

On $\pi g(\alpha g)^*$ – continuous maps and $\pi g(\alpha g)^*$ - irresolute maps in Topological Spaces

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Abstract : In this paper, we have introduced the concept of continuous, irresolute and homeomorphism maps of $\pi g(\alpha g)^*$ closed set. Some of the fundamental properties of this set are studied. And their application also given namely, $\pi g(\alpha g)^* - T_{1/2}$ - space.

Keywords : $\pi g(\alpha g)^*$ - closed set, $\pi g(\alpha g)^*$ - continuous map, $\pi g(\alpha g)^*$ -irresolute map, $\pi g(\alpha g)^* - T_{1/2}$ - space

1.Introduction

Levine[5] introduced the class of g-closed sets, a super class of closed sets in 1970. Dontchev and Noiri [19] have introduced the concept of πg -closed sets and studied their most fundamental properties in topological spaces. Also, Ekici and Noiri [21] have introduced a generalization of πg -closed sets and πg -open sets. Recently , a new class of $\pi g(\alpha g)^*$ -closed sets in topological spaces introduced and studied by R.Savithiri , A.Manonmani and M.Anandhi [29]. In this paper , we have made a study on $\pi g(\alpha g)^*$ - continuous map, $\pi g(\alpha g)^*$ -irresolute map and $\pi g(\alpha g)^*$ -homeomorphism. Also, Applications of $\pi g(\alpha g)^*$ -closed sets are analyzed.

2.Preliminaries

For a subset H of a space (X, τ) , $cl(H)$ and $int(H)$ denote the closure and the interior of H respectively. The class of all closed subsets of a space (X, τ) is denoted by $C(X, \tau)$. The smallest closed (resp. α -closed) set containing a subset H of (X, τ) is called the closure (resp. α -closure) of H and is denoted by $cl(H)$ (resp. $\alpha cl(H)$).

Definition 2.1 : 1) A π open set [21] of X is a finite union of all r-open sets in (X, τ) .

2) A subset H of a space X is called α -generalized closed (briefly αg -closed) [13] if $\alpha cl(H) \subseteq U$ whenever $H \subseteq U$ and U is open in X.

3) A subset H of a space X is called π - generalized closed set [21] (briefly πg -closed) if $cl(H) \subseteq U$ whenever $H \subseteq U$ and U is π -open in (X, τ) .

4) A subset H of a space X is called **π -generalized- $(\alpha g)^*$ closed set [29] (briefly $\pi g(\alpha g)^*$ - closed set)** if $\alpha g cl(H) \subseteq U$, whenever $H \subseteq U$ and U is π open in X.

Diagram-I

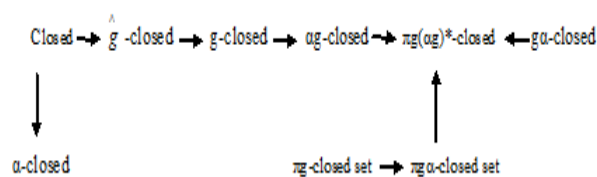


Diagram-I is obvious (see related papers).

Remark 2.2 :

$\pi g(\alpha g)^*$ -closed set is independent with the following closed sets:gp-closed set, rg-closed set, s-closed set, wg-closed set,w πg -closed set, b*-closed set,b-closed set, gs-closed set, gs-closed set, gb-closed set and πgp -closed set .

