Grey Multi Criteria Decision Making Methods

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Abstract- Multi-Criteria Decision Making is the most well-known branch of decision making. In some cases, determining precisely the exact value of attributes is difficult and their values can be considered as Uncertain data. This paper presents two different Multi Criteria Decision Making methods based on grey numbers. The two methods are used to obtain the final ranking of the alternatives and select the best one under grey numbers. Finally, an illustrative example is presented and the results are analyzed.

Index Terms- Grey Numbers, Multi-Criteria Decision Making Methods, subjective weights, objective weights.

I. Introduction

Multi-Criteria Decision Making (MCDM) is a branch of a general class of Operations Research (OR) models which deal with decision problems under the presence of a number of conflicting decision criteria. The MCDM is considered as a complex decision making (DM) tool involving both quantitative and qualitative factors. Some of the most widely used MCDM methods are Simple Additive Weighting (SAW), Weighting Product (WP), Analytic Hierarchy Process (AHP), Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE), COmplex PRoportional ASsessment (COPRAS) , ELimination and Choice Expressing Reality (ELECTRE), and the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method [1].

Grey theory, which was proposed by Deng[2]. Many authors investigated grey system theory in decision making. Turskis et al. [3] presented a new additive ratio analysis method with grey criteria scores . Tavana et al. [4] proposed a novel hybrid approach that is also based on fuzzy ANP and COPRAS-G for multiple attribute decision making under uncertainty. Oztays [5] proposed A decision model built by integrating AHP and TOPSIS-grey method. Kuang et al. [6] presented The grey-based PROMETHEE II methodology and analyse decision problems under uncertainty. Zavadskas et al. [7] proposed a novel aggregated WASPAS-G Method. Nyaoga et al. [8] introduced a hybrid MCDM approach based on grey theory and TOPSIS to evaluate the value chain performance. Vatansever et al. [9] proposed a hybrid model to combine Entropy Weight Method and Grey Relational Analysis for determining and evaluating the quality of airline websites with the sample of eleven airline websites.

This paper presents two novel different Multi-Criteria Decision Making (MCDM) methods with grey numbers; grey Operational Competitiveness Rating Analysis (GOCRA) and Grey multi-objective optimization on the basis of simple ratio analysis (GMOOSRA). The two methods are used to choose the best alternative among the various alternatives under the grey numbers. The Subjective and Objective Weight Integrated Approach (SOWIA) method was presented to determine the weight of each evaluation criterion.

I. Grey Theory

Grey theory, which was proposed by Deng [2], is one of the new mathematical theories born out of the concept of the grey set. It is an effective method used to solve uncertainty problems with discrete data and incomplete information [2].

Basic Operations of Grey Numbers

Let $\otimes x_1 = [\underline{x}_1, \bar{x}_1]$ and $\otimes x_2 = [\underline{x}_2, \bar{x}_2]$ be two interval grey numbers. The basic operations of grey numbers $\otimes x_1$ and $\otimes x_2$ are defined as follows [10]:

$\otimes x_1 + \otimes x_2 = [\underline{x}_1 + \underline{x}_2, \bar{x}_1 + \bar{x}_2]$ addition, (1)

$\otimes x_1 - \otimes x_2 = [\underline{x}_1 - \underline{x}_2, \bar{x}_1 - \bar{x}_2]$ substraction, (2)

$\otimes x_1 \times \otimes x_2 = [\underline{x}_1 \times \underline{x}_2, \bar{x}_1 \times \bar{x}_2]$ multiplication, $\otimes x$ (3)

$\otimes x_1 \div \otimes x_2 = \left[\frac{\underline{x}_1}{\bar{x}_2}, \frac{\bar{x}_1}{\underline{x}_2}\right]$ division. (4)

Whitened Value [11]:

