The Impact of Chemical Reaction and Heat source on MHD Free Convection Flow over an Inclined Porous Surface.

Ekakitie Omamoke
Department of Mathematics, Bayelsa Medical University, Bayelsa State, Nigeria.
omamoke.ekakitie@bmu.edu.ng; ekakitieomamoke@gmail.com.

Amos Emeka
Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.
amos.emeka@ust.edu.ng

DOI: 10.29322/IJSRP.10.05.2020.p10103
http://dx.doi.org/10.29322/IJSRP.10.05.2020.p10103

Abstract: This study analyzes the chemical reaction and heat source effects on MHD free convection flow, of an incompressible fluid that is viscous over an inclined porous surface. With the application of the perturbation technique, the solution of a set of ordinary differential equations are gotten as a result of reducing the non-linear partial differential equations of motion, energy and diffusion, which are solved analytically for velocity, temperature and the concentration distribution. The effect of Chemical reaction and heat source on the velocity, temperature, concentration, skin friction, heat transfer and rate of mass flux distribution is plotted graphically and discussed.

Keywords: **Free Convection, Chemical Reaction, Heat Source, Inclined Plate, Porous Surface, Magneto Hydrodynamic (MHD).**

1.0 Introduction

Free convection flow with Chemical reaction and heat source effect has significant effect on the boundary layer control and fluid flow which affects the performance of many engineering devices and Medical systems with fluids that conducts electricity. This fluid flow finds application in MHD power generation, plasma studies, nuclear reactor, Human Blood flow, Water flows etc. MHD free convective heat and mass transfer flow for vertical, horizontal and inclined surfaces have attracted many researchers because of its rich application in Medcine, engineering and technology, agriculture, space technology and many more. Paras *et al.* (2013) studied the effect of porosity on unsteady MHD flow past a semi-infinite moving vertical plate with time dependent suction. The equations for motion and energy was solved into ordinary differential equations and then analytically solved using the flex partial differential flex method. Rajakumar *et al.* (2017) studied chemical reaction and viscous dissipation effects on MHD free convective flow past a semi-infinite moving vertical porous plate with radiation absorption. Dimensionless governing equations were solved analytically using multiple perturbation technique. Isreal-Cookey *et al.* (2003) investigated the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction. Mebine (2007) did a study on the thermal solutal MHD flow with radiative heat transfer over oscillating plate in a fluid that is chemically active and got the solutions for the unsteady velocity, temperature and concentration with Laplace transform on the assumption of an optically thin medium, and linear differential approximation model for the radiative flux. Sandeep *et al*. (2012) analyzed the Magneto hydrodynamic, Radiation and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate through porous media. The porous plate was subjected to a transverse variable suction velocity. Sugunamma *et al.* (2013) studied MHD Radiation and chemical reaction effects on unsteady flow, heat and mass transfer in a viscous, incompressible and electrically conducting fluid over a semi-infinite vertical porous plate through porous media in presence of inclined magnetic field with porous plate subjected to a transverse variable suction velocity. Veeresh *et al.* (2015) did an analysis on heat and mass transfer in MHD free convection of a chemically reactive and radiative flow in a moving inclined porous plate with temperature dependent heat source and joule heating using regular perturbation technique. Venkateswarlu *et al.* (2015) analyzed the MHD unsteady flow of viscous, incompressible electrical conduction fluid over a vertical porous plate under the influence of thermal radiation and chemical reaction. Tripathy *et al.* (2015) studied heat and mass transfer effect in a boundary layer flow of an electrically conducting viscous fluid subject to transverse magnetic field past over a moving vertical plate through porous medium in the presence of heat source and chemical reaction. Ramana Rendy *et al.* (2016) analyzed the effects of magneto hydrodynamic force and buoyancy on convective heat and mass transfer flow past a moving vertical porous plate in the presence of thermal radiation and chemical reaction. Amos Emeka *et al.* (2018) studied MHD free convective flow over an inclined porous surface with variable suction and radiation effect. Pertinent parameters such as suction and radiation had an effect on velocity profile, temperature profile, concentration profile and heat transfer.

2.0 Mathematical Formulation

A free convective unsteady flow of an incompressible hydro magnetic fluid that is viscous and chemically reactive over a porous surface inclined at an angle α in the vertical direction is considered. The magnetic field of constant intensity B_0 applied in the y – direction of the plate moves uniformly with velocity V_0 in the x – direction. The temperature $T_w > T_\infty$ and concentration $C_w > C_\infty$ are conditions at the wall and ambient regions respectively.

Thus the flow equations are as follows.

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} = v\frac{\partial^2 u'}{\partial y'} - \left[\frac{\delta_c B_0^2}{\rho} + \frac{v}{k}\right]u' + gB_T(T' - T_\infty)cos \propto + gB_C(C' - C_\infty)cos \propto \quad (2)$$

$$\frac{\rho C_p}{K_T}\left[\frac{\partial T'}{\partial t'}\right] = \frac{\partial^2 T'}{\partial y'^2} + \frac{Q}{K_T}(T' - T_\infty) \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r'(C' - C_\infty) \quad (4)$$

The boundary conditions are

$$u' = U'\,;\ T' = T_w';\ C' = C_w'; \qquad at\ y = 0 \quad (5)$$
$$u' = 0\ T' = T_\infty';\ C' = C_w' \qquad as\ y \to \infty$$

The non-dimensional quantities are introduced in order to write the governing equations and boundary conditions in dimensionless form.

