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Magnetohydrodynamic Flow over a Flat Plate

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Abstract- The hydrodynamic flow over a flat plate in the presence of magnetic field is investigated. The governing non linear partial differential equations of momentum and energy are transformed into non linear ordinary differential equations using the appropriate similarity transformation variables. The transformed non linear differential equations are solved using shooting method along side with finite difference method. The effect of magnetic field, Prandtl number and some other important parameters encountered were discussed. From the result it is observed that the parameters have significant influence on the flow.

Index Terms- Magnet-hydrodynamic flow, Prandtl Number, Nusselt Number, shooting method

I. Introduction

Over the years, fluid flow over a flat plate has been of utmost interest in the study of fluid dynamics. When a fluid flows over a flat plate, the fluid particles adjacent to the wall adhere to the plate and eventually come to rest. The flow velocity increases in the direction normal to the plate up to a certain extent, beyond which the flow velocity is the free stream velocity. Thus, there exist a region called the boundary layer were velocity gradient is acting. The boundary layer is the layer adjacent to a surface where viscous effects are important.

Magneto-hydrodynamics (MHD) is the branch of fluid dynamics that studies the interaction of electrically conducting fluids and magnetic fields. Examples of such fluids include hot ionized gases (plasma), strong electrolytes and liquid metals. The term Magneto-hydrodynamics (MHD) is a part of Magneto-fluid dynamics (MFD) that involves applications in water or incompressible fluids. The word Magneto-hydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning liquid and dynamics-meaning movement.

The applications of Magneto-hydrodynamics covers a wide range of areas from liquid metals to cosmic plasma. Plasma physicist use the Magneto-hydrodynamics (MHD) techniques for controlling the stability of plasmas confined by magnetic fields in thermonuclear fusion reaction (Khan,2009).

The study of fluid flows is of great importance in engineering applications. The material processing industry employs Magneto hydrodynamics (MHD), where Lorentz force provides a non-intrusive means of controlling the flow of metals. Also, Magneto hydrodynamics (MHD) provides a unique means of controlling the casting and refining processes, therefore has made it possible to produce fine quality material with minimal cost (Khan,2009).

Different researchers have worked on steady flow of an electrically conducting incompressible flow. Jat (2010) worked on two dimensional steady flow of an electrically conducting, viscous incompressible fluid past a continuously moving surface in the presence of uniform transverse magnetic field. The governing boundary layer equations were transformed to coupled differential equation and then solved numerically using the Newton's Shooting method with Fourth-Order Runge-Kutta integration scheme.
Patel and Timol (2012) worked on the Similarity equations for steady two dimensional laminar incompressible boundary layer flows past a moving continuous flat surface in the presence of transverse magnetic field. The governing equations were transformed and solved using the Maple ODE solver.

Krishnendu et al(2011) studied magneto-hydrodynamic (MHD)boundary layer flow and heat transfer over a plate with slip condition. A complete self- similar set of equations were obtained from the governing equations using the similarity transformation and were solved numerically using shooting method.

Damseh et al(2006) worked on electrically conducting fluid in the presence of a uniform magnetic field. The governing equations were transformed using similarity transformation.

Adel et al(2003) investigated heat and mass transfer along a semi-infinite vertical flat plate under the combined buoyancy force effects of thermal and species diffusion in the presence of a strong non-uniform magnetic field. The similarity equations were solved numerically by using a fourth-order Runge-Kutta scheme with the shooting method.

Hayat et al(2011) worked on electrically conducting fluid in the presence of a uniform magnetic field. The governing equations were transformed using similarity transformation.

Makinde(2010) considered magneto-hydrodynamic (MHD) boundary layer flow with heat and mass transfer over a vertical plate.
Magua(2013) worked on heat and mass transfer along a semi-infinite vertical plate under the combined buoyancy force effect. The non linear boundary layer equations were linearized and solved using finite difference method.

Gupta et al(2001) considered three- dimensional flow past a porous plate and established the effects of Hartmann number and suction parameter on velocity and skin friction.

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Youn(2000) worked on the unsteady two- dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. The effects of parameters were discussed.

Hazem(2006) worked on asymptotic solution for hyromagnetic rotating disk flow.The governing equations were transformed using the scaled von Karman's similarity coordinates .The non linear ordinary differential equations were solved by a two- point finite difference technique.

Hazem et al(2011)considered the steady hydromagnetic laminar flow of an incompressible non-Newtonian micropolar fluid. The equations were solved under the boundary conditions using central differences for the derivatives and Thomas algorithm for the solution of the set of discretized equations.

(Bhattachrjee et al,2013)considered laminar incompressible viscous flow over a flat plate.The governing equations were transformed and solved numerically.

(Ramana,et al,2011) worked on the Magneto-hydrodynamic (MHD) effects on the unsteady heat transfer convective flow past an infinite vertical porous plate with variable suction and soret effect. The governing equation were then solved analytically.