The whitened value of an interval grey number, ⊗x, is a deterministic number with its value lying between the upper and lower bounds of interval. For a given interval grey number ⊗x = $[\underline{x}, \bar{x}]$ the whitened value $x(\lambda)$ can be determined as follows :

$$x(\lambda) = \lambda\underline{x} + (1 - \lambda)\,\bar{x} \tag{5}$$

Here λ as whitening coefficient and λ€ [0, 1]. Because of its similarity with a popular λ function formula is often shown in the following form:

$$x(\lambda) = \lambda\underline{x} + (1 - \lambda)\,\bar{x} \tag{6}$$

For λ =0.5, formula gets the following form:

$$x(\lambda{=}0.5) = \frac{1}{2}(\underline{x} + \bar{x}) \tag{7}$$

II. USED METHODS

This section is made for illustrating and analyzing the methods and techniques used in this paper.

A. *Subjective and objective weight integrated approach (SOWIA) method:*

SOWIA method was used in this study to determine the weight of each criterion. This method based on a combination of objective and subjective weights that gives its superiority over the subjective and objective standalone techniques well known in literature. In this study, a novel combination of the SOWIA method by employing the Standard deviation (as the objective part) and Rank Sum method (as the subjective part). Both components were never combined before in previous studies.

The SOWIA Method procedure consists of the following steps:

Step 1: The decision matrix D = [x_{ij}] is prepared

Step 2: The values associated with attributes (x_{ij}) may be in different units. To avoid the influence of various dimensions of elements of (x_{ij}), the matrix is normalized using the following equation [12]:

$$x^*_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{8}$$

Where x^*_{ij} is the normalized value of x_{ij} and $\sum_{i=1}^{m} x_{ij}$ is the total of the values of j_{th} attribute for 'm' number of alternatives.

Step 3: This step deals with determination of weights of importance of the attributes. The same can be determined by three ways, namely:

a. Objective weight (O_j) of importance of attributes [13]:

Standard deviation (SD) method is considered to determine the objective weight (O_j) of the attributes. This is determined in the following way

$$\sigma_j = \sqrt{\frac{\sum_i^m (x_{ij} - \bar{x}_j)^2}{m}} \qquad j = 1,\dots, n \tag{9}$$

The following equation is used to calculate O_j .

$$O_j = \sigma_j \Big/ \sum_j^n \sigma_j \qquad j = 1,\dots, n \tag{10}$$

σ_j = standard deviation

O_j = Objective weight of importance of criteria

b. Subjective weight (Sj) of importance of attributes [12]:

Using Rank Sum weight method (RS) to determine the Subjective weight. In the rank sum (RS) procedure the weights, are the individual ranks normalized by dividing by the sum of the ranks. The formula producing the weights is the following :

$$w_j(RS) = \frac{n - r_j + 1}{\sum_{k=1}^{n} n - r_k + 1} = \frac{2(n + 1 - r_j)}{n(n+1)} \tag{11}$$

Where w_j is the normalized weight for dimension j, N is the number of dimensions, and r_j is the rank of the *j*- th criterion, *j* =1,2,..., *n* .

c. Integrated weight (W_j^{int}) of importance of attributes [12]:

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If the decision maker wants to utilize the combined effect of objective and subjective weight as described in earlier steps he/she may use the integrated weight as defined in the following equation:

$$W_j^{int} = \alpha . O_j + (1-\alpha) . S_j \quad (12)$$

Where α is known as objective factor decision weightage. The value of α lies in the interval (0, 1). O_j and S_j are objective and subjective weight respectively. As α can be assigned any value in the interval (0, 1).

B. ***The multi-objective optimization on the basis of simple ratio analysis (MOOSRA) method***

First MOOSRA method has been developed by Das et al.[15]. Generally, the MOOSRA methodology starts with the formulation of decision matrix which has in general four parameters, namely: alternatives, criteria or attributes, individual weights or significance coefficients of each criteria and measure of performance of alternatives with respect to the criteria. Considers both beneficial and non-beneficial objectives (criteria) for ranking or selecting one or more alternatives from a set of available options.

