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Comparative Study on Image Compression Using Various Principal Component Analysis Algorithms

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Abstract- Principal Component analysis (PCA) is one of the statistical methods employed in image compression. Presented paper deals with four different types of PCA algorithms those are 2D-PCA, 3D-PCA, 2D -Kernel PCA (2D-KPCA) and 3D-KPCA. A comparative study is made for all four types of PCA based on their PSNR values. These algorithms are also tested on several standard test images. It has been found that the quality of reconstructed image of 3DKPCA is better than other types of PCA based image compression.

Index Terms- Image compression; 2D-PCA; 3D-PCA; 2D-KPCA; 3D-KPCA.

I. Introduction

Image compression deals with minimizing the number of bits required to represent an image. The aim of image compression is to reduce the number of bits while keeping the quality and resolution of reconstructed data close to the original data. By using this process the data/image can be easily transmitted and stored. Several algorithms may be used for the compression process. By choosing a proper compression algorithm not only the storage requirement is reduced but the overall execution time also reduces. PCA can be approached by using a mathematical tool called Singular Value Decomposition (SVD). In the next sections the relation between PCA and SVD has been explained. Principal Component Analysis also known as Karhunen-Loeve or Hotelling transform [2] - PCA is a simple linear transform type technique. This method reduces the data size. A reduced data makes computation easier and faster. So, this is an efficient technique in data compression. PCA has been used over several areas s.a. signal processing, image processing, statistics, neural networks, and data mining etc. PCA is an optimal dimension reduction technique that minimizes the mean square error between the original images and their reconstructions for any given level of compression. [2]. By reducing dimension computational cost and noise reduces. The traditional PCA works on 1-dimensional vectors. In this scheme the 2D data must be converted into a 1D vector form. This results in the problem of dealing with high dimensional vector space. This will also cause over-compression of original data and degradation in image quality. To solve this dimensionality problem, *Jian Yang et al.* [3] proposed an approach known as 2-Dimensional Principal Component Analysis (2D-PCA). This is another linear dimensionality reduction method used for image compression. In this method the data sample matrix is not converted to a set of vectors but a covariance matrix is constructed with the help of 2D data sample matrices directly. So, the eigenvectors of the covariance matrix is directly computed. This reduces the size of covariance matrix and easily an accurate covariance matrix is evaluated. This also improves the computational efficiency and takes less time to determine the corresponding eigenvectors. By using this type of approach the problem of over-compression can be alleviated. However, there still remains variety of problems in 2D-PCA which can be eliminated by using 3D-PCA. 2D-PCA uses generalized singular value decomposition which is limited to two matrices while 3D-PCA utilizes the concept of multidimensional array. However, if the image has complicated structures which cannot be solved in a linear subspace, non linear dimensionality reduction technique can be used. Kernel principal component analysis (KPCA) is a true nonlinear extension of PCA.

In this paper, four different PCA algorithms are compared. The paper is organized as follows: In section 2, 2D-PCA and 3D-PCA are reviewed. In section 3, 2D-KPCA and 3D-KPCA are discussed. Section 4, contains experiments and results. Finally, conclusions are presented in section 5.

II. 2D and 3D-PCA , A BREIF REVIEW

2.1 1D-PCA

2D-PCA (Yang et al., 2004), is performed on the two dimensional image. 2D-PCA is used to overcome the limitations of classical PCA. 2D-PCA is PCA performed on the rows of all the images, which considers the spatial information of the rows of the images but ignores the spatial information of the columns of the images [4]. This technique can be achieved better performance than conventional PCA in image compression when the number of principal component is big. The ortho-normal vector of 2D-PCA is obtained by using SVD. This will cause a major reduction in the size of the reconstructed image. So, it can be said that, SVD and PCA are the common techniques for analysis of data reduction. Based on its name SVD can be applied to a singular matrix. PCA is intimately related to this type of mathematical tool. This tool is used to extract principal components [6]. So, it will describe how to apply PCA in image compression. But the drawback of 2D-PCA is that more coefficients are required to represent the image in the 2D-PCA subspace. Furthermore, it can be noted that 2D-PCA gives good result as compared to 1D-PCA still it is not giving a satisfied result and it cannot support the multidimensional data set or image. Since 2D-PCA is using singular value decomposition and

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it only deals with two dimensional tensors and so, it cannot support multidimensional tensors or data. Also, higher order tensors of the image are neglected due to the linearity of 2D-PCA.

