$$
\begin{aligned}
&= \lambda \mathbf{E}\left[y(t-1)y(t-\tau)\right] + \mathbf{E}\left[e(t)y(t-\tau)\right] \\
&\quad + \mathbf{E}\left[v(t)y(t-\tau)\right] - \lambda \mathbf{E}\left[v(t-1)y(t-\tau)\right] \\
\Psi_y(0) &= \lambda\Psi(1) + \delta^2 + \xi^2 \\
\Psi_y(1) &= \lambda\Psi(0) - \lambda\xi^2 \\
\Psi_y(2) &= \lambda\Psi(1)
\end{aligned}
$$

Appendix D* - Covariance $\Psi_y(\tau) \longrightarrow \tau = 0, 1, 2$

$$
\begin{aligned}
\Psi_y(1) &\coloneqq \frac{\Psi(1)}{\lambda} + \xi^2 = \lambda\Psi(1) + \delta^2 + \xi^2 \\
&\coloneqq \frac{\lambda\delta^2}{1-\lambda^2} \\
\Psi_y(0) &\coloneqq \lambda\frac{\lambda\delta^2}{1-\lambda^2} + \delta^2 + \xi^2 \\
&\coloneqq \frac{\delta^2}{1-\lambda^2} + \xi^2 \longrightarrow \Phi_y \\
\Psi_y(2) &\coloneqq \lambda\Psi(1) \\
&\coloneqq \frac{\lambda^2\delta^2}{1-\lambda^2} \longrightarrow \frac{\lambda^\tau\delta^2}{1-\lambda^2} \text{ for } \Psi_y(\tau > 1)
\end{aligned}
$$

Appendix E - $\Gamma(\theta_1) \longrightarrow \phi_1$

$$
\begin{aligned}
\Gamma(\theta_1) &= \mathbf{E}\left\{\left[y(t) - \hat{y}(t|t-1)\right]^2\right\} \\
&= \mathbf{E}\left\{\left[y(t) - \phi_1 y(t-1)\right]^2\right\} \\
&= \mathbf{E}\left[y(t)^2 - 2\phi_1 y(t)y(t-1) + \phi_1^2 y(t-1)^2\right] \\
&= \Psi_y(0) - 2\phi_1\Psi_y(1) + \phi_1^2\Psi_y(0) \\
&= \left(1+\phi_1^2\right)\Psi_y(0) - 2\phi_1\Psi_y(1) \\
\frac{\partial\Gamma(\theta_1)}{\partial\phi_1} &= 2\phi_1\Psi_y(0) - 2\Psi_y(1) = 0 \\
\phi_1 &= \frac{\Psi_y(1)}{\Psi_y(0)} \longrightarrow \frac{\lambda\Psi_y(0) - \lambda\xi^2}{\Psi_y(0)} \\
&= \frac{\lambda\delta^2}{\delta^2 + (1-\lambda^2)\xi^2}
\end{aligned}
$$

Appendix F - $\Gamma(\theta_2) \longrightarrow \theta_2^\star \coloneqq [\phi_1, \phi_2]^\top$

