ISSN 2250-3153

Application on Non Linear Waves Traffic Flow

Ghada A. Ahmed

* Department of mathematics faculty of science
Al-Baha university – Kingdom of Saudia Arabia & university of medical science and technology-sudan

Abstract- Traffic flow wave theory was studied by many authors , This article presents survey with references about various topics by using the kinematic waves theory to evaluate flows $q(x,t)$ past any Point x by time t. It is shown here how a formal solution can be evaluated directly from boundary or initial conditions. If there are shocks we have discontinuities in slope describing the passage of a shock and the solution is multiple valued, the solution can be evaluated directly from the boundary conditions by method of characteristic.

Index Terms- Traffic flow $q(x,t)$, Traffic velocity $q(x,t)$, Traffic density $\rho(x,t)$, non linear waves .

INTRODUCTION

The traffic flow wave in mathematics is the study of the dynamic properties of traffic on road sections which represent the interactions between vehicles, drivers, and traffic control devices, with the aim of understanding and developing an optimal road network with efficient movement of traffic and minimal traffic congestion problems. The Kinematic Waves described a theory of one-dimensional wave motion of highway traffic flow. The key postulate of the theory was that there exists some functional relation between the flow q and the density ρ . The flow q and the concentration ρ have no significance except as mean.the purpose of the theory is to ask how they vary in space and time. An account of the experimental methods employed in this field has been given by the head of the traffic-flow section at the Road Research Laboratory " Charlesworth I950 ". They include methods for measuring the means and standard deviations of vehicle speed at a point or journey time over a stretch of road, and for measuring the flow (number of vehicles passing a given point per unit of time). Attempts to correlate these variables for roads of particular mean width, mean curvature, etc., are made. Also, traffic performance is studied before and after some change in road conditions, and statistical technique is used to find out whether the change significantly reduces journey times or accidents.

II. FORMULATION OF THE PROBLEM

2.1: Basic notation on waves motion

In this section we present some basic concepts about one dimension hyperbolic waves motion.

Definition (2.1.1) [Wave]

A wave is any recognizable signal that travels from one location of the medium to another `with a recognizable velocity of propagation.

Definition (1.2.2) [Velocity of Propagation]

Velocity of propagation is a measure of how fast a signal travels over time , or the speed of the transmitted signal as compared to the speed of light

Definition (2.1.3)[Linear hyperbolic Waves]

The one dimensional equation wave plane

(2.1) $$\varphi_{tt} - C^2{}_0\, \varphi_{xx} = 0$$

is particularly simple it can be written in new variables $\alpha = x - c_0\, t$, $\beta = x + c_0\, t$ *as* $\varphi_{\alpha\beta} = 0$ and its general solution is $\varphi = f(\alpha) + g(\beta)$. Where f *and* g are arbitrary functions , the solution is combination of two waves one with shape described by the function f moving to the write with speed c_0 , and the other with the shape g moving to the left with speed c_0 where c_0 is a constant , which represent the hyperbolic equation If we retain only

(2.2) $$\varphi_t + c^2{}_0\, \varphi_x = 0$$

And the general solution is $\varphi = f\left(x - c_0 t\right)$, this is the simplest hyperbolic wave problem.

Definition (2.1.4) [Non Linear hyperbolic Waves]

The general non linear first order equation for $\varphi(x,t)$ is any functional relation between $\varphi, \varphi_{x,} \varphi_t$, and the simplest equation is

(2.3) $$\varphi_t + c(\varphi)\ \varphi_x = 0$$

Where the propagation speed $c(\varphi)$ is a function of local disturbance φ equation (2.3) is called quasi linear equation

2.2: Formulation of Traffic Flow Wave

The traffic flow on high way is one of most common real world problems . We consider the traffic flow on along high way under the assumption that cars do not enter or exit the high way at any one of it is point and those individual cars are replaced by continuous density function (i.e.) for stretch of high way with no entries or exits cars are conserved , we take x-axis along the high way and assume the traffic flow in the positive direction . This problem can be described by three fundamental traffic variables .