2.2 3D-PCA

The above mentioned difficulties can be avoided by introducing three dimensional PCA (3D-PCA). This type of PCA scheme is then compiled by using the Higher Order SVD (HO-SVD). HO-SVD is the generalization of SVD into higher order tensors. For clarity of 3D-PCA the concept of HO-SVD has to be understood and then the 3-dimensional PCA scheme was formulated [3]. HO-SVD is higher order decomposition with similar properties of SVD. This technique is applied to each dimension of vector but not to the dimension of sample number, so that the principal eigenvector matrices are generated. In this paper HOSVD has been successfully applied to image compression. This technique provides efficient result than the 1D-PCA and also 2D-PCA.

III. KERNEL PCA AND IT'S VARIANTS

This subsection describes the principles of nonlinear type PCA. Various types of kernel techniques are generalized in order to form major types of nonlinear PCA known as Kernel Principal Component Analysis (KPCA). This has been proposed by *Schölkopf et al., 1998*. [5]. KPCA maps the sample data set onto a high- dimensional feature space using the kernel function. Then, KPCA performs a classical linear PCA on the kernel function. In contrast to linear PCA, KPCA is capable of capturing part of the higher-order statistics which are particularly important for encoding image structure [kpca2]. This technique avoids explicitly constructing the covariance matrix in feature space. KPCA algorithm is summarized below in few steps. In the first step the dot product of the matrix A using kernel function is determined

$$A_{ij} = \mathrm{k}\,(x_i, x_j) \qquad (3.1)$$

Then the Eigen vectors from the resultant A matrix is calculated and then normalize with another function. In the final step the test point projection on to Eigen vectors using kernel function is calculated.

3.1 Kernel Based 2D-PCA

Almost similar to KPCA, a nonlinear mapping without explicit function is performed which is known as Kernel based 2D-PCA (2D- KPCA). This type of mapping is performed on each row of all the image matrices [8]. The implementation of kernel based method works more efficiently than other types of above explained methods. Just like KPCA, 2D-KPCA also uses the dot product but of the high dimensional feature vector. This makes the method different from KPCA. The kernel method achieves this type of mapping implicitly. Also, this method incurs very limited computational overhead. Kernel based 2D-PCA is implemented by using KPCA. The kernelization of 2D-PCA will give a great help to model the nonlinear structures in the input data. This method makes use of the nonlinearity of principle components [8]. The mathematical intuition behind the construction of the 2D-KPCA matrix is SVD. Using SVD, the reduction of data is well suited to analyze 2D-KPCA. SVD is also known as a low-rank approximation. Similar to 2D-PCA, 2D-KPCA also does not support multidimensional data. This is the main limitation of 2D-KPCA.

3.2 Kernel Based 3D-PCA

The limitation of 2D-KPCA is eliminated by introducing the concept of Kernel based 3D-PCA. 3D-KPCA effectively extracts the nonlinear features instead of projecting the image on to the subspace. Both 2D-KPCA and 3D-KPCA are belongs to unsupervised feature extraction technique. As discussed earlier in 3D-PCA, the same HO-SVD concept is also used in this part. This HO-SVD method computes the othonormal spaces associated with the different modes of a tensor. In this work HO-SVD is built from several SVDs. As previously discussed HO-SVD also possess strikingly analogous properties than SVD. Now, we came to know that this explained technique provides more prominent result than the rest of the prior PCA based image compression that has been described in the above sections.

IV. EXPERIMENTS AND RESULTS

To be able to carry out the implementation and evaluation of the algorithms we use MATLAB codes. The experiments are performed on the standard gray test 8-bit images like Lena, Cameraman, Pepper, Barbara. The size of all four images is 512×512.The results are discussed and to know the quality of image the peak signal-to-noise ratio (PSNR) is taken as a measure. The PSNR is the ratio between a signal's maximum power and the power of the signal's noise. By using the PSNR values the quality of reconstructed images is best described. Mathematically, PSNR is defined as:

$$PSNR = 10\log_{10}\left[\!\left[N^2 255^2 \Big/ \sum_{i=1}^{N}\sum_{j=1}^{N}(F_{ij} - \widetilde{F_{ij}})^2 \right]\!\right]$$

(4.1)

where 255 is the peak signal value, F_{ij} and $\widetilde{F_{ij}}$ are the pixel intensities for the original and the reconstructed image. Here the image is of size $N \times N$ where $\mathrm{N} = 512$ for each image. All tables and graphs from the experiments are given below.