$$
\begin{aligned}
\Gamma(\theta_2) &= \mathbf{E}\left\{\left[y(t) - \hat{y}(t|t-1)\right]^2\right\} \\
&= \mathbf{E}\left\{\left[y(t) - \phi_1 y(t-1) - \phi_2 y(t-2)\right]^2\right\} \\
&= \mathbf{E}\left[y(t)^2 + \phi_1^2 y(t-1)^2 + \phi_2^2 y(t-2)^2\right. \\
&\quad - 2\phi_1 y(t)y(t-1) - 2\phi_2 y(t)y(t-2) \\
&\quad \left. + 2\phi_1\phi_2 y(t-1)y(t-2)\right] \\
&= \Psi_y(0) + \phi_1^2\Psi_y(0) + \phi_2^2\Psi_y(0) - 2\phi_1\Psi_y(1) \\
&\quad - 2\phi_2\Psi_y(2) + 2\phi_1\phi_2\Psi_y(1) \\
&= \left(1+\phi_1^2+\phi_2^2\right)\Psi_y(0) + 2\phi_1\left(\phi_2 - 1\right)\Psi_y(1) \\
&\quad - 2\phi_2\Psi_y(2) \\
\frac{\partial\Gamma(\theta_2)}{\partial\phi_1} &= 2\phi_1\Psi_y(0) + 2\phi_2\Psi_y(1) - 2\Psi_y(1) = 0 \\
\phi_1 &= \frac{\Psi_y(1) - \phi_2\Psi_y(1)}{\Psi_y(0)} \\
&= \frac{\Psi_y(1) - \left(\dfrac{\Psi_y(2)\Psi_y(0) - \Psi_y(1)^2}{\Psi_y(0)^2 - \Psi_y(1)^2}\right)\Psi_y(1)}{\Psi_y(0)} \\
&= \frac{\Psi_y(1)\Psi_y(0) - \Psi_y(2)\Psi_y(1)}{\Psi_y(0)^2 - \Psi_y(1)^2} \\
&= \frac{\lambda\delta^2\left(\delta^2+\xi^2\right)}{\left(\delta^2+\xi^2\right)^2 - \lambda^2\xi^4} \\
\frac{\partial\Gamma(\theta_2)}{\partial\phi_1} &= 2\phi_2\Psi_y(0) + 2\phi_1\Psi_y(1) - 2\Psi_y(2) = 0 \\
\phi_2 &= \frac{\Psi_y(2) - \phi_1\Psi_y(1)}{\Psi_y(0)} \\
&= \frac{\Psi_y(2) - \left(\dfrac{\Psi_y(1)\Psi_y(0) - \Psi_y(2)\Psi_y(1)}{\Psi_y(0)^2 - \Psi_y(1)^2}\right)\Psi_y(1)}{\Psi_y(0)} \\
&= \frac{\Psi_y(2)\Psi_y(0) - \Psi_y(1)^2}{\Psi_y(0)^2 - \Psi_y(1)^2} \\
&= \frac{\lambda^2\delta^2\xi^2}{\left(\delta^2+\xi^2\right)^2 - \lambda^2\xi^4}
\end{aligned}
$$

Appendix G - $\Phi_{\theta_1^\star}$ and $\Phi_{\theta_2^\star}$

$$
\begin{aligned}
\Phi_{\theta_1^\star} &= \Gamma(\theta_1) = \mathbf{E}\left\{\left[\varepsilon_{\theta_1}(t) - \bar{\varepsilon}_{\theta_1}\right]^2\right\} \longrightarrow \bar{\varepsilon}_{\theta_1} = 0 \\
&= \left[1+\phi_1^2\right]\Psi_y(0) - 2\phi_1\Psi_y(1) \longrightarrow \phi_1 \text{ from Eq. (21)} \\
&= \left[1 + \left(\frac{\lambda\delta^2}{\delta^2 + (1-\lambda^2)\xi^2}\right)^2\right]\frac{\delta^2 + (1-\lambda^2)\xi^2}{1-\lambda^2} \\
&\quad - 2\left(\frac{\lambda\delta^2}{\delta^2 + (1-\lambda^2)\xi^2}\right)\frac{\lambda\delta^2}{1-\lambda^2} \\
&= \frac{\left[\delta^2 + \left(1-\lambda^2\right)\xi^2\right]^2 - \lambda^2\delta^4}{\left[\delta^2 + \left(1-\lambda^2\right)\xi^2\right]\left[1-\lambda^2\right]} \\
&= \frac{\delta^4\left(1-\lambda^2\right) + 2\delta^2\xi^2\left(1-\lambda^2\right) + \xi^4\left(1-\lambda^2\right)^2}{\left[\delta^2 + \left(1-\lambda^2\right)\xi^2\right]\left[1-\lambda^2\right]} \\
&= \frac{\left(\delta^2+\xi^2\right)^2 - \lambda^2\xi^4}{\delta^2 + \left(1-\lambda^2\right)\xi^2} \\
\Phi_{\theta_2^\star} &= \Gamma(\theta_2) = \mathbf{E}\left\{\left[\varepsilon_{\theta_2}(t) - \bar{\varepsilon}_{\theta_2}\right]^2\right\} \longrightarrow \bar{\varepsilon}_{\theta_2} = 0 \\
&= \left[1+\phi_1^2+\phi_2^2\right]\Psi_y(0) - \left[2\phi_1 - 2\phi_1\phi_2\right]\Psi_y(1) \\
&\quad - 2\phi_2\Psi_y(2) \longrightarrow [\phi_1, \phi_2] \text{ from Eq. (22)} \\
&= \Psi_y(0)^3 - \Psi_y(2)^2\Psi_y(0) - 2\Psi_y(1)^2\Psi_y(0) \\
&\quad + 2\Psi_y(1)^2\Psi_y(2) \longrightarrow \text{after some complex-algebra} \\
&= \left(\frac{\delta^2 + (1-\lambda^2)\xi^2}{1-\lambda^2}\right)^3 - \left(\frac{\lambda^2\delta^2}{1-\lambda^2}\right)^2\frac{\delta^2 + (1-\lambda^2)\xi^2}{1-\lambda^2} \\
&\quad - 2\left(\frac{\lambda\delta^2}{1-\lambda^2}\right)^2\frac{\delta^2 + (1-\lambda^2)\xi^2}{1-\lambda^2} + 2\lambda\left(\frac{\lambda\delta^2}{1-\lambda^2}\right)^3 \\
&= \frac{\left(\delta^2+\xi^2\right)^3 - \lambda^2\left(\delta^2\xi + \xi^3\right)^2 + 2\lambda\delta^4\xi^2}{1-\lambda^2}
\end{aligned}
$$

