

Radius, Diameter and Center of a Directed Fuzzy Graph Using Algorithm

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Abstract- In this paper, the length of an edge sequence is said to be the number of edges contained in the edge sequence. The length of a fuzzy chain is the number of edges that makes up the fuzzy chain. The distance between vertex i and vertex j is s_{ij} , which is defined as the length of the smallest fuzzy chain between v_i and v_j . The concept of radius and diameter are associated with connected fuzzy graph. The radius of a fuzzy graph is defined as the set of maximum values selected from each row of the distance matrix is in a sense a measure of the closeness of a fuzzy graph. The diameter of a graph is the greatest distance that can be found in the fuzzy graph. To determine the diameter, select the maximum value from each row in the distance matrix. The largest of these numbers is the diameter.

Index Terms- Connected Directed fuzzy graph, Vertex strings, Radius, Diameter.

I. INTRODUCTION

Fuzzy set is one of the branches of modern mathematics having experienced a most impressive development in recent years. The notion of fuzzy sets was introduced by L.A. Zadeh in 1965. It involves the concept of a membership function defined on a universal set. The value of the membership function lies in $[0,1]$. Using the concept of fuzzy subsets, the concept of fuzzy graph was introduced by A. Rosenfeld in 1975. We present a taxonomy of fuzzy graphs that treats fuzziness in vertex existence, edge existence, edge connectivity, and edge weight. Within that framework, we formulate some standard graph-theoretic problems (Radius, Diameter and centre) for fuzzy graphs using a unified approach distinguished by its uniform application of guiding principles such as the construction of membership grades via the ranking of fuzzy numbers, the preservation of membership grade normalization, and the "collapsing" of fuzzy sets of graphs into fuzzy graphs. In the case of directed fuzzy graphs, the terms radius, center and diameter are defined in an analogous manner but apply only to strongly connected fuzzy graphs. To find radius, diameter and center of a problem from a specified source node to the other nodes appears in several applications. The works developed to find radius, diameter and center of a problem have been initiated in the contribution of my paper. Finally, we provide algorithmic solutions to these problems, with examples.

II. DEFINITION

Definition 2.1: A fuzzy graph $G=(X,F)$ is a pair of functions $F :V \rightarrow [0,1]$ and $X:V \times V \rightarrow [0,1]$, where for all $u,v \in V$, such that $X(u,v) \leq \min\{F(u), F(v)\}$.

Definition 2.2: A directed fuzzy graph is said to be strongly connected fuzzy graph if there is at least one directed fuzzy path from every vertex to every other vertex.

Definition 2.3: The **radius** of a fuzzy graph is defined as the minimum of the row distances, (i.e) the minimax distance of the fuzzy graph.

Definition 2.4: The **center** is defined as the point in a connected fuzzy graph which has the minimal separation. It need not always be a single point.

III. ALGORITHM TO FIND RADIUS OF DIRECTED FUZZY GRAPH

Let A be the adjacency matrix of a directed fuzzy graph, S_d be the shortest distance of directed fuzzy graph and S the distance matrix with elements s_{ij} . Initially let all s_{ij} be undefined.

Step 1

For all i and j , if $a_{ij} > 0$, then for $i \neq j$, $s_{ij}=1$ and $s_{ij}=0$ for all i . If any element of S is not defined go to step 2. Otherwise go to step 7.

Step 2

Define a $n \times n$ matrix $[M]^1$ from the fuzzy graph using the following

- If there is an arc from vertex v_i to vertex v_j (i.e) $a_{ij} > 0$, $i \neq j$, put v_{ij} in the (i, j) location in the matrix.
- Put 0 elsewhere.
- The non-zero entries are called strings.

Step 3

Define a $n \times n$ matrix $\{N\}^1$. $\{N\}^k$ is obtained from $\{M\}^k$ by deleting the first vertex in each non zero entry of $\{M\}^k$.

Step 4

$\{M\}^k \bullet \{N\}^m = \{M\}^{k+m}$

- zero multiplied by anything equals zero.
- strong multiplication concatenates the vertex strings.

Example : $v_1v_2 \bullet v_5v_6 = v_1v_2v_5v_6$ and $v_1v_2 \bullet v_1v_6 = 0$.

- Any string that has a vertex more than once equals zero.

Step 5

Find $[M]^1$. Define $k=1$.

Step 6

- a) Using step 2 and step 3 , find $[M]^{k+1}$ from $[M]^k$.
- b) For every s_{ij} that is not defined and the (i,j) element of $[M]^{k+1}$ is non zero , define $s_{ij}=k+1$
- c) If, for all i and j , s_{ij} is defined , then go to step 7. Otherwise, increase k by 1 return to step 6a.

Step 7

Radius = $\min_i \max_j s_{ij}$,

Diameter= $\max_i \max_j s_{ij}$.

Vertex v_k is center with k such that radius = $\max_j s_{kj}$.

IV. PROBLEM DEFINITION

We shall illustrate the technique with a simple **example**.

To find the radius, diameter and center of the connected fuzzy graph as shown in figure-1.

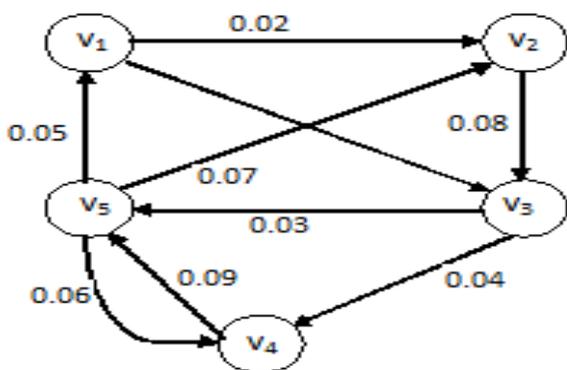


Figure-1.

Step 1

Let A be an adjacent matrix of directed Fuzzy graph.

$$A = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & 1 & 1 & 0 & 0 \\ V_2 & 0 & 0 & 1 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 1 & 1 \\ V_4 & 0 & 0 & 0 & 0 & 1 \\ V_5 & 1 & 1 & 0 & 1 & 0 \end{matrix}$$

Let S_d be the shortest distance of directed Fuzzy graph.

$$S_d = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & 0.02 & 0.06 & 0.10 & 0.09 \\ V_2 & 0.16 & 0 & 0.08 & 0.12 & 0.11 \\ V_3 & 0.08 & 0.10 & 0 & 0.04 & 0.03 \\ V_4 & 0.14 & 0.16 & 0.24 & 0 & 0.09 \\ V_5 & 0.05 & 0.07 & 0.15 & .06 & 0 \end{matrix}$$

$$S = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & \infty & \infty & \infty & \infty & \infty \\ V_2 & \infty & \infty & \infty & \infty & \infty \\ V_3 & \infty & \infty & \infty & \infty & \infty \\ V_4 & \infty & \infty & \infty & \infty & \infty \\ V_5 & \infty & \infty & \infty & \infty & \infty \end{matrix}$$

For all i and j , if $a_{ij} > 0$, then for $i \neq j$, $s_{ij}=1$ and $s_{ij}=0$ for all i . comparing the matrix A and S_d we get D_1

$$D_1 = \begin{matrix} & \sqrt{} & \sqrt{} & \infty & \infty \\ \infty & 0 & \sqrt{} & \infty & \infty \\ \infty & \infty & 0 & \sqrt{} & \sqrt{} \\ \infty & \infty & \infty & 0 & \sqrt{} \\ \sqrt{} & \sqrt{} & \infty & \sqrt{} & 0 \end{matrix}$$

Step 2

Let $[M]^1$ be the vertex representation of D_1

$$[M]^1 = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & V_1 V_2 & V_1 V_3 & 0 & 0 \\ V_2 & 0 & 0 & V_2 V_3 & 0 & 0 \\ V_3 & 0 & 0 & 0 & V_3 V_4 & V_3 V_5 \\ V_4 & 0 & 0 & 0 & 0 & V_4 V_5 \\ V_5 & V_5 V_1 & V_5 V_2 & 0 & V_5 V_4 & 0 \end{matrix}$$

Step 3

$[N]^k$ is obtained from $[M]^k$ by deleting the first vertex in each entry of $[M]^k$. Hence

$$[N]^1 = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & V_2 & V_3 & 0 & 0 \\ V_2 & 0 & 0 & V_3 & 0 & 0 \\ V_3 & 0 & 0 & 0 & V_4 & V_5 \\ V_4 & 0 & 0 & 0 & 0 & V_5 \\ V_5 & V_1 & V_2 & 0 & V_4 & 0 \end{matrix}$$

