Research of one problem of pursuit with different constraints on controls

[1]Nargiza Siddiqova, [2]Shakhnoza Muksimova, [3]Asqar Rakhmanov

[1]Department of Systematic and Practical Programming, Tashkent University of Information Technologies, Uzbekistan
snargiza0727@gmail.com
[2]Department of Systematic and Practical Programming, Tashkent University of Information Technologies, Uzbekistan
muksimova1986@gmail.com
[3]Department of Systematic and Practical Programming, Tashkent University of Information Technologies, Uzbekistan
asqartr1.2.3dipu@gmail.com

***Abstract* -** We consider the pursuit problem for the so-called game “boy and crocodile”. “Boy” is moving in according to the equation $\dot{y} = v$, “crocodile” moving according to the equation $\ddot{x} = u$ and “boy” can not leave the closed ball of radius R > 0. Control of the pursuer is subjected to integral constraint, and the evader’s control is subjected to geometric constraint. Sufficient conditions of completion of pursuit from all initial positions are obtained.

Key words: differential games, boy and crocodile, Cauchy problems, integral constraint, geometric constraint, phase vector.

I. INTRODUCTION

The problem of pursuit and escaping in differential games with different constraints on controls are investigated in many works, for example [1-24]. From them in [10, 13] describes the main approaches for solving differential pursuit-evasion games and obtained basic results. In recent years one of the low-studied areas of the theory of differential games - differential games with integral [3,9,10,13,14,18-24] and different [3-6,21] constraints on controls is widely investigated. In works [1,2,4-8,9,11,13,16,17,24] the differential games of pursuit-evasion of different structure are studied with phase constraints on the state of players. This article is sanctified to the decision of problem of pursuit for the so-called game "boy and crocodile" with different constraints on the controls of players. In the present we investigated the case with integral constraints on the control of the pursuer (crocodile) and geometric constraints on the control of the evader (boy), in the case of phase constraints on the state of the evader. In this case, at the decision of problem of pursuit from all initial positions of phase space, a pursuer runs into the problem of the reasonable use of limit resources of pursuit. For the decision of this problem, the idea of work [4,17,18] in-process is used, according to that at first the phase vector of pursuer is driven to the certain set, taking no notice on behavior of escaping player, and then using the control of escaping and initial positions of players, the control of pursuer, guaranteeing completion of

ISSN 2250-3153
pursuit, is built. The sufficient conditions of completion of pursuit from all initial positions are obtained. Work joins to researches [4-6, 17, 21]

Consider the so-called differential game "boy and crocodile" described by the equations:

$$\text{a) } \ddot{x} = u, \qquad \text{b) } \dot{y} = v \tag{1.1}$$

where $x, y \in R^n$ - the phase vectors, $u, v \in R^n$, $n \geq 1$ - the control parameters of pursuit and evasion, respectively. The pursier (crocodile) moves according to the equation (1a), evader (boy) moves according to the equation (1b). Control parameters u, v selected from the class of measurable functions satisfying according to integral (2) and geometric (3) constraints, respectively:

$$\int_0^\infty |u(t)|^2 dt \leq \rho^2, \tag{1.2}$$

$$|v| \leq \sigma. \tag{1.3}$$

The boy is trapped crocodiles, if at some moment $t_1 > 0$ the inequality $|y(t_1) - x(t_1)| \leq \ell$ is hold , where $\ell > 0$ – the given positive constant number, while the boy all the time must be in the closed ball RS where, R -given positive number, $S = \{z \in R^n : |z| \leq 1\}$. By setting the initial point $x(0) = x^0$, $\dot{x}(0) = x_1^0$, $y(0) = y^0$, defining measurable functions $u = u(t)$, $v = v(t), 0 \leq t < \infty$, and introducing the notation $z_1 = x$, $z_2 = \dot{x}$, from (1) will get the next linear Cauchy problems:

$$\begin{cases} \dot{z}_1 = z_2, & z_1(0) = z_{10} = x^0, \\ \dot{z}_2 = u(t), & z_2(0) = z_{20} = x_1^0. \end{cases} \tag{1.4}$$

$$\dot{y} = v(t), \qquad y(0) = y^0. \tag{1.5}$$

For the solution of Cauchy problems (4), (5) the following formulas is hold

$$z_2(t) = z_{20} + \int_0^t u(\tau)dt, \quad z_1(t) = z_{10} + tz_{20} + \int_0^t (t-\tau)u(\tau)dt, \tag{1.6}$$

$$y(t) = y^0 + \int_0^t v(\tau)dt. \tag{1.7}$$

Definition 1. Measurable functions $u = u(t)$, $v = v(t)$, $0 \leq t < \infty$, satisfying to constraints (2), (3) are called admissible control of pursuers and evader respectively.

