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Ratio Estimators in two Stage Sampling Using Auxiliary Information

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Abstract- In this paper certain ratio estimators in two-stage sampling set up with unequal first stage units, using information on two auxiliary variables (x and z) are considered and their efficiencies are compared with estimator without using second auxiliary variables z.

Index Terms- Ratio Estimator, auxiliary information, two stage sampling

AMS Classification: 62D05.

I. Introduction

In sample surveys using multi-stage sampling the survey practitioner comes across more than one auxiliary variable either positively or negatively correlated with main variable (y) under study at different stages. Under these circumstances we consider in the following sections certain estimates of the population mean of the study variable (y) in two-stage sampling setup with unequal first stage units, using information on two auxiliary variables (x and z) which may be available at the first stage or at the second stage or at both the stages.

In the following sections we make use of first stage information on second auxiliary variable z to improve estimators formed with the help of first stage or second stage information at both the stage on the main auxiliary variable x. Further, z may be either positively or negatively correlated with x.

II. Sampling method, definitions and Notations

Consider a finite population U partitioned into N first stage units (fsu) denoted by $U_1, U_2, \ldots, U_N$. Let M_i be the number of second stage units in U_i (i = 1, 2, …, N). Define $M = \sum_{i=1}^{N} M_i$ and $\bar{M} = \frac{1}{N}\sum_{i=1}^{N} M_i$. Let y_{ij}, x_{ij} and z_{ij} denoted values of the study variable y and the auxiliary variable x, z respectively for the j^{th}ssu of U_i, (j = 1, 2, …, M_i, i = 1, 2, …, N).

Define $\bar{Y}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} y_{ij}$, $\bar{X}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} x_{ij}$, and $\bar{Z}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} z_{ij}$ (i = 1, 2, …, N)

The population mean of y, x, z are respectively

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} u_i \bar{Y}_i$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} u_i \bar{X}_i$$

$$\bar{Z} = \frac{1}{N}\sum_{i=1}^{N} u_i \bar{Z}_i, \quad \text{where} \quad u_i = \frac{M_i}{\bar{M}}$$

Further, define $R_1 = \frac{\bar{Y}}{\bar{X}}$, $R_2 = \frac{\bar{Y}}{\bar{Z}}$, $R_1 R_2 = \frac{\bar{Y}^2}{\bar{X}\,\bar{Z}}$ and $R_i = \frac{\bar{Y}_i}{\bar{X}_i}$ (i = 1, 2, …, N)

$$S'^2_{by} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i \bar{Y}_i - \bar{Y})^2$$

$$S'^2_{bx} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i \bar{X}_i - \bar{X})^2$$

$$S'^2_{bz} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i\bar{Z}_i - \bar{Z})^2$$

$$S'_{bxy} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i\bar{Y}_i - \bar{Y})\left(u_i\bar{X}_i - \bar{X}\right)$$

$$S'_{bxz} = \frac{1}{N-1}\sum_{i=1}^{N}\left(u_i\bar{X}_i - \bar{X}\right)\left(u_i\bar{Z}_i - \bar{Z}\right)$$

$$S'_{byz} = \frac{1}{N-1}\sum_{i=1}^{N}\left(u_i\bar{Y}_i - \bar{Y}\right)\left(u_i\bar{Z}_i - \bar{Z}\right)$$

$$S^2_{iy} = \frac{1}{M_i-1}\sum_{j=1}^{M_i}\left(y_{ij} - \bar{Y}_i\right)^2 \qquad i = 1, 2, \ldots, N.$$

$$S^2_{ix} = \frac{1}{M_i-1}\sum_{j=1}^{M_i}\left(x_{ij} - \bar{X}_i\right)^2 \qquad i = 1, 2, \ldots, N.$$

$$S_{ixy} = \frac{1}{M_i-1}\sum_{j=1}^{M_i}\left(x_{ij} - \bar{X}_i\right)\left(y_{ij} - \bar{Y}_i\right) \qquad i = 1, 2, \ldots, N.$$