Appendix H - $\Gamma(\theta_1) \longrightarrow \phi_1$; (non) zero-mean

$$
\begin{aligned}
\Gamma(\theta_1) &= \mathbf{E}\left[y(t)^2 - 2y(t)y(t|t-1) + y(t|t-1)^2\right] \\
&= \mathbf{E}\left\{\left[\breve{y}(t) + \bar{y}\right]^2\right\} - 2\phi_1\mathbf{E}\left\{\left[\breve{y}(t) + \bar{y}\right]\left[\breve{y}(t-1) + \bar{y}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \phi_1^2 \mathbf{E}\left\{[\breve{y}(t-1)+\bar{y}]^2\right\} \\
&= \left[1+\phi_1^2\right]\Psi_y(0) - 2\phi_1\Psi_y(1) + (1-\phi_1)^2\,\bar{y}^2 \\
\frac{\partial\Gamma(\theta_1)}{\partial\phi_1} &= -2\Psi_y(1) + 2\phi_1\Psi_y(0) - 2\,(1-\phi_1)\,\bar{y}^2 = 0 \\
\theta_1^\star &:= \frac{\Psi(1)+\bar{y}^2}{\Psi(0)+\bar{y}^2}
\end{aligned}
$$

Appendix I - $\Gamma(\theta_2) \longrightarrow \theta_2^\star := [\phi_1, \phi_2]^\top$; (non) zero-mean

$$
\begin{aligned}
\Gamma(\theta_2) &= \mathbf{E}\left[y(t)^2 - 2y(t)y(t|t-1) + y(t|t-1)^2\right] \\
&= \mathbf{E}\left\{[\breve{y}(t)+\bar{y}]^2\right\} - 2\mathbf{E}\Big\{[\breve{y}(t)+\bar{y}]\,[\phi_1\breve{y}(t-1) \\
&\quad + \phi_2\breve{y}(t-2) + (\phi_1+\phi_2)\,\bar{y}]\Big\} + \phi_1^2\mathbf{E}\left\{[\breve{y}(t-1)]^2\right\} \\
&\quad + \phi_2^2\mathbf{E}\left\{[\breve{y}(t-2)]^2\right\} + (\phi_1+\phi_2)\,\mathbf{E}\left\{[\bar{y}]^2\right\} \\
&\quad + 2\phi_1\,(\phi_1+\phi_2)\,\mathbf{E}\left\{[\breve{y}(t-1)\bar{y}]\right\} \\
&\quad + 2\phi_2\,(\phi_1+\phi_2)\,\mathbf{E}\left\{[\breve{y}(t-2)\bar{y}]\right\} \\
&\quad + 2\phi_1\phi_2\mathbf{E}\left\{[\breve{y}(t-1)\breve{y}(t-2)]\right\} \\
&= \Psi_y(0) + \phi_1^2\Psi_y(0) + \phi_2^2\Psi_y(0) - 2\phi_1\Psi_y(1) \\
&\quad - 2\phi_2\Psi_y(2) + 2\phi_1\phi_2\Psi_y(1) + [1-(\phi_1+\phi_2)]^2\,\bar{y}^2
\end{aligned}
$$