$$y = \frac{V_O y'}{v};\ u = \frac{u'}{U_O};\ t = \frac{V_O^2 t'}{v};\ \theta = \frac{T' - T_\infty}{T_W' - T_\infty};\ C = \frac{C' - C_\infty}{C_W' - C_\infty} \quad (6)$$

$$M = \frac{v\delta_c B_0^2}{\rho V_O^2};\ K = \frac{KV_O^2}{v^2};\ G_T = \frac{gvB_T\theta(T_W' - T_\infty)}{U_O V_O^2};\ G_C = \frac{gvB_C C(C_W' - C_\infty)}{U_O V_O^2};\ v = \frac{\mu}{\rho};\ P_r = \frac{\mu C_p}{K_T};\ S_c = \frac{v}{D};\ H = \frac{QV}{\rho C_p V_O^2} \quad (7)$$

The momentum, energy and diffusion equation in dimensionless form is written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M - \frac{1}{K}\right)u + G_T\theta cos \propto + G_C C cos \propto \quad (8)$$

$$P_r\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + HP_r\theta \quad (9)$$

$$S_c\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - S_c K_r C \quad (10)$$

The corresponding boundary conditions in non-dimensional form are:

$$u = 1, \theta = 1, C = 1, \quad at\ y = 0$$
$$u \to 0, \theta \to 0, C \to 0, \quad at\ y \to 0 \quad (11)$$

In the equations above, u and v are the velocities, x and y are the Cartesian coordinates, α is the angle of inclined porous surface in the vertical direction, t' is the time, T_W' is the temperature at the wall, T_∞' is the ambient temperature and g is the acceleration due to gravity. B_0 is the constant magnetic field, K is the porosity parameter, C_p is the specific heat capacity and M is the magnetic parameter. G_T is the Grashof temperature number, Pr is the Prandtle number, , ε is the small positive constant, ρ is the density, β_T is the coefficient of volume expansion due to temperature and β_C is the coefficient of volume expansion due to concentration, ω is the free stream frequency oscillation, δ_c is the electrical conductivity, G_C is the modified grashoffs number, K_r is the chemical reaction parameter, θ is the temperature and C_w is the concentration at the wall.

3.0 Method of Solution

ISSN 2250-3153
The nonlinear partial differential equations in (2) – (4) are solved analytically and then reduced to a set of ordinary differential equations with the expressions for velocity (u), temperature (θ) and concentration (C) of the fluid in dimensionless form as follows:

$$u(y,t) = u_o(y) + \varepsilon e^{i\omega t} u_1 + 0(\varepsilon^2) \quad (12)$$
$$\theta(y,t) = \theta_o(y) + \varepsilon e^{i\omega t} \theta_1 + 0(\varepsilon^2) \quad (13)$$
$$C(y,t) = C_o(y) + \varepsilon e^{i\omega t} C_1 + 0(\varepsilon^2) \quad (14)$$

Substituting equation (12) to (14) in the set of equations (8) **-** (9) and equating non – harmonic and harmonic terms neglecting the higher order terms of $0(\varepsilon^2)$ where $\varepsilon \ll 1$, the ordinary differential equations are obtained with their boundary conditions.

$$u_0'' - \left(M + \frac{1}{K}\right) u_0 = -G_T \theta_0 cos \propto - G_C C_0 cos \propto \quad (15)$$

$$\theta_0'' + HP_r \theta_0 = 0 \quad (16)$$

$$C_0'' - S_c K_r C_o = 0 \quad (17)$$

$$u_1'' - \left(M + \frac{1}{K} + i\omega\right) u_1 = -G_T \theta_1 cos \propto - G_C C_1 cos \propto \quad (18)$$

$$\theta_1'' - P_r(H - i\omega)\theta_1 = 0 \quad (19)$$

$$C_1'' - S_c(K_r + i\omega)C_1 = \quad (20)$$

Boundary conditions

$$u_0 = 1; u_1 = 1;\ \theta_0 = 1;\ ,\theta_1 = 1;\ C_0 = 1;\ C_1 = 1 \qquad at\ y = 0 \quad (21)$$
$$u_0 \to 0; u_1 \to 0;\ \theta_0 \to 0;\ ,\theta_1 \to 0;\ C_0 \to 0;\ C_1 \to 0 \qquad as\ y \to \infty$$

Substituting equation (15) – (20) into equation (12) – (14) we obtain the velocity, temperature and concentration profiles respectively as

$$u(y,t) = e^{-m_6 y} - A_3 e^{-m_4 y} - A_4 e^{-m_2 y} + \varepsilon e^{i\omega t}[e^{-m_5 y} - A_1 e^{-m_3 y} - A_2 e^{-m_1 y}] \quad (22)$$

$$\theta(y,t) = e^{-m_4 y} + \varepsilon e^{i\omega t}[e^{-m_3 y}] \quad (23)$$

$$C(y,t) = e^{-m_2 y} + \varepsilon e^{i\omega t}\{e^{-m_1 y}\} \quad (24)$$

The physical quantities of interest are the wall shear stress τ_w is given by

$$\tau_w = \mu \frac{\partial u^I}{\partial y^I} = \rho v_0^2 u^I(0)$$

The local skin friction factor

$$C_{fx} = \frac{\tau_w}{\rho v_0^2} = u^I(0) = -m_6 + m_4 A_3 + m_2 A_4 + \varepsilon[-m_5 + m_3 A_1 + m_1 A_2] \quad (25)$$

Heat transfer is given by

$$\frac{N_{ux}}{R_{ex}} = \frac{\partial \theta}{\partial y} = \theta^I(0) = -m_4 + \varepsilon[-m_3] \quad (26)$$