(Oahimire et al, 2013) considered the hydromagnetic flow of a viscous fluid near a stagnation point on a linearly stretching sheet with variable thermal conductivity and heat source. The governing non-linear partial differential equations were transformed into non-linear ordinary differential equations using the usual similarity variables. The resulting equations were then solved analytically.

(Yasser et al, 2010) considered the flow of a viscous incompressible fluid over a stretching sheet considering the variation of viscosity and thermal conductivity with temperature. The governing conservation equations of mass, momentum and energy are non-dimensionalized by using appropriate transformation. The resulting systems of coupled ordinary differential equations are solved numerically by using the shooting method.

II. MATHEMATICAL FORMULATION OF MODEL

We consider the steady two dimensional laminar flow of an electrically conducting viscous incompressible fluid over a flat plate in the presence of magnetic field.

Continuity equation $\nabla.V = 0$ (1)

Momentum equation $\rho(u.\nabla)u = -\nabla p + \mu\nabla^2 + \mathbf{J} \times \mathbf{B}$ (2)

Energy equation $\rho C_p(u.\nabla)T = K\nabla^2 T + \emptyset$ (3)

Where $\emptyset$ is dissipation.Maxwell's equation

$$\left.\begin{aligned} &\nabla x H = J \\ &\nabla . B = 0 \\ &\nabla x E = -\partial B/\partial t \\ &\nabla . J = 0 \end{aligned}\right\} \quad (4)$$

The equation of conservation of electric charge

$\nabla.J = 0$ gives J_y=constant, where the current density is J= (J_x J_y,) Since the current density J_y = constant, the constant is assumed to be zero and so J_y=0 everywhere in the flow.The magnetic field is written as B_y which is in the Y-direction and affects the motion while the current density J_x is in the x-direction. The Ohm's law is given by

$J=\sigma(\mathbf{E}+\mathbf{V}\mathbf{x}\mathbf{B})$,

from Maxwell's equation $\nabla x E$=0,we conclude that E_x is a constant which will arbitrarily be taken as zero, Since no electric field is applied in the x-direction. Therefore,

$J= \sigma(VxB)$

$J_x= \sigma(VxB_y)= = \sigma B_0 u$ (5)

Hence the Lorentz force is given by

$F=- (J_x \times B_y) =- \sigma B_0^2 u$ (6)

Also, from the energy equation

$\rho C_P(u.\nabla)T = K\nabla^2 T + \emptyset$

$\emptyset$ is assumed to be negligible. $\rho C_P(u.\nabla)T = K\nabla^2 T$ (7)

The equations become :

- $\nabla.V = 0$ (8)
- $\rho(u.\nabla)u = \mu\nabla^2 - \sigma B_0^2 u$ (9)
- $\rho C_p(u.\nabla)T = K\nabla^2 T$ (10)

Transforming the equations to Cartesian coordinate form the governing equations are:

- $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (11)
- $\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=\mu\frac{\partial^2 u}{\partial y^2}-\sigma B_0^2 u$ (12)
- $\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}$ (13)

With boundary conditions:

At y=0 u=U_∞ v =0 T = T_w (14)

As y ∞ u=0 $\longrightarrow$ T = T_∞

SIMILARITY TRANSFORMATION

Similarity transformation involves the use of similarity transformation variables to transform partial differential equations into ordinary differential equations.

$$\left.\begin{aligned}
\eta &= y\sqrt{\frac{u_\infty}{2\upsilon x}}\\
\psi &= \sqrt{2\upsilon x u_\infty f(\eta)}\\
u &= \frac{\partial\psi}{\partial y}\\
v &= -\frac{\partial\psi}{\partial x}\\
\theta &= \frac{T-T_\infty}{T_W-T_\infty}
\end{aligned}\right\} \tag{15}$$

SIMILARITY TRANSFORMATION OF MOMENTUM EQUATION

Using the similarity variables in equation (15), we obtain the following.

1.
$$u=\frac{\partial}{\partial y}\left[\sqrt{2\upsilon x U_\infty f(\eta)}\right]$$

$$u=\sqrt{2\upsilon x U_\infty}\frac{\partial}{\partial y}f(\eta)$$

$$u=\sqrt{2\upsilon x U_\infty}\frac{\partial\eta}{\partial y}*\frac{\partial f(\eta)}{\partial\eta}$$

$$u=\sqrt{2\upsilon x U_\infty}(\sqrt{\frac{U_\infty}{2\upsilon x}}*f')$$

$$u=\frac{\sqrt{2\upsilon x U_\infty}}{\sqrt{2\upsilon x U_\infty}}(U_\infty*f')$$

$$u=U_\infty f'$$

2.
$$v=-\frac{\partial}{\partial x}\left[\sqrt{2\upsilon x U_\infty f(\eta)}\right]$$

$$v=-\left[\sqrt{\frac{U_\infty\upsilon}{2x}}f-\frac{y}{2x}\sqrt{\frac{U_\infty}{2\upsilon x}}f'*\sqrt{2\upsilon x U_\infty}\right]$$