Define,

$$\bar{y}_i = \frac{1}{m_i}\sum_{j=1}^{m_i} y_{ij}, \qquad \bar{x}_i = \frac{1}{m_i}\sum_{j=1}^{m_i} x_{ij}, \qquad \bar{z}_i = \frac{1}{m_i}\sum_{j=1}^{m_i} z_{ij}$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} u_i\bar{y}_i, \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} u_i\bar{x}_i, \qquad \bar{z} = \frac{1}{n}\sum_{i=1}^{n} u_i\bar{z}_i$$

$$s'^2_{by} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{y}_i - \bar{y}\right)^2, \qquad s'^2_{bx} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{x}_i - \bar{x}\right)^2, \qquad s'^2_{bz} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{z}_i - \bar{z}\right)^2$$

$$s'_{bxy} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{y}_i - \bar{y}\right)\left(u_i\bar{x}_i - \bar{x}\right), \qquad s'_{bxz} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{x}_i - \bar{x}\right)\left(u_i\bar{z}_i - \bar{z}\right)$$

$$s'_{byz} = \frac{1}{n-1}\sum_{i=1}^{n}\left(u_i\bar{y}_i - \bar{y}\right)\left(u_i\bar{z}_i - \bar{z}\right)$$

$$s^2_{iy} = \frac{1}{m_i-1}\sum_{j=1}^{m_i}\left(y_{ij} - \bar{y}_i\right)^2, \qquad s^2_{ix} = \frac{1}{m_i-1}\sum_{j=1}^{m_i}\left(x_{ij} - \bar{x}_i\right)^2$$

$$s_{ixy} = \frac{1}{m_i-1}\sum_{j=1}^{m_i}\left(y_{ij} - \bar{y}_i\right)\left(x_{ij} - \bar{x}_i\right)^2, \quad (i = 1,2, \ldots, n)$$

$$\rho_{bxy} = \frac{S'_{bxy}}{S'_{by}S'_{bx}}, \quad \rho_{ixy} = \frac{S_{ixy}}{S_{iy}S_{ix}}, \qquad (i = 1,2, \ldots, N).$$

$$\rho_{byz} = \frac{S'_{byz}}{S'_{by}S'_{bz}}, \quad \rho_{bxz} = \frac{S'_{bxz}}{S'_{bx}S'_{bz}}$$

$$C_{bx} = \frac{S'_{bx}}{\bar{X}}, \quad C_{by} = \frac{S'_{by}}{\bar{Y}}, \quad C_{bz} = \frac{S'_{bz}}{\bar{Z}}$$

$$C_{ix} = \frac{S_{ix}}{\overline{X}_i}, \quad C_{iy} = \frac{S_{iy}}{\overline{Y}_i}, \quad C_{iz} = \frac{S_{iz}}{\overline{Z}_i}, \qquad (i = 1,2, \ldots, N).$$

1. Proposed Ratio Estimators

Several ratio estimators using auxiliary variable x and first stage information on second auxiliary variable z may be formulated as follows:

(i) $$T_1' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{y}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{x}_i} . \overline{X} . \left(\frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i}{\overline{Z}} \right)$$

(ii) $$T_1'' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{y}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{x}_i} . \overline{X} . \left(\frac{\overline{Z}}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i} \right)$$

(iii) $$T_2' = \frac{1}{n}\sum_{i=1}^{n} u_i \frac{\overline{y}_i}{\overline{x}_i} \overline{X}_i . \left(\frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i}{\overline{Z}} \right)$$

(iv) $$T_2'' = \frac{1}{n}\sum_{i=1}^{n} u_i \frac{\overline{y}_i}{\overline{x}_i} \overline{X}_i \left(\frac{\overline{Z}}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i} \right)$$

(v) $$T_3' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{y}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{X}_i} . \overline{X} . \left(\frac{\frac{1}{n}\sum_{i-1}^{n} u_i \overline{Z}_i}{\overline{Z}} \right)$$