The local surface mass flux is given by

$$\frac{Sh_x}{R_{ex}} = -\frac{\partial C}{\partial y} = C^I(0) = -m_2 + \varepsilon(-m_1) \quad (27)$$

4. RESULTS AND DISCUSSIONS

In this session we will be looking at the effect of viscous parameters on the velocity, temperature, concentration, skin friction, heat transfer and mass flux rate. From figure 4.1 it is observed that an increase in the magnetic field causes reduction in the velocity of the fluid as a result of Lorentz force which makes the boundary layer thick resisting flow of fluid. Figure 4.2 shows a slight increase in the chemical reaction that results to an increase in the velocity more at the boundary layer. Similar effect of increase in heat source in figure 4.3 causes an increase in the velocity of the fluid. An in the porosity causes the velocity to reduce in figure 4.4 while the velocity increased in figure 4.5 and 4.6 as a result of increase in both Grashof temperature number and Grashof concentration number causing the plate to become cooler. In figure 4.7 there is a mixed effect in the velocity of the fluid as the angle of inclination increases. An increase in the heat source and Prandtl number reduces the temperature in figure 4.8 and 4.9. Furthermore an increase in chemical reaction and Schmidt number increases the concentration in figure 4.10 and 4.11. Figure 4.12 shows an increase in heat source increase the skin friction while an increase in magnetic field reduces the skin friction in figure 4.14, while an increase in chemical reaction causes a mixed effect on the skin friction in figure 4.13. Finally, figure 4.15 shows that increase in heat source increases the heat transfer while the increase in chemical reaction increases the mass flux rate in figure 4.16

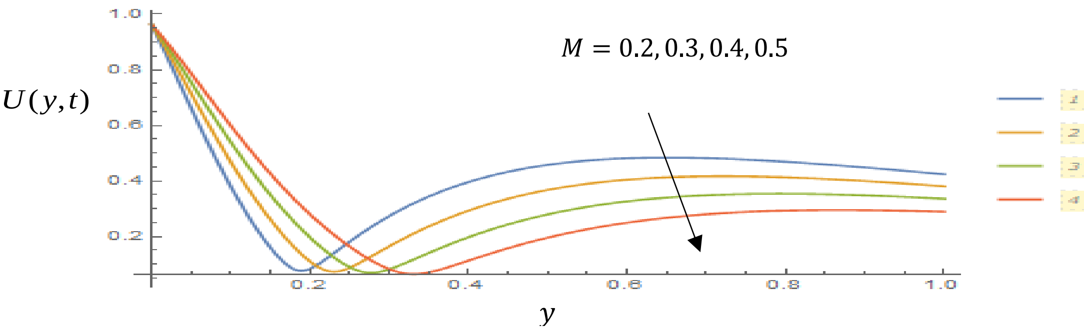


Figure 4.1 **Velocity Profile with Variation of Magnetic Field Parameter *M* for**
$K_r = 0.2, H = 0.1, P_r = 0.71, G_r = 15, G_c = 15, \ S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

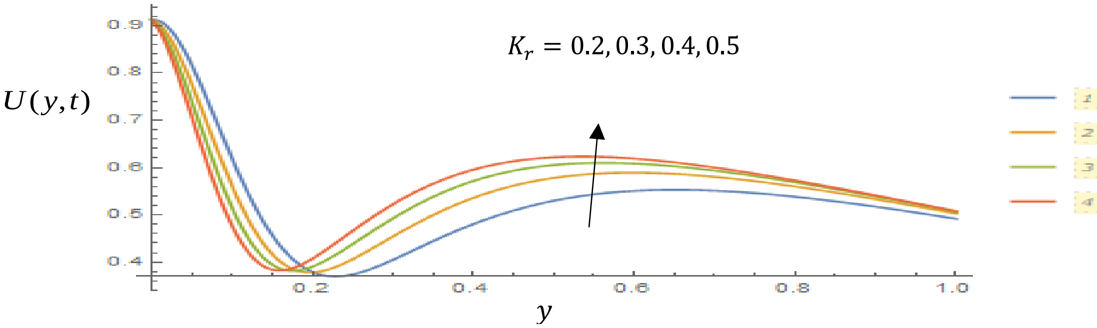


Figure 4.2 **Velocity Profile with Variation of Chemical Reaction Parameter for** K_r
$M = 0.2, H = 0.1, P_r = 0.71, G_r = 15, G_c = 15, \ S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