Definition 2.2.1 [Traffic velocity] $u(x,t)$

Let the velocity of vehicles j be u_j and its position is $x_j(t)$ at time t thus

(2.4) $$u_j = \left(\frac{dx_j}{dt}\right),\ j = 1,2,3,\ldots, M$$

Any discussion of traffic on our single-lane road must deal with a collection of vehicles , with positions $x_j(t),\ j=1,2,3,\ldots M$ and velocities $u_j = \dfrac{dx_j}{dt}$, $j = 1,2,3,\ldots, M$ and so we may define a velocity field by a function $u(x,t)$ thus the value of $u(x,t)$ at a certain time t^* and a certain position x^* on the road should be the velocity of cars on that particular part of the road at that time t.

Definition 2.2.2 [Traffic Density] $\rho(x,t)$

The traffic density $\rho(x,t)$ associated to given position x and time t is the average number of vehicles per unit length of road at position and time specified .

Definition 2.2.3 [Traffic Flow] $q(x,t)$

The traffic flow $q(x,t)$ is the number of vehicles per unit time which cross a given point on the road . This means that the flow will depend on x and t thus

(2.5) $$q(x,t) = \rho(x,t)\ u(x,t)$$

We assume the theory of " Kinematic waves " to formulate the problem in term of first – order – non liner partial differential equation on the basis of conservation of cars and experimental relationship between the car velocity and traffic density . We consider our assumption that no inter or exit car (i.e.) the material is conserved if we select some stretch of the road between the points $x = x_1 = R$ and $x = x_2 = S$ where $x_2 \succ x_1$. We know that the number of cars found to lie between x_1 and x_2 at some time t will in general depend upon the time t. After moment the number of cars within the segment RS will increase and the flow out of the segment RS will decrease this can be express in terms of flow at R and S , that the rate of change of the number of vehicles in the segment with respect to time should equal the difference in flow rate . If N_{RS} is this number of vehicles , then

(2.6) $$\left(\frac{dM_{RS}}{dt}\right) = -q(x_1,t) + q(x_2,t)$$

Also we can computed N_{RS} from the density by

(2.7) $$M_{RS}(t) = \int_{x_1}^{x_2} \rho(x,t)\, dx$$

Thus we can rewrite (2.6) as

(2.8) $$\frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)\, dx = -q(x_2,t) + q(x_1,t)$$

ISSN 2250-3153

This last relation represents the conservation law for the cars on the road. If $q(x_2,t) \succ q(x_1,t)$, M_{RS} will decrease in time because the more calls flow out than in. From conservation law (2.5) that $[-q(x_2,t)+q(x_1,t)]$ whenever ρ becomes independent of time this is because

(2.9) $$\frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)\,dx = \int_{x_1}^{x_2} \frac{\partial \rho}{\partial t}\,dx$$

And from the fundamental theorem of calculus we have

(2.10) $$\int_{x_1}^{x_2} \left(\frac{\partial q\ (x,t)}{\partial x}\right) dx = q(x_2,t) - q(x_1,t)$$

Since q depends on both x and t

And from (2.8), (2.9), (2.10) we have

(2.11) $$\int_{x_1}^{x_2} \left[\left(\frac{\partial \rho(x,t)}{\partial t}\right) + \left(\frac{\partial q(x,t)}{\partial x}\right)\right] dx = 0$$

The equation (2.11) hold in the interval $x_1\ x_2$, If $\rho(x,t)$ has a continuous derivatives , we may take the limit as $x_2 \to x_1$ the conservation equation (2.11) implies that (with no entering or exiting traffic)

(2.12) $$\left(\frac{\partial \rho(x,t)}{\partial t}\right) + \left(\frac{\partial q(x,t)}{\partial x}\right) = 0$$

Where q is some function of ρ determined by

(2.13) $$q = q(\rho)$$

By using (2.5), we can write (2.12) as

(2.14) $$\left(\frac{\partial \rho}{\partial t}\right) + \left(\frac{\partial (\rho u)}{\partial x}\right) = 0$$

We make a basis simplifying a assumption that the velocity of car at any point along the high way depend only on the traffic density alone that is $u = u(\rho)$ and hence the traffic flow

(2.15) $$q = \rho u(\rho)$$

(2.16) $$\left(\frac{\partial \rho}{\partial t}\right) + \left(\frac{\partial (\rho u(\rho))}{\partial x}\right) = 0$$

The traffic velocity $u(\rho)$ clearly must be monotonically decreasing function of density ρ . It follows that $\left(\frac{du}{d\rho}\right) = u'(\rho) \le 0$ the decreasing feature of traffic velocity is shown in Figure (1) , (2)

