
$= 2.55 + 0.1(15 - 3(2.55)) = 2.55 + 0.735 = 3.285$ [6.14]
We now move on to get the fourth point in the solution
$I_4 = I_3 + hf(t_3, I_3)$
$= 3.285 + 0.1(15 - 3(3.285)) = 3.285 + 0.5145 = 3.7995$ [6.15]
We now move on to get the fifth and the last point in the solution
$I_5 = I_4 + hf(t_4, I_4)$
$= 3.7995 + 0.1(15 - 3(3.7995)) = 3.7995 + 0.36015 = 4.15965$ [6.16]
The current in the circuit half a second after the switch is closed is equal :≈ 4.16 A Here's a quick table that gives the approximations as well as the exact value of the solutions at the given points :

t	Exact	Euler with h = 0.1
1	4.7511	1.50000
2	4.9876	2.55000
3	4.9994	3.28500
4	4.99997	3.79950
5	4.99999	4.15965

Euler's method does not produce the exact solution to an initial-value problem. It gives approximations.

Here is the graph of the solution and a tangent line approximation for the initial value problem:

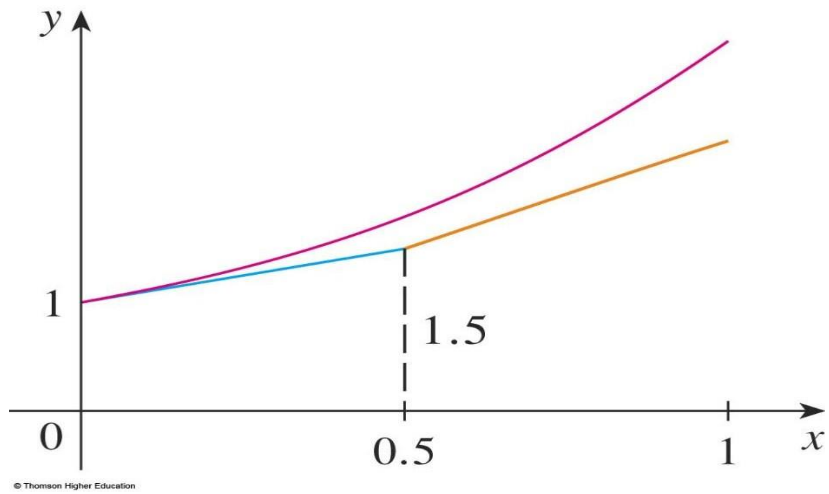

Figure : 4

The same problem above with a **small values** .

 Use Euler's method with step size 0.1 to construct a table of approximate values for the solution of the initial-value problem with simple electric circuit

contains from : resistance 6 Ω , inductance 1 H.A battery gives a constant voltage of 12 V.

$$L \frac{dI}{dt} + RI = E(t) \quad [6.17]$$

which is a first-order differential equation that models the current I at time t with a series circuit .

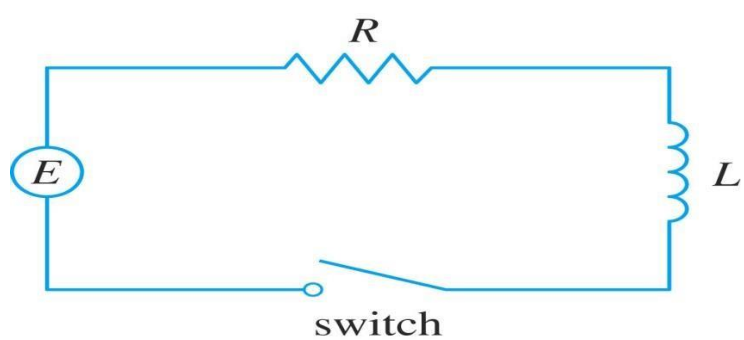

Figure : 5

If the switch is closed when $t = 0$ so the current starts with $I(0) = 0$ Estimate the current in the circuit half a second after the switch is closed.

Solution :

To use Euler's method, note that : $\frac{dI}{dt} + 6I = 12$

So $\frac{dI}{dt} = 12 - 6I$ [6.18]

The equation can be written in the form :

$\frac{dI}{dt} + 6I = 12$ [6.19]

Firstly

The exact solution to the initial value problem by using the method of integrating factors is :

Put : $\mu(t) = e^{6t}$

Multiplying the original equation by the integrating factor to obtain

$e^{6t}\frac{di}{dt} + e^{6t}(6I) = 12e^{6t}$ [6.20]

So $(e^{6t}I)' = 12e^{6t}$

Integrating both sides we have :

$e^{6t}I = \int 12e^{6t}dt$ [6.21]

$e^{6t}I = 2e^{6t} + C$

So $I = 2 + Ce^{-6t}$ [6.22]

we must to use the condition above so as to get the value of C :

