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Profit Evaluation of a Two-Unit System with Instructions and Post Repair Inspection

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Abstract- In this paper, we analyse two-unit cold standby system with instructions and post-repair inspection. On the failure of a unit, an expert repairman comes with his assistant and gives instructions to his assistant for repairing the unit. After getting instructions, the assistant repairman immediately starts the repair of the failed unit. When the assistant completes the repair, the inspection is carried out by the expert to see whether the unit repaired by his assistant is perfectly repaired or not. If not, the expert himself repairs it perfectly. Various measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made.

I. Introduction

Two unit standby system has been discussed that on the failure of a unit, an ordinary repairman is called who may or may not be able to repair the unit or sometimes he may damage the unit while repairing it. If he is unable to repair or damages the unit then expert repairman is called immediately. However, there may be situations when on the failure of a unit, the ordinary repairman is given instructions by the expert before starting the repair. This is done to avoid the damages or to minimize the inability of the ordinary repairman. Kumar, Gupta and Taneja (1995, 96) analysed two unit systems introducing the concepts of instruction time and assistant repairman. In these papers, it has been assumed that on the failure of a unit, the assistant repairman is given instruction and he repairs the unit perfectly after getting the instructions. However, it may happen that even after getting instructions, the repair done by the assistant repairman may not be perfect. So, there comes an idea of introducing the post repair inspection which is carried out by the expert to see whether the repair done by the assistant is perfect or not.

The system is analysed by making use of semi-Markov process and regenerative point technique. Various measures of system effectiveness such as mean time to system failure (MTSF), steady state availability, expected busy period of the assistant, expected instruction time, expected repair time of the expert, expected inspection time by the expert, expected number of visits by the expert repairman and expected profit earned by the system are obtained. Graphs are also plotted for a particular case.

Notations

O	operative unit
Cs	cold standby
Fi	expert is giving instructions to the assistant for repairing the failed unit.
FI	instructions are going on from previous state
Fra	failed unit under repair of the assistant repairman.
Fre	failed unit under repair of the expert repairman
Fw	failed unit waiting for repair
Fpri	failed unit is under post repair inspection by the expert
FPRI	post repair inspection is going on from previous state
FRa	repair by assistant is continuing from the previous state.
FRe	repair by expert is continuing from previous state
λ	constant failure rate of operative unit
$g_1(t)$, $G_1(t)$	p.d.f. and c.d.f. of repair time by assistant
$g_2(t)$, $G_2(t)$	p.d.f. and c.d.f. of repair time by expert
$i(t)$, $I(t)$	p.d.f. and c.d.f. of instruction time
$h(t)$, $H(t)$	p.d.f. and c.d.f. of post repair inspection.

II. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The state transition diagram is shown as in Fig. 1. The epochs of entry into states 0, 1, 3, 4, 5, 7, 8 and 10 are regeneration points and so these are regeneratives states, 2, 4, 6, 7, 9, 10 and 11 are down states. The transition probabilities are

$$dQ_{01}(t)=\lambda e^{-\lambda t}dt$$
$$dQ_{12}(t)=\lambda e^{-\lambda t}\ \bar{I}(t)dt$$
$$dQ_{14}^{(2)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]\ i(t)dt$$
$$dQ_{13}(t)=e^{-\lambda t}i(t)dt$$
$$dQ_{35}(t)=e^{-\lambda t}g_1(t)dt$$
$$dQ_{36}(t)=\lambda e^{-\lambda t}\overline{G_1(t)}dt$$
$$dQ_{37}^{(6)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]g_1(t)\ dt$$
$$dQ_{47}(t)=g_1(t)\ dt$$
$$dQ_{50}(t)=p\,e^{-\lambda t}h(t)dt$$
$$dQ_{58}(t)=q\,e^{-\lambda t}\ h(t)dt$$
$$dQ_{59}(t)=\lambda\, e^{-\lambda t}\bar{H}(t)\ dt$$
$$dQ_{51}^{(9)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]p\,h(t)dt$$
$$dQ_{5,10}^{(9)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]q\,h(t)dt$$