Definition 2. We say that pursuit can be completed in differential game (1)- (3) from initial positions z_{10}, z_{20}, y^0, $|z_{10} - y^0| > \ell$, at the time $t_1 > 0$, if for any admissible control of the evader

$v = v(t),\ 0 \le t < t_1$, it is possible to find an admissible control $u = u(t),\ 0 \le t < t_1$, of the pursuer, such that for the solutions $z_1(t), y(t)$, $0 \le t \le t_1$, corresponding Cauchy problems (4), (5) the inclusion $|y(t_1) - z_1(t_1)| \le \ell$ is holds and $y(t) \in RS$, $0 \le t \le t_1$.

Problem. Find sufficient conditions of pursuit.

Condition 1. There are numbers $\ell_1 > 0, \ell_2 > 0$, , $\rho_3 > 0$ such, that $\ell_1 + \ell_2 = \ell$ and $\rho - \rho_3 > \frac{\sigma}{\alpha c}\sqrt{t_3(R, \rho_3)}$, where $t_3(R, \rho_3)$ first positive root of the equation

$$R + tR = \frac{t\sqrt{t}}{2}\rho_3\ ,\ \alpha = \frac{R}{R - \ell_2}\ ,\ c = \frac{2\alpha\ell_1}{\sigma}\ .$$

Theorem 1. If condition 1 is hold true, then pursuit can be completed in the game (1)-(4) from all points $z_{10},\ z_{20},\ y^0,\ |z_{10} - y^0| > \ell,$ for a finite time.

Proof. A. In this point A we will prove that choosing a measurable control $u = u(t),\ t \ge 0$, $\int_0^{t_2} |u(t)|^2 dt \le \rho_1^2 + \rho_2^2$, phase vector $z(t) = (z_1(t), z_2(t)),\ t \ge 0$, going out at $t = 0$ from a point $z_0 = (z_{10}, z_{20}),\ |z_0| > R,$ it can lead to the set RS at some time $t_2,\ 0 < t_2 < \infty$, where $\rho_1 > 0, \rho_2 > 0$, are chosen from the condition $\rho - \rho_3 - \rho_1 - \rho_2 > \frac{\sigma}{\alpha c}\sqrt{t_3(R, \rho_3)}$. By the continuity of the condition 1 should be the choice of such numbers $\rho_1 > 0\ , \rho_2 > 0$.

We will put $t_1 = \frac{|z_{20}|^2}{\rho_1^2}$, $u(t) = u_1(t) = \frac{u_0}{\sqrt{t_1}} = -\frac{z_{20}}{\sqrt{t_1}|z_{20}|}\rho_1,\ 0 \le t \le t_1,$ and from the second equality of formulas (6) we have

$$z_2(t_1) = z_{20} - \sqrt{t_1}\frac{z_{20}}{|z_{20}|}\rho_1 = z_{20} + \sqrt{t_1}u_0 = 0. \tag{1.8}$$

$$z_1(t_1) = z_{20} + t_1 z_{20} + \frac{t_1^2 u_0}{2\sqrt{t_1}} = z_{10} + t_1 z_{20} + \frac{t_1\sqrt{t_1}u_0}{2};$$

Agree (8) $\sqrt{t_1}u_0 = -z_{20}$, therefore from the last equation we have equality

$$z_1(t_1) = z_{10} + t_1 z_{20} - \frac{t_1}{2}z_{20} = z_{10} + \frac{t_1}{2}z_{20};$$

Now moment $t = t_1$, is taken as the initial time for the game and we will put

$$z_1^0 = z_1(t_1),\ t_2 = \sqrt[3]{\frac{4|z_1^0|^2}{\rho_2^2}}\ ,\quad u(t) = u_2(t) = \frac{u_1^0}{\sqrt{t_2}} = -\frac{z_1^0}{\sqrt{t_2}|z_1^0|}\rho_2,\ 0 \le t \le t_2. \tag{1.9}$$