$$
\frac{\partial\Gamma(\theta_2)}{\partial\phi_1} = 0 \text{ and } \frac{\partial\Gamma(\theta_2)}{\partial\phi_2} = 0
$$

after some complex-algebra

$$
\begin{aligned}
\phi_1 &= \frac{\Psi_y(1)\Psi_y(0) - \Psi_y(2)\Psi_y(1) + \bar{y}^2\,[\Psi(0)-\Psi(2)]}{\Psi_y(0)^2 - \Psi_y(1)^2 + 2\bar{y}^2\,[\Psi(0)-\Psi(1)]} \\
\phi_2 &= \frac{\Psi_y(2)\Psi_y(0) - \Psi_y(1)^2 + \bar{y}^2\,[\Psi(0)-2\Psi(1)+\Psi(2)]}{\Psi_y(0)^2 - \Psi_y(1)^2 + 2\bar{y}^2\,[\Psi(0)-\Psi(1)]}
\end{aligned}
$$

Appendix J - $\Phi_{\theta_1^\star}$ and $\Phi_{\theta_2^\star}$; (non) zero-mean

$$
\begin{aligned}
\bar{\varepsilon}_{\theta_1} &= \mathbf{E}\,[\breve{y}(t)] + \mathbf{E}\,[\bar{y}] - \phi_1\mathbf{E}\,[\breve{y}(t-1)] - \phi_1\mathbf{E}\,[\bar{y}] \\
&= (1-\phi_1)\bar{y} \\
\Phi_{\theta_1^\star} &= \Gamma(\theta_1) = \mathbf{E}\left\{[\varepsilon_{\theta_1}(t) - \bar{\varepsilon}_{\theta_1}]^2\right\} \\
&= \mathbf{E}\left\{[\breve{y}(t) + \bar{y} - \phi_1\breve{y}(t-1) - \phi_1\bar{y} - (1-\phi_1)\,\bar{y}]^2\right\} \\
&= \mathbf{E}\left\{[\breve{y}(t) - \phi_1\breve{y}(t-1)]^2\right\} \longrightarrow \textbf{Appendix E} \\
&= \frac{\left(\delta^2+\xi^2\right)^2 - \lambda^2\xi^4}{\delta^2 + (1-\lambda^2)\,\xi^2} \longrightarrow \textbf{Appendix G} \\
\bar{\varepsilon}_{\theta_2} &= \mathbf{E}\,[\breve{y}(t)] + \mathbf{E}\,[\bar{y}] - \phi_1\mathbf{E}\,[\breve{y}(t-1)] - \phi_2\mathbf{E}\,[\breve{y}(t-2)] \\
&\quad - (\phi_1+\phi_2)\,\mathbf{E}\,[\bar{y}] \\
&= (1-\phi_1-\phi_2)\bar{y} \\
\Phi_{\theta_2^\star} &= \Gamma(\theta_2) = \mathbf{E}\left\{[\varepsilon_{\theta_2}(t) - \bar{\varepsilon}_{\theta_2}]^2\right\} \\
&= \mathbf{E}\Big\{[\breve{y}(t) + \bar{y} - \phi_1\breve{y}(t-1) - \phi_2\breve{y}(t-2) \\
&\quad - (\phi_1+\phi_2)\,\bar{y} - (1-\phi_1-\phi_2)\,\bar{y}]^2\Big\} \\
&= \mathbf{E}\left\{[\breve{y}(t) - \phi_1\breve{y}(t-1) - \phi_2\breve{y}(t-2)]^2\right\} \\
&= \frac{\left(\delta^2+\xi^2\right)^3 - \lambda^2\left(\delta^2\xi+\xi^3\right)^2 + 2\lambda\delta^4\xi^2}{1-\lambda^2}
\end{aligned}
$$

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