Recall that
$$\eta=y\sqrt{\frac{U_\infty}{2\upsilon x}}$$

$$v=-(\sqrt{\frac{U_\infty\upsilon}{2x}}f-\eta f'\sqrt{\frac{U_\infty\upsilon}{2x}})$$

$$v = -\sqrt{\frac{U_\infty \upsilon}{2x}} f + \eta f' \sqrt{\frac{U_\infty \upsilon}{2x}}$$

$$v = \sqrt{\frac{U_\infty \upsilon}{2x}} (\eta f' - f')$$

3. $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(U_\infty f')$$

$$\frac{\partial u}{\partial x} = U_\infty \frac{\partial}{\partial x}(f')$$

$$= -\frac{u_\infty y}{2x}\sqrt{\frac{u_\infty}{2\upsilon x}} f''$$

Recall that $\eta = y\sqrt{\dfrac{U_\infty}{2\upsilon x}}$

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta f''$$

4. $$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(U_\infty f')$$

$$\frac{\partial u}{\partial y} = U_\infty \frac{\partial}{\partial y}(f')$$

$$\frac{\partial u}{\partial y} = U_\infty \frac{\partial}{\partial y}(f')$$

$$= U_\infty \left(\frac{\partial \eta}{\partial y} * \frac{\partial f'(\eta)}{\partial \eta} \right)$$

$$= U_\infty \left(\sqrt{\frac{U_\infty}{2\upsilon x}} j'' \right)$$

$$= \sqrt{\frac{U^3{}_\infty}{2\upsilon x}} f''$$

5. $$u\frac{\partial u}{\partial x} = (U_\infty f')\left(-\frac{U_\infty}{2x}\eta f''\right)$$

$$= -\frac{U^2{}_\infty}{2x}\eta f f''$$

6. $$v\frac{\partial u}{\partial y} = \sqrt{\frac{U_\infty \upsilon}{2x}}(\eta f' - f)\left(\sqrt{\frac{U^3}{2\upsilon x}} f''\right)$$

$$= \left(\eta f' \sqrt{\frac{U_\infty \upsilon}{2x}} - f\sqrt{\frac{U_\infty \upsilon}{2x}}\right)\left(\sqrt{\frac{U^3}{2\upsilon x}} f''\right)$$

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$$= (\eta f f'' \sqrt{\frac{U^4{}_\infty \upsilon}{4x^2\upsilon}} - ff''\sqrt{\frac{U^4{}_\infty \upsilon}{4x^2\upsilon}})$$

$$= \eta f f'' \frac{U_\infty^2}{2x} - ff'' \frac{U_\infty^2}{2x}$$

7. $$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\sqrt{\frac{U_\infty^2}{2\upsilon x}} f''\right)$$

$$= \sqrt{\frac{U_\infty^3}{2\upsilon x}} \cdot \frac{\partial}{\partial y} f''$$

$$= \sqrt{\frac{U_\infty^3}{2\upsilon x}} \cdot \frac{\partial \eta}{\partial y} \cdot \frac{\partial f''(\eta)}{\partial \eta}$$

$$= \sqrt{\frac{U_\infty^4}{4\upsilon^2 x^2}} f''$$

$$= \frac{U_\infty^2}{2\upsilon x} f'''$$

Therfore, we have

$$\left.\begin{aligned}
u &= U_\infty f' \\
v &= \sqrt{\frac{U_\infty \upsilon}{2x}}(\eta f' - f') \\
\frac{\partial u}{\partial x} &= -\frac{u_\infty}{2x}\eta f'' \\
\frac{\partial u}{\partial y} &= \sqrt{\frac{U^3{}_\infty}{2\upsilon x}} f'' \\
u\frac{\partial u}{\partial x} &= -\frac{U^2{}_\infty}{2x}\eta f f'' \\
v\frac{\partial u}{\partial y} &= \eta f f'' \frac{U_\infty^2}{2x} - ff''\frac{U_\infty^2}{2x} \\
\frac{\partial^2 u}{\partial y^2} &= \frac{U_\infty^2}{2\upsilon x} f'''
\end{aligned}\right\} \quad (16)$$

We now substitute equation (16) into equation (12)