The original gray images are given here:

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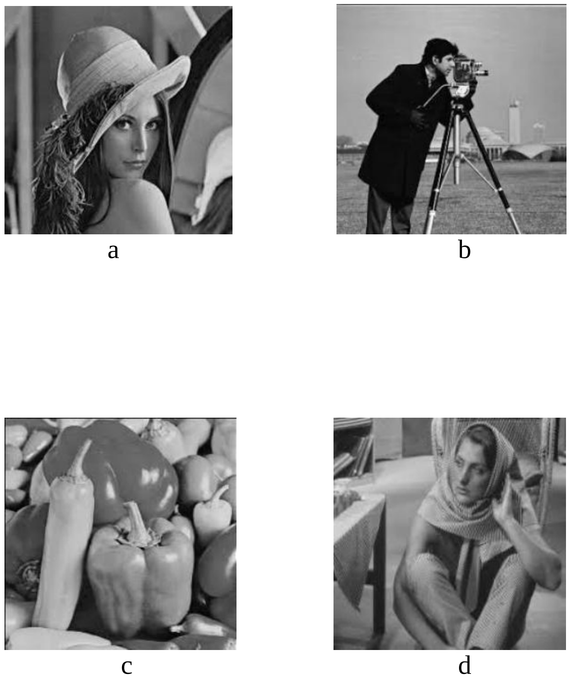

Fig.1. a) Lena, b) Cameraman, c) Pepper, d) Barbara

Tables I-IV compare different PCA algorithms (2D-PCA; 3D-PCA; 2D-KPCA; 3D-KPCA***)*** under consideration, applied to Lena, Cameraman, Pepper and Barbara images. Out of the four images, in all four cases 3D-KPCA algorithm showed highest PSNR values (in db).

TABLE I. PSNR VALUES FOR COMPRESSED IMAGES USING 2D-PCA

PCs	***Lena***	***Camera man***	***Pepper***	***Barbar a***
30	28.54	33.07	27.57	31.32
60	29.23	34.11	28.33	31.93
90	29.87	35.30	29.17	32.58
100	30.08	35.73	29.47	32.81

TABLE II. PSNR VALUES FOR COMPRSSED IMAGES USING 3D-PCA

PCs	***Lena***	***Camera man***	***Pepper***	***Barbar a***
30	43.23	43.34	38.40	43.50
60	46.32	47.43	42.49	47.58
90	51.19	55.38	50.45	55.54
100	53.68	61.40	56.47	61.56

TABLE III. PSNR VALUES COMPRESSED IMAGES USING 2D-KPCA

PCs	***Lena***	***Camera man***	***Pepper***	***Barbar a***
30	66.75	64.14	57.91	64.36
60	72.77	68.16	59.64	68.51
90	85.78	75.87	61.81	76.74
100	90.86	81.48	62.67	83.23

TABLE IV. PSNR VALUES FOR COMPRESSED IMAGES USING 3D-KPCA

PCs	***Lena***	***Camera man***	***Pepper***	***Barbar a***
30	67.42	73.60	89.93	93.96
60	64.52	67.62	71.93	74.58
90	56.98	59.99	63.83	64.41
100	64.32	68.88	73.54	75.63

In tables I-IV, the effect for principal components has been studied. It can be seen that the PSNR values performance improves, on the whole, with the increase in the size of the principal components. Next a graphical variation of these PSNR values is presented. Fig. 4.2 depicts the PSNR values achieved by the four algorithms for the Lena image, Fig. 4.3 for the Cameraman image, Fig. 4.4 for the Pepper image, and Fig. 4.5 for the Barbara image.

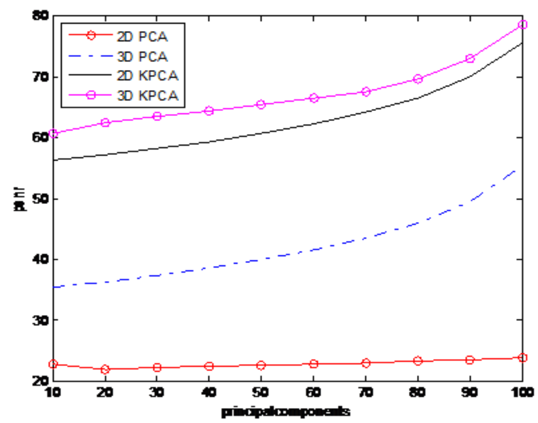

Fig.4.2 The peak signal-to-noise ratio (PSNR) for different PCA algorithms as a function of the principal components for the Lena image.