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$$dQ_{71}(t) = p e^{-\lambda t} h(t) dt$$
$$dQ_{7,10}(t) = q\, e^{-\lambda t} h(t) dt$$
$$dQ_{80}(t) = e^{-\lambda t} g_2(t) dt$$
$$dQ_{8,11}(t) = \lambda e^{-\lambda t} \overline{G_2(t)} dt$$
$$dQ_{81}^{(11)}(t) = \left[\lambda e^{-\lambda t} © 1\right] g_2(t)\, dt$$
$$dQ_{10,1}(t) = g_2(t)\, dt$$

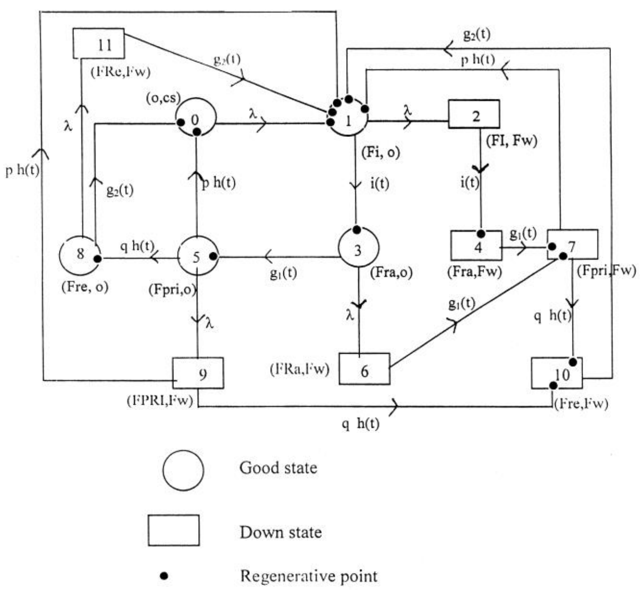

Fig. 1 : State Transition Diagram

The non-zero elements p_{ij} of the transition probability matrix for the system are found out as $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$.
Mean Sojourn times (μ_i) in state i are as follows:

$$\mu_0 = 1/\lambda, \qquad \mu_1 = \frac{1 - i^*(\lambda)}{\lambda}$$
$$\mu_3 = \frac{1 - g_i^*(\lambda)}{\lambda}, \qquad \mu_4 = \int_0^\infty \overline{G_1(t)}\, dt$$
$$\mu_5 = \frac{1 - h^*(\lambda)}{\lambda}, \qquad \mu_8 = \frac{1 - g_2^*(\lambda)}{\lambda}$$
$$\mu_7 = \int_0^\infty \overline{H}(t) dt$$
$$\mu_{10} = \int_0^\infty \overline{G_2(t)}\, dt$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into the state is mathematically stated as

$$m_{ij} = \int t\, dQ_{ij}(t) = -\lim_{s \to 0} q_{ij}^{*'}(s)$$

Thus,

$$m_{01} = \mu_0$$
$$m_{12} + m_{13} = \mu_1$$
$$m_{14}^{(2)} + m_{13} = k_1 = \int_0^\infty \bar{I}(t) dt$$
$$m_{35} + m_{36} = \mu_3$$
$$m_{35} + m_{36}^{(7)} = m_{47} = \mu_4$$
$$m_{50} + m_{58} + m_{59} = \mu_5$$
$$m_{50} + m_{58} + m_{5,10}^{(9)} + m_{51}^{(9)} = m_{71} + m_{7,10} = \mu_7$$
$$m_{80} + m_{8,11} = \mu_8$$
$$m_{80} + m_{81}^{(11)} = m_{10,1} = \mu_{10}$$

III. MEAN TIME TO SYSTEM FAILURE

To determine the MTSF of the system, we regard the failed states of the system absorbing. By probabilistic arguments, we have:

$$\phi_0(t) = Q_{01}(t)(s)\ \ \phi_1(t)$$
$$\phi_1(t) = Q_{12}(t) + Q_{13}(t)(s)\phi_3(t)$$
$$\phi_3(t) = Q_{35}(t)(s)\phi_5(t) + Q_{36}(t)$$
$$\phi_5(t) = Q_{50}(t)(s)\phi_0(t) + Q_{58}(t)(s)\phi_8(t) + Q_{59}(t)$$
$$\phi_8(t) = Q_{80}(t)(s)\phi_0(t) + Q_{8,11}(t)$$

Taking Laplace-Stieltjes transform of these equations and solving them for $\phi_0^{**}(s)$.