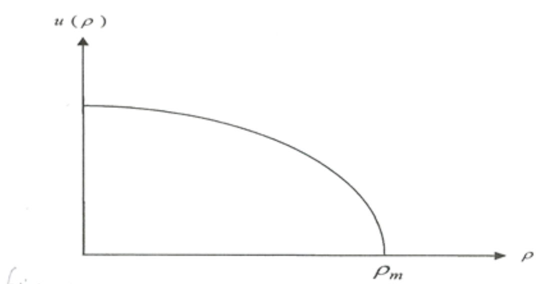

Fig. (1) : A VELOCITY DENSITY CURVE

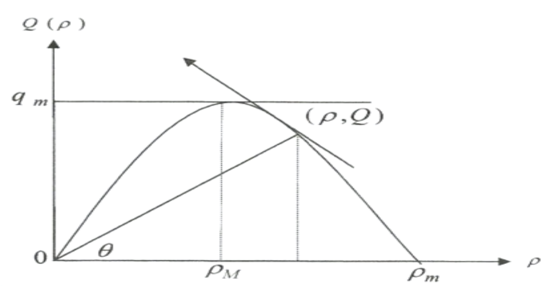

Fig. (2) : FLOW DENSITY CURVE

And the traffic flow $q(\rho)$ is an increasing function of density ρ until attains a maximum value $(q)_{\max} = (q)_m$ for some $0 \prec \rho \prec \rho_m$, in general the mean speed of car $u = \left(\frac{q}{\rho}\right) = \tan\theta$, represents the slope of the chord from the origin as in

Figure (2) , The propagation speed

(2.17) $$c = \frac{dq}{d\rho}$$

Hence we can write

(2.18) $$c = \left(\frac{dq}{d\rho}\right) = \left(\frac{d}{d\rho}\right)(\rho u) = u + \rho\left(\frac{du}{d\rho}\right)$$

Evidently $c \prec u$, if $\frac{du}{d\rho} \prec 0$ and $c = u$ only at a very low density. Substituting (3.13) in (3.14) we have

(2.19) $$\left(\frac{\partial \rho}{\partial t}\right) + c(\rho)\left(\frac{\partial \rho}{\partial x}\right) = 0$$

Where

(2.20) $$c(\rho) = q'(\rho) = u(\rho) + \rho u'(\rho)$$

Represent the velocity of the density waves , since $u'(\rho) \prec 0, c(\rho) \prec u(\rho)$ that is the velocity of these waves is always less than that of cars. The Figure (2) shows that $c(\rho) = q'(\rho) \succ 0$ for all ρ in $[0, \rho_M]$ and zero at $\rho = \rho_M$ and it is negative in (ρ_M, ρ_m). All these mean that wave propagate forward relative to the high way in $[0, \rho_M]$ and are stationary at $\rho = \rho_M$ and then travel backward in $[\rho_m, \rho_M]$. Further discontinuous wave are likely to occur on any segment of high way when the traffic is very heavy in front and light behind.

2.3 : Shocks wave in traffic flow

In general we mean by a shock a strong pressure wave. Shock wave exists whenever the traffic conditions change. The equation that is used to estimate the shock velocity waves is given by.

(3.17) $$U = \left[\frac{(q_1 - q_2)}{(\rho_1 - \rho_2)}\right]$$

Where.

U : Propagation velocity of shock wave (miles / hour).	
q_2 : Flow after change in conditions (vehicles / hour).	ρ_2 :Traffic density after change in conditions (vehicles / miles)
q_1 : Flow prior to change in condition (vehicles / hour).	ρ_1 : Traffic density prior to change in conditions (vehicles / miles)

Note 3.2.1 [The magnitude and direction of the shock wave]

(+) shock wave is travelling in same direction as traffic stream.	(-) shock wave is travelling up stream or against the traffic stream.

Since $q = Q(\rho)$ in the continuous part, we have $q_{2} = Q(\rho_2)$ *and* $q_1 = Q(\rho_1)$ on two sides of any shock and the shock condition (3.17) may be written as $U = \frac{Q(\rho_1) - Q(\rho_2)}{\rho_1 - \rho_2}$

III. SOLUTION OF THE TRAFFIC FLOW PROBLEM

3.1. The continuous solution

We are going to formal a solution of the traffic flow wave according to the boundary conditions by using the method of characteristic.