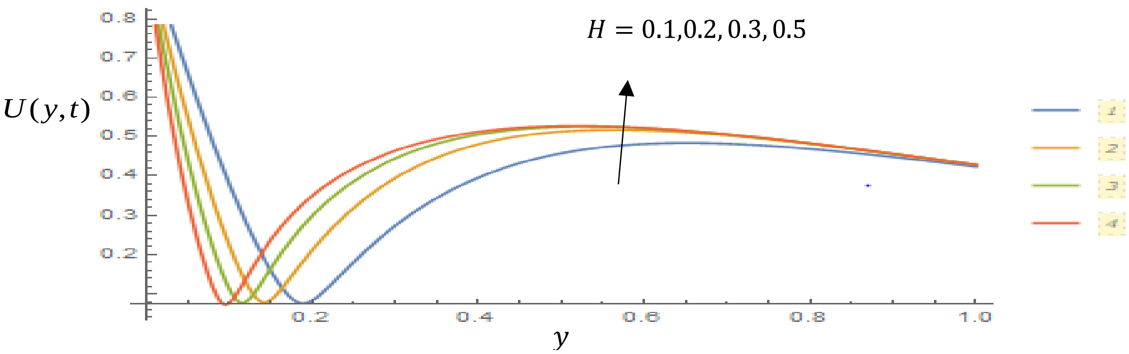
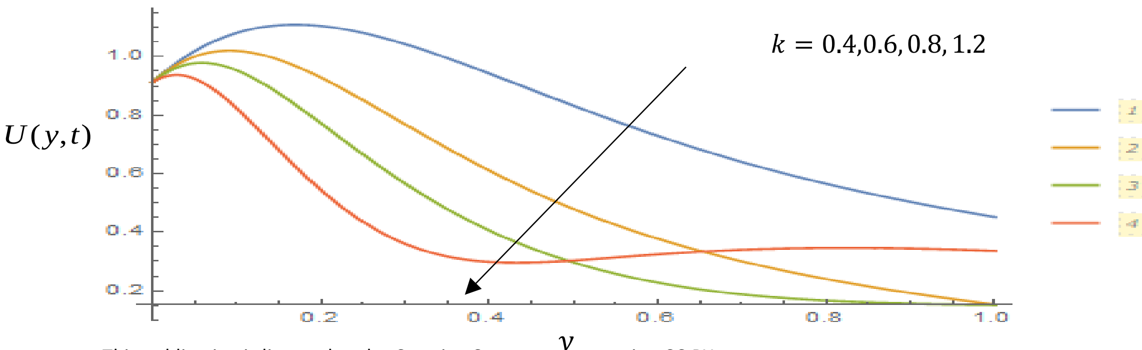


Figure 4.3 **Velocity Profile with Variation of Heat Source Parameter for H**
$M = 0.2, K_r = 0.2, P_r = 0.71, G_r = 15, G_c = 15, \ S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$



ISSN 2250-3153

Figure 4.4 **Velocity Profile with Variation of Porosity Parameter for k**

$M = 0.2, K_r = 0.2\ H = 0.1, P_r = 0.71, G_r = 15, G_c = 15,\ S_c = 0.22, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

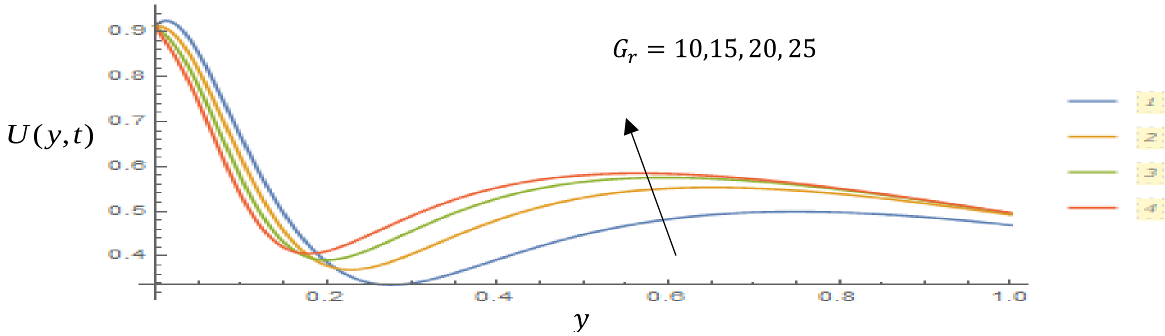


Figure 4.5 **Velocity Profile with Variation of Grashof Temperature Parameter for** G_r

$M = 0.2, K_r = 0.2\ H = 0.1, P_r = 0.71, G_c = 15,\ S_c = 0.22, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

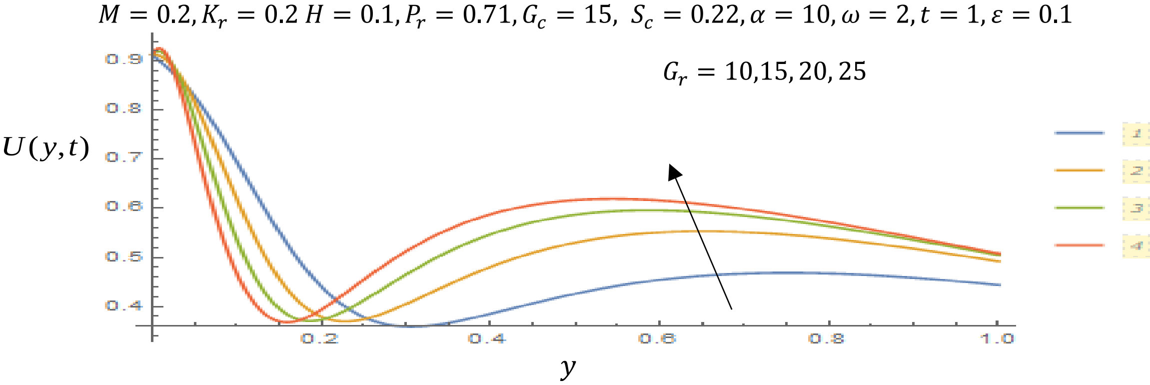


Figure 4.6 **Velocity Profile with Variation of Grashof Temperature Parameter for** G_C

$M = 0.2, K_r = 0.2\ H = 0.1, P_r = 0.71, G_T = 15,\ S_c = 0.22, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