Now the MTSF, given that the system started at the beginning of state 0, is

$$T = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 + p_{13}\mu_3 + p_{13}p_{35}\mu_5 + p_{13}p_{35}p_{58}\ \mu_8$$

and

$$D = 1 - p_{13}p_{35}\left(p_{50} + p_{58}p_{80}\right)$$

IV. AVAILABILITY ANALYSIS

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$
$$A_1(t) = M_1(t) + q_{13}(t) © A_3(t) + q_{14}^{(2)}(t) © A_4(t)$$
$$A_3(t) = M_3(t) + q_{35}(t) © A_5(t) + q_{37}^{(6)}(t) © A_7(t)$$
$$A_4(t) = q_{47}(t) © A_7(t)$$
$$A_5(t) = M_5(t) + q_{50}(t) © A_0(t) + q_{58}(t) © A_8(t)$$
$$+ q_{5,10}^{(9)}(t) © A_{10}(t) + q_{51}^{(9)}(t) © A_1(t)$$
$$A_7(t) = q_{71}(t) © A_1(t) + q_{7,10}(t) © A_{10}(t)$$
$$A_8(t) = M_8(t) + q_{80}(t) © A_0(t) + q_{81}^{(11)}(t) © A_1(t)$$
$$A_{10}(t) = q_{10,1}(t) © A_1(t)$$

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where

$M_0(t) = e^{-\lambda t}$; $\quad M_1(t) = e^{-\lambda t}\overline{I(t)}$

$M_3(t) = e^{-\lambda t}\overline{G_1(t)}$; $\quad M_5(t) = e^{-\lambda t}\overline{H(t)}$

$M_8(t) = e^{-\lambda t}\overline{G_2(t)}$

Taking Laplace transform of these equations and solving them for $A_0^{**}(s)$.

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0}\left(sA_0^*(s)\right) = N_1 / D_1$$

where

$$N_1 = \mu_0\left[1 - p_{13}p_{35}\left(p_{58}p_{81}^{(11)} + p_{51}^{(9)} + p_{5,10}^{(9)}\right) - p_{14} - p_{13}p_{37}^{(6)}\right] + \mu_1 + \mu_3 p_{13} + \mu_3 p_{13}p_{35} + \mu_8 p_{13}p_{35}p_{58}$$

and

$$D_1 = k_1 + \mu_0 p_{13}p_{35}p_{58}p_{80} + \mu_4 + \mu_7 + \mu_{10}\left[p_{13}p_{35}\left(p_{58} + p_{5,10}^{(9)}\right) + p_{13}p_{37}^{(6)} + p_{14}^{(2)}\right]$$

In steady state, the total fraction of the time for which the assistant is busy, is given by

$$B_0 = \lim_{s\to 0} sB_0^{(a)*}(s) = \frac{N_2}{D_1}$$

where

$N_2 = \mu_4$.

and D_1 is already specified.

In a steady state, the number of visits per unit time of expert is given by

$$V_0 = \lim_{s\to 0} sV_0^{**}(s) = \frac{N_3}{D_1}$$

where

$$N_3 = 1 - p_{13}\left[p_{35}\left(p_{58}p_{81}^{(11)} + p_{51}^{(9)} + p_{5,10}^{(9)}\right) + p_{37}^{(6)} - 1\right]$$

and D_1 is already specified.

In a steady state, the total fraction of time for which the expert is busy in repairing, is given by

$$BR_0 = \lim_{s\to 0} sBR_0^*(s) = \frac{N_4}{D_1}$$

where

$$N_4 = \mu_{10}\left[p_{13}p_{35}\left(p_{58} + p_{5,10}^{(9)}\right) + p_{7,10}\left(p_{14}^{(2)} + p_{13}p_{37}^{(6)}\right)\right]$$

and D_1 is already specified.