Definition (3.1.1) [Method of characteristic]

The method of characteristics is a numerical method for solving evolutionary partial differential equation problems by transforming them into a set of ordinary differential equations. The (O.D. Es) is solved along particular characteristics, using standard methods and the initial and boundary conditions of the problem . One approach to the solution of (2.19) is to consider the function $\rho(x,t)$ at each point of (x,t) plane and to note that $\rho_t + c(\rho)\rho_x = 0$ is the total derivative of ρ along curve which has slope

(3.1) $$\left(\frac{dx}{dt}\right) = c(\rho)$$

At every point of it , therefore along any curve in (x,t) plane consider x and ρ to be a function of t , then the total derivative of ρ is $\frac{d\rho}{dt} = \left(\frac{\partial\rho}{\partial t}\right) + \left(\frac{dx}{dt}\right)\left(\frac{\partial\rho}{\partial x}\right)$. We now consider a curve C in the (x,t) plane which satisfies (3. 1) . of course such curve cannot be determined explicitly since the defining equation(3.1) involves the unknown values ρ on the curve , we deduce from (2.19) and the total derivative relation and from (2.19) that

(3.2) $$\frac{d\rho}{dt} = 0 \ , \frac{dx}{dt} = c(\rho)$$

We first observe that ρ remains constant on C then it follows that $q = q(\rho)$ remains constant on C , there for the curve C must be straight line in the (x,t)plane with slope $c(\rho) = \frac{dx}{dt}$ thus the general solution of (2.19) depends on the construction of family of

Straight lines in (x,t)plane, each line with slope $c(\rho) = \frac{dx}{dt}$ corresponding to the value of ρ on it. If we take the boundary condition that $\rho = f(x), t = 0 \ -\infty \prec x \prec \infty$ and refer to the (x,t)diagram in Figure (3) .if one of the curves C intersects $t = 0$ at $x = \alpha$, then $\rho = f(x) = f(\alpha)$ on the whole of that curve. The corresponding slope of the curve is $c(\rho) = c(f(\alpha)) = F(\alpha)$. The equation of the curve then is $x = \alpha + F(\alpha)\,t$. This determines one typical curve and the value of ρ on it is $f(\alpha)$ allowing α we obtain the whole family:

(3.3) $$\rho = f(\alpha) \ , \quad c = F(\alpha) = c(f(\alpha))$$

On

(3.4) $$x = \alpha + F(\alpha)\,t$$

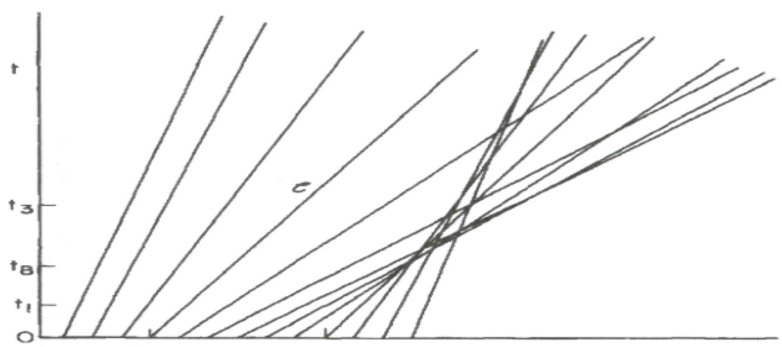

Fig.(3) :CHARACTERASTIC DIAGRAM FOR NONLINEAR WAVES

Use (3.3) and (3.4) as an analytic expression for the solution , free for particular construction . That is ρ is given by (3.3) where $\alpha(x,t)$ is defined implicitly by (3.4) . From (3.3) $\rho_t = f'(\alpha)\alpha_t \ , \rho_x = f'(\alpha)\alpha'(t)$ by differentiate (3.4) with respect to t and x respectively $0 = F(\alpha) + t F'(\alpha)\alpha_t + \alpha_t = F(\alpha) + (1 + F'(\alpha)t)\,\alpha_t$, $1 = (1 + F'(\alpha))\alpha_x$, Therefore

(3.5) $$\rho_t = -\frac{F(\alpha) f'(\alpha)}{1 + F'(\alpha)\,t} \ , \quad \rho_x = \frac{f'(\alpha)}{1 + F'(\alpha)\,t}$$