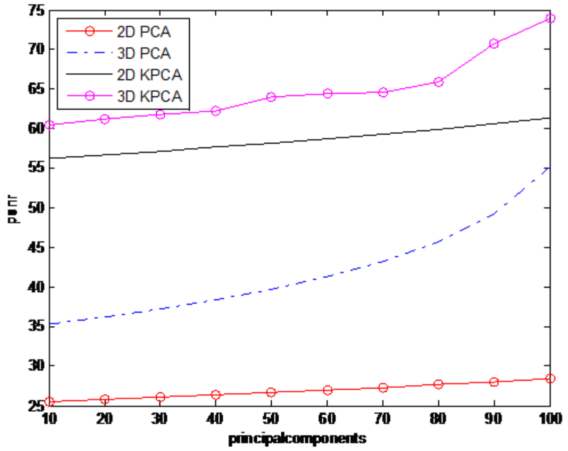

Fig.4.3 The peak signal-to-noise ratio (PSNR) for different PCA algorithms as a function of the principal components for the Cameraman image.

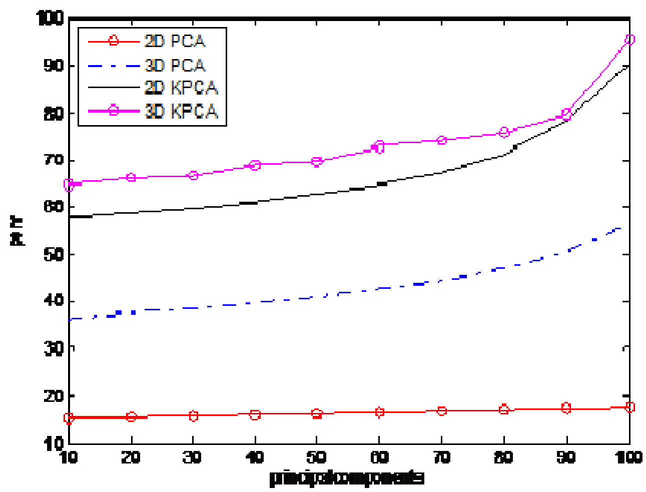

Fig.4.4 The peak signal-to-noise ratio (PSNR) for different PCA algorithms as a function of the principal components for the Pepper image.

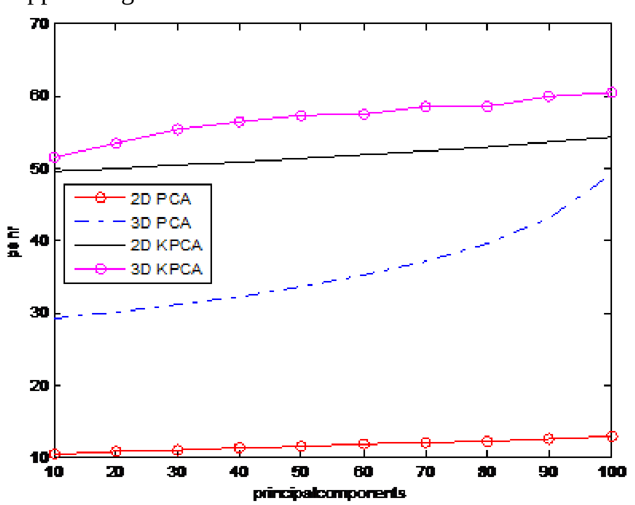

Fig.4.5 The peak signal-to-noise ratio (PSNR) for different PCA algorithms as a function of the principal components for the Barbara image.

V. CONCLUSION

PCA is basically a technique to represent the future vector in the lower dimensionality space. So, by considering all the pixel values of image as the feature vector, we are getting better representation of image. The result obtained by using the different PCA algorithms and KPCA algorithm for image compression are found impressive. By looking at the PSNR values of these algorithms, it is clear that 3D KPCA performs better than all of these algorithms. PSNR values for Cameraman image gives better result in all the linear PCA algorithms whereas these values for Barbara image gives better result in case of KPCA algorithms.

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