$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u$

$$\rho\left[-\frac{U^2{}_\infty}{2x}\eta f f'' + \eta f f'' \frac{U_\infty^2}{2x} - ff''\frac{U_\infty^2}{2x}\right] = \mu\frac{U_\infty^2}{2\upsilon x} f''' - \sigma B_0^2 U_\infty f'$$

$$\rho\frac{U_\infty^2}{2x}\left[-\eta f f'' + \eta f f'' - ff''\right] = \mu\frac{U_\infty^2}{2\upsilon x} f''' - \sigma B_0^2 U_\infty f'$$

$$\rho\frac{U_\infty^2}{2x}\left[-ff''\right] = \mu\frac{U_\infty^2}{2\upsilon x} f''' - \sigma B_0^2 U_\infty f' \quad (17)$$

Divide by $\dfrac{2x}{U_\infty^2 \rho}$

$\left[-ff''\right] = \dfrac{\mu}{\rho \upsilon} f''' - \dfrac{2x\sigma B_0^2 U_\infty}{\rho U_\infty} f'$

For $\upsilon = \dfrac{\mu}{\rho}$,we have

$$f''' + ff'' - Nf' = 0 \qquad (18)$$

Where N= $\dfrac{2x\sigma B_0^2 U_\infty}{\rho U_\infty} = \dfrac{R_m}{M_m^2}$ is the Interaction parameter

μ=absolute viscosity

υ =kinematic viscosity

U_∞ =free stream velocity

SIMILARITY TRANSFORMATION OF THE ENERGY EQUATION

Transforming the energy equation in equation (13) using the similarity variables in equation (15).The energy equation from equation(13) is

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \frac{\partial^2 T}{\partial y^2}$$

From equation (15),we have

$$\theta = \frac{T - T_\infty}{T_W - T_\infty}$$

$$T - T_\infty = (T_W - T_\infty)\,\theta$$

$$T = (T_W - T_\infty)\theta(\eta) + T_\infty$$

1. $\dfrac{\partial T}{\partial x} = \dfrac{\partial}{\partial x}\left((T_w - T_\infty)\vartheta(\eta)\right) + \dfrac{\partial T_\infty}{\partial x}$

$$= \frac{\partial \eta}{\partial x} \cdot \frac{\partial\left((T_w - T_\infty)\theta(\eta)\right)}{\partial \eta} + 0$$

$$= \frac{-y}{2\sqrt{x^3}} \sqrt{\frac{U_\infty}{2\upsilon}} \cdot \left((T_w - T_\infty)\theta'(\eta)\right)$$

$$= \frac{-y\left((T_w - T_\infty)\theta'(\eta)\right)}{2\sqrt{x^3}} \sqrt{\frac{U_\infty}{2\upsilon}}$$

2. $\dfrac{\partial T}{\partial y} = \dfrac{\partial}{\partial y}\left((T_w - T_\infty)\vartheta(\eta)\right) + \dfrac{\partial T_\infty}{\partial y}$

$$= \frac{\partial \eta}{\partial y} \cdot \frac{\partial\left((T_w - T_\infty)\theta(\eta)\right)}{\partial \eta} + 0$$

$$= \sqrt{\frac{U_\infty}{2\upsilon x}} \cdot (T_w - T_\infty)\theta'(\eta)$$

3. $$u\frac{\partial T}{\partial x}=U_{\infty}f'\cdot\frac{-y\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)}{2\sqrt{x^3}}\sqrt{\frac{U_{\infty}}{2\upsilon}}$$

$$=\frac{-y\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)f'}{2\sqrt{x^3}}\sqrt{\frac{U^3_{\infty}}{2\upsilon}}$$

$$=\frac{\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)f'U_{\infty}}{2x}\left(-y\sqrt{\frac{U_{\infty}}{2\upsilon x}}\right)$$

Since $\eta=y\sqrt{\frac{u_{\infty}}{2\upsilon x}}$, we have

$$u\frac{\partial T}{\partial x}=\frac{-\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)\eta f'U_{\infty}}{2x}$$

4. $$\frac{v\partial T}{\partial y}=\sqrt{\frac{U_{\infty}\upsilon}{2x}}\left(\eta f'-f\right)\sqrt{\frac{U_{\infty}}{2\upsilon x}}\cdot\left(T_w-T_{\infty}\right)\theta'(\eta)$$

$$=\frac{U_{\infty}}{2x}\left(\eta f'\left(T_w-T_{\infty}\right)\theta'(\eta)-\left(T_w-T_{\infty}\right)\theta' f\right)$$

$$=\frac{U_{\infty}}{2x}f'\left(T_w-T_{\infty}\right)\theta'(\eta)-\frac{U_{\infty}}{2x}\left(T_w-T_{\infty}\right)\theta' f$$