C. ***Operational Competitiveness Rating Analysis (OCRA) Method***

In OCRA method, in the first step, the preference ratings with respect to nonbeneficial or input criteria are determined; in the second step, the preference ratings of the output criteria are determined and in the last step, the overall preference ratings of the available alternatives are evaluated where both the cardinal and ordinal data are used. OCRA uses an intuitive method for incorporating the decision maker's preferences about the relative importance of the criteria [16].

III. PROPOSED TECHNIQES

All the proposed methods are using SOWIA method. The two methods proposed are GMOOSRA, and GOCRA for performance evaluation of alternatives.

D. ***Grey multi-objective optimization on the basis of simple ratio analysis (GMOOSRA) method***

The aims of this study extend the MOOSRA method [15] for solving decision making problems with grey data. By extending the MOOSRA method, an algorithm to determine the most preferable alternative among all possible alternatives, when performance ratings are given as grey numbers, is presented.

The procedure of applying the MOOSRA-G method consists of the following steps:

Step 1: Selecting the set of the most important attributes, describing the alternatives.

Step 2: Constructing the decision-making matrix ⊗X. Grey number matrix ⊗X can be defined as [17]:

$$\otimes X = \begin{bmatrix} \otimes x_{11} & \otimes x_{12} & \cdots & \otimes x_{1m} \\ \otimes x_{21} & \otimes x_{22} & \cdots & \otimes x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{n1} & \otimes x_{n2} & \cdots & \otimes x_{nm} \end{bmatrix}, \quad (13)$$

$W = [w_1, w_2, \ldots, w_n]$

Where $\otimes x_{ij}$ denotes the grey evaluations of the i-th alternative with respect to the j-th attribute; $[\otimes x_{i1}, \otimes x_{i2}, \ldots, \otimes x_{im}]$ is the grey number evaluation series of the i-th alternative, i=$\overline{1, n}$, i=$\overline{1, m}$.

Step 3 : Constructing the normalized grey decision matrices**.**

For the normalization of responses of alternatives expressed in the form of interval grey numbers, Jahanshahloo et al. [18] suggested the use of the following formula:

$$\otimes X_{ij}^* = \frac{\otimes x_{ij}}{\sqrt{\sum_{j=1}^{m} \underline{x}_{ij}^2 + \overline{x}_{ij}^2}} \quad (14)$$

To calculate the lower and the upper bounds of an interval grey number can be determined using the following formulae [18]:

$$\overline{X}_{ij}^* = \frac{\overline{x}_{ij}}{\sqrt{\frac{1}{2}\sum_{j=1}^{m} \underline{x}_{ij}^2 + \overline{x}_{ij}^2}} \quad \text{and} \quad (15)$$

$$\underline{X}_{ij}^* = \frac{\underline{x}_{ij}}{\sqrt{\frac{1}{2}\sum_{j=1}^{m} \underline{x}_{ij}^2 + \overline{x}_{ij}^2}} . \quad (16)$$

Step 4: Weighted normalized grey decision matrix $\otimes\hat{X}$ is formed by using W^{int} criteria significance calculated in SOWIA method. The weighted normalized values $\otimes\hat{x}_{ij}$ are calculated as follows:

$$\otimes\hat{x}_{ij} = \otimes x^*_{ij} \times w_j \quad i=\overline{0,m}, \tag{17}$$

Where w_j is the weight (importance) of the j criterion and $\otimes x^*_{ij}$ is the normalized rating of the j criterion.

Step 5: Determination of performance of the Alternatives

For optimization based on the MOOSRA method, the performance score $\otimes y_i^*$ of all the alternatives are computed as:

$$\otimes y_i^* = \frac{\sum_{j=1}^{g} \otimes\hat{x}_{ij}}{\sum_{j=g+1}^{n} \otimes\hat{x}_{ij}} \tag{18}$$

Here;

$\sum_{j=1}^{g} \otimes\hat{x}_{ij}$, benefit criteria for 1, …, *g*

$\sum_{j=g+1}^{n} \otimes\hat{x}_{ij}$, cost criteria for *g*+1,…,*n*

g, maximum number of criteria to be done. (*n*–*g*), minimum number of criteria to be done.