3. On $\pi g(\alpha g)^*$ - continuous map.

Definition 3.1:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called

- 1) continuous[5] if inverse image of every closed set of (Y, τ_2) is a closed set of (X, τ_1) .
- 2) g-continuous[5] if the inverse image of every closed set of (Y, τ_2) is g- closed set of (X, τ_1) .
- 3) \hat{g} -continuous[4] if the inverse image of every closed set of (Y, τ_2) is \hat{g} - closed set of (X, τ_1) .
- 4) αg -continuous[13] if the inverse image of every closed set of (Y, τ_2) is αg - closed set of (X, τ_1) .
- 5) $g\alpha$ -continuous[13] if the inverse image of every closed set of (Y, τ_2) is $g\alpha$ - closed set of (X, τ_1) .
- 6) πg -continuous[21] if the inverse image of every closed set of (Y, τ_2) is πg - closed set of (X, τ_1) .
- 7) $\pi g\alpha$ -continuous [25] if the inverse image of every closed set of (Y, τ_2) is $\pi g\alpha$ - closed of (X, τ_1) .
- 8) s-continuous [6] if the inverse image of every closed set of (Y, τ_2) is s- closed set of (X, τ_1) .
- 9) gp-continuous[23] if the inverse image of every closed set of (Y, τ_2) is gp- closed set of (X, τ_1) .
- 10) rg-continuous[28] if the inverse image of every closed set of (Y, τ_2) is rg- closed set of (X, τ_1) .
- 11) wg-continuous[15]if the inverse image of every closed set of (Y, τ_2) is wg-closed set of (X, τ_1) .
- 12) w πg -continuous[15]if the inverse image of every closed set of (Y, τ_2) is w πg -closed of (X, τ_1) .

- 13) b*-continuous[26]if the inverse image of every closed set of (Y, τ_2) is b*- closed set of (X, τ_1) .
- 14) b-continuous[16] if the inverse image of every closed set of (Y, τ_2) is b- closed set of (X, τ_1) .
- 15) gs-continuous[29]if the inverse image of every closed set of (Y, τ_2) is gs- closed set of (X, τ_1) .
- 16) gb-continuous[27] if the inverse image of every closed set of (Y, τ_2) is gb-closed set of (X, τ_1) .
- 17) πgp -continuous[24] if the inverse image of every closed set of (Y, τ_2) is πgp - closed of (X, τ_1) .
- 18) α -continuous [11] if the inverse image of every closed set of (Y, τ_2) is α - closed set of (X, τ_1) .

Definition 3.2:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a $\pi g(\alpha g)^*$ -continuous if the inverse image of every closed set of (Y, τ_2) is $\pi g(\alpha g)^*$ - closed set of (X, τ_1) .

Theorem 3.3:

Every continuous map, g-continuous map, α -continuous map and αg -continuous map is $\pi g(\alpha g)^*$ -continuous.

Proof:

- (i) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since closed $\rightarrow \pi g(\alpha g)^*$ -closed. Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .
- (ii) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be g-continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is g-closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since g-closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .
- (iii) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be α -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is α -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since α -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .
- (iv) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be αg -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is αg -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) ,

since αg -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .

The converse of the above theorem need not be true from the following example.

Example 3.4:

Take $X = Y = \{a,b,c\}$ and $\tau_1 = \{X, \Phi, \{a\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{b,c\}\}$. Define $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ as $\theta(a) = a$, $\theta(b) = b$, $\theta(c) = c$. Here inverse image of all τ_2^c are $\pi g(\alpha g)^*$ -closed of (X, τ_1) but not closed, g -closed, α -closed and αg -closed of (X, τ_1) . This implies converse not true.

Theorem 3.5:

Every \hat{g} -continuous map and $g\alpha$ -continuous map is $\pi g(\alpha g)^*$ -continuous.

Proof:

(i) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be \hat{g} -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is \hat{g} -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since \hat{g} -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .

(ii) Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be $g\alpha$ -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is $g\alpha$ -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since $g\alpha$ -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .

The converse of the above theorem need not be true from the following example.

Example 3.6:

Take $X = Y = \{a,b,c\}$ and $\tau_1 = \{X, \Phi, \{c\}, \{b,c\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{b,c\}\}$. Define $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ as $\theta(a) = a$, $\theta(b) = b$, $\theta(c) = c$. Here inverse image of all τ_2^c are $\pi g(\alpha g)^*$ -closed of (X, τ_1) but not $g\alpha$ -closed and \hat{g} -closed of (X, τ_1) .

This implies converse not true.