(vi) $$T_3'' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{y}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{X}_i} . \overline{X} \left(\frac{\overline{Z}}{\frac{1}{n}\sum_{i-1}^{n} u_i \overline{Z}_i} \right)$$

(vii) $$T_4' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \frac{\overline{y}_i}{\overline{x}_i} \overline{X}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{X}_i} \overline{X} \left(\frac{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i}{\overline{Z}} \right)$$

(viii) $$T_4'' = \frac{\frac{1}{n}\sum_{i=1}^{n} u_i \frac{\overline{y}_i}{\overline{x}_i} \overline{X}_i}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{X}_i} \overline{X} \left(\frac{\overline{Z}}{\frac{1}{n}\sum_{i=1}^{n} u_i \overline{Z}_i} \right)$$

III. BIASAND MEAN SQUARE ERRORS OF ESTIMATORS

To first order of approximations the biases and mean square errors of estimators are given below

$$B\left(T_1'\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{\bar{X}}^2}-\frac{S_{bxy}'}{\bar{\bar{Y}}.\bar{\bar{X}}}-\frac{S_{bxz}'}{\bar{\bar{X}}\bar{\bar{Z}}}+\frac{S_{byz}'}{\bar{\bar{Y}}\bar{\bar{Z}}}\right]$$

$$+\bar{Y}.\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[\frac{S_{ix}^2}{\bar{X}^2}-\frac{S_{ixy}}{\bar{Y}.\bar{X}}\right]$$

$$MSE(T_1')=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2+R_1^2S_{bx}'^2+2R_2S_{byz}'-2R_1S_{bxy}'-2R_1R_2S_{bxz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_1^2S_{ix}^2-2R_1S_{ixy}\right]$$

$$B\left(T_1''\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{\bar{X}}^2}-\frac{S_{bxy}'}{\bar{\bar{Y}}\bar{\bar{X}}}-\frac{S_{byz}'}{\bar{\bar{Y}}\bar{\bar{Z}}}+\frac{S_{bxz}'}{\bar{\bar{X}}\bar{\bar{Z}}}\right]$$

$$+\bar{Y}.\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[\frac{S_{ix}^2}{\bar{X}^2}-\frac{S_{ixy}}{\bar{Y}.\bar{X}}\right]$$

$$MSET_1'')=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2+R_1^2S_{bx}'^2-2R_2S_{byz}'-2R_1S_{bxy}'+2R_1R_2S_{bxz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_1^2S_{ix}^2-2R_1S_{ixy}\right]$$

$$MSE\left(T_2'\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2+2R_2S_{byz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_{1i}^2S_{ix}^2-2R_{1i}S_{ixy}\right]$$

$$MSE(T_2'')=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2-2R_2S_{byz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_{1i}^2S_{ix}^2-2R_{1i}S_{ixy}\right]$$

$$B\left(T_3'\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{\bar{X}}^2}-\frac{S_{bxy}'}{\bar{\bar{Y}}.\bar{\bar{X}}}-\frac{S_{bxz}'}{\bar{\bar{X}}.\bar{\bar{Z}}}+\frac{S_{byz}'}{\bar{\bar{Y}}.\bar{\bar{Z}}}\right]$$

$$MSE\left(T_3'\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2+R_1^2S_{bx}'^2+2R_2S_{byz}'-2R_1R_2S_{bxz}'-2R_1S_{bxy}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)S_{iy}^2$$

$$B\left(T_3''\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{\bar{X}}^2}-\frac{S_{bxy}'}{\bar{\bar{Y}}.\bar{\bar{X}}}-\frac{S_{byz}'}{\bar{\bar{Y}}\bar{\bar{Z}}}+\frac{S_{bxz}'}{\bar{\bar{X}}.\bar{\bar{Z}}}\right]$$

$$MSE\left(T_3''\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_1^2S_{bx}'^2+R_2^2S_{bz}'^2-2R_1S_{bxy}'-2R_2S_{byz}'+2R_1R_2S_{bxz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)S_{iy}^2$$