Step 4

$[M]^2 = [M]^1 \bullet [N]^1$

$$[M]^2 = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & 0 & V_1 V_2 & V_1 V_3 & V_1 V_3 \\ & & & V_3 & V_4 & V_5 \\ V_2 & 0 & 0 & 0 & V_2 V_3 & V_2 V_3 \\ & & & & V_4 & V_5 \\ V_3 & V_3 & V_3 V_5 & 0 & V_3 V_4 & V_3 V_5 \\ & & & & V_5 V_4 & V_4 V_5 \\ V_4 & V_4 & V_4 V_5 & 0 & 0 & 0 \\ & & & & V_5 & V_5 \\ V_5 & 0 & V_5 V_2 & V_5 V_3 & 0 & 0 \end{matrix}$$

Define $[N]^2$ from $[M]^2$

$$[N]^2 = \begin{matrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ V_1 & 0 & 0 & V_2 V_3 & V_3 V_4 & V_3 V_5 \\ V_2 & 0 & 0 & 0 & V_3 V_4 & V_3 V_5 \\ V_3 & V_5 & V_5 V_2 & 0 & V_5 V_4 & V_4 V_5 \\ & & & & V_5 & V_5 \\ V_4 & V_5 & V_5 V_2 & 0 & 0 & 0 \\ & & & & V_1 & V_1 \\ V_5 & 0 & V_1 V_2 & V_1 V_3 & 0 & 0 \end{matrix}$$

Similarly proceeding we get the value of $[M]^3$ and $[M]^4$.

	V_1	V_2	V_3	V_4	V_5
$[M]^4 =$	0	$V_1 V_3 V_4 V_5 V_2$	0	0	0
	$V_2 V_3 V_4 V_5 V_1$	0	0	0	0
	0	$V_3 V_4 V_5 V_1 V_2$	0	0	0
	0	0	$V_4 V_5 V_1 V_2 V_3$	0	0
	0	0	0	$V_1 V_2 V_3 V_4$	0

V. CONCLUSION

The radius of fuzzy directed graph is 2 and diameter is 3. The recursive procedure described here was carried out for different directed fuzzy graphs. This same procedure is often utilized for undirected fuzzy graphs which leads to a solution. It is useful for solving several different types of network problems.

Step 5

Find $[M]^1$ for $k=1$ (using step 2).

$[M]^1 =$	0	\sqrt	\sqrt	0	0
	0	0	\sqrt	0	0
	0	0	0	\sqrt	\sqrt
	0	0	0	0	\sqrt
	\sqrt	\sqrt	0	\sqrt	0

Step 5a

Repeat step 2,3,and 4 as in algorithm procedure , we get $s_{14}, s_{15}, s_{24}, s_{25}, s_{31}, s_{32}, s_{41}, s_{42}, s_{43}$ all equal to $\sqrt\sqrt$. $k=2$ the results of these steps are shown in D_2 .

$D_2 =$	0	\sqrt	\sqrt	$\sqrt\sqrt$	$\sqrt\sqrt$
	∞	0	\sqrt	$\sqrt\sqrt$	$\sqrt\sqrt$
	$\sqrt\sqrt$	$\sqrt\sqrt$	0	\sqrt	\sqrt
	$\sqrt\sqrt$	$\sqrt\sqrt$	∞	0	\sqrt
	\sqrt	\sqrt	$\sqrt\sqrt$	\sqrt	0

Similarly proceeding we get s_{21} and s_{43} equal to $\sqrt\sqrt\sqrt$. It is shown in D_3

$D_3 =$	0	\sqrt	\sqrt	$\sqrt\sqrt$	$\sqrt\sqrt$
	$\sqrt\sqrt\sqrt$	0	\sqrt	$\sqrt\sqrt$	$\sqrt\sqrt$
	$\sqrt\sqrt$	$\sqrt\sqrt$	0	\sqrt	\sqrt
	$\sqrt\sqrt$	$\sqrt\sqrt$	$\sqrt\sqrt\sqrt$	0	\sqrt
	\sqrt	\sqrt	$\sqrt\sqrt$	\sqrt	0

Go to step 7.Observing the values in the column marked maximum of s_{ij} in D_3 , we find that

$$\text{Radius} = \min_i \max_j s_{ij} = 2$$

$$\text{Diameter} = 3$$

$$\text{Center} = v_1, v_3 \text{ and } v_5.$$

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