Theorem 3.7:

Every πg -continuous map is $\pi g(\alpha g)^*$ -continuous.

Proof:

Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be πg -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is πg -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) ,

since πg -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .

The converse of the above theorem need not be true from the following example.

Example 3.8:

Take $X = Y = \{a,b,c,d\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{a,b,d\}\}$. Define $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ as $\theta(a) = a$, $\theta(b) = b$, $\theta(c) = c$, $\theta(d) = d$. Here inverse image of all τ_2^c are $\pi g(\alpha g)^*$ -closed of (X, τ_1) but not πg -closed of (X, τ_1) . This implies converse not true.

Theorem 3.9:

Every $\pi g\alpha$ -continuous map is $\pi g(\alpha g)^*$ -continuous but not converse.

Proof:

Take $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ be $\pi g\alpha$ -continuous map. Let W be closed set of (Y, τ_2) then inverse image of W is $\pi g\alpha$ -closed of (X, τ_1) . Inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , since $\pi g\alpha$ -closed $\rightarrow \pi g(\alpha g)^*$ -closed . Hence θ is $\pi g(\alpha g)^*$ -continuous of (X, τ_1) .

Remark 3.10:

The composition of two $\pi g(\alpha g)^*$ -continuous map is need not be a $\pi g(\alpha g)^*$ -continuous map.

Example:3.11

Take $X = Y = Z = \{a,b,c\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{a,b\}\}$ and $\tau_3 = \{Z, \Phi, \{a\}, \{b,c\}\}$. Define $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ and $h : (Y, \tau_2) \rightarrow (Z, \tau_3)$ be an identity maps. Let θ and h be a $\pi g(\alpha g)^*$ -continuous maps. . But $(h \circ \theta)^{-1}(\{a\}) = \theta^{-1}(h^{-1}(\{a\})) = \{a\}$ is not $\pi g(\alpha g)^*$ -closed of (X, τ_1) . Hence $h \circ \theta$ is not $\pi g(\alpha g)^*$ -continuous.

Theorem 3.12:

A map $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ is $\pi g(\alpha g)^*$ -continuous and $h : (Y, \tau_2) \rightarrow (Z, \tau_3)$ is continuous , then $h \circ \theta : (X, \tau_1) \rightarrow (Z, \tau_3)$ is $\pi g(\alpha g)^*$ -continuous.

Proof:

Take W be any closed set in (Z, τ_3) and so h^{-1} of W is closed of (Y, τ_2) , Since h is continuous. $(h \circ \theta)^{-1}(W) = \theta^{-1}(h^{-1}(W))$ is $\pi g(\alpha g)^*$ -closed (X, τ_1) , Since θ is $\pi g(\alpha g)^*$ -continuous . Hence $h \circ \theta$ is $\pi g(\alpha g)^*$ -continuous.

Theorem 3.13:

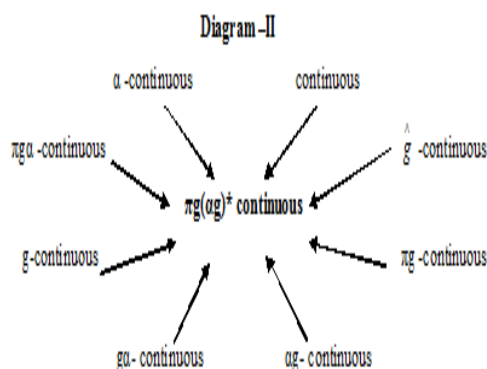
A map $\theta : (X, \tau_1) \rightarrow (Y, \tau_2)$ is $\pi g(\alpha g)^*$ -continuous if and only if inverse image of every open set in

(Y, τ_2) is $\pi g(\alpha g)^*$ - open set in (X, τ_1) .

Proof:

Take $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ be $\pi g(\alpha g)^*$ -continuous map and W be open set in (Y, τ_2) then W^c is closed in (Y, τ_2) . Inverse image of W^c is $\pi g(\alpha g)^*$ -closed of (X, τ_1) since θ is $\pi g(\alpha g)^*$ -continuous. But $\theta^{-1}(W^c) = (\theta^{-1}(W))^c$. Hence inverse image of W is $\pi g(\alpha g)^*$ -open of (X, τ_1) .