Further, using equalities $z_2(t_1) = 0$ and (6) we have

$$z_1(t_2) = z_1^0 - \frac{t_2\sqrt{t_2}\rho_2}{2z_1^0} z_1^0 = z_1^0 + \frac{t_2\sqrt{t_2}}{2} u_1^0 = 0$$

or $\sqrt{t_2}\, u_1^0 = -\frac{2}{t_2} z_1^0.$

Then for $z_2(t_2)$ using equality $z_2(t_1) = 0$ from (6) we have

$$z_2(t_2) = \sqrt{t_2} u_1^0 = \frac{2}{t_2} z_1^0 = \frac{2}{t_2}(z_{10} + t_1 z_{20}) = \frac{2}{t_2} z_{10} + \frac{2t_1}{t_2} z_{20} .$$

If it is necessary, in (9) we decrease $\rho_2 > 0$, what means increase t_2, and we try to get implementation of inequality $|z_2(t_2)| = \left|\frac{2}{t_2} z_{10} + \frac{2t_1}{t_2} z_{20}\right| \le R$. Thus at the moment t_2 we have $z_1(t_2) = 0, |z_2(t_2)| \le R,$ what means $|z(t_2)| \le R.$

B. Consider the equation $R + tR = \frac{t\sqrt{t}}{2}\rho_3$ with respect to unknown $t, t > 0$. As $R > 0$, $\rho_3 > 0$ - the finite number and the right side of the equation is growing faster than the left side with respect to t , that obviously this equation has a positive root, depending only on R, ρ_3. The first positive root of this equation we will denote through $t_3(R, \rho_3)$. Further, if t_4 first positive root of the equation

$$|a - tz_{20}| = \frac{t\sqrt{t}}{2}\rho_3 \tag{1.10}$$

according to $a, z_{20} \in R^n, |a| \le R, |z_{20}| \le R$, that is clear that for such vectors a, z_{20} have a place of inequalities: $|a - tz_{20}| \le R + Rt$,

$$t_4 \le t_3(R, \rho_3) . \tag{1.11}$$

We will consider inclusion

$$a - tz_{20} \in \int_0^t (t-\tau)\frac{\rho_3}{\sqrt{t}} S dt . \tag{1.12}$$

The radius of the ball on the right side (12) is equal to $r(t) = \frac{t\sqrt{t}}{2}\rho_3,$ so comparing it with equation (10) we find that for $t = t_4$ inclusion (12) is hold first runs.

Now consider the equation

$$a - t_4 z_{20} = \int_0^{t_4} (t_4 - \tau)\frac{\rho_3}{\sqrt{t_4}}\omega dt\,, \tag{1.13}$$

with respect to unknown vector $\omega \in S$. At $t = t_4$ inclusion (12) is executed that provides existence of the solution of the equation (13). The solution of equation (13) is denoted by ω_0. **C.** We now describe the method of pursuit and estimate the $|y(t) - z_1(t)|$ at a some moment of time. Let given initial points: $z_{10}, z_{20}, y^0 \in R^n$, $|y^0 - z_{10}| > \ell$, $|y^0| \le R$. Then possible the following cases:

1) $\left|\frac{1}{\alpha}y^0 - z_{10}\right| \le R$, $|z_{20}| \le R$,
2) $\left|\frac{1}{\alpha}y^0 - z_{10}\right| > R$, $|z_{20}| > R$,
3) $\left|\frac{1}{\alpha}y^0 - z_{10}\right| > R$, $|z_{20}| \le R$,
4) $\left|\frac{1}{\alpha}y^0 - z_{10}\right| \le R$, $|z_{20}| > R$.