$$B\left(T_4'\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{X}^2}-\frac{S_{bxy}'}{\bar{Y}.\bar{X}}-\frac{S_{bxz}'}{\bar{X}.\bar{Z}}+\frac{S_{byz}'}{\bar{Y}.\bar{Z}}\right]$$

$$MSE\left(T_4'\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_2^2S_{bz}'^2+R_1^2S_{bx}'^2+2R_2S_{byz}'-2R_1S_{bxy}'-2R_1R_2S_{bxz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_{i1}^2S_{ix}^2-2R_{1i}S_{ixy}\right]$$

$$B\left(T_4''\right)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{S_{bx}'^2}{\bar{X}^2}-\frac{S_{bxy}'}{\bar{Y}.\bar{X}}-\frac{S_{byz}'}{\bar{Y}.\bar{Z}}+\frac{S_{bxz}'}{\bar{X}.\bar{Z}}\right]$$

$$MSE\left(T_4''\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{by}'^2+R_1^2S_{bx}'^2+R_2^2S_{bz}'^2-2R_1S_{bxy}'-2R_2S_{byz}'+2R_1R_2S_{bxz}'\right]$$

$$+\frac{1}{nN}\sum_{i=1}^{N}u_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left[S_{iy}^2+R_{1i}^2S_{ix}^2-2R_{1i}S_{ixy}\right]$$

IV. COMPARISONOF EFFICIENCIES

(i) Comparison of T_1' and T_2'' with estimator without using second auxiliary variable z.

MSE (T_1') – MSE(T_1)

$$=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{bz}'^2\left(\frac{\bar{Y}^2}{\bar{Z}^2}\right)+2S_{byz}'\left(\frac{\bar{Y}}{\bar{Z}}\right)-2S_{bxz}'\left(\frac{\bar{Y}^2}{\bar{Z}\,\bar{X}}\right)\right]$$

$$MSE(T_1'')-MSE(T_1)$$

$$=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{bz}'^2\left(\frac{\bar{Y}^2}{\bar{Z}^2}\right)-2S_{byz}'\left(\frac{\bar{Y}}{\bar{Z}}\right)+2S_{bxz}'\left(\frac{\bar{Y}^2}{\bar{Z}\,\bar{X}}\right)\right]$$

Thus T_1' will be more efficient than T_1 if

$$C_{bz}+2\rho_{byz}C_{by}<2\rho_{bxz}C_{bx}$$

and further T_1'' will be more efficient than T_1 is

$$C_{bz}+2\rho_{bxz}C_{bx}<2\rho_{byz}C_{by}$$

(ii) Comparison of T_2' and T_2'' with estimator without using second auxiliary variable z.

$$MSE(T_2')-MSE(T_2)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{bz}'^2.\frac{\bar{Y}^2}{\bar{Z}^2}+2S_{byz}'\frac{\bar{Y}}{\bar{Z}}\right]$$

$$MSE(T_2'')-MSE(T_2)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_{bz}'^2.\frac{\bar{Y}^2}{\bar{Z}^2}-2S_{byz}'\frac{\bar{Y}}{\bar{Z}}\right]$$

Thus T_2' will be more efficient than T_2 if

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$$\rho_{byz} < \frac{1}{2}\frac{C_{bz}}{C_{by}}$$

Further, T_2'' will be more efficient than T_2 is

$$\rho_{byz} < \frac{1}{2}\frac{C_{bz}}{C_{by}}$$

(iii) Comparison of T_3' and T_3'' with estimators without using second auxiliary variable z.

$$MSE(T_3') - MSE(T_3) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[S_{bz}'^2 . \frac{\bar{Y}^2}{\bar{\bar{Z}}^2} + 2S_{byz}' . \frac{\bar{Y}}{\bar{\bar{Z}}} - 2S_{bxz}' . \frac{\bar{Y}^2}{\bar{\bar{Z}}\,\bar{\bar{X}}}\right]$$

$$MSE(T_3'') - MSE(T_3) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[S_{bz}'^2 . \frac{\bar{Y}^2}{\bar{\bar{Z}}^2} - 2S_{byz}' . \frac{\bar{Y}}{\bar{\bar{Z}}} + 2S_{bxz}' . \frac{\bar{Y}^2}{\bar{\bar{Z}}\,\bar{\bar{X}}}\right]$$

Thus T_3' will be more efficient than T_3 is

$$C_{bz} + 2\rho_{byz}C_{by} < 2\rho_{bxz}\, C_{bx}$$

and further T_3'' will be more efficient than T_3 if

$$C_{bz} + 2\rho_{bxz}C_{bx} < 2\rho_{byz}\, C_{by}$$

(iv) Comparison of T_4' and T_4'' with estimators without using second auxiliary variable z.

$$MSE(T_4') - MSE(T_4)$$

$$= \left(\frac{1}{n} - \frac{1}{N}\right)\left[S_{bz}'^2 . \frac{\bar{Y}^2}{\bar{\bar{Z}}^2} + 2S_{byz}' . \frac{\bar{Y}}{\bar{\bar{Z}}} - 2S_{bxz}' . \frac{\bar{Y}^2}{\bar{\bar{Z}}\,\bar{\bar{X}}}\right]$$

$$MSE(T_4'') - MSE(T_4)$$

$$= \left(\frac{1}{n} - \frac{1}{N}\right)\left[S_{bz}'^2 . \frac{\bar{Y}^2}{\bar{\bar{Z}}^2} - 2S_{byz}' . \frac{\bar{Y}}{\bar{\bar{Z}}} + 2S_{bxz}' . \frac{\bar{Y}^2}{\bar{\bar{Z}}\,\bar{\bar{X}}}\right]$$

Thus T_4' will be more efficient than T_4 is

$$C_{bz} + 2\rho_{byz}C_{by} < 2\rho_{bxz}C_{bx}$$

and further T_4'' will be more efficient than T_4 is

$$C_{bz} + 2\rho_{bxz}C_{bx} < 2\rho_{byz}C_{by}$$

V. NUMERICAL STUDY

This population is MU 284 population available in Sarndal, Swensson and Wretman (1992, P- 660, Appendix –C). It consists of 284 Municipalities (ssu) divided into 15 clusters (fsu) with three variables i.e. Revenues from the 1985 municipal taxation as y, 1975 population as x and 1985 population as z. For comparison of mean square error of T_0, T_1, T_2, T_3, T_4, T_5,. T_1', T_1'', T_2', T_2'', T_3', T_3'', T_4', T_4'', $\hat{\bar{Y}}_1$, $\hat{\bar{Y}}_C$ we consider two-stage sampling with n = 5 and m_i (i = 1, 2, … 15) are assumed to be 2, 2, 2, 2, 2, 2, 3, 2, 3, 2, 2, 3, 2, 3, 2.

Table 1: Comparison of Mean Square Errors (MSE)

Estimator	MSE	Estimator	MSE	Estimator	MSE

T_0	35564.168	T'_1	12139.82	T''_1	8124.30
T_1	548.375	T'_2	45093.81	T''_2	438.25
T_2	13182.34	T'_3	34302.88	T''_3	30287.36
T_3	22711.44	T'_4	11921.06	T''_4	7905.54
T_4	329.61				

Remarks

From the above numerical illustration,

(i) MSE(T4) < MSE(T1) < MSE(T2) < MSE(T3) < MSE(T0)
(ii) MSE(T'4) < MSE(T'1) < MSE(T'3) < MSE(T'2)
(iii) MSE(T"2) < MSE(T"4) < MSE(T"1) < MSE(T"3)

Also, T4 is more efficient than all other estimators under comparison. However, the results obtained through numerical illustration is not conclusive because of limitations of data. Moreover, the efficiency of an estimator using second auxiliary variable z at the primary stage depends on the correlation structure between x and z at the primary stage.

VI. Summary and Conclusion

Using first stage information on second auxiliary variable z, a number of ratio type estimators in two-stage sampling have been suggested and it is seen that the proposed estimators are more efficient than the competitive estimators using auxiliary information on x only under certain sufficient conditions.

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