Conversely, Assume, For every open set W of (Y, τ_2) , inverse image of W is $\pi g(\alpha g)^*$ -open of (X, τ_1) . If W of (Y, τ_2) be a closed set, then W^c of (Y, τ_2) be a open set. By assumption, inverse image of W^c of (X, τ_1) be a $\pi g(\alpha g)^*$ -open set of (X, τ_1) . But $\theta^{-1}(W^c) = (\theta^{-1}(W))^c$. Hence inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) . This implies θ is $\pi g(\alpha g)^*$ -continuous



4. On $\pi g(\alpha g)^*$ - open map.

Definition 4.1:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a $\pi g(\alpha g)^*$ -open map if the image of every open set of (X, τ_1) is $\pi g(\alpha g)^*$ - open set of (Y, τ_2) .

Theorem 4.2:

Every open map, g -open map, \hat{g} -open map, α -open map, αg -open map, $g\alpha$ -open map, πg -open map and $\pi g\alpha$ -open map is $\pi g(\alpha g)^*$ -open map.

Proof:

Follows from the fact that “Every open set, g -open set, \hat{g} -open set, α -open set, αg -open set, $g\alpha$ -open set, πg -open set and $\pi g\alpha$ -open set is $\pi g(\alpha g)^*$ -open set”.

The converse of the above theorem need not be true from the following example.

Example 4.3:

Take $X = Y = \{a,b,c\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b,c\}\}$, $\tau_2 = \{Y, \Phi, \{a\}\}$. Define $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ as $\theta(a) = a$, $\theta(b) = b$, $\theta(c) = c$. Here $W = \{b,c\}$ be a open set of (X, τ_1) . But image of W is $\pi g(\alpha g)^*$ -open set of (Y, τ_2) but not open set, g -open set, α -open set, αg -open set of (Y, τ_2) . This implies converse of above theorem not true.

5. On $\pi g(\alpha g)^*$ - irresolute map.

Definition 5.1:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a $\pi g(\alpha g)^*$ -irresolute if the inverse image of every $\pi g(\alpha g)^*$ -closed set of (Y, τ_2) is $\pi g(\alpha g)^*$ - closed set of (X, τ_1) .

Theorem 5.2:

Every $\pi g(\alpha g)^*$ -irresolute map is $\pi g(\alpha g)^*$ -continuous map.

Proof:

Take a map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ be $\pi g(\alpha g)^*$ -irresolute map. Let W be closed set of (Y, τ_2) then W be a $\pi g(\alpha g)^*$ -closed set of (Y, τ_2) , since closed $\rightarrow \pi g(\alpha g)^*$ -closed. But inverse image of W is $\pi g(\alpha g)^*$ -closed of (X, τ_1) . Hence θ is $\pi g(\alpha g)^*$ -continuous.

The converse of the above theorem need not be true from the following example.

Example 5.3:

Take $X = Y = \{a,b,c\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{a,b\}\}$. Define $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ as $\theta(a) = a$, $\theta(b) = b$, $\theta(c) = c$, $\theta(d) = d$. Here inverse image of all τ_2^c are of $\pi g(\alpha g)^*$ -closed of (X, τ_1) so θ is $\pi g(\alpha g)^*$ -continuous. But inverse image of all $\pi g(\alpha g)^*$ -closed of (Y, τ_2) are not $\pi g(\alpha g)^*$ -closed of (X, τ_1) so θ is not $\pi g(\alpha g)^*$ -irresolute map.

Theorem 5.4:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is $\pi g(\alpha g)^*$ -irresolute and $h: (Y, \tau_2) \rightarrow (Z, \tau_3)$ is $\pi g(\alpha g)^*$ -continuous, then $h \circ \theta: (X, \tau_1) \rightarrow (Z, \tau_3)$ is $\pi g(\alpha g)^*$ -continuous.