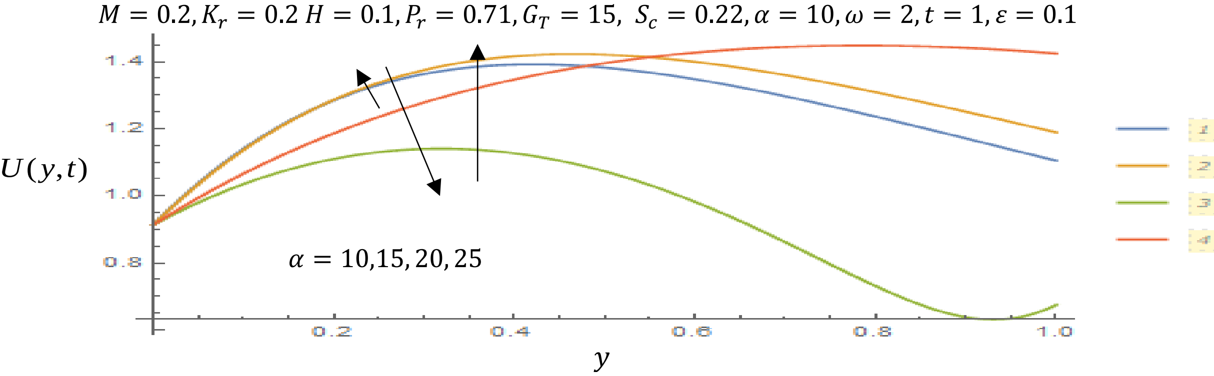


Figure 4.7 **Velocity Profile with Variation of angle of inclination Parameter for** α

$M = 0.2, K_r = 0.2\ H = 0.1, P_r = 0.71, G_T = 15,\ S_c = 0.22, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

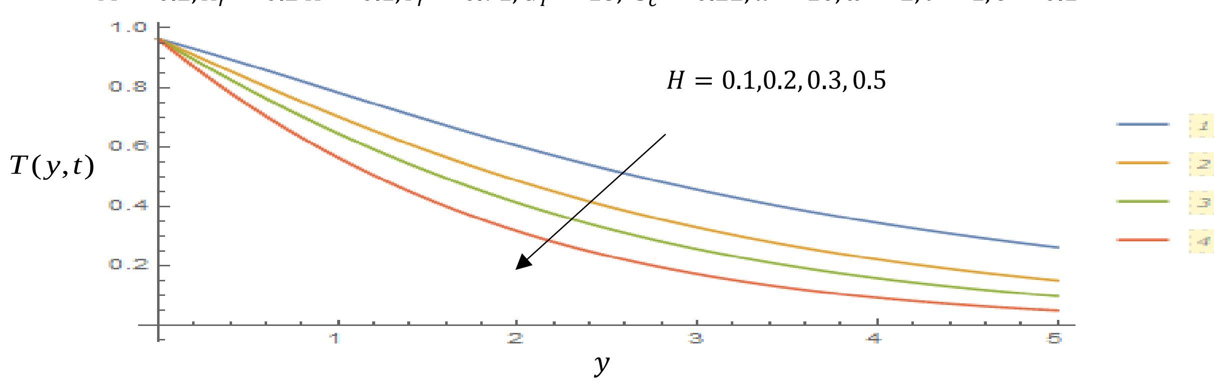


Figure 4.8 **Temperature Profile with Variation of Heat Source Parameter for** ***H***

$H = 0.1, P_r = 0.71, \omega = 2, t = 1, \varepsilon = 0.1$

ISSN 2250-3153

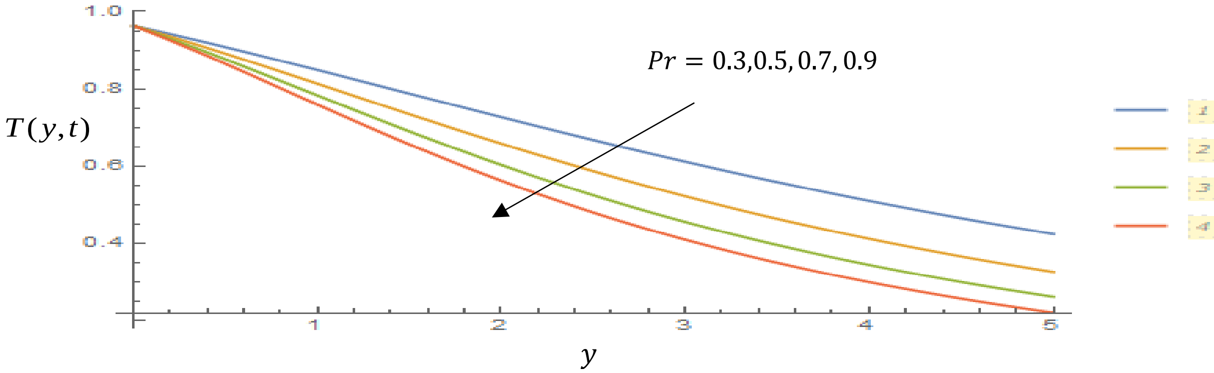


Figure 4.9 **Temperature Profile with Variation of Prandtl number Parameter for *Pr***
$H = 0.1, \omega = 2, t = 1, \varepsilon = 0.1$

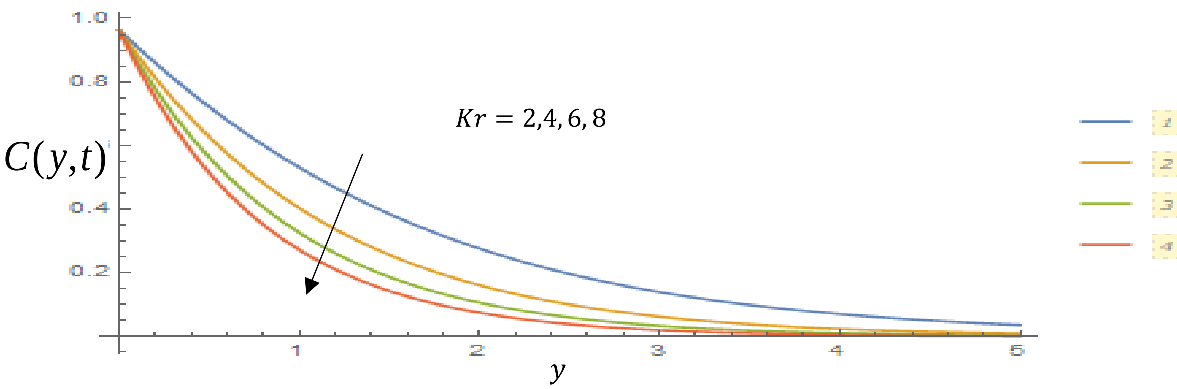


Figure 4.10 **Concentration Profile with Variation of Chemical Reaction Parameter *Kr***
$Sc = 0.22, \omega = 2, t = 1, \varepsilon = 0.1$

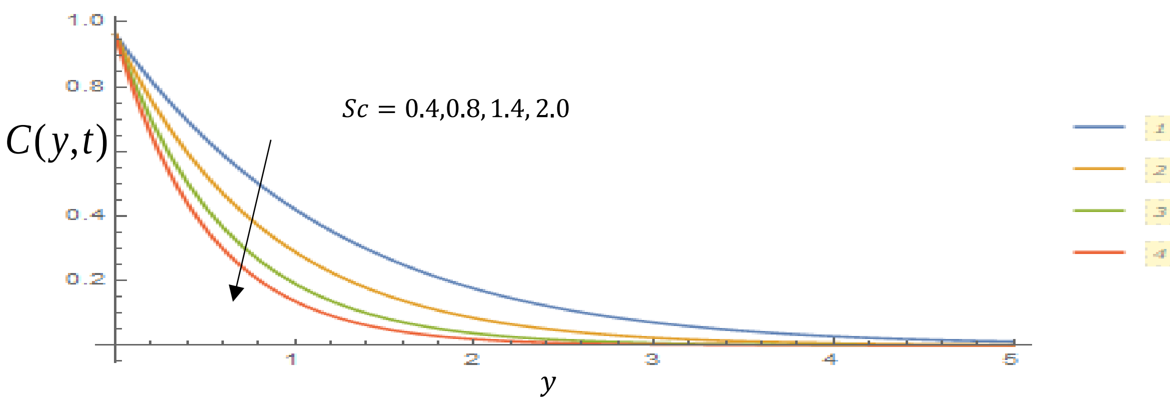


Figure 4.11 **Concentration Profile with Variation of Schmidt number Parameter *Sc***
$Sc = 0.22, \omega = 2, t = 1, \varepsilon = 0.1$

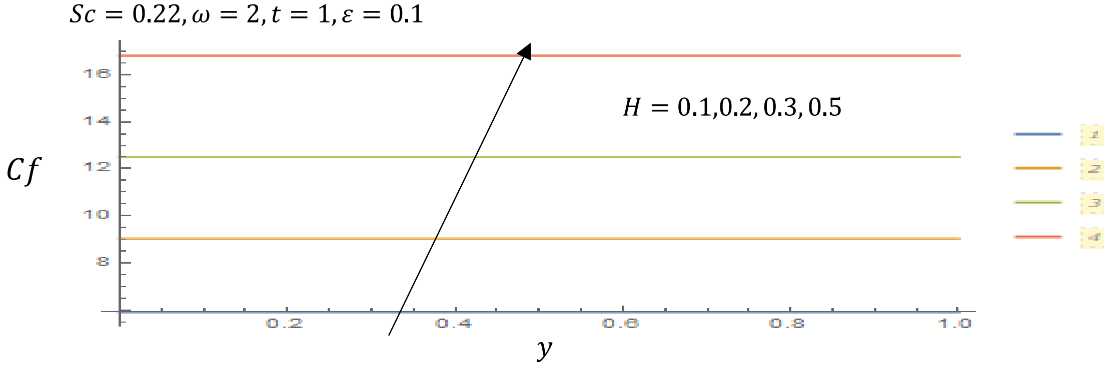


Figure 4.12 **Skin Friction Profile with Variation of Heat Source Parameter for H**
$M = 0.2, K_r = 0.2, P_r = 0.71, G_r = 15, G_c = 15, \; S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

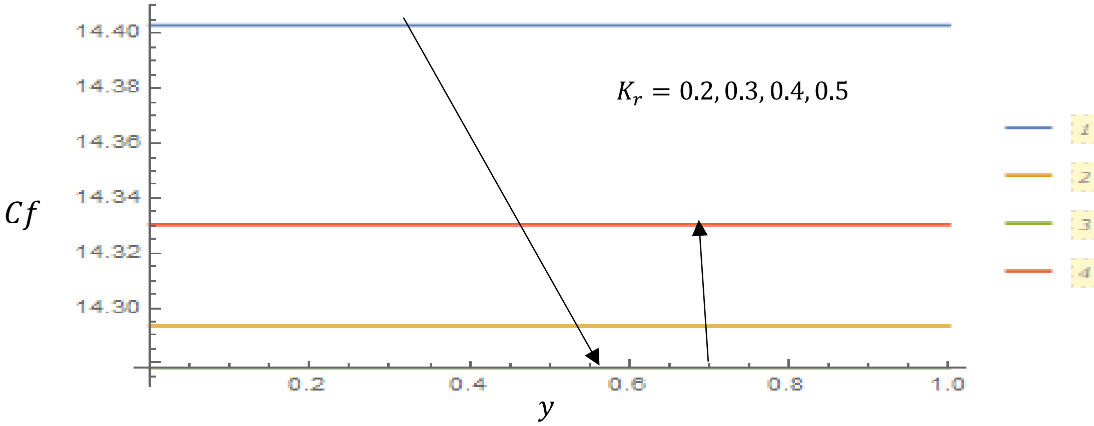


Figure 4.13 **Skin Friction Profile with Variation of Chemical Reaction Parameter for** K_r
$M = 0.2, H = 0.1, P_r = 0.71, G_r = 15, G_c = 15, S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

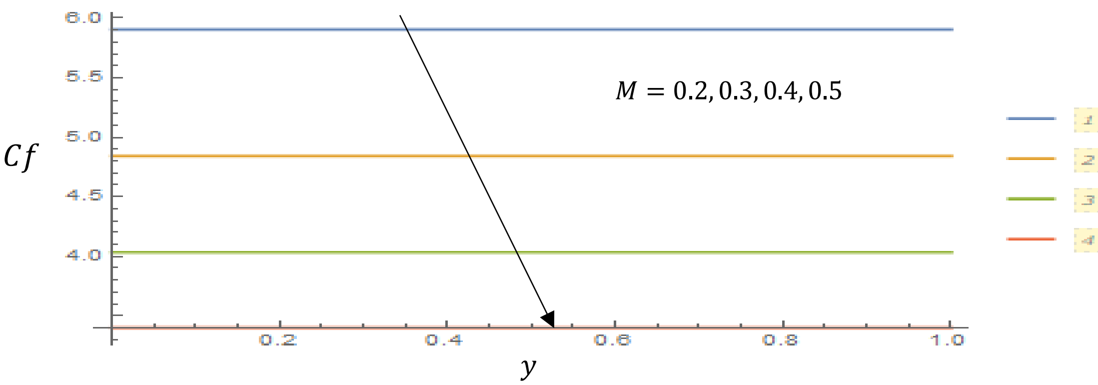


Figure 4.14 **Skin Friction Profile with Variation of Magnetic Field Parameter *M* for**
$K_r = 0.2, H = 0.1, P_r = 0.71, G_r = 15, G_c = 15, S_c = 0.22, \mathrm{k} = 2, \alpha = 10, \omega = 2, t = 1, \varepsilon = 0.1$

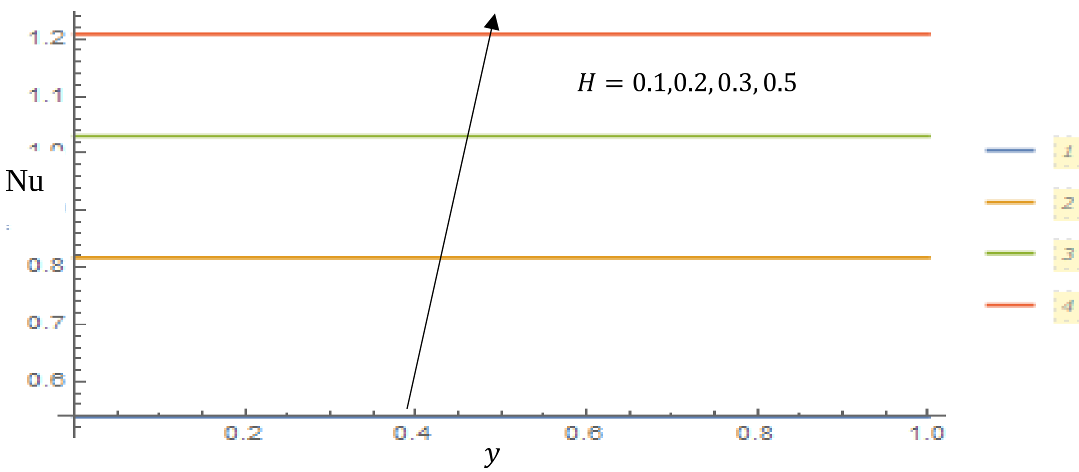


Figure 4.15 **Heat Transfer Profile with Variation of Heat Source Parameter for *H***
$H = 0.1, P_r = 0.71, \omega = 2, t = 1, \varepsilon = 0.1$

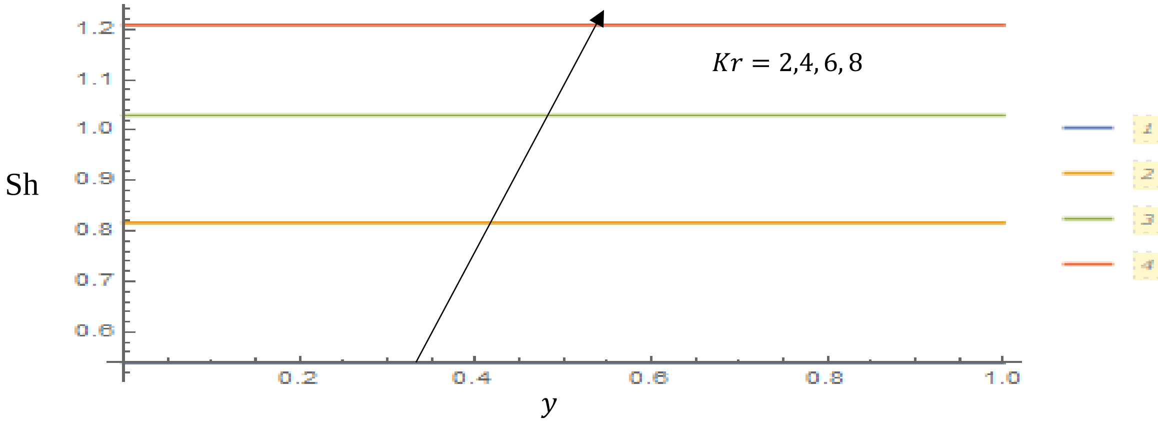


Figure 4.16 **Mass Transfer Profile with Variation of Chemical Reaction Parameter *Kr***
$Sc = 0.22, \omega = 2, t = 1, \varepsilon = 0.1$

5. CONCLUSION

Chemical reaction and heat source effect on MHD free convective flow over an inclined porous surface and the effects of other parameters are summarized as follows,

- Increased Magnetic field reduces the velocity of the fluid because of a thickened boundary layer caused by Lorentz force which opposes the flow of the fluid,
- Increase in chemical reaction causes an increase in the velocity of the fluid and increase in the concentration of the fluid by increasing the rate of mass flux,
- Increase in heat source caused the velocity of the fluid to increase with an effect of reduced temperature due to rise in skin friction and heat transfer.
- Increase in porosity of the plate reduces the velocity of the fluid.

References.

Amos E. and Omamoke E. (2018). MHD Free Convetive Flow over an Inclined Porous Surface with Variable Suction and Radiation Effects. International Journal of Applied Science and Mathematical Theory, ISSN 2489-009X, Vol. 4, No. 3.

Israel-Cookey C., Ogulu A. and V.B. Omubo-Pepple. (2002). Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction, International Journal of Heat and Mass Transfer 46 (2003) 2305–2311 Received 1 August 2002; received in revised form 9.

Mebine P. (2007). Thermosolutal MHD Flow with Radiative Heat Transfer past an Oscillating Plate. Advances in Theoritical and Applied Mathematics, 2,3, pp 217-231.

Paras R., Ashok K. and Hawa S. (2013). Effect of Porosity on Unsteady MHD Flow Past a Semi-infinite Moving Plate Past a Moving Vertical Plate with Time Dependent Suction. Indian Journal of Pure and Applied Physics. Volume 51, pp. 461 - 470

Ramana Reddy G. V., N. BhaskarBeddy and Gorla, R.S.R. (2016). Radiation and Chemical Reaction Effects on MHD Flow along a Moving Vertical Porous Plate. International Journal of Applied Mechanics and Engineering, 2016, vol.21, no.1, pp.157-168 Doi: 10.1515/ijame-2016-0010.

Rajakumar K. V. B., Balamurugan K. S., Ramana, M. C.V. and Umasenkara Reddy M. (2017). Chemical Reaction and Viscous Dissipation Effects on MHD free Convective flow Past a Semi-Infinite Moving Vertical Porous Plate with Radiation Absorption.
Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 Volume 13, Number 12, pp. 8297-8322.

Sandeep N., VijayaBhaskar A. and Reddy V. S. (2012). Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate through Porous Media. S.V.U College of Mathematical and Physical Sciences, ISSN 2224-7467 (Paper) ISSN 2225-0913 (Online) Volume 2.

Singh K. D. and Kumar R. (2011). "Fluctuating Heat and Mass Transfer on Unsteady MHD Free Convection Flow of Radiating and Reacting Fluid past a Vertical Porous Plate in Slip- Flow Regime," vol. 4, no. 4, pp. 101–106.

Sugunamma V., Sandeep N., Mohan P. K. and Ramana B. (2013). Inclined Magnetic field and Chemical Reaction Effects on Flow over a Semi Infinite Vertical Porous Plate through Porous Medium Communications in Applied Sciences ISSN 2201-7372 Volume 1, Number 1-24.

Tripathy R. S., Dash G. C., Mishra S. R. and Baag S. Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium April (2015) Alexandria Engineering Journal (2015) 54, 673–679

Veeresh C., Praveen S. V. D. and Varma K. (2015). Heat and Mass Transfer in MHD Free Convection Chemically Reactive and Radiative Flow in a Moving Inclined Porous Plate with Temperature Dependent Heat Source and Joule Heating, International Journal of Management, Information Technology and Engineering (BEST: IJMITE) ISSN (P): 2348-0513, ISSN (E): 2454-471X, Vol. 3, Issue 11, Nov 2015, 63-74

Venkateswarlu M., (2015), Unsteady MHD free convective Heat and Mass Transfer in a Boundary Layer Flow Past a vertical Permeable plate with Thermal Radiation and Chemical Reaction. Internal Conference of Computational Heat and Mass Transfer, Procedia Engineering 127, 791 – 799

Appendix

$m_1 = \sqrt{ScKr + i\omega};\ m_2 = \sqrt{ScKr}\ ;\ m_3 = \sqrt{\Pr(H - i\omega)}\ ;\ m_4 = \sqrt{HPr};\ m_5 = \sqrt{M + \frac{1}{K} + i\omega};$

$m_6 = \sqrt{M + \frac{1}{K}}\ ;\ A_1 = \frac{G_T cos\propto}{m_3^2 - m_5^2};\ A_2 = \frac{G_c cos\propto}{m_1^2 - m_5^2};\ A_3 = \frac{G_T cos\propto}{m_4^2 - m_6^2};\ A_4 = \frac{G_c cos\propto}{m_2^2 - m_6^2};$