And we see that $\rho_t + c(\rho)\rho_x = 0$ *i.e* $\rho = f(\alpha)$ is the solution on $x = \alpha + F(\alpha)\,t$, since $c(\rho) = F(\alpha)$ .the initial condition $\rho = f(x)$ is satisfies because $\alpha = x$ when $t = 0$. the curves used in the construction of the solution are the characteristic curves for this special problem.since by the theory of characteristic the equation $\rho_t + c(\rho)\rho_x = 0$ have the characteristics

$$\frac{dx}{1}=\frac{dt}{c(\rho)}=\frac{d\rho}{0}\ ,then\ d\rho=0\ ,\ \frac{dx}{dt}=c(\rho)\ and\ \rho=\text{ Constant on }x=c(\rho)t.$$

3.2 The discontinuous solution

Any compressive part of the wave where the propagation velocity is a decreasing function of x ultimately breaks to give a triple - valued solution for $\rho(x,t)$ the breaking starts at the time $t=t_\beta=-\dfrac{1}{F'(\alpha)}$ for which $F'(\alpha)\prec 0\ and \left|F'(\alpha)\right|$ is maximum that is the breaking occurs first at the characteristic $\alpha=\alpha_\beta$

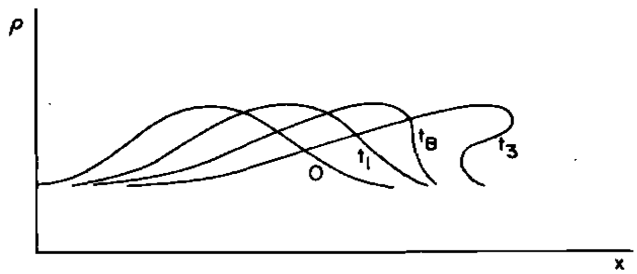

Fig.(4) : BREAKING WAVE SUCCESSIVE PROFILE CORRESPONDING TO THE TIME $(0,t_1,t\beta,t_3)$ in Fig(3)

The simplest case is the problem

(3.6)
$$\left.\begin{cases}\rho=\rho_2\ , & c=c(\rho_2)=c_2, \quad x\succ 0\\ \rho=\rho_1\ , & c=c(\rho_1)=c_1\ , \quad x\prec 0\end{cases}\right\}t=0 \text{ with } c_1\succ c_2$$

For the case $c'(\rho)\succ 0\,,\rho_2\succ\rho_1$ the multi-valued region starts right at the Origin and is bounded by the characteristics $x=c_1t\ and\ x=c_2t$, Where ρ_1 and ρ_2 are constants $0\le\rho_2\prec\rho_1\le\rho_{\max}$ hence $U_1\succ U_2$.

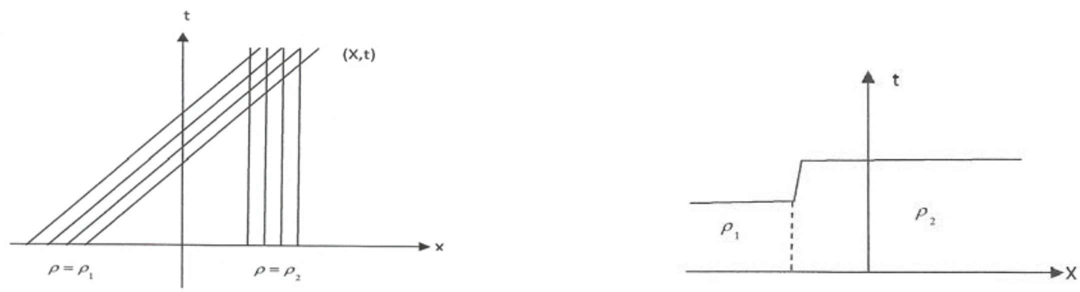

Fig.(5) : COMPAESSIVE WAVES

(3.6)

The density wave for the lighter traffic travel with velocity $c(\rho_1)=q'(\rho_1)$ which is greater than $c(\rho_2)=q'(\rho_2)$ of the heavy traffic density wave that is $c(\rho_1)\succ c(\rho_2)$ a set of characteristic (3.6).In any situation traffic becomes denser further along road characteristic intersect $\rho=\rho_1\ and\ \rho=\rho_2$. Note : But in physically impossible that density is multiple valued

A jump occurs across a curve as a shock. The behavior of the hump, in the density $\rho\ in(x,t)$ plane indicate that a shock should be inserted between two regions of constant density ρ separated by constant shock velocity which given above. The equations (3.6) are satisfied in both side of shock evidently $\rho=\rho_1$ = constant similarly $\rho=\rho_2$

The shock condition is satisfied if

(3.7)
$$U=\frac{q_1-q_2}{\rho_1-\rho_2}$$

That is if the shock propagates at the constant velocity given by (3.7).