Proof:

Take W be any closed set of (Z, τ_3) and so h^{-1} of W is $\pi g(\alpha g)^*$ -closed of (Y, τ_2) , Since h is $\pi g(\alpha g)^*$ -continuous. $(h \circ \theta)^{-1}(W) = \theta^{-1}(h^{-1}(W))$ is $\pi g(\alpha g)^*$ -closed of (X, τ_1) , Since θ is $\pi g(\alpha g)^*$ -irresolute. Hence $h \circ \theta$ is $\pi g(\alpha g)^*$ -continuous.

Theorem 5.5:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is $\pi g(\alpha g)^*$ -irresolute and $h: (Y, \tau_2) \rightarrow (Z, \tau_3)$ is $\pi g(\alpha g)^*$ -irresolute, then $h \circ \theta: (X, \tau_1) \rightarrow (Z, \tau_3)$ is $\pi g(\alpha g)^*$ -irresolute.

Proof:

Take W be any $\pi g(\alpha g)^*$ -closed set of (Z, τ_3) and so h^{-1} of W is $\pi g(\alpha g)^*$ -closed of (Y, τ_2) . Since h is $\pi g(\alpha g)^*$ -irresolute. $(h \circ \theta)^{-1}(W) = \theta^{-1}(h^{-1}(W))$ is $\pi g(\alpha g)^*$ -closed of (X, τ_1) . Since θ is $\pi g(\alpha g)^*$ -irresolute. Hence $h \circ \theta$ is $\pi g(\alpha g)^*$ -irresolute.

Theorem 5.6:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is $\pi g(\alpha g)^*$ -irresolute if and only if inverse image of every $\pi g(\alpha g)^*$ -open set in (Y, τ_2) is $\pi g(\alpha g)^*$ -open set in (X, τ_1) .

Proof:

Similar to Theorem 3.13.

6. On $\pi g(\alpha g)^*$ -homeomorphisms.

Definition 6.1:

A bijective map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a $\pi g(\alpha g)^*$ -homeomorphism if a map is both $\pi g(\alpha g)^*$ -continuous and $\pi g(\alpha g)^*$ -open.

Remark 6.2:

For a bijective map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$, the following statement are equivalent:

- (i) θ^{-1} is $\pi g(\alpha g)^*$ -continuous, (ii) θ is $\pi g(\alpha g)^*$ -open,
- (iii) θ is $\pi g(\alpha g)^*$ -closed.

Theorem 6.3:

Every homeomorphism is a $\pi g(\alpha g)^*$ -homeomorphism.

Proof:

Follows from the fact that “Every continuous map is $\pi g(\alpha g)^*$ -continuous map and Every open map is $\pi g(\alpha g)^*$ -open map”.

The converse of the above theorem need not be true from the following example.

Example 6.4:

Take $X = Y = \{a, b, c\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{a, b\}\}$.

Define $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ as bijective map. Now θ is $\pi g(\alpha g)^*$ -homeomorphism but not a homeomorphism, Since $\theta(\{b\}) = \{b\}$ is not in open set of (Y, τ_2) .

Remark 6.5:

The composition of two $\pi g(\alpha g)^*$ -homeomorphism map is need not be a $\pi g(\alpha g)^*$ -homeomorphism map.

Example 6.6:

Take $X = Y = Z = \{a, b, c\}$ and $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{Y, \Phi, \{a\}, \{a, b\}\}$ and $\tau_3 = \{Y, \Phi, \{a\}, \{b, c\}\}$. Define $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ and $h: (Y, \tau_2) \rightarrow (Z, \tau_3)$ be an identity maps. Let θ and h be a $\pi g(\alpha g)^*$ -homeomorphism maps. But $(h \circ \theta)^{-1}(\{a\}) = \theta^{-1}(h^{-1}(\{a\})) = \{a\}$ is not $\pi g(\alpha g)^*$ -closed of (X, τ_1) . Hence $h \circ \theta$ is not $\pi g(\alpha g)^*$ -homeomorphism, Since $h \circ \theta$ is not a $\pi g(\alpha g)^*$ -continuous map.