Step 6: the crisp values of the overall performance of the Alternatives

Performance values are also grey numbers ,these values should be transformed into performance crisp values .

Step 7: Determining the Whitened Value

Convert the interval grey number to Whitened Value can be determined as the follows:

$$x(\lambda) = \lambda\underline{x} + (1 - \lambda)\bar{x} \tag{19}$$

For λ =0.5, formula gets the following form:

Here as whitening coefficient and λ€ [0, 1],

$$as\ \otimes y_i^* = (\underline{y}_i\ ,\ \bar{y}_i) \tag{20}$$

$$y_i^* = \frac{1}{2}(\underline{y}_i + \bar{y}_i) \tag{21}$$

Calculated y_i values are ranked from the biggest to the smallest and an evaluation is done among the alternatives.

Step8: Ranking of the Alternatives

The alternative with the highest overall performance index is the most favorable choice.

E. *Grey Operational Competitiveness Rating Analysis Method (GOCRA)*

This study extends the OCRA method [16] for dealing with grey data, and an algorithm for determining the best choice among all possible choices when the data are grey number is also presented.

The approach for extending the OCRA method grey environment is as follows:

Step 1: construct the grey decision matrix. In the grey decision matrix $\otimes X$ [17],

$$\otimes X = \begin{bmatrix} \otimes x_{11} & \otimes x_{12} & \cdots & \otimes x_{1m} \\ \otimes x_{21} & \otimes x_{22} & \cdots & \otimes x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{n1} & \otimes x_{n2} & \cdots & \otimes x_{nm} \end{bmatrix}, \tag{22}$$

$$W = [w_1, w_2, \ldots, w_n]$$

Where $\otimes x_{ij}$ is determined by $\otimes\underline{x}_{ij}$ (the smallest value, the lower limit) and $\otimes\bar{x}_{ij}$ (the biggest value, the upper limit). $\otimes x_{ij}$ − grey value representing the performance value of the i alternative in terms of the j criterion and wj is the weight of criterion C_j.

In this method, the j^th^ criterion of the i^th^ alternative is represented by grey number:

$\otimes x_{ij} = [\underline{x}_{ij}\ ,\ \bar{x}_{ij}]$.

Step 2: Compute the preference ratings with respect to the non-beneficial attribute

The arrogate performance of i^{th} alternative with respect to all the input attribute is calculated using the following equation:

$$\otimes \bar{I}_i = \sum_{j=1}^{n} W_j \frac{min(\otimes x_j^m) - \otimes x_i^j}{min(\otimes x_j^m)} \quad \text{(i= 1,2,…,m; j=1,2,…n; i≠m)} \qquad (23)$$

Where $\otimes \bar{I}_i$ is the measure of the relative performance of i^{th} alternative and $\otimes X_i^j$ is the performance score of i^{th} alternative with respect to j^{th} input criterion. The calibration constant w_j (relative importance of j^{th} criterion) is used to increase or reduce the impact of this difference on the rating $\otimes \bar{I}_i$ with respect to j^{th} criterion.

Step 3: Calculate the linear preference rating for the input criteria.

$$\otimes \bar{\bar{I}}_i = \otimes \bar{I}_i - \min(\otimes \bar{I}_i) \qquad (24)$$

Step 4: Compute the preference ratings with respect to the beneficial criterion.