5. $$\frac{\partial^2 T}{\partial y^2}=\frac{\partial\left(T_w-T_{\infty}\right)\theta'}{\partial y}\sqrt{\frac{U_{\infty}}{2\upsilon x}}$$

$$=\frac{\partial\eta}{\partial y}\cdot\frac{\partial}{\partial\eta}\left(\partial\left(T_w-T_{\infty}\right)\theta'\sqrt{\frac{U_{\infty}}{2\upsilon x}}\right)$$

$$=\sqrt{\frac{U_{\infty}}{2\upsilon x}}\cdot\sqrt{\frac{U_{\infty}}{2\upsilon x}}\left(T_w-T_{\infty}\right)\theta''$$

$$=\frac{U_{\infty}}{2\upsilon x}\left(T_w-T_{\infty}\right)\theta''$$

Therefore we have

$$\left.\begin{aligned}
u\frac{\partial T}{\partial x}&=\frac{-y\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)}{2\sqrt{x^3}}\sqrt{\frac{U_{\infty}}{2\upsilon}}\\
u\frac{\partial T}{\partial y}&=\sqrt{\frac{U_{\infty}}{2\upsilon x}}\cdot\left(T_w-T_{\infty}\right)\theta'(\eta)\\
u\frac{\partial T}{\partial x}&=\frac{-\left(\left(T_w-T_{\infty}\right)\theta'(\eta)\right)\eta f'U_{\infty}}{2x}\\
v\frac{\partial T}{\partial y}&=\frac{U_{\infty}}{2x}\eta f'\left(T_w-T_{\infty}\right)\theta'(\eta)-\frac{U_{\infty}}{2x}\left(T_w-T_{\infty}\right)\theta' f
\end{aligned}\right\}\quad(19)$$

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$$\frac{\partial^2 T}{\partial y^2} = \frac{U_\infty}{2\upsilon x}(T_w - T_\infty)\theta''$$

Substituting equation (19) into the energy equation (13) we have

$$\frac{-((T_w - T_\infty)\theta'(\eta))\eta f' U_\infty}{2x} + \frac{U_\infty}{2x}\eta f'(T_w - T_\infty)\theta'(\eta) - \frac{U_\infty}{2x}(T_w - T_\infty)\theta' f$$

$$= \frac{kU_\infty}{\rho C_p 2\upsilon x}(T_w - T_\infty)\theta''$$

$$-\frac{U_\infty}{2x}(T_w - T_\infty) f\theta' = \frac{kU_\infty}{\rho C_p 2\upsilon x}(T_w - T_\infty)\theta''$$

Multiply through by $\frac{2x}{U_\infty (T_w - T_\infty)}$

$$-f\theta' = \frac{k}{\rho C_p}\theta'' \qquad \frac{k}{\rho C_p}\theta'' + f\theta' = 0$$

$$\frac{1}{pr}\theta'' + f\theta' = 0$$

$$\theta'' + \Pr f\theta' = 0 \qquad (20)$$

Where Pr is the Prandtl number.

The transformed equation s for Model 1

$$f''' + ff'' - Nf' = 0 \qquad (21)$$

$$\theta'' + \Pr f\theta' = 0 \qquad (22)$$

With boundary conditions $f(0) = 0 \quad f(\infty) = 1$

$$f'(0) = 0 \quad f'(\infty) = 1 \qquad (23)$$

SKIN FRICTION COEFFICIENT

Skin friction coefficient is defined as the non dimensional wall shear stress given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} \qquad (24)$$

Where $\tau_w = \mu \frac{\partial u}{\partial y}$ (25)

And $\frac{\partial u}{\partial y} = \sqrt{\frac{U_\infty^3}{2\upsilon x}} f''(0)$

$$\tau_w = \mu\sqrt{\frac{U_\infty^3}{2\upsilon x}} f''(0)$$

Recall that $\upsilon = \frac{\mu}{\rho}$

$$\tau_w = U_\infty \mu \sqrt{\frac{\rho U_\infty}{2\mu x}} f''(0)$$

$$\tau_w = U_\infty \sqrt{\frac{\rho U_\infty \mu^2}{2\mu x}} f''(0)$$

$$\tau_w = U_\infty \sqrt{\frac{\rho U_\infty \mu}{2x}} f''(0)$$

Since $\tau_w = U_\infty \sqrt{\frac{\rho U_\infty \mu}{2x}} f''(0)$

then $C_f = \left(U_\infty \sqrt{\frac{\rho U_\infty \mu}{2x}} f''(0) \right) \cdot \frac{2}{\rho U_\infty^2}$

$$C_f = \left(\sqrt{\frac{\rho U_\infty \mu}{2x}} f''(0) \right) \cdot \frac{2}{\sqrt{\rho^2 U_\infty^2}}$$

$$C_f = \left(2 \cdot \sqrt{\frac{\mu}{2x\rho U_\infty}} f''(0) \right)$$

Recall that $\mathrm{Re} = \frac{\rho U_\infty 2x}{\mu}$

$$C_f = \left(2 \cdot (\mathrm{Re})^{-\frac{1}{2}} f''(0) \right) \tag{26}$$

NUSSELT NUMBER

The rate of heat transfer in terms of the nusselt number at the plate is given by

$$Nu = \frac{q}{k(T_w - T_\infty)} \tag{27}$$

Where $q = -k(\frac{\partial T}{\partial y})_{y=0}$

And $\frac{\partial T}{\partial y} = (T_w - T_\infty)\theta'(\eta)$

$$Nu = \frac{-k((T_w - T_\infty)\theta'(\eta))}{k(T_w - T_\infty)}$$

$$Nu = -\theta'(\eta) \tag{28}$$

SHOOTING METHOD

We now use the shooting method to convert the boundary value problem to an initial value problem by estimating the missing initial condition to a desired degree of accuracy by an iterative scheme.

ALGORITHM

Step1: Rewrite the equation to obtain initial value problem with initial conditions. Let s_k and s_{k-1} be two guessed values for the missing initial condition.

Step II: Integrate the equation with initial conditions and $f''(0) = s_k$ from $\eta = a$ and $\eta = b$.Where [a,b] is the interval.

Step III: Let f(s_k;b) be the values of f obtained on integration in step II

Step IV: Integrate the equation with initial conditions and $f''(0) = s_{k-1}$.

Step V: compute $\delta s_k = \dfrac{(f(s_k;b) - f(b))\,(s_k - s_{k-1})}{f(s_k;b) - f(s_{k-1};b)}$

Step VI : Obtain next approximation from

$s_{k+1}=s_k - \delta s_k$ for k=1,2,3..........

Step VII: stop when $(f(s_k;b) \approx f'(\infty) = 1$, then go to step two.
Otherwise choose another value for s_k.

Using the mathematical software tool MATlab, the following results were obtained.

III. RESULTS AND ANALYSIS

To study the physical behaviour of the flow, the non dimensional velocity, temperature and all the results obtained are presented in tables and profiles respectively.

Table 1 .Effect of magnetic parameter on velocity

η	f'(N=.5)	f'(N=1)	f'(N=1.5)
0	0	0	0
1	0.8934	0.8327	0.7779
2	0.9765	0.9327	0.8997
3	0.9947	0.9767	0.9593
4	1.0065	1.0005	1.0002
5	1	1	1

Table 2 .Effect of prandtl number on temperature

η	θ(Pr=.72)	θ(Pr=1.5)	θ(Pr=2.5)	Pr=3
0	1	1	1	1
1	1.0443	0.98594	0.89524	0.84108
2	0.91029	0.83556	0.72	0.65154
3	0.64923	0.58907	0.49619	0.44127
4	0.32461	0.29454	0.2481	0.22064
5	0	0	0	0

Table 3: Effect of Prandtl number on Nusselt number

N	-θ(Pr=0.72)	-θ'(Pr=3)	-θ'(Pr=5)	-θ'(Pr=10)
0	1.1541	1.0000	0.8487	0.3892
0.5	1.1602	1.0272	0.8992	0.5493
1	1.1634	1.0415	0.9250	0.5862

0

0.5

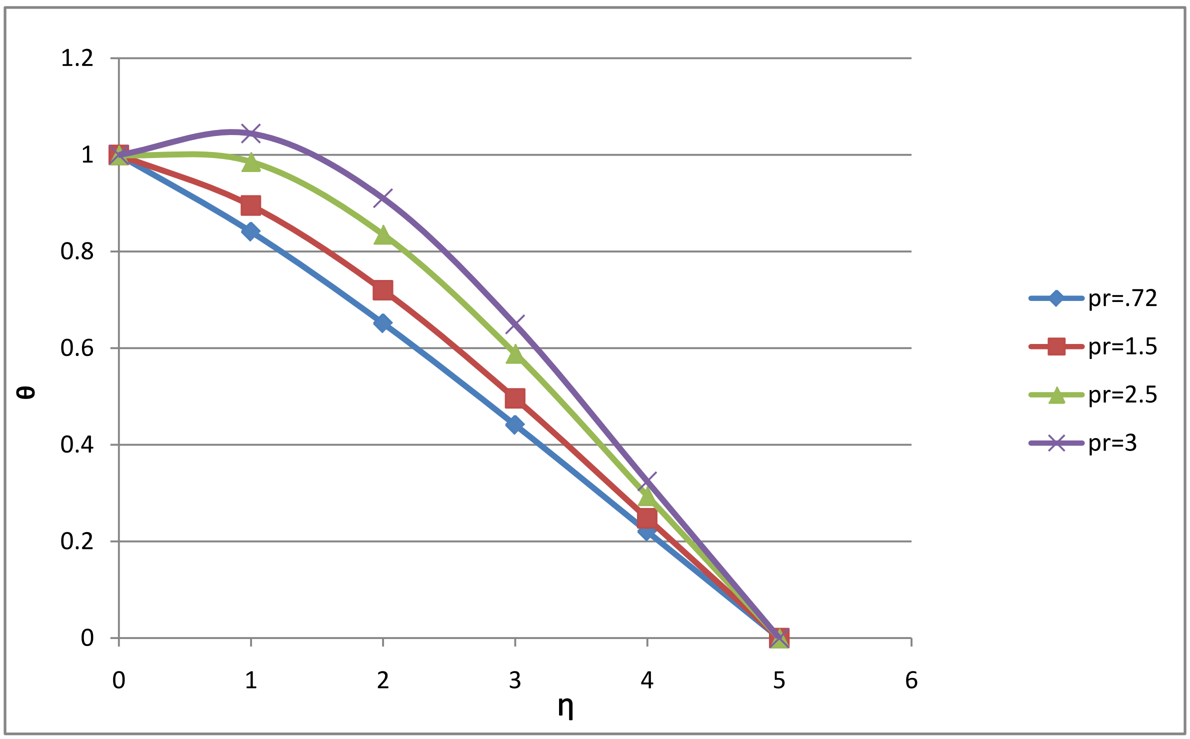

Fig.1 Temperature profile for different values of the prandtl Number pr

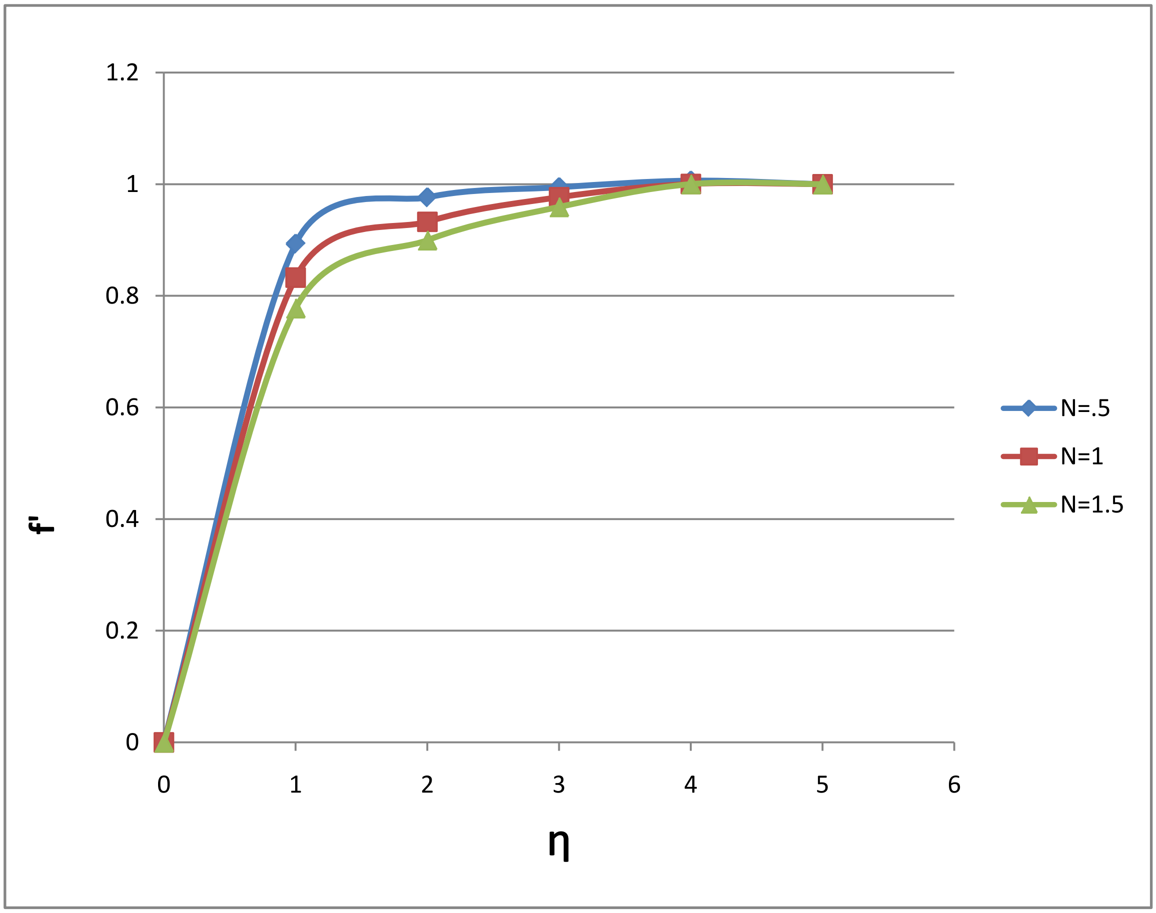

Fig.2 Velocity profile for different values of the interaction parameter N

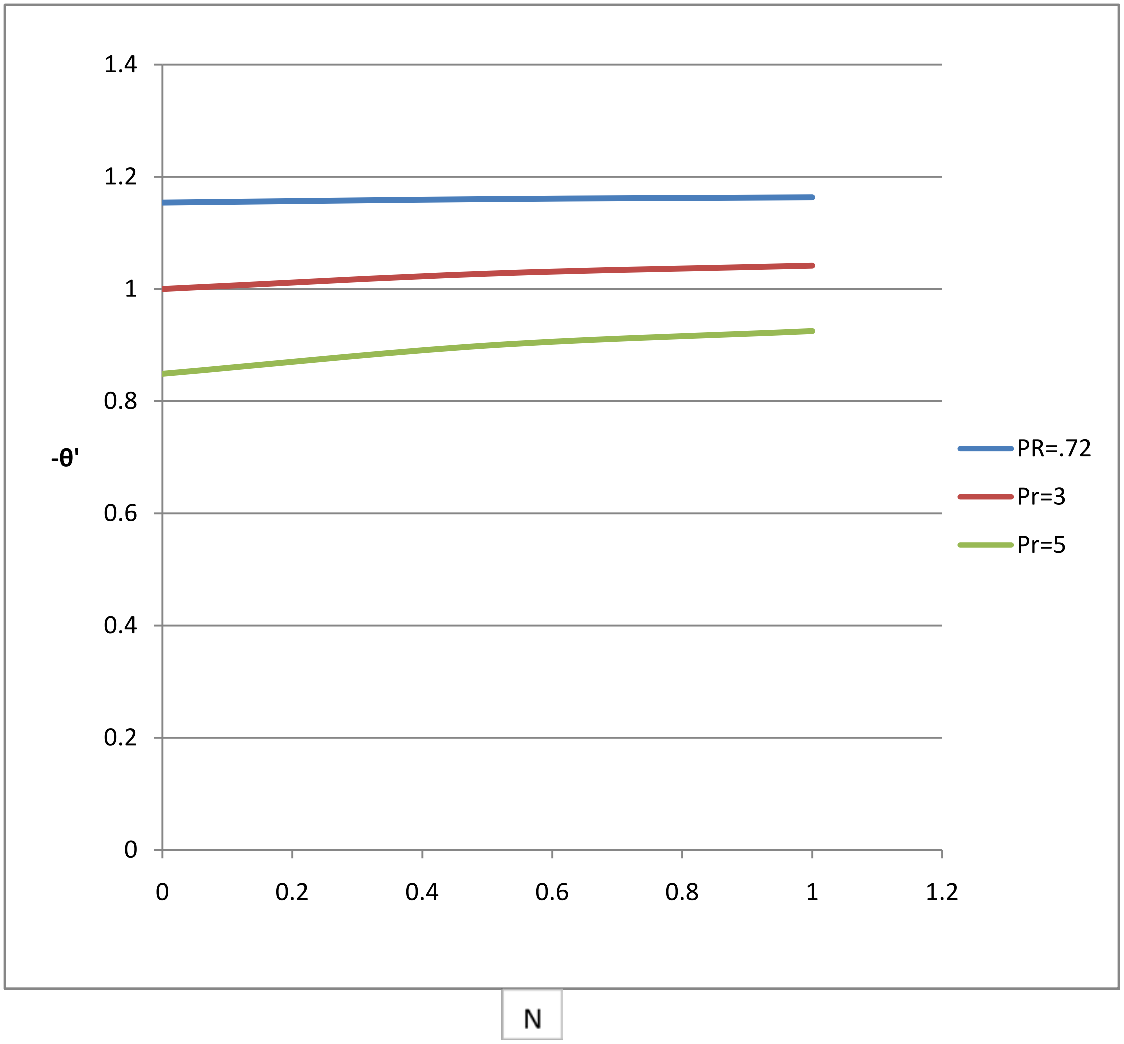

FIG3. EFFECT OF PRANDTL NUMBER AND MAGNETIC PARAMETER ON HEAT TRANSFER

Fig1represents the temperature profile for different values of prandtl. Increase in the prandtl number decreases the temperature.Figure 2 represent the velocity profile for different values of N which is the magnetic field parameter From the profile it is seen that increasing the parameter N decreases the velocity. Figure 3 represents the heat transfer profile Nusselt number. Nusselt number decreases as the Prandtl number increases.

IV. Conclusion

Steady, two dimensional hydromagnetic f flow over a flat plate has been investigated . The equations were transformed using appropriate transformation variables.The resulting governing equations were solved numerically. The effects of the different parameters(magnetic field parameter,prandtl number, and Nusselt number) obtained from the transformation are also studied. From analysis ,it was found that increase in prandtl number results in decrease in temperature and increase in magnetic parameter decreases velocity of the flow. Thus all parameters have significant influence on the flow velocity and temperature.

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