At 1) case the control of pursuit we will choose in the form

$$u(\tau) = u_3(\tau) = \begin{cases} \dfrac{1}{\alpha(t_4 - \tau)}v(\tau) + \dfrac{\rho_3}{\sqrt{t_4}}\omega_0, 0 \le \tau \le t_4 - c,\ t_4 > c, \\ \dfrac{1}{\alpha c}v(\tau) + \dfrac{\rho_3}{\sqrt{t_4}}\omega_0, t_4 - c \le \tau \le t_4,\ t_4 > c, \\ \dfrac{1}{\alpha c}v(\tau) + \dfrac{\rho_3}{\sqrt{t_4}}\omega_0, 0 \le \tau \le t_4,\ t_4 \le c, \end{cases} \tag{1.14}$$

where ω_0, t_4 are defined according to point B, ω_0 - solution of equation (13), t_4 the first positive root of the equation (10) according to vector $a = \frac{1}{\alpha}y^0 - z_{10}$. Then considering, that in process game "a boy" does not can to leave a sphere RS, for the $|y(t) - z_1(t)|$ of the solutions (6), (7) , we have

$$\left|\frac{1}{\alpha}y(t_4) - z_1(t_4)\right| = \left|\frac{1}{\alpha}y^0 - z_{10} - t_4 z_{20} - \int_0^{t_4}(t_4 - \tau)\frac{\rho_3}{\sqrt{t_4}}\omega_0 dt + \frac{1}{\alpha}\int_{t_4 - c}^{t_4}\left(1 - \frac{t_4 - \tau}{c}\right)v(\tau)dt\right| =$$

$$= \frac{1}{\alpha}\int_{t_4 - c}^{t_4}\left(1 - \frac{t_4 - \tau}{c}\right)v(\tau)dt \le \frac{\sigma c}{2\alpha} = \ell_1.$$

$$|y(t_4) - z_1(t_4)| = \left|y(t_4) - \frac{1}{\alpha}y(t_4) + \frac{1}{\alpha}y(t_4) - z_1(t_4)\right| \le$$

$$\leq (1-\frac{1}{\alpha})\left|y(t_4)\right| + \left|\frac{1}{\alpha}y(t_4) - z_1(t_4)\right| \leq (1-\frac{1}{\alpha})R + \ell_1 = \ell_2 + \ell_1 = \ell .$$

In this way, it is proven that the moment t_4 game is over. Further, we prove the admissibility of pursuit control (14):

$$\int_0^{t_4}\left|u_3(\tau)\right|^2 d\tau = \int_0^{t_4-c}\left|\frac{1}{\alpha(t_4-\tau)}v(\tau) + \frac{\rho_3}{\sqrt{t_4}}\omega_0\right|^2 d\tau + \int_{t_4-c}^{t_4}\left|\frac{1}{\alpha c}v(\tau) + \frac{\rho_3}{\sqrt{t_3}}\omega_0\right|^2 d\tau \leq$$

$$\leq \int_0^{t_4}\left|\frac{1}{\alpha c)}v(\tau) + \frac{\rho_3}{\sqrt{t_4}}\omega_0\right|^2 d\tau \leq \left(\left(\int_0^{t_4}\left|\frac{1}{\alpha c}v(\tau)\right|^2 d\tau\right)^{\frac{1}{2}} + \left(\int_0^{t_4}\left|\frac{\rho_3}{\sqrt{t_4}}\omega_0\right|^2 d\tau\right)^{\frac{1}{2}}\right)^2$$

$$\leq (\frac{\sigma}{\alpha c}\sqrt{t_4} + \rho_3)^2 \leq (\frac{\sigma}{\alpha c}\sqrt{t_3(R,\rho_3)} + \rho_3)^2 < \rho^2 .$$

In the 1) case, proof of the theorem1 is completed. In other cases 2), 3), 4) proposed the following method of pursuit: at the beginning of the game, we will control the phase vectors $z_1(t), z_2(t)$ according to point A, and define the time $t_2 > 0$, such that $z_1(t_2) = 0, \left|z_2(t_2)\right| \leq R$. Further, according to the constraint on the state of the evader we have inequality $\left|y(t_2)\right| \leq R$. Therefore, if at the beginning of the game we will take time t_2 and put $y^0 = y(t_2),\ z_{10} = z_1(t_2), z_{20} = z_2(t_2)$, we are in the conditions of application of case 1). Further, using the control method of the case 1) argue that there $t_4' \leq t_3(R,\rho_3)$, for which $\left|y(t_4^{'}) - z_1(t_4^{'})\right| \leq \ell.$ Unlike the case of 1) in the three other cases, we choose the numbers $\rho_1, \rho_2 > 0$ satisfying the inequality (see point. A)

$$\rho - \rho_3 - \rho_1 - \rho_2 > \frac{\sigma}{\alpha c}\sqrt{t_4(R,\rho_3)}$$