The shock path in the straight line through the origins in (x,t) plane, thus $\rho(x,t)$ satisfy all equations for all x and t except across shock.

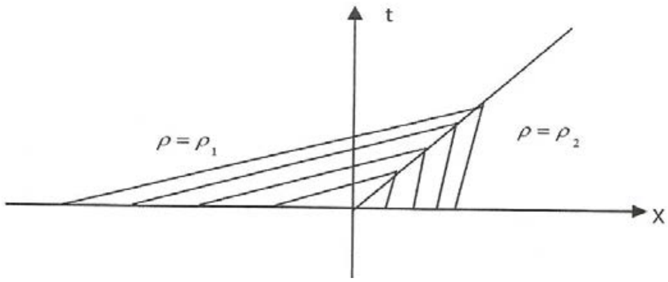

Fig. (6) : GRAPHICAL REPRESENTATION

There is another case corresponds to $\rho_1 \succ \rho_2$ hence $U_1 \prec U_2$ and $c(\rho_1) \prec c(\rho_2)$, which physically correspond to the situation in which the Traffic initially become light further along high way. The characteristic shown in Figure (6).

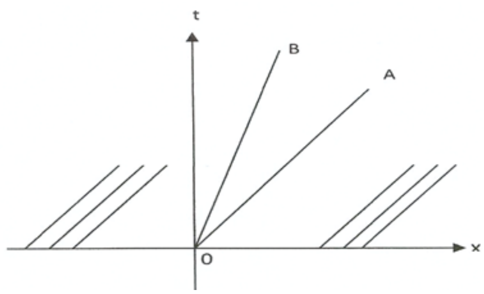

Fig.(7) : THE SOLUTION OF FAN CENTER ON THE ORIGIN

There are no Characteristics with infinite sector $(\mathrm{A} \circ \mathrm{B})$, hence no solution for Traffic density $\rho(x,t)$.

There are two regions; each constant density ρ separated by a fan centered at the origin.

The solution is given by

(3.8)
$$\rho(x,t)=\begin{cases}\rho_1 & : \textit{to the left of} \quad OA \\ \rho_2 & : \textit{to the right of} \quad \mathrm{OB} \\ \rho_0 & : \textit{constant}, \quad \rho_2 < \rho_0 < \rho_1 \quad \textit{on} \quad \mathrm{X} = q'(\rho) \quad \textit{in sector} \quad \mathrm{AOB}\end{cases}$$

A cross OA and OB $\rho(x,t)$ is continuous but ρ_t , ρ_x are discontinuous.

2.2. Traffic light problem:

The traffic light also known as traffic lamp , stop light , are signaling devices posionalat road intersection , pedestrian crossings and another location to control competing flows of traffic . And it has been installed in most cities around the world to control the flow of traffic. They assign the right of way to road users by the use of lights in standard colors (Red , yellow , green) using universal color code .they are used at busy intersections to more evenly apportion delay to the various users. The most common traffic light consists of a set of three lights: red, yellow (official amber) and green . When illuminated, the red light indicates for vehicles facing the light stop, the amber indicate caution, either because lights are about to turn red and green light to proceed, if it is safe to dos . There are many variations is use and legislation of traffic lights depending on the customs of country and the special needs of particular intersection. We construct the characteristic in the (x,t) diagram –these are lines of constant density and their slope $c(\rho)$ determine the corresponding values of ρ On them so the problem is solved once the (x,t) diagram has been obtained. Suppose first that the red period of lights is long enough to allow the incoming traffic to flow freely at some value $\rho_i \prec \rho_m$. Then we may start with characteristics of slope $c(\rho)$ intersecting the t axis in the interval AB as in Figure (7) , is AB a part of green period , just below the red period , BC the cars are stationary with $\rho=\rho_j$, hence the characteristic have the negative slope $c(\rho_j)$. The line of separation between the stopped queue at the traffic light and the free flow must be shock B_p , and from the shock condition (3.7) its velocity is $\dfrac{-q(\rho_i)}{\rho_i-\rho_j}$

When the light turn green at C, the leading cars can get at maximum speed since $\rho=0$ a head of them. This is represented by the characteristic CS with maximum slope $c(0)$. Between CS and CP have an expansion fan with all values of C being taken. Exactly at the intersection CQ , the slope c must be zero but this corresponds to the maximum $q=q_m$. The total incoming flow for the time BQ is $(t_r+t_s)q_i$ where t_r the red period BC and t_s is the part of green period before the shock gets through .The flow across the intersection in this time is $t_s\, q_m$ there for $(t_r+t_s)\, q_i=t_s\, q_m$, $t=\dfrac{t_r\, q_i}{q_m-q_i}$ For the shock to get through and the light to operate freely, the green period must exceed this critical value . Green light problem Stream of car stopped at red signal at x = 0 , the road is jammmed intially behind the signal and rhere is no traffic behind signal and there is no traffic behind signal . As soon a signal turn green, the stream of car start moving across $x=0$, the initial state of the traffic is given by

$$\rho(x,0)=\rho_m\cdot H(-x)=\begin{cases}\rho_m & x\prec 0\\ 0 & x\succ 0\end{cases}$$

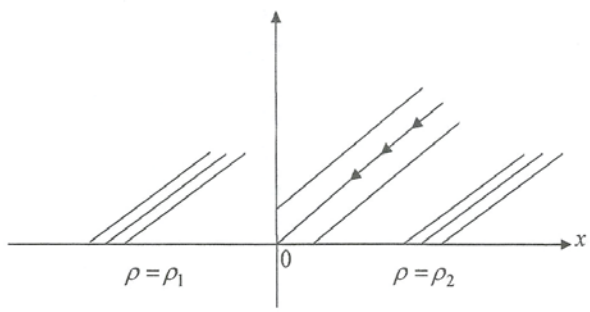

Fig. (8): CHARACTERISTIC AND DISCONTINUOUS SOLUTION

Where ρ_m is maximum density and $H(x)$ is Heaviside unit step function. Assume that the traffic flow $q(\rho)$ is quadratic in region $0\prec\rho\prec\rho_m$ and zero otherwise, so that $q=\dfrac{q_m}{\rho_m}\left(1-\dfrac{\rho}{\rho_m}\right)\rho$, $0\prec\rho\prec\rho_m$ the velocity of traffic flow is $u=\dfrac{q}{\rho}$, That is

(3.9) $$u=u_m\left(1-\frac{\rho}{\rho_m}\right).$$

Where $u_m=\dfrac{q_m}{\rho_m}$ is the maximum speed, when $\rho=0$ the traffic flow equation for the density is given by

(3.10) $$\rho_t+c(\rho)\,\rho_x=0$$

Where

(3.11) $$c(\rho)=q'(\rho)=u_m\left(1-\frac{2\rho}{\rho_m}\right)$$

Thus the characteristic equation of (3.10) is $\dfrac{dx}{1}=\dfrac{dx}{c(\rho)}=\dfrac{d}{0}$,where

(3.12) $$d\rho=0,=\frac{dx}{dt}=c(\rho)=u_m\left(1-\frac{2\rho}{\rho_m}\right)$$

Thus the density ρ along characteristic given by

(3.13) $$\frac{dx}{dt}=c(\rho)=u_m\left(1-\frac{2\rho}{\rho_m}\right)$$

Any characteristic that intersect the positive x – axis at $x=x_0\succ 0$ has the slope

(3.14) $$\frac{dx}{dt}=u_m\left(1-\frac{2\rho}{\rho_m}\right)=u_m\left(1-\frac{2\rho}{\rho_m}(x,0)\right)=u_m$$

This gives a family of characteristic .as the right leading straight lines as shown in figure.(8)

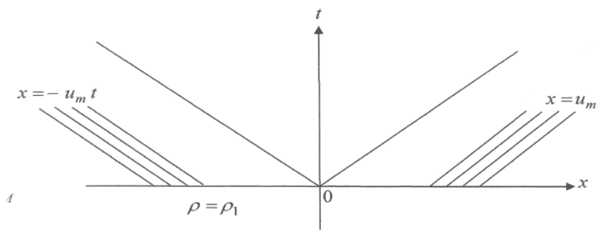

Fig.(9) : CHARACTERISTIC – DUE TO DISCONTINIOUS DENSITY

On other hand the characteristic that intersects the negative $x-axis$ at $x=x_0\prec 0$ has the slope

(3.15) $$\frac{dx}{dt}=u_m\left(1-\frac{2\rho_m}{\rho_m}\right)=-u_m$$

Equation (3.15)gives a family of characteristics .represent a left leading straight line $x=-u_m t+x_0$ in the fan link region all characteristics must pass through the origin and must be straight line, hence $C=\left(\frac{dx}{dt}\right)\left(\frac{x}{t}\right)$ it follows from (3.13) that

(3.16) $$\frac{x}{t}=u_m\left(1-\frac{2\rho}{\rho m}\right)$$

Solving for gives

(3.17) $$\rho=\frac{1}{2}\rho_m\left(1-\frac{x}{u_m t}\right) \quad , \quad -u_m t\prec x\prec u_m t$$

$t=0\ \ at\ x=\alpha\,,\ \ x=\alpha+F(\alpha)t$ In the fan take $\alpha=0\,,\ x=F\,t\ ,C=\frac{x}{t}$ at any instant the density varies linearly in x from $\rho_m\ \ at\ \ x=-u_m t$ to zero at $x=u_m t$. Thus the solution of (3.16) for $\rho(x,t)$ for all x at any time t is drawn in figure (10)

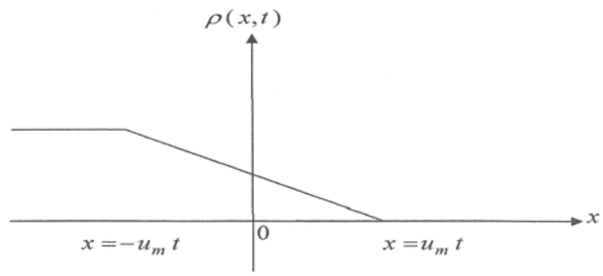

Fig.(10) : TRAFFIC DENSITIES AFTER THE SIGNAL TURN GREEN

VI. CONCLUSION

I. In this paper we assume the theory of Kinematic waves to formulate the traffic flow wave in term of first – order – non liner partial differential equation on the basis of conservation of cars and experimental relationship between the car velocity and traffic density.

II. We consider our assumption that no inter or exit car (i.e.) the material is conserved. We formal a solution of the problem according to the boundary conditions by method of characteristic.

REFERENCES

[1] G.B. Witham , Linear and nonlinear waves, John willey & sun ,-1974 .
[2] M. J. LIGHTHILL, F.R.S. AND G. B. WHITHAM- A theory of traffic flow on long crowded roads,www.jstor.org1955 .
[3] Cathleen S. Morawetz Tata , lecture on Nonlinear Waves and Shocks , Institute Of Fundamental ResearchBombay-1981.
[4] Debnath –lokenath , Nonlinear partial differential equation for scientist and engineers , 2nd ed .Abirkhauser, boston book , (2005).
[5] G. F. NEWELL , asimplified theory of kinematic waves in high way traffic , Institute of Transportation Studies, University of California,Berkeley, CA 9472U.S.A.-1992.
[6] L.H. Immers .S. Logghe,Traffic Flow Theory , Leni Hurley and Jim Stada (January 2005).
[7] Stephen Childress , Notes on traffic flow ,www.math.nyu.edu/faculty/childres, March 22, 2005.
[8] Boris S.kerner , introduction to modern traffic flow , springer , (2009).
[9]Komentani,Eand T.Sasaki , On the stability of traffic flow , Oper.Res.(japan).(1958).

AUTHORS

First Author –
Dr:Ghada Awad Elkarim Mohammed Ahmed-(phd)
Assistant professor of university of Al-Baha –kingdom of Saudia Arabia & university of medical science and technology-Sudan
Al-Baha P.O.Box (1998)-Tel.Fax:00966-7-7274111
Mathematics Department-Faculty of Science
1-Ghada.ahmed832@yahoo.com
2-Ghada.ahmed 151171@gmail.com
3-Ghada.ahmed112233@hotmail.com
Tel. : 00966531247180