Theorem 6.7:

If map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ is bijective $\pi g(\alpha g)^*$ -continuous map, then following statement are equivalent:

- (i) θ is $\pi g(\alpha g)^*$ -homeomorphism, (ii) θ is $\pi g(\alpha g)^*$ -open,
- (iii) θ is $\pi g(\alpha g)^*$ -closed.

Proof:

Follows from Remark 6.2.

7. Applications of $\pi g(\alpha g)^*$ -closed set.

Definition 7.1:

- 1) A topological space (X, τ) is called $T_{1/2}$ -space [5] if every g -closed in (X, τ) is closed in (X, τ) .
- 2) A topological space (X, τ) is called T_b -space [14] if every g_s -closed in (X, τ) is closed in (X, τ) .
- 3) A topological space (X, τ) is called ${}_a T_b$ -space [17] if every αg -closed in (X, τ) is closed in (X, τ) .

Definition 7.2:

A topological space (X, τ) is called $\pi g(\alpha g)^*$ - $T_{1/2}$ -space if every $\pi g(\alpha g)^*$ -closed set of (X, τ) is closed of (X, τ) .

Theorem 7.3:

Every $\pi g(\alpha g)^*$ - $T_{1/2}$ -space is a $T_{1/2}$ -space.

Proof:

Assume that (X, τ) is a $\pi g(\alpha g)^*$ - $T_{1/2}$ -space. Let H be a g -closed set. But every g -closed set is a $\pi g(\alpha g)^*$ -closed. By assumption, (X, τ) is a $T_{1/2}$ -space.

Theorem 7.4:

Every $\pi g(\alpha g)^*$ - $T_{1/2}$ -space is a ${}_a T_b$ -space.

Proof:

Assume that (X, τ) is a $\pi g(\alpha g)^*$ - $T_{1/2}$ -space. Let H be a αg -closed set. But every αg -closed set is a $\pi g(\alpha g)^*$ -closed. By assumption, (X, τ) is a ${}_a T_b$ -space.

Remark 7.5:

$\pi g(\alpha g)^*$ - $T_{1/2}$ -space and T_b -space are independent spaces

Theorem 7.6:

A space (X, τ) is a $\pi g(\alpha g)^* - T_{1/2}$ - space if and only if every singleton of X is either π -closed set or αg -open.

Proof :

Assume that (X, τ) is a $\pi g(\alpha g)^* - T_{1/2}$ - space. Let y be an element in X and $\{y\}$ is not in π -closed, then $X - \{y\}$ is not in π -open and then $X - \{y\}$ is $\pi g(\alpha g)^* -$ closed. By assumption , $X - \{y\}$ is αg -closed. Hence $\{y\}$ is αg -open. The converse is similar.

Theorem 7.7:

A map $\theta: (X, \tau_1) \rightarrow (Y, \tau_2)$ and $h: (Y, \tau_2) \rightarrow (Z, \tau_3)$ be two maps and if θ is αg -irresolute and h is a $\pi g(\alpha g)^* -$ continuous and Y is a $\pi g(\alpha g)^* - T_{1/2}$ - space, then $h \circ \theta: (X, \tau_1) \rightarrow (Z, \tau_3)$ is αg -continuous.

Proof :

Take W be any closed set in (Z, τ_3) . Here (Y, τ_2) is a $\pi g(\alpha g)^* - T_{1/2}$ - space and h^{-1} of W is $\pi g(\alpha g)^* -$ closed of (Y, τ_2) , Since h is $\pi g(\alpha g)^* -$ continuous. But $(h \circ \theta)^{-1}(W) = \theta^{-1}(h^{-1}(W))$ is αg -closed (X, τ_1) , Since θ is αg - irresolute. Hence $h \circ \theta$ is αg -continuous.

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