$0 = 2 + C \Rightarrow C = -2$ [6.23]

Now the solution of the equation is :

$I(t) = 2 - 2e^{-6t}$ [6.24]

Secondly

To use Euler's method , note that $f(t, I) = 12 - 6I$ $I_1 = I_0 + hf(t_0, I_0)$ [6.25]

The first step is :

$I_1 = 0 + 0.1(12 - 6(0)) = 0 + 1.2 = 1.2$ [6.26]

Then let us to move so as get the second point :

$I_2 = I_1 + hf(t_1 , I_1)$

$= 1.2 + 0.1(12 - 6(1.2)) = 1.2 + 0.1(4.8) = 1.68$ [6.27]

Now the third point should be :

$I_3 = I_2 + hf(t_2 , I_2)$

$= 1.68 + 0.1(12 - 6(1.68)) = 1.68 + 0.1(1.92) = 1.872$ [6.28]

We now move on to get the fourth point in the solution

$I_4 = I_3 + hf(t_3 , I_3)$

$= 1.872 + 0.1(12 - 6(1.872)) = 1.872 + 0.0768 = 1.9488$ [6.29]

We now move on to get the fifth and the last point in the solution

$I_5 = I_4 + hf(t_4 , I_4)$

$= 1.9488 + 0.1(12 - 6(1.9488)) = 1.9488 + 003072 = 1.97952$ [6.30]

The current in the circuit half a second after the switch is closed is equal ≈ 1.98 A Here's a quick table that gives the approximations as well as the exact value of the solutions at the given points :

t	Exact	Euler with $h = 0.1$
1	1.9950	1.20000
2	1.99998	1.68000
3	1.99999	1.87200
4	2	1.94880
5	2	1.97952

Here is the graph of the solution and a tangent line approximation for the initial value problem: *y* Euler with *h* = 0.1

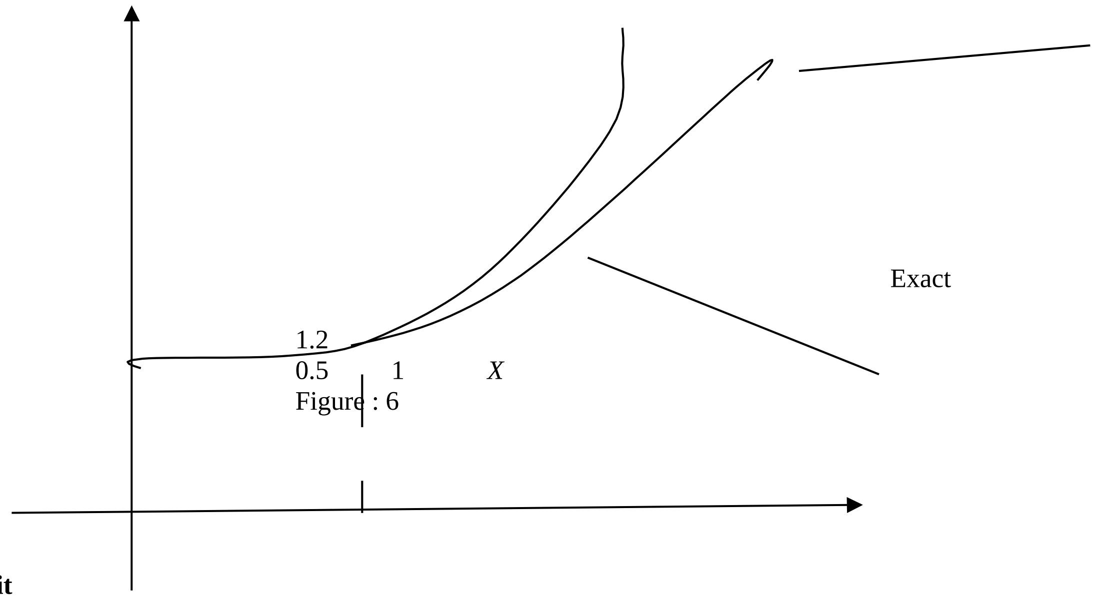

Figure : 6

Using Parallel Circuit

When resistors are connected in parallel(or any other electric circuit) the start and end points for all the resistors are the same. These points have the same potential energy and so the potential difference between them is the same no matter what is put in between them. You can have one, two or many resistors between the two points, the potential difference will not change. You can ignore whatever components are between two points in a circuit when calculating the difference between the two points.
Now for all we must to find the formula of the equation that used for solving parallel circuit and the exact solution of the equation by using the method of integrating factor so as to compare between the Euler solution and the exact solution of the equation.

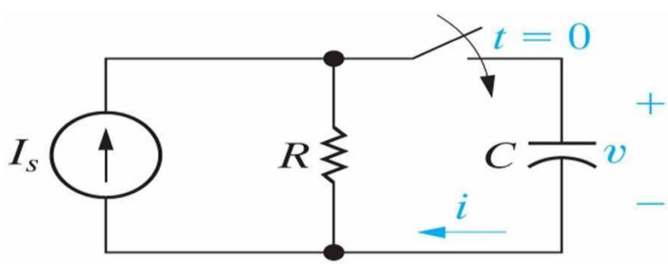

Figure : 7

From the circuit above we have :

$I_s = I_R + I_C$ [6.31]

$$\frac{I}{s} = \frac{V_R}{R} + C\frac{dV_R}{dt} \quad [6.32]$$

So the equation is : $\frac{dV}{dt} = \frac{1}{C}\left[I_S - \frac{V}{R}\right]$ [6.33]

In the parallel circuit we know that :

$V_R = V_C = V$

To find the exact equation of the solution we must to use the method of the integrating factor as shown here .

The equation can be written in the form :

$$\frac{dV}{dt} + \frac{V}{CR} = \frac{I_S}{C} \quad [6.34]$$

Put : $\mu(t) = e^{\frac{1}{RC}}$

Multiplying the original equation by the integrating factor to obtain

$$e^{\frac{1}{RC}}\frac{dV}{dt} + e^{\frac{1}{RC}}\frac{V}{CR} = \frac{I_S}{C}e^{\frac{1}{RC}} \quad [6.35]$$

So $$\left(e^{\frac{1}{RC}}V\right)' = \int \frac{I_S}{C} e^{\frac{1}{RC}}\,dt \quad [6.36]$$

$$\Rightarrow e^{\frac{1}{RC}}V = I_S \frac{RC}{C} e^{\frac{t}{RC}} + C_1 \quad [6.37]$$

$$\Rightarrow V = I_S R + C_1 e^{\frac{-t}{RC}} \quad [6.38]$$

Using the condition above we get

$V(0) = 0 \Rightarrow 0 = I_S R + C_1$ [6.39]

$\Rightarrow C_1 = -I_S R$

$$V(0) = I_S R - I_S R e^{\frac{-t}{RC}} \quad [6.40]$$

$$V(t) = IR\left(1 - e^{\frac{-t}{RC}}\right) \quad [6.41]$$

So the equation is using to find the exact solution.

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Solving some problems using Euler's Method

When Iam substituting with : t = 0, 1 , 2 , 3 , 4 , 5 in the solution of exact the values is getting so big and being divergent as seen her :

Use Euler’s method with step size 0.2 to construct a table of approximate values for the solution of the initial-value problem with simple electric parallel
circuit contains from : resistance 10 Ω , inductance 4 H.A battery gives a constant voltage of 60 V.

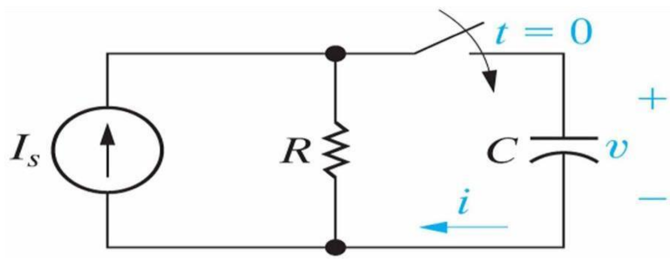

Figure : 8

Solution :

$IS = IR + IC$ [6.42] R = 10 , C = 4 , E = 60 V

So : $I_S = \frac{V}{R} + \frac{V}{C}$ [6.43]

$I_S = \frac{60}{10} + \frac{60}{4} = 21$ [6.44]

$I_1 = I_0 + hf(t_0 , I_0)$

$I_1 = 0 + 0.2(0.25) (21 - \frac{0}{10})$ [6.45]

$= 0 + 0.05(21)$

$= 1.05$ [6.46]

Now we move to get a second point

$I_2 = I_1 + hf(t_1 , I_1)$

$I_2 = 1.05 + 0.05(21- \frac{1.05}{10})$ [6.47]

= 1.05 + 0.05(21- 0.105)
= 2.09475 [6.48]
So the third point should be in form :
$I_3 = I_2 + hf(t_2 , I_2)$
I_3 = 2.09475 + 0.05(21 – 0.209475) [6.49]
= 2.09475 + 0.05(20.790525)
= 3.13427625 [6.50]
We now move on to get the fourth point in the solution
$I_4 = I_3 + hf(t_3 , I_3)$
I_4 = 3.13427625 + 0.05(21 – 0.313427625) [6.51]
= 3.13427625 + 0.05(20.686572)
= 4.1686086 [6.52]
We now move on to get the fifth and the last point in the solution
$I_5 = I_4 + hf(t_4 , I_4)$
I_5 = 4.1686086 + 0.05(21 - 0. 41686086) [6.53]
= 4.1686086 + 0.05(20.58313914)
= 5.197765557 [6.54]
Here's a quick table that gives the approximations as well as the exact value of the solutions at the given points :

t	Exact	Euler with $h = 0.1$
1	5.18492	1.05
2	10.24182	2.09475
3	15.173868	3.13427625
4	19.984114	4.1686086
5	24.675651	5.197765557

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There is big difference between the Euler solution method and exact one so its divergent .
Solving another problem using $t = 0.1 , 0.2 , 0.3 , 0.4 , 0.5$.
First of all Iam using big values , secondly small values and see whether is divergent or convergent solution .
 Consider the circuit when the switch is closed at $t = 0$, solve for $i(t)$ for the circuit given that $v(t) = 60$ v , R = 15 Ω , L = 8H .
Solving in parallel circuit with Euler's Method that step size is 0.1 .

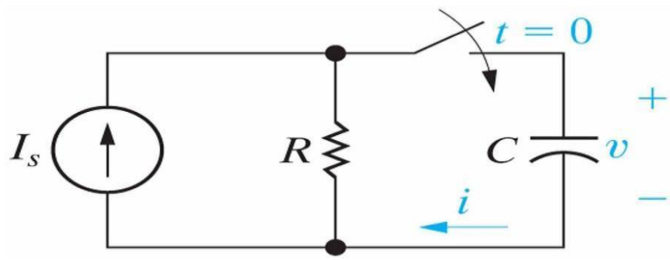

Figure : 9
Solution :
We use Kirchhoff's Law

$$\frac{dV}{dt} = \frac{1}{C}\left[IS - \frac{V}{R} \right] \qquad [6.55]$$

$$Is = IR + IC \qquad [6.56]$$

$$\frac{I}{R} = \frac{V}{R} = \frac{60}{15} = 4$$

$$IC = \frac{60}{8} = 7.5$$

So $Is = 4 + 7.5 = 11.5$ [6.57]
We can get the first point as follows :
$I_1 = I_0 + hf(t_0 , I_0)$
$I_1 = 0 + (0.1)(0.125)(11.5 – 0)$ [6.58]
$= 0 + (0.0125)(11.5)$
$= 0.14375$ [6.59]
Now we move to get a second point
$I_2 = I_1 + hf(t_1 , I_1)$

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$I_2 = 0.14375 + (0.0125)(11.5 - \frac{0.14375}{15})$ [6.60]

= 0.28738 [6.61]

So the third point should be in form :

$I_3 = I_2 + hf(t_2 , I_2)$

$I_3 = 0.28738 + (0.0125)(11.5 - \frac{0.28738}{15})$ [6.62]

= 0.430891 [6.63]

We now move on to get the fourth point in the solution

$I_4 = I_3 + hf(t_3 , I_3)$

$I_4 = 0.430891 + (0.0125)(11.5 - \frac{0.430891}{15})$ [6.64]

= 0.574282 [6.65]

Summarizing, the fifth point in our numerical solution is

$I_5 = I_4 + hf(t_4 , I_4)$

$I_5 = 0.574282 + (0.0125)(11.5 - \frac{0.574282}{15})$ [6.66]

= 0.7175534 [6.67]

So we can get the exact solution from the equation

$V(t) = IR(1 - e^{\frac{-t}{RC}})$ [6.68]

Here's a quick table that gives the approximations as well as the exact value of the solutions at the given points :

Time , t_n	Approximation	Exact	Error
$t_1 = 0.1$	$I_1 = 0.14375$	V(0.1) = 0.1436326	2.79 %
$t_2 = 0.2$	$I_2 = 0.28738$	V(0.2) = 0.2861124	4.11 %
$t_3 = o.3$	$I_3 = 0.430891$	V(0.3) = 0.4307114	4.42 %
$t_4 = 0.4$	$I_4 = 0.574282$	V(0.4) = 0.5683177	4.16 %
$t_5 = 0.5$	$I_5 = 0.7175534$	V(0.5) = 0.7161095	3.63 %

We've also included the error as a percentage. It's often easier to see how an approximation does if you look at percentages. The formula for this is,

$$\text{Percent error} = \frac{|\text{exact} - \text{approximate}|}{\text{exact}} \times 100$$

From the table we can see just how poorly our numerical approximation solution did .

VII. Conclusion :

In a Series circuit when we use big values in the equation the result of exact solution by using the method of integrating factor and numerical method (Euler's Method) , the solutions are divergent . But in the Parallel one when we use big values the result of the solution is convergent , so I replaced the big values with small one . In this case the result is getting divergent .
Series circuit big values , Parallel one small values to get divergent in Solution of Electrical Circuit using Euler's Method

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