The aggregate performance for i^{th} alternative on all the beneficial or output criteria is measure using the following expression:

$$\otimes \bar{O}_i = \sum_{h}^{H} W_h \frac{\otimes x_h^j - min(\otimes x_h^m)}{min(\otimes x_h^m)} \qquad (25)$$

Where h=1,2,…, H indicates the number of beneficial attributes or output attribute and w_h is calibration constant or weight importance of h^{th} output criteria. The higher an alternative's score for an output criterion, the higher is the preference for that alternative. It can be mentioned that

$$\sum_{j=1}^{n} w_j + \sum_{h=1}^{H} w_h = 1 \qquad (26)$$

Step 5: Calculate the linear preference rating for the output criteria:

$$\otimes \bar{\bar{O}}_i = \otimes \bar{O}_i - \min(\otimes \bar{O}_i) \qquad (27)$$

Step 6: Compute the overall preference ratings.

For each alternative ($\otimes P_i$) is calculated by scaling the sum ($\otimes \bar{\bar{I}}_i + \otimes \bar{\bar{O}}_i$) so that the least preferable alternative receives a rating of zero.

$$\otimes P_i = (\otimes \bar{\bar{I}}_i + \otimes \bar{\bar{O}}_i) - \min(\otimes \bar{\bar{I}}_i + \otimes \bar{\bar{O}}_i) \qquad (28)$$

Step 7: the crisp values of the overall performance of the Alternatives

The result of grey decision making for each alternative is grey number $\otimes P_i$. The center-of-area is the most practically and simple to apply:

$$\otimes P_i = [\underline{P}_i , \bar{P}_i], \qquad (29)$$

$$P_i = \frac{1}{2}(\underline{P}_i + \bar{P}_i). \qquad (30)$$

Step 8: The alternative with the highest overall performance index is the most favorable choice

II. A numerical example with Grey data

We consider a numerical example in order to explain the proposed approaches. By using the set of four criteria: x_1, x_2 and x_3 are the beneficial type and x_4 is the cost or non-beneficial type criteria, and the set of five alternatives: A_1, A_2, A_3, A_4, A_5 .The data are tabulated in Table 1.

Table 1
Grey decision making matrix

	Criteria							
Optimization	**max**		**max**		**max**		**min**	
	$\otimes x_1$		$\otimes x_2$		$\otimes x_3$		$\otimes x_4$	
Alternatives	$\underline{x}$	$\bar{x}$	$\underline{x}$	$\bar{x}$	$\underline{x}$	$\bar{x}$	$\underline{x}$	$\bar{x}$
A_1	64	85	50	55	60	80	80	75
A_2	57	81	52	56	62	76	75	70

A_3	61	78	55	58	53	61	80	70
A_4	59	93	54	62	55	72	90	80
A_5	63	89	61	68	54	63	75	65

Source: Zavadskas et al. [19]

A . SOWIA Method Computations

First, applying the SOWIA method to determine the weights of criteria.

The steps of the SOWIA method:

1. To begin with this method, we prepare the normalized decision matrix from the original decision matrix.
2. Determine the objective weights (O_j) of the attributes by the Standard deviation (SD) method, finally the objective weights (O_j) is computed by dividing the j[th] deviation with the sum of all 'n' number of Standard deviation.

Table 2
Objective weights of criteria

Criteria	x_1	x_2	x_3	x_4
$\bar{x}_j$	0.20	0.20	0.20	0.20
σ_j	0.00954	0.016306976	0.018584	0.015146
$\sum \sigma_j$	0.059577	0.059576923	0.059577	0.059577
O_j	0.160129	0.273712964	0.311934	0.254224

3. Determine the Subjective weights (S_j) of importance of attributes by Rank Sum weight method (RS):

a. The importance ranking of the criteria.

Table 3
Rank the importance of the changes in the criteria from the best levels

Criteria	Importance ranking
x_2	1
x_4	2
x_3	3
x_1	4

b. Calculate the RS weight value for each criterion.