According to point A such a choice is possible. In these cases, to complete the game pursuer need used $t_1 + t_2 + t_4'$ time, and control of the pursuer has the form

$$u(\tau) = \begin{cases} u_1(\tau),\ 0 \leq \tau \leq t_1, \\ u_2(\tau), t_1 \leq \tau \leq t_2, \\ u_3(\tau), t_2 \leq \tau \leq t_4'. \end{cases}$$

Therefore, from the construction of the point A and C for implementations $u_1(\tau),\ 0 \leq \tau \leq t_1,$ $u_2(\tau), t_1 \leq \tau \leq t_2,\ u_3(\tau), t_2 \leq \tau \leq t_4',$ of the pursuit control $u(\tau),\ 0 \leq \tau \leq t_4'$, we have:

$$\int_0^{t_1}\left|u_1(\tau)\right|^2 d\tau \leq \rho_1^2,\ \int_{t_1}^{t_2}\left|u_2(\tau)\right|^2 d\tau \leq \rho_2^2,\ \int_0^{t_4'}\left|u_3(\tau)\right|^2 d\tau \leq \left(\rho_3 + \frac{\sigma}{\alpha c}\sqrt{t_4(R,\rho_3)}\right)^2 ,$$

that proved the admissibility of pursuit control in cases 2), 3), 4). Theorem 1 has been proved.

Condition 2. There are numbers $\ell_1 > 0$, $\ell_2 > 0$ such that $\ell_1 + \ell_2 = \ell$ and

$$\rho > \rho_0 + \frac{\sigma}{\alpha c}\sqrt[3]{\frac{4R^2}{\rho_0^2}} \text{ at } \rho_0 = \sqrt[5]{\frac{32\sigma^3 R^2}{27\alpha^3 c^3}} \text{ , where } \alpha = \frac{R}{R-\ell_2} \text{ , } c = \frac{2\alpha\ell_1}{\sigma} \text{ .}$$

Theorem 2. *If the condition 2 is hold true, then pursuit can be completed in the game (1)- (3) from all initial points* $z_{10}, z_{20}, y^0, \left|z_{10} - y^0\right| > \ell,$ *for a finite time. The proof of theorem 2 is carried out by the scheme of proof of theorem 1 it is easy to change. It should be noted that theorem 1,2 are complementary, i.e., if we apply the theorem to the game (1) - (3) for one and the same starting point, then, by theorem 1, the game can be completed in less time than by theorem 2, but uses more resources than theorem 2.*

Comment 1. The proof of theorem 1 it follows that if the condition 1 is hold true, then condition 2 is hold true.

Comment 2. The same problem for the general linear games research and solved in [6]. But to research this game does not use the method of [6], as unfulfilled assumption 6 of [6].

ACKNOWLEDGEMENT

I wish to thank Professor Asqar Rakhmanov under Department of Systematic and Practical Programming for his support over the period in which this paper was written. I also express my gratitude to my parents for their encouragement and my colleagues for their constant cooperation. I would like to express my gratitude towards my college, Hanyang University, for giving me the opportunity to make its facilities available for me.

CONCLUSION

This paper proposed a new method based on a differential game. In recent years one of the low-studied areas of the theory of differential games with integral and different constraints on controls are widely investigated. According to the above section, this system can better highlight to the decision of the problem of pursuit for the so-called game "boy and crocodile" with different constraints on the controls of players. In the present, we investigated the case with integral constraints on the control of the pursuer (crocodile) and geometric constraints on the control of the evader (boy), in the case of phase constraints on the state of the evader.

Our future work is to develop more accurate and efficient differential games. We are going to implement our application for visually impaired people to assist them in easily realizing natural scene games.

REFERENCES

[1] Azamov A .A. On the evasion problem for a given curve, Applied Mathematics and Mechanics, no. 4, pp. 694–697, 1982.

[2] Azamov A. A. On the problem of kind for the simple motion game with restriction. Serdika. Blgarsko Mat.call list . 1986 vol.12. S.38-43.

[3] Azamov A.A, Kuchkarov A.Sh, B.T Samatov. The relation between problems of pursuit, controllability and stability in the large in linear systems with different types of constraints. / PMM 2007 , t.71 , issue 2, p.259-263 .