In a steady state, the total fraction of time, for which the expert is busy in giving instructions, is given by

$$BI_0 = \lim_{s\to 0} s\,BI_0^*(s) = \frac{N_5}{D_1}$$

where

$N_5 = k_1$

and D_1 is already specified.

In the steady state, the total fraction of time for which the expert is busy in inspection, is given by

$$BT_0 = \lim_{s\to 0} s\,BT_0^*(s) = \frac{N_6}{D_1}$$

Where $N_6 = \mu_7$.

V. COST-BENEFIT ANALYSIS

The total expected profit, in steady state, is given by

$$P = C_0A_0 - C_1B_0^{(a)} - C_2BR_0 - C_3BI_0 - C_4BT_0 - C_5V_0$$

where

C_0 = the revenue per unit up time of the system

C_1 = cost per unit time for which the assistant repairman is busy in repairing the units.

C_2 = Cost per unit time for which the expert repairman is busy in repairing the units.

C_3 = cost per unit time for which the expert is busy is giving instructions to the assistant.

C_4 = cost per unit time for which the expert is busy in doing the post repair inspection.

C_5 = cost per visit by the expert repairman.

VI. GRAPHICAL STUDY

Let us assume that the repair times, instruction and inspection time are exponentially distributed i.e.,

$$i(t) = \alpha e^{-\alpha t},\ h(t) = \beta e^{-\beta t},\ g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t}$$

The behaviour of the MTSF and the profit with respect to repair rate (α_1) for different values of instruction rate (α) is shown as in fig 2 and 3 respectively. From the graphs, we see that the MTSF and the profit increase with the increase in repair rate (α_1) and also with the increase in instruction rate.

Figs. 4 and 5 study the behaviour of the MTSF and the profit respectively with respect to failure rate (λ) for different values of inspection rate (β). From the graphs, it is interpreted that both the MTSF and the profit decreases as failure rate (λ) increases. However, their values increase with increase in the values of inspection rate (β).

The graphs for the MTSF and the profit with respect to probability that the assistant repairman successfully completes the repair (p) for different values of instruction rate (α) are shown as in Figs. 6 and 7 respectively. We conclude from the graphs that as the values of p and α increase, both the MTSF and the profit increase.