Table 4
Subjective weights of criteria

Criteria	x_2	x_4	x_3	x_1
S_j	0.4	0.3	0.2	0.1

4. Combined the objective and subjective weights W_j^{int}

$$W_j^{int} = \alpha . O_j + (1 - \alpha) \cdot S_j$$

W_j^{int} ($\alpha = 0.5$) As α can be assigned value 0.5

Table 5
Integrated weight (W_j^{int}) using SOWIA method

Criteria	x_1	x_2	x_3	x_4
O_j	0.160129	0.273712964	0.311934	0.254224
S_j	0.1	0.4	0.2	0.3
W_j^{int}(α=0.5)	0.130064	0.336856482	0.255967	0.277112

B. GOCRA computations

Applying the further steps of the GOCRA method. Grey the overall preference $\otimes P_i$ for each of the alternative is calculated. The computation details of this method are shown in Table 6.

Table 6
Computational details and ranking using (GOCRA) method

Alternatives	$\otimes P_i$	**P_i**	**Rank**
A_1	(-0.54249 , 0.547092)	0.002301	4
A_2	(-0.51505 , 0.611607)	0.048281	2
A_3	(-0.54624 , 0.546241)	0	5
A_4	(-0.58426 , 0.615665)	0.015704	3
A_5	(-0.48349 , 0.668042)	0.092277	1

Then the final ranking of alternatives is obtained with GOCRA method as $A_5 \succ A_2 \succ A_4 \succ A_1 \succ A_3$,the best alternative is A_5 and the worst alternative is A_3 .

C. GMOOSRA computations

By applying the procedure of GMOOSRA. In Table 7, the beneficial, non-beneficial, and composite scores are listed for all alternatives. The five alternative should be selected because it has the maximum composite score.

Table 7
Ranking using GMOOSRA method

Materials	$\oplus y_i$	y_i	**Rank**
A_1	(2.220476 , 2.904206)	2.562341	3
A_2	(2.395828 , 3.04446)	2.720144	2
A_3	(2.207465 , 2.834855)	2.52116	4
A_4	(1.957866 , 2.802825)	2.380345	5
A_5	(2.511563 ,3.417314)	2.964439	1

According to the result of GMOOSRA method ,the final ranking of alternatives is obtained as $A_5 \succ A_2 \succ A_1 \succ A_3 \succ A_4$, the best alternative is A_5 and the worst alternative is A_4 .

IV. Results and Discussions

Table 8
The final results of the proposed methods

Materials	**GOCRA**	**GMOOSRA**
A_1	4	3
A_2	2	2
A_3	5	4
A_4	3	5
A_5	1	1

According to the GMOOSRA and GOCRA the order of alternative ranks is the different . The results obtained using proposed two methods suggest A_5 is the best alternative. The discrepancy that appears between the rankings obtained by different two methods is only due to the difference in their mathematical modeling while solving a decision problem . Compared with the different MCDM methods the proposed methods are very simple to understand and easy to implement.

Table 9. Comparison of the proposed methods

MCDM methods	Computational Time	Simplicity	Problem structure	flexibility
MOOSRA	Very Less	Very Simple	Many criteria and alternatives	Very High
OCRA	Less	Moderate	Many criteria and alternatives	Very

The proposed methods are also newly proposed MCDM methods and successfully are applied to grey numbers under incomplete data, the calculation processes are always effective when a new alternative is added or removed.
The proposed methods are applicable when large numbers of alternatives and criteria are involved because the proposed methods are direct and causes no complication in calculation despite the large-scale data [20].

V. Conclusions

This paper proposed two different methods based on grey numbers technique to determine the most preferable choice among all possible choices. The proposed methods are based on the SOWIA technique with modification for each proposed method .An illustrative example is given to show the results and to compare the performance of the two methods.

Hence the GMOOSRA and GOCRA methods are more practical, realistic, comprehensive and applicable for any grey numbers multi-criteria decision making problems and very simple to understand and easy to implement in comparison to other methods.

The decision maker can easily apply GMOOSRA method because of its several advantages such as highly stable, simple and easy to implement decision making approach with less mathematical calculations over the other GOCRA method.

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