[4] Asqar Raxmanov., Gafurjan Ibragimov. Linear Discrete Pursuit Game with Phase Constraints. Hindawi Publishing Corporation. The Scientific World Journal, Volume 2014, Article ID 435103, 5 pages http://www.hindawi.com/journals/tswj/2014/435103/abs/.

[5] Asqar Rakhmanov and Gafurjan Ibragimov. A simple motion differential game with different constraints on controls and under phase constraint on the state of the evader. AIP Conference Proceedings 1739, 020063 (2016); doi: 10.1063/1.4952543

[6] Askar Rakhmanov, Gafurjan Ibragimov, Massimiliano Ferrara. Linear pursuit differential game under phase constraint on the state of evader. Discrete Dynamics in Nature and Society, vol. 2016, Article ID 1289456, 6 pages, 2016. doi:10.1155/2016/1289456

[7] Ivanov R.P. The simple pursuit - evasion on the compact . Reports of the Academy of Sciences of the USSR , 1980 , 254 (6) : 1318-1321.

[8] Ibragimov G.I. Game problem on a convex closed set. Mathematic . Tr. 2001 , 4 : 2, 96-112 .

[9] Ibragimov G.I., Azamov A., and Khakestari M .. Solution of a linear pursuit- evasion game with integral constraints. ANZIAM J., 52 (E): E59-E75. 2011.

[10] Ibragimov G.I, The problem of group pursuit with integral constraints on the player controls . Mat . Notes, 2001 , t.70 , №2, p.201-212 .

[11] Ibragimov G.I. Game Problem on a Closed Convex Set. Siberian Advances in Mathematics. 2002 , V.12, No. 3 , p. 16-31 .

[12] N.N Krasovsky , Control of Dynamic Systems, Nauka, Moscow, Russia, 1985., M .: Science, 1985 .

[13] KuchkarovA.SH , About one differential game with integral and state constraints . Automation and Remote Control, 2013 , №1, p.12-25

[14] Nicolskiy M.C., Linear differential games with integral constraints. Differ. equation. 1992 T.28, №2, p.219-223.

[15] Pontryagin L.S, Selected Works. M .: Nauka, 1988.

[16] Pshenichniy B.N, Chikrii A.A, Rappoport I.S, Pursuit several controlled objects with phase constraints. USSR 1981. T.259. №4. p. 785-789.

[17] Rakhmanov A.T. The linear differential game of pursuit with phase constraints on the state of the evader. Collection of articles VII international conference "Priority directions in the field of science and technology in the twenty-first century», II chapter, p.123-127, Tashkent 2014.

[18] Rakhmanov A.T. On the method of persecution in linear differential games with integral constraints on the controls. Differ. equation. 1989 , 25 (5): p. 785-790.

[19] Satimov N.Y. Methods for solving problems pursuit in the theory of differential games .Tashkent : Publishing NU of UZ, 2003.

[20] Satimov N.Y., Rikhsiev B. B., Hamdamov A.A. On a Pursuit Problem for n-person linear differential and discrete games with integral constraints. Math. USSR Sb. Vol.46, p. 456-469 . 1983. Doi: 10.1070 / Sm1983v046n04abeh002946.

[21] Samatov B.T. The problem of pursuit, escape with integral-geometric constraints on the pursuer's control. "Automation and Remote Control", 2013, №7, p.17-28.

[22] Chikrii A.A, Belousov A.A. On linear differential games with integral constraints. Proceedings of the Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences. 2009, 15, number 4, p. 290-301.

[23] Chikrii A.A, Belousov A.A On linear differential games with integral constraints. Proceedings of the Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences. 2013, 19, number 4, p. 308-319.

[24] Yugay L.P. "On differential games with constraints on phase state and controls of players," Doklady Academy of Sciences of UZSSR, n.4,p.5–6,1977.

CORRESPONDING AUTHOR

Nargiza Siddiqova received her B.S. and M.S degree in Informatics and Information Technologies, Computer Systems and Software from Tashkent University of Information Technologies, Tashkent, Uzbekistan, in June 2011 and 2013. She is currently working as a Senior Lecturer at Systematic and Practical Programming Department. Her current research interest includes object extraction, image processing, practical programming and information technology.