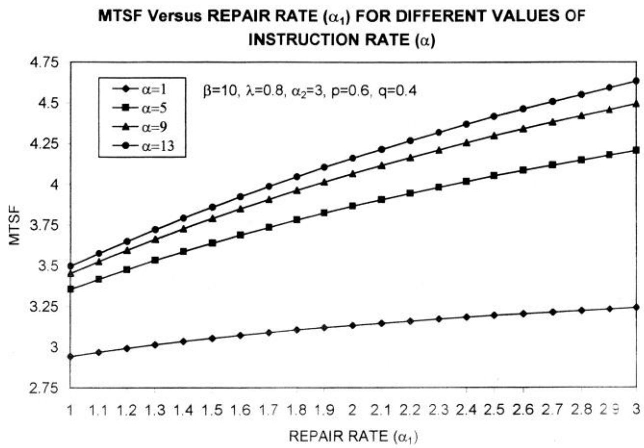

Fig. 2

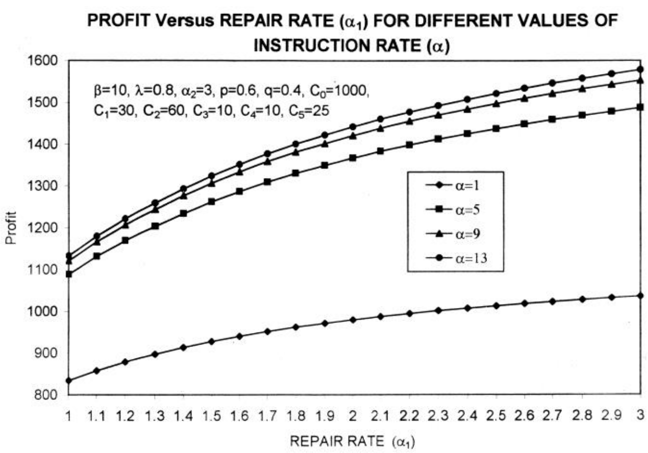

Fig. 3

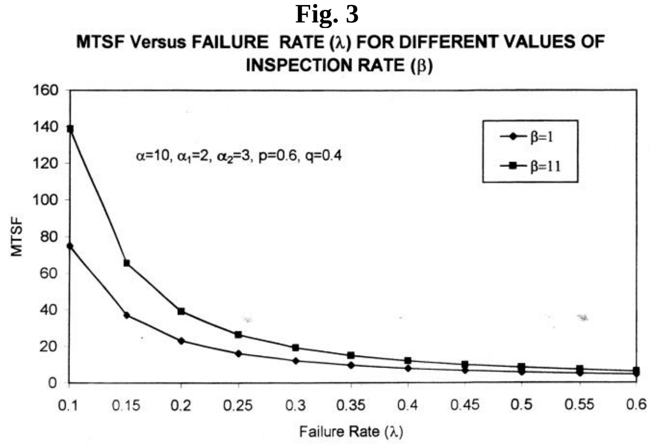

Fig. 4

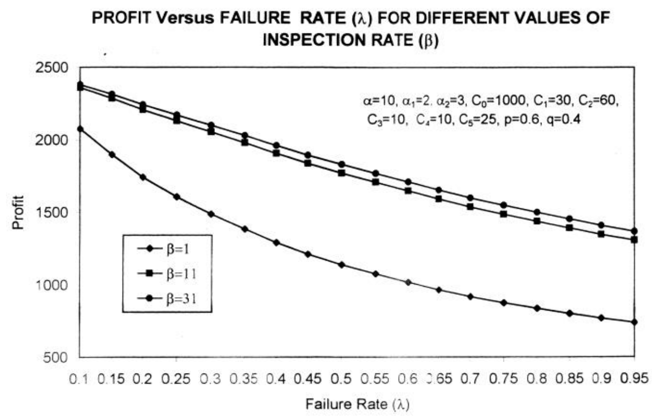

Fig. 5

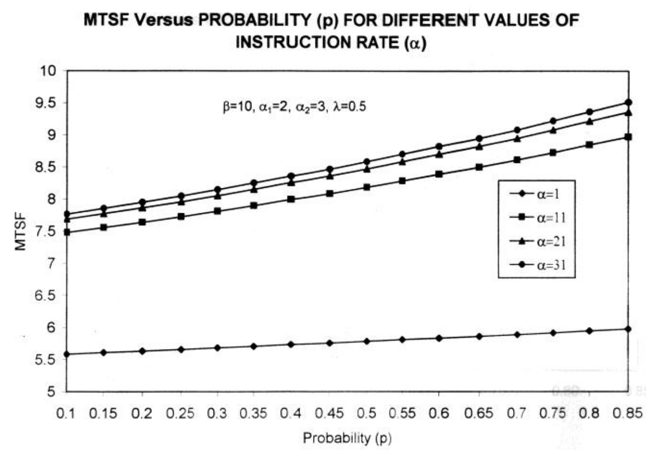

Fig. 6

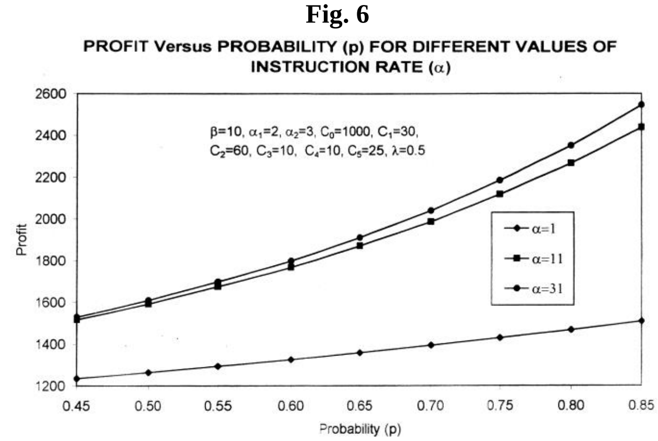

Fig. 7

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