ISSN 2250-3153

Fuzzy Soft Contra Continuous and Fuzzy Soft Almost Contra Continuous Mapping

A.Ponselvakumari*, R.Selvi **

*Dept of Mathematics Anna University, Tuticorin Campus ,Tuticorin Tamil Nadu, India. – 628 008
**Dept of Mathematics , Sri Parasakthi College for women,.Courtallam Tamil Nadu India -627 802.

Abstract- This paper is devoted to the concepts of fuzzy soft contra continuous and fuzzy soft almost contra continuous mapping in fuzzy soft topological spaces.

Index Terms- Fuzzy soft set, Fuzzy soft topological space, Fuzzy soft mapping, Fuzzy soft identity mapping.

I. Introduction

Molodtsov[8] initiated the novel concept of soft set as a new mathematical tool for dealing with uncertainties in 1999. Maji et al[7] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union , intersection, complement of a fuzzy soft set Demorgan Law etc. These results were further revised and improved by Ahmad and Kharal[1]. Kharal and Ahmad [1]have been introduced the concepts of fuzzy soft bijective mapping, fuzzy soft identity mapping and fuzzy soft continuous mapping. BanashreeBora[2] studied some propositions and examples related to fuzzy soft continuous mapping . In 1996,Dontchev [3] introduced contra continuous functions. Almost contra continuous functions were introduced by Joseph and Kwack[5]. Ekici[4] introduced and studied almost contra pre continuous functions. A.ponselvakumari and R.selvi [11 & 12 & 13] have been introduced the concepts of fuzzy soft almost continuous mapping and fuzzy soft pre continuous mapping and fuzzy soft semi continuous mapping. In this paper we studied the notions related to Fuzzy soft contra continuous mapping and Fuzzy soft almost contra continuous mapping and some related theorems along with examples have been cited.

II. Preliminaries

Throughout our discussion, U would refer to an initial universe, E ,the set of all parameters for U and $\tilde{P}(U)$, the set of all fuzzy subset of U and also by "(U,E)",we mean the universal set U and the parameter set E .

Definition 2.1 [4]

Let $f: X \to Y$ be a single valued function from a topological space X to a topological space Y. Then f is contra continuous if $f^{-1}(V)$ is closed (open) set in X for every open (closed) set V in Y .

Definition 2.2 [4]

A function $f: X \to Y$ is said to be almost contra continuous if $f^{-1}(V)$ is closed (open) in X for every regular open (regular closed) set V of Y .

Definition 2.3 [17]

A fuzzy set A in U is a set of ordered pairs $A = \{(x, \mu_A(x)) : x \in U\}$ where $\mu_A(x): U \to [0,1] = I$ is a mapping and $\mu_A(x)\ (or\ A(x))$. The family of all fuzzy sets in U is denoted by I^U .

Definition 2.4 [2]

A pair (F, A) is called a fuzzy soft set over U where $F: A \to \tilde{P}(\mathrm{U})$ is a mapping from A into $\tilde{P}(U)$

Definition 2.5 [2]

Let $A \subseteq E$. Then the mapping $F_A : E \to \tilde{P}(U)$, defined by $F_A(e) = \mu^e F_A$ (a fuzzy subset of U) is called fuzzy soft set over (U,E), where $\mu^e F_A = \overline{0}$ if $e \in E - A$ and $\mu^e F_A \neq \overline{0}$ if $e \in A$. The set of all fuzzy soft set over (U,E) is denoted by $FS(U,E)$.

Definition 2.6 [2]
Let $(U,E,\Im)$ and ($U^*, E^*, \Im^*$) be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and f $= (\rho,\psi) : (U,E) \to (U^*,E^*)$ be a fuzzy soft mapping

Then $f = (\rho,\psi)$ is said to be fuzzy soft continuous if the inverse image under $f = (\rho,\psi)$ of any $G_B \in (\Im^*)$ is a fuzzy soft set F_A $\in \Im$, that is $f^{-1}(G_B) \in \Im$ whenever $G_B \in (\Im^*)$

Definition 2.7 [2]
Let (U,E) and (U^*,E^*) be classes of fuzzy soft sets over U and U^* with attributes from E and E^* respectively. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings. Then a fuzzy soft mapping $f = (\rho,\psi) : (U,E) \to (U^*,E^*)$ would be defined as follows-

For a fuzzy soft set F_A in (U,E), $f(F_A)$ is a fuzzy soft set in (U^*,E^*) obtained as follows: for $\beta \in \psi(E) \subseteq E^*$ and $y \in E^*$,

$$f(F_A)(\beta)(y) = \begin{cases} \bigcup_{x\in\rho^{-1}(y)} \left(\bigcup_{\alpha\in\psi^{-1}(\beta)\cap A} F_A(\alpha) \right)(x), \text{if } \rho^{-1}(y) \neq \varphi, \psi^{-1}(\beta)\cap A \neq \varphi \\ 0, otherwise \end{cases}$$

$f(F_A)$ is called fuzzy soft image of the fuzzy soft set F_A.

III. Fuzzy Soft Contra Continuous Mapping

Definition 3.1
Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and $f = (\rho,\psi) : (U,E) \to (U^*,E^*)$ be a fuzzy soft mapping.

Let $H_B \in \Im^*$. Then $f = (\rho,\psi)$ is said to be fuzzy soft contra continuous if the inverse image under $f = (\rho,\psi)$ of any $H_B \in \Im^*$ is a fuzzy soft set closed in $(U,E,\Im)$
That is $f^{-1}(H_B)$ is closed in $(U,E,\Im)$ whenever $H_B \in \Im^*$.

EXAMPLE 3.2
Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $U = \{a,b,c\}$ and $U^* = \{x,y,z\}$ $E = \{e_1,e_2,e_3,e_4\}$. $E^* = \{e'_1,e'_2,e'_3\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings defined as $\rho(a) = z, \rho(b) = x, \rho(c) = y$ $\psi(e_1) = e'_2, \psi(e_2) = e'_3, \psi(e_3) = e'_1$. Let $A_1 = \{e_1,e_2\}$, $A_2 = \{e_1,e_2,e_3\}$ $B_1 = \{e'_1,e'_2\}$, $B_2 = \{e'_1,e'_2,e'_3\}$

Let us consider the fuzzy soft sets F_A, F_B in (U,E) and G_A, G_B in (U^*,E^*) as,

ISSN 2250-3153

$$F_A = \{(e_1,\{a/0.9, b/0.4, c/0.6\}),(e_2,\{a/0.6, b/0.9, c/0.8\}),(e_3,\{a/0.7, b/0.5, c/0.8\})\}$$
$$F_A^C = \{(e_1,\{a/0.1, b/0.6, c/0.4\}),(e_2,\{a/0.4, b/0.1, c/0.2\}),(e_3,\{a/0.3, b/0.5, c/0.2\})\}$$
$$F_B = \{(e_1,\{a/0.7, b/0.6, c/0.3\}),(e_2,\{a/0.9, b/0.1, c/0.5\}),(e_3,\{a/0.4, b/0.2, c/0.8\})\}$$
$$F_B^C = \{(e_1,\{a/0.3, b/0.4, c/0.7\}),(e_2,\{a/0.1, b/0.9, c/0.5\}),(e_3,\{a/0.6, b/0.8, c/0.2\})\}$$
$$G_A = \{(e_1^{/},\{x/0.5, y/0.2, z/0.3\}),(e_2^{/},\{x/0.6, y/0.4, z/0.1\}),(e_3^{/},\{x/0.1, y/0.2, z/0.4\})\}$$
$$G_B = \{(e_1^{/},\{x/0.8, y/0.2, z/0.6\}),(e_2^{/},\{x/0.4, y/0.7, z/0.3\}),(e_3^{/},\{x/0.9, y/0.5, z/0.1\})\}$$

Let $\Im = \{\tilde{\varphi}, \tilde{E}, F_A, F_B\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, G_A, G_B\}$

where $\tilde{\varphi}^* = \{(e_1^{/},\{x/0, y/0, z/0\}),(e_2^{/},\{x/0, y/0, z/0\}),(e_3^{/},\{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e_1^{/},\{x/1, y/1, z/1\}),$ $(e_2^{/},\{x/1, y/1, z/1\})(e_3^{/},\{x/1, y/1, z/1\})\}$

$$\tilde{\varphi} = \{(e_1,\{a/0, b/0, c/0\}), (e_2,\{a/0, b/0, c/0\})\ (e_3,\{a/0, b/0, c/0\})\}.$$
$$\tilde{E} = \{(e_1,\{a/1, b/1, c/1\}),\ (e_2,\{a/1, b/1, c/1\}),\ (e_3,\{a/1, b/1, c/1\})\}.$$

Here f is fuzzy soft contra continuous.

THEOREM 3.3

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and $f = (\rho, \psi) : (U, E) \to (U^*, E^*)$ be a fuzzy soft mapping.

Then the following statements are equivalent:

(i) The fuzzy soft function $f = (\rho, \psi) : (U, E) \to (U^*, E^*)$ is contra continuous.

(ii) Inverse image of every closed subset of (U^*, E^*) is an open subset of (U, E).

(iii) Inverse image of every open subset of (U^*, E^*) is a closed subset of (U, E).

(iv) For each point $e(F_A)$ in (U, E) and for each open neighbourhood G_B of $f(e(F_A))$ there is a neighbourhood H_B of $e(F_A)$ such that $f(H_B) \subseteq G_B$.

(v)

Proof:

(i) $\Rightarrow$ (ii),It is obvious.

(ii) $\Rightarrow$ (iii)

Let F_A be a open subset of (U^*, E^*) .

Then $(U^*, E^*) \setminus F_A$ is closed and Therefore $f^{-1}(U^*, E^*) \setminus F_A$ is open.

That is $(U, E) \setminus f^{-1}(F_A)$ is open.

Hence $f^{-1}(F_A)$ is closed.

(iii) $\to$ (iv)

ISSN 2250-3153

Let F_B be closed & G_B be a neighbourhood of $f(e(F_A))$.

Then $(U^*,E^*) \square F_B$ is open and consequently $f^{-1}[(U^*,E^*) \square F_B]$ is closed.

That is $f^{-1}(F_B)$ is open. Also $e(F_A) \in f^{-1}(F_B) = H_B$ .Then H_B is a neighbourhood of $e(F_A)$ such that $f(H_B) \subseteq G_B$.
(iv) $\Rightarrow$ (i)

By hypothesis, f is fuzzy soft contra continuous.

EXAMPLE 3.4

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let U ={ a,b,c } and $U^* = \{x,y,z\}$ $E = \{e_1, e_2, e_3, e_4\}$. $E^* = \{e'_1, e'_2, e'_3\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho : \mathrm{U} \rightarrow \mathrm{U}^*$ and $\psi : \mathrm{E} \rightarrow \mathrm{E}^*$ be mappings defined as $\rho(a) = z, \rho(b) = x, \rho(c) = y$ $\psi(e_1) = e'_2, \psi(e_2) = e'_3, \psi(e_3) = e'_1$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e'_1, e'_2\}$, $B_2 = \{e'_1, e'_2, e'_3\}$

Let us consider the fuzzy soft sets F_A, F_B in (U,E) and G_A, G_B in (U^*,E^*) as,

$$F_A = \{(e_1,\{a/0.9, b/0.4, c/0.6\}),(e_2,\{a/0.6, b/0.9, c/0.8\}),(e_3,\{a/0.7, b/0.5, c/0.8\})\}$$
$$F_A^C = \{(e_1,\{a/0.1, b/0.6, c/0.4\}),(e_2,\{a/0.4, b/0.1, c/0.2\}),(e_3,\{a/0.3, b/0.5, c/0.2\})\}$$
$$F_B = \{(e_1,\{a/0.7, b/0.6, c/0.3\}),(e_2,\{a/0.9, b/0.1, c/0.5\}),(e_3,\{a/0.4, b/0.2, c/0.8\})\}$$
$$F_B^C = \{(e_1,\{a/0.3, b/0.4, c/0.7\}),(e_2,\{a/0.1, b/0.9, c/0.5\}),(e_3,\{a/0.6, b/0.8, c/0.2\})\}$$
$$G_A = \{(e'_1,\{x/0.5, y/0.2, z/0.3\}),(e'_2,\{x/0.6, y/0.4, z/0.1\}),(e'_3,\{x/0.1, y/0.2, z/0.4\})\}$$
$$G_B = \{(e'_1,\{x/0.8, y/0.2, z/0.6\}),(e'_2,\{x/0.4, y/0.7, z/0.3\}),(e'_3,\{x/0.9, y/0.5, z/0.1\})\}$$

Let $\Im = \{\tilde{\varphi}, \tilde{E}, F_A, F_B\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, G_A, G_B\}$

where $\tilde{\varphi}^* = \{(e'_1,\{x/0, y/0, z/0\}),(e'_2,\{x/0, y/0, z/0\}),(e'_3,\{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e'_1,\{x/1, y/1, z/1\}),$ $(e'_2,\{x/1, y/1, z/1\})(e'_3,\{x/1, y/1, z/1\})\}$

$$\tilde{\varphi} = \{(e_1,\{a/0, b/0, c/0\}), (e_2,\{a/0, b/0, c/0\})\ (e_3,\{a/0, b/0, c/0\})\}.$$

$$\tilde{E} = \{(e_1,\{a/1, b/1, c/1\}),\ (e_2,\{a/1, b/1, c/1\}),\ (e_3,\{a/1, b/1, c/1\})\}.$$

Let $e(F_A) = F_A^C = \{(e_1,\{a/0.1, b/0.6, c/0.4\}),(e_2,\{a/0.4, b/0.1, c/0.2\}),(e_3,\{a/0.3, b/0.5, c/0.2\})\}$ be a fuzzy soft point in (U,E) and G_B be a neighbourhood of $f(e(F_A))$.

Then there is a $G_A \in \Im^*$ such that $f(e(F_A)) \tilde{\subseteq} G_A \tilde{\subseteq} G_B$, Where $G_B = \{(e_1,\{x/0.7, y/0.3, z/0.5\}),(e_2,\{x/0.8, y/0.6, z/0.4\}),(e_3,\{x/0.5, y/0.6, z/0.9\})\}$

And $f(e(F_A)) \in (U^*, E^*)$,

$f(e(F_A)) = G_A = \{(e_1^/,\{x/0.5, y/0.2, z/0.3\})(e_2^/,\{x/0.6, y/0.4, z/0.1\})(e_3^/,\{x/0.1, y/0.2, z/0.4\})\}$

Therefore $f(e(F_A)) \tilde{\subseteq} G_A \tilde{\subseteq} G_B$

That implies G_B is a neighbourhood of $f(e(F_A))$

THEOREM 3.5

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and $f = (\rho,\psi):(U,E) \to (U^*,E^*)$ be a fuzzy soft mapping.

If f is fuzzy soft contra continuous & open then f is fuzzy soft continuous.

Proof:

Let f be fuzzy soft contra continuous and let $H_B \in \Im^*$,where $\Im^*$ is an open set.

Since f is fuzzy soft contra continuous , $f^{-1}(H_B)$ is closed in $(U,E,\Im)$.

$\Rightarrow (f^{-1}(H_B))^C$ is open.

$\Rightarrow f^{-1}(H_B^C)$ is open.

$\Rightarrow f(f^{-1}(H_B^C))$ is open, since f is open.

$\Rightarrow H_B^C$ is open.

$\Rightarrow H_B$ is closed.

That is $f^{-1}(H_B)$ is closed implies H_B is closed

Therefore f is fuzzy soft continuous.

EXAMPLE 3.6

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let U ={ a,b,c } and $U^* = \{x,y,z\}$ $E = \{e_1,e_2,e_3,e_4\}$. $E^* = \{e_1',e_2',e_3'\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho : \text{U} \to \text{U}^*$ and $\psi : \text{E} \to \text{E}^*$ be mappings defined as $\rho(a) = z, \rho(b) = x, \rho(c) = y$ $\psi(e_1) = e_2^/, \psi(e_2) = e_3^/, \psi(e_3) = e_1^/$. Let $A_1 = \{e_1,e_2\}$, $A_2 = \{e_1,e_2,e_3\}$ $B_1 = \{e_1^/,e_2^/\}$, $B_2 = \{e_1^/,e_2^/,e_3^/\}$

Let us consider the fuzzy soft sets F_A, F_B in (U,E) and G_A, G_B in (U^*,E^*) as,

$$F_A = \{(e_1,\{a/0.9, b/0.4, c/0.6\}),(e_2,\{a/0.6, b/0.9, c/0.8\}),(e_3,\{a/0.7, b/0.5, c/0.8\})\}$$
$$F_A^C = \{(e_1,\{a/0.1, b/0.6, c/0.4\}),(e_2,\{a/0.4, b/0.1, c/0.2\}),(e_3,\{a/0.3, b/0.5, c/0.2\})\}$$
$$F_B = \{(e_1,\{a/0.7, b/0.6, c/0.3\}),(e_2,\{a/0.9, b/0.1, c/0.5\}),(e_3,\{a/0.4, b/0.2, c/0.8\})\}$$

$F_B^C = \{(e_1,\{a/0.3,b/0.4,c/0.7\}),(e_2,\{a/0.1,b/0.9,c/0.5\}),(e_3,\{a/0.6,b/0.8,c/0.2\})\}$
$G_A = \{(e_1^/,\{x/0.5,y/0.2,z/0.3\}),(e_2^/,\{x/0.6,y/0.4,z/0.1\}),(e_3^/,\{x/0.1,y/0.2,z/0.4\})\}$
$G_B = \{(e_1^/,\{x/0.8,y/0.2,z/0.6\}),(e_2^/,\{x/0.4,y/0.7,z/0.3\}),(e_3^/,\{x/0.9,y/0.5,z/0.1\})\}$

Let $\Im = \{\tilde{\varphi},\tilde{E},F_A,F_B\}$ and $\Im^* = \{\tilde{\varphi}^*,\tilde{E}^*,G_A,G_B\}$

where $\tilde{\varphi}^* = \{(e_1^/,\{x/0,y/0,z/0\}),(e_2^/,\{x/0,y/0,z/0\}),(e_3^/,\{x/0,y/0,z/0\})\}$ $\tilde{E}^* = \{(e_1^/,\{x/1,y/1,z/1\}),$
$(e_2^/,\{x/1,y/1,z/1\})(e_3^/,\{x/1,y/1,z/1\})\}$

$\tilde{\varphi} = \{(e_1,\{a/0,b/0,c/0\})$, $(e_2,\{a/0,b/0,c/0\})$ $(e_3,\{a/0,b/0,c/0\})\}$. $\tilde{E} = \{(e_1,\{a/1,b/1,c/1\}),$
$(e_2,\{a/1,b/1,c/1\}),\ (e_3,\{a/1,b/1,c/1\})\}$.

Here f is fuzzy soft contra continuous then fuzzy soft continuous.

NOTE
Fuzzy soft continuous does not implies fuzzy soft contra continuous.

EXAMPLE 3.7

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $U = \{a,b,c\}$ and $U^* = \{x,y,z\}$ $E = \{e_1,e_2,e_3,e_4\}$. $E^* = \{e_1',e_2',e_3'\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho: U \to U^*$ and $\psi: E \to E^*$ be mappings defined as $\rho(a)=z,\rho(b)=x,\rho(c)=y$ $\psi(e_1)=e_2^/,\psi(e_2)=e_3^/,\psi(e_3)=e_1^/$. Let $A_1=\{e_1,e_2\}$, $A_2=\{e_1,e_2,e_3\}$ $B_1=\{e_1^/,e_2^/\}$, $B_2=\{e_1^/,e_2^/,e_3^/\}$

Let us consider the fuzzy soft sets F_A,F_B in (U,E) and G_A,G_B in (U^*,E^*) as,

$F_A = \{(e_1,\{a/0.9,b/0.4,c/0.6\}),(e_2,\{a/0.6,b/0.9,c/0.8\}),(e_3,\{a/0.7,b/0.5,c/0.8\})\}$
$F_A^C = \{(e_1,\{a/0.1,b/0.6,c/0.4\}),(e_2,\{a/0.4,b/0.1,c/0.2\}),(e_3,\{a/0.3,b/0.5,c/0.2\})\}$
$F_B = \{(e_1,\{a/0.7,b/0.6,c/0.3\}),(e_2,\{a/0.9,b/0.1,c/0.5\}),(e_3,\{a/0.4,b/0.2,c/0.8\})\}$
$F_B^C = \{(e_1,\{a/0.3,b/0.4,c/0.7\}),(e_2,\{a/0.1,b/0.9,c/0.5\}),(e_3,\{a/0.6,b/0.8,c/0.2\})\}$
$G_A = \{(e_1^/,\{x/0.5,y/0.2,z/0.3\}),(e_2^/,\{x/0.6,y/0.4,z/0.1\}),(e_3^/,\{x/0.1,y/0.2,z/0.4\})\}$
$G_B = \{(e_1^/,\{x/0.8,y/0.2,z/0.6\}),(e_2^/,\{x/0.4,y/0.7,z/0.3\}),(e_3^/,\{x/0.9,y/0.5,z/0.1\})\}$

Let $\Im = \{\tilde{\phi},\tilde{E},F_{A_1},F_{B_1}\}$ & $\Im^* = \{\tilde{\phi}^*,\tilde{E}^*,G_{A_1},G_{B_1}\}$.

where $\tilde{\varphi}^* = \{(e_1^/,\{x/0,y/0,z/0\}),(e_2^/,\{x/0,y/0,z/0\}),(e_3^/,\{x/0,y/0,z/0\})\}$ $\tilde{E}^* = \{(e_1^/,\{x/1,y/1,z/1\}),$
$(e_2^/,\{x/1,y/1,z/1\})(e_3^/,\{x/1,y/1,z/1\})\}$

$\tilde{\varphi} = \{(e_1, \{a/0, b/0, c/0\}), (e_2, \{a/0, b/0, c/0\})\ (e_3, \{a/0, b/0, c/0\})\}$. $\tilde{E} = \{(e_1, \{a/1, b/1, c/1\}),$
$(e_2, \{a/1, b/1, c/1\}), (e_3, \{a/1, b/1, c/1\})\}$.

Here f is fuzzy soft continuous does not implies fuzzy soft contra continuous.

THEOREM 3.8
Every restriction of an fuzzy soft contra continuous mapping is fuzzy soft contra continuous.

Proof:
Let f be a fuzzy soft contra continuous mapping of (U, E) into (U^*, E^*).

Let F_A be any subset of (U, E) .

For any open subset G_A of (U^*, E^*) .

$$(f / F_A)^{-1}(G_A) = (F_A) \cap f^{-1}(G_A)$$

But f being fuzzy soft contra continuous , and hence $f^{-1}(G_A)$ is closed and

Therefore $F_A \cap f^{-1}(G_A)$ is relatively closed subset of F_A .

That implies $(f / F_A)^{-1}(G_A)$ is closed subset of F_A.

Hence $f / (F_A)$ is fuzzy soft contra continuous.

EXAMPLE 3.9
Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $U = \{a, b, c\}$ and $U^* = \{x, y, z\}$ $E = \{e_1, e_2, e_3, e_4\}$. $E^* = \{e'_1, e'_2, e'_3\}$ and $(U, E), (U^*, E^*)$ classes of fuzzy soft sets.

Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings defined as $\rho(a) = z, \rho(b) = x, \rho(c) = y$ $\psi(e_1) = e'_2, \psi(e_2) = e'_3, \psi(e_3) = e'_1$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e'_1, e'_2\}$, $B_2 = \{e'_1, e'_2, e'_3\}$

Let us consider the fuzzy soft sets F_A, F_B in (U, E) and G_A, G_B in (U^*, E^*) as,

$$F_A = \{(e_1, \{a/0.9, b/0.4, c/0.6\}), (e_2, \{a/0.6, b/0.9, c/0.8\}), (e_3, \{a/0.7, b/0.5, c/0.8\})\}$$
$$F_A^C = \{(e_1, \{a/0.1, b/0.6, c/0.4\}), (e_2, \{a/0.4, b/0.1, c/0.2\}), (e_3, \{a/0.3, b/0.5, c/0.2\})\}$$
$$F_B = \{(e_1, \{a/0.7, b/0.6, c/0.3\}), (e_2, \{a/0.9, b/0.1, c/0.5\}), (e_3, \{a/0.4, b/0.2, c/0.8\})\}$$
$$F_B^C = \{(e_1, \{a/0.3, b/0.4, c/0.7\}), (e_2, \{a/0.1, b/0.9, c/0.5\}), (e_3, \{a/0.6, b/0.8, c/0.2\})\}$$
$$G_A = \{(e'_1, \{x/0.5, y/0.2, z/0.3\}), (e'_2, \{x/0.6, y/0.4, z/0.1\}), (e'_3, \{x/0.1, y/0.2, z/0.4\})\}$$

$G_B = \{(e_1^/, \{x/0.8, y/0.2, z/0.6\}), (e_2^/, \{x/0.4, y/0.7, z/0.3\}), (e_3^/, \{x/0.9, y/0.5, z/0.1\})\}$

Let $\Im = \{\tilde{\varphi}, \tilde{E}, F_A, F_B\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, G_A, G_B\}$

where $\tilde{\varphi}^* = \{(e_1^/, \{x/0, y/0, z/0\}), (e_2^/, \{x/0, y/0, z/0\}), (e_3^/, \{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e_1^/, \{x/1, y/1, z/1\}),$ $(e_2^/, \{x/1, y/1, z/1\})(e_3^/, \{x/1, y/1, z/1\})\}$

$\tilde{\varphi} = \{(e_1, \{a/0, b/0, c/0\})$, $(e_2, \{a/0, b/0, c/0\})$ $(e_3, \{a/0, b/0, c/0\})\}$. $\tilde{E} = \{(e_1, \{a/1, b/1, c/1\}),$ $(e_2, \{a/1, b/1, c/1\}),$ $(e_3, \{a/1, b/1, c/1\})\}$.

The restriction of function f is $f/(F_A) : (F_A) \to (U^*, E^*)$ defined by the inverse image under $f/(F_A)$ of any $G_C \in \Im^*$ is a fuzzy soft set $F_S \in \Im$, that is $(f/(F_A)^{-1}(G_C) \in \Im$

Whenever $G_C \in \Im^*$. Here $(f/(F_A))^{-1}(G_A) = F_A$.

Hence restriction of f is fuzzy soft contra continuous.

IV. FUZZY SOFT ALMOST CONTRA CONTINUOUS MAPPING

DEFINITION 4.1

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and $f = (\rho, \psi) : (U, E) \to (U^*, E^*)$ be a fuzzy soft mapping. Let $G_B \in RO(\Im^*)$. Then .

$f = (\rho, \psi)$ is said to be fuzzy soft almost contra continuous if the inverse image under $f = (\rho, \psi)$ of any $G_B \in RO(\Im^*)$ is a fuzzy soft closed set in $(U, E, \Im)$.

That is $f^{-1}(G_B)$ is closed in $(U, E, \Im)$ whenever $G_B \in RO(\Im^*)$

EXAMPLE 4.2

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $U = \{a, b, c\}$ and $U^* = \{x, y, z\}$ $E = \{e_1, e_2, e_3, e_4\}$.

$E^* = \{e_1', e_2', e_3'\}$ and $(U, E), (U^*, E^*)$ classes of fuzzy soft sets.

Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings defined as $\rho(a) = z, \rho(b) = y, \rho(c) = y$

$\psi(e_1) = e_1^/, \psi(e_2) = e_1^/, \psi(e_3) = e_3^/$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e_1^/, e_2^/\}$, $B_2 = \{e_1^/, e_2^/, e_3^/\}$

Let us consider the fuzzy soft sets C_1, C_2 in (U, E) and $F_{B_1}^1, F_{B_2}^2$ in (U^*, E^*) as,

$C_1 = \{(e_1, \{a/0.7, b/0.7, c/0.7\}), (e_2, \{a/0.7, b/0.7, c/0.7\}), (e_3, \{a/0.4, b/0.4, c/0.4\})\}$

$C_1^C = \{(e_1,\{a/0.3,b/0.3,c/0.3\}),(e_2,\{a/0.3,b/0.3,c/0.3\}),(e_3,\{a/0.6,b/0.6,c/0.6\})\}$
$C_2 = \{(e_1,\{a/0.8,b/0.8,c/0.8\}),(e_2,\{a/0.8,b/0.8,c/0.8\}),\{e_3,\{a/0.9,b/0.9,c/0.9\})\}$
$C_2^C = \{(e_1,\{a/0.2,b/0.2,c/0.2\}),(e_2,\{a/0.2,b/0.2,c/0.2\}),\{e_3,\{a/0.1,b/0.1,c/0.1\})\}$
$F_{B_1}^1 = \{(e_1^/,\{x/0.1,y/0.2,z/0.2\}),(e_2^/,\{x/0.1,y/0.2,z/0.2\}),(e_3^/,\{x/0.1,y/0.1,z/0.1\})\}$
$F_{B_2}^2 = \{(e_1^/,\{x/0.6,y/0.7,z/0.7\}),(e_2^/,\{x/0.6,y/0.7,z/0.7\}),(e_3^/,\{x/0.6,y/0.4,z/0.4\})\}$

Let $\Im = \left\{\tilde{\varphi},\tilde{E},C_1,C_2\right\}$ and $\Im^* = \{\tilde{\varphi}^*,\tilde{E}^*,F_{B_1}^1,F_{B_2}^2\}$ $\mathrm{RO}(\Im^*) = \left\{\tilde{\varphi}^*,\tilde{E}^*,F_{B_1}^1\right\}$

where $\tilde{\varphi}^* = \{(e_1^/,\{x/0,y/0,z/0\}),(e_2^/,\{x/0,y/0,z/0\}),(e_3^/,\{x/0,y/0,z/0\})\}$ $\tilde{E}^* = \{(e_1^/,\{x/1,y/1,z/1\}),$ $(e_2^/,\{x/1,y/1,z/1\})(e_3^/,\{x/1,y/1,z/1\})\}$ $\tilde{\varphi} = \{(e_1,\{a/0,b/0,c/0\})$, $(e_2,\{a/0,b/0,c/0\})$ $(e_3,\{a/0,b/0,c/0\})\}$.

$\tilde{E} = \{(e_1,\{a/1,b/1,c/1\}),\ (e_2,\{a/1,b/1,c/1\}),\ (e_3,\{a/1,b/1,c/1\})\}$.

Here f is fuzzy soft almost contra continuous.

THEOREM 4.3

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $\rho:U\to U^*$ and $\psi:E\to E^*$ be mappings and $f=(\rho,\psi):(U,E)\to(U^*,E^*)$ be a fuzzy soft mapping .

Then the following statements are equivalent.

(i) The fuzzy soft function $f=(\rho,\psi):(U,E)\to(U^*,E^*)$ is almost contra continuous.
(ii) Inverse image of every regular closed subset of (U^*,E^*) is an open subset of (U,E)
(iii) Inverse image of every regular open subset of (U^*,E^*) is an closed subset of (U,E)
(iv) For each point $e(F_A)$ in (U,E) and for each open neighbourhood G_A of $f(e(F_A))$ there is a neighbourhood H_B of $e(F_C)$ such that $f(H_B)\subseteq G_A$.
(v)

Proof:

(i) $\Rightarrow$ (ii), It is obvious.

(ii) $\Rightarrow$ (iii)

Let F_B be a regular open subset of (U^*,E^*).

Then $(U^*,E^*) \square F_B$ is regular closed and Therefore $f^{-1}(U^*,E^*) \square F_B$ is open.

That is $(U,E) \square f^{-1}(F_B)$ is open.

Hence $f^{-1}(F_B)$ is closed.

(iii) $\Rightarrow$ (iv)

Let F_B be regular closed and G_A be a neighbourhood of $f(e(F_A))$

Then $(U^*,E^*) \square F_B$ is regular open and consequently $f^{-1}[(U^*,E^*) \square F_B]$ is closed.

That is $f^{-1}(F_B)$ is open. Also $e(F_A) \in f^{-1}(F_B) = G_A$.
Then G_A is a neighbourhood of $e(F_A)$ such that $f(H_B) \subseteq G_A$.

(iv) $\Rightarrow$ (i)
By hypothesis f is fuzzy soft almost contra continuous.

EXAMPLE 4.4

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let U ={ a, b, c } and $U^* = \{x, y, z\}$ $E = \{e_1, e_2, e_3, e_4\}$. $E^* = \{e'_1, e'_2, e'_3\}$ and $(U, E), (U^*, E^*)$ classes of fuzzy soft sets.

Let $\rho : \mathrm{U} \to \mathrm{U}^*$ and $\psi : \mathrm{E} \to \mathrm{E}^*$ be mappings defined as $\rho(a) = z, \rho(b) = y, \rho(c) = y$ $\psi(e_1) = e'_1, \psi(e_2) = e'_1, \psi(e_3) = e'_3$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e'_1, e'_2\}$, $B_2 = \{e'_1, e'_2, e'_3\}$

Let us consider the fuzzy soft sets C_1 , C_2 in (U, E) and $\mathrm{F}^1_{B_1}, \mathrm{F}^2_{B_2}$ in (U^*, E^*) as,

$$C_1 = \{(e_1, \{a/0.7, b/0.7, c/0.7\}), (e_2, \{a/0.7, b/0.7, c/0.7\}), (e_3, \{a/0.4, b/0.4, c/0.4\})\}$$
$$C_1^C = \{(e_1, \{a/0.3, b/0.3, c/0.3\}), (e_2, \{a/0.3, b/0.3, c/0.3\}), (e_3, \{a/0.6, b/0.6, c/0.6\})\}$$
$$C_2 = \{(e_1, \{a/0.8, b/0.8, c/0.8\}), (e_2, \{a/0.8, b/0.8, c/0.8\}), \{e_3, \{a/0.9, b/0.9, c/0.9\})\}$$
$$C_2^C = \{(e_1, \{a/0.2, b/0.2, c/0.2\}), (e_2, \{a/0.2, b/0.2, c/0.2\}), \{e_3, \{a/0.1, b/0.1, c/0.1\})\}$$
$$F^1_{B_1} = \{(e'_1, \{x/0.1, \mathrm{y}/0.2, \mathrm{z}/0.2\}), (e'_2, \{x/0.1, \mathrm{y}/0.2, \mathrm{z}/0.2\}), (e'_3, \{x/0.1, \mathrm{y}/0.1, \mathrm{z}/0.1\})\}$$
$$F^2_{B_2} = \{(e'_1, \{x/0.6, \mathrm{y}/0.7, \mathrm{z}/0.7\}), (e'_2, \{x/0.6, \mathrm{y}/0.7, \mathrm{z}/0.7\}), (e'_3, \{x/0.6, \mathrm{y}/0.4, \mathrm{z}/0.4\})\}$$

Let $\Im = \{\tilde{\varphi}, \tilde{E}, \mathrm{C}_1, C_2\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, F^1_{B_1}, F^2_{B_2}\}$ $\mathrm{RO}(\Im^*) = \{\tilde{\varphi}^*, \tilde{E}^*, \mathrm{F}^1_{B_1}\}$
where $\tilde{\varphi}^* = \{(e'_1, \{x/0, \mathrm{y}/0, \mathrm{z}/0\}), (e'_2, \{x/0, y/0, z/0\}), (\mathrm{e}'_3, \{\mathrm{x}/0, \mathrm{y}/0, \mathrm{z}/0\})\}$ $\tilde{E}^* = \{(\mathrm{e}'_1, \{x/1, y/1, z/1\}),$ $(e'_2, \{x/1, y/1, z/1\})(e'_3, \{x/1, y/1, z/1\})\}$ $\tilde{\varphi} = \{(e_1, \{\mathrm{a}/0, \mathrm{b}/0, \mathrm{c}/0\})$, $(e_2, \{a/0, b/0, c/0\})$ $(e_3, \{a/0, b/0, c/0\})\}$.
$\tilde{E} = \{(\mathrm{e}_1, \{\mathrm{a}/1, \mathrm{b}/1, \mathrm{c}/1\}), (\mathrm{e}_2, \{\mathrm{a}/1, \mathrm{b}/1, \mathrm{c}/1\}), (\mathrm{e}_3, \{\mathrm{a}/1, \mathrm{b}/1, \mathrm{c}/1\})\}$.

$F^1_{B_1}$ is a subset of $RO(\Im^*)$ and C_2 is a subset of $\Im$. Then $f^{-1}(F^1_{B_1}) = C_2^C$.

Here inverse image of a regular open subset is closed and hence f is fuzzy soft almost contra continuous.

THEOREM 4.5

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\psi : E \to E^*$ be mappings and $f = (\rho, \psi) : (U, E) \to (U^*, E^*)$ be a fuzzy soft mapping.

If f is fuzzy soft almost contra continuous then f is fuzzy soft continuous.

Proof:

Let f be fuzzy soft almost contra continuous and $H_B \in RO(\Im^*)$. Since every regular open set is open and since f is fuzzy soft almost contra continuous, $f^{-1}(H_B)$ is closed in $(U,E,\Im)$.

This theorem follows from (Theorem 3.5)

EXAMPLE 4.6

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let U ={ a,b,c } and $U^*=\{x,y,z\}$ $E=\{e_1,e_2,e_3,e_4\}$. $E^*=\{e'_1,e'_2,e'_3\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho: \mathrm{U} \to \mathrm{U}^*$ and $\psi: \mathrm{E} \to \mathrm{E}^*$ be mappings defined as $\rho(a)=z, \rho(b)=y, \rho(c)=y$ $\psi(e_1)=e'_1, \psi(e_2)=e'_1, \psi(e_3)=e'_3$. Let $A_1=\{e_1,e_2\}$, $A_2=\{e_1,e_2,e_3\}$ $B_1=\{e'_1,e'_2\}$, $B_2=\{e'_1,e'_2,e'_3\}$

Let us consider the fuzzy soft sets C_A, C_B in (U,E) and $F^1_{B_1}, F^2_{B_2}$ in (U^*,E^*) as,

$$C_1=\{(e_1,\{a/0.7,b/0.7,c/0.7\}),(e_2,\{a/0.7,b/0.7,c/0.7\}),(e_3,\{a/0.4,b/0.4,c/0.4\})\}$$
$$C_1^C=\{(e_1,\{a/0.3,b/0.3,c/0.3\}),(e_2,\{a/0.3,b/0.3,c/0.3\}),(e_3,\{a/0.6,b/0.6,c/0.6\})\}$$
$$C_2=\{(e_1,\{a/0.8,b/0.8,c/0.8\}),(e_2,\{a/0.8,b/0.8,c/0.8\}),\{e_3,\{a/0.9,b/0.9,c/0.9\})\}$$
$$C_2^C=\{(e_1,\{a/0.2,b/0.2,c/0.2\}),(e_2,\{a/0.2,b/0.2,c/0.2\}),\{e_3,\{a/0.1,b/0.1,c/0.1\})\}$$
$$F^1_{B_1}=\{(e'_1,\{x/0.1,y/0.2,z/0.2\}),(e'_2,\{x/0.1,y/0.2,z/0.2\}),(e'_3,\{x/0.1,y/0.1,z/0.1\})\}$$
$$F^2_{B_2}=\{(e'_1,\{x/0.6,y/0.7,z/0.7\}),(e'_2,\{x/0.6,y/0.7,z/0.7\}),(e'_3,\{x/0.6,y/0.4,z/0.4\})\}$$

Let $\Im=\{\tilde{\varphi},\tilde{E},C_1,C_2\}$ and $\Im^*=\{\tilde{\varphi}^*,\tilde{E}^*,F^1_{B_1},F^2_{B_2}\}$ $\mathrm{RO}(\Im^*)=\{\tilde{\varphi}^*,\tilde{E}^*,F^1_{B_1}\}$

where $\tilde{\varphi}^*=\{(e'_1,\{x/0,y/0,z/0\}),(e'_2,\{x/0,y/0,z/0\}),(e'_3,\{x/0,y/0,z/0\})\}$ $\tilde{E}^*=\{(e'_1,\{x/1,y/1,z/1\}),$ $(e'_2,\{x/1,y/1,z/1\})(e'_3,\{x/1,y/1,z/1\})\}$ $\tilde{\varphi}=\{(e_1,\{a/0,b/0,c/0\})$, $(e_2,\{a/0,b/0,c/0\})$ $(e_3,\{a/0,b/0,c/0\})\}$. $\tilde{E}=\{(e_1,\{a/1,b/1,c/1\}),(e_2,\{a/1,b/1,c/1\}),(e_3,\{a/1,b/1,c/1\})\}$.

Here f is fuzzy soft almost contra continuous and fuzzy soft continuous.

NOTE:
Fuzzy soft continuous does not implies fuzzy soft almost contra continuous.

EXAMPLE 4.7

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let U ={ a,b,c } and $U^*=\{x,y,z\}$ $E=\{e_1,e_2,e_3,e_4\}$. $E^*=\{e'_1,e'_2,e'_3\}$ and $(U,E),(U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho: \mathrm{U} \to \mathrm{U}^*$ and $\psi: \mathrm{E} \to \mathrm{E}^*$ be mappings defined as $\rho(a)=z, \rho(b)=x, \rho(c)=y$ $\psi(e_1)=e'_2, \psi(e_2)=e'_3, \psi(e_3)=e'_1$. Let $A_1=\{e_1,e_2\}$, $A_2=\{e_1,e_2,e_3\}$ $B_1=\{e'_1,e'_2\}$, $B_2=\{e'_1,e'_2,e'_3\}$

Let us consider the fuzzy soft sets F_{A_1}, F_{B_1} in (U,E) and G_{A_1}, G_{B_1} in (U^*,E^*) as,

$$F_{A_1} = \{(e_1,\{a/0.5, b/0.3, c/0.4\}), (e_2,\{a/0.9, b/0.5, c/0.7\}), (e_3,\{a/0.4, b/0.7, c/0.5\})\}$$
$$F_{B_1} = \{(e_1,\{a/0.7, b/0.9, c/0.6\}), (e_2,\{a/0.6, b/0.8, c/0.5\}), (e_3,\{a/0.4, b/0.7, c/0.5\})\}$$
$$G_{A_1} = \{(e_1^{/},\{x/0.7, y/0.5, z/0.4\}), (e_2^{/},\{x/0.3, y/0.4, z/0.5\}), (e_3^{/},\{x/0.5, y/0.7, z/0.9\})\}$$
$$G_{B_1} = \{(e_1^{/}\{x/0.7, y/0.5, z/0.4\}), (e_2^{/},\{0.9, y/0.6, z/0.7\}), (e_3^{/},\{x/0.8, y/0.5, z/0.6\})\}$$

Let $\Im = \{\tilde{\phi}, \tilde{E}, F_{A_1}, F_{B_1}\} \,\&\, \Im^* = \{\tilde{\phi}^*, \tilde{E}^*, G_{A_1}, G_{B_1}\}$.

where $\tilde{\varphi}^* = \{(e_1^{/},\{x/0, y/0, z/0\}), (e_2^{/},\{x/0, y/0, z/0\}), (e_3^{/},\{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e_1^{/},\{x/1, y/1, z/1\}),$
$(e_2^{/},\{x/1, y/1, z/1\})(e_3^{/},\{x/1, y/1, z/1\})\}$

$$\tilde{\varphi} = \{(e_1,\{a/0, b/0, c/0\}), (e_2,\{a/0, b/0, c/0\})\ (e_3,\{a/0, b/0, c/0\})\}.$$
$$\tilde{E} = \{(e_1,\{a/1, b/1, c/1\}), (e_2,\{a/1, b/1, c/1\}), (e_3,\{a/1, b/1, c/1\})\}.$$

Here f is fuzzy soft continuous does not implies fuzzy soft almost contra continuous.

THEOREM 4.8

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $\rho: U \to U^*$ and $\psi: E \to E^*$ be mappings and $f = (\rho,\psi): (U,E) \to (U^*,E^*)$ be a fuzzy soft mapping.

If f is fuzzy soft almost contra continuous then f is fuzzy soft contra continuous.

Proof:

Let f be fuzzy soft almost contra continuous and let $H_B \in RO(\Im^*)$.

since regular open set is open, H_B is open.

Since f is fuzzy soft almost contra continuous, $f^{-1}(H_B)$ is closed in $(U,E,\Im)$.

Therefore f is fuzzy soft contra continuous.

EXAMPLE 4.9

Let $(U,E,\Im)$ and $(U^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $U = \{a,b,c\}$ and $U^* = \{x,y,z\}$ $E = \{e_1, e_2, e_3, e_4\}$. $E^* = \{e_1', e_2', e_3'\}$ and $(U,E), (U^*,E^*)$ classes of fuzzy soft sets.

Let $\rho: U \to U^*$ and $\psi: E \to E^*$ be mappings defined as $\rho(a) = z, \rho(b) = y, \rho(c) = y$ $\psi(e_1) = e_1^{/}, \psi(e_2) = e_1^{/}, \psi(e_3) = e_3^{/}$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e_1^{/}, e_2^{/}\}$, $B_2 = \{e_1^{/}, e_2^{/}, e_3^{/}\}$

Let us consider the fuzzy soft sets C_1, C_2 in (U,E) and $F_{B_1}^1, F_{B_2}^2$ in (U^*,E^*) as,

$$C_1 = \{(e_1,\{a/0.7, b/0.7, c/0.7\}), (e_2,\{a/0.7, b/0.7, c/0.7\}), (e_3,\{a/0.4, b/0.4, c/0.4\})\}$$

$C_1^C = \{(e_1,\{a/0.3, b/0.3, c/0.3\}), (e_2,\{a/0.3, b/0.3, c/0.3\}), (e_3,\{a/0.6, b/0.6, c/0.6\})\}$
$C_2 = \{(e_1,\{a/0.8, b/0.8, c/0.8\}), (e_2,\{a/0.8, b/0.8, c/0.8\}), \{e_3,\{a/0.9, b/0.9, c/0.9\})\}$
$C_2^C = \{(e_1,\{a/0.2, b/0.2, c/0.2\}), (e_2,\{a/0.2, b/0.2, c/0.2\}), \{e_3,\{a/0.1, b/0.1, c/0.1\})\}$
$F_{B_1}^1 = \{(e_1^/,\{x/0.1, y/0.2, z/0.2\}), (e_2^/,\{x/0.1, y/0.2, z/0.2\}), (e_3^/,\{x/0.1, y/0.1, z/0.1\})\}$
$F_{B_2}^2 = \{(e_1^/,\{x/0.6, y/0.7, z/0.7\}), (e_2^/,\{x/0.6, y/0.7, z/0.7\}), (e_3^/,\{x/0.6, y/0.4, z/0.4\})\}$

Let $\Im = \left\{\tilde{\varphi}, \tilde{E}, C_1, C_2\right\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, F_{B_1}^1, F_{B_2}^2\}$ $\quad RO(\Im^*) = \left\{\tilde{\varphi}^*, \tilde{E}^*, F_{B_1}^1\right\}$

where $\tilde{\varphi}^* = \{(e_1^/,\{x/0, y/0, z/0\}), (e_2^/,\{x/0, y/0, z/0\}), (e_3^/,\{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e_1^/,\{x/1, y/1, z/1\}),$
$(e_2^/,\{x/1, y/1, z/1\})(e_3^/,\{x/1, y/1, z/1\})\}$ $\quad \tilde{\varphi} = \{(e_1,\{a/0, b/0, c/0\})$, $(e_2,\{a/0, b/0, c/0\})$

$\tilde{E} = \{(e_1, \{a/1, b/1, c/1\}), (e_2, \{a/1, b/1, c/1\}), (e_3, \{a/1, b/1, c/1\})\}$.

Here f is fuzzy soft almost contra continuous and fuzzy soft contra continuous.

NOTE
Fuzzy soft contra continuous does not implies fuzzy soft almost contra continuous.

EXAMPLE 4.10

Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let $U = \{a, b, c\}$ and $U^* = \left\{x, y, z\right\}$ $E = \left\{e_1, e_2, e_3, e_4\right\}$. $E^* = \left\{e_1', e_2', e_3'\right\}$ and $(U, E), (U^*, E^*)$ classes of fuzzy soft sets.

Let $\rho: U \to U^*$ and $\psi: E \to E^*$ be mappings defined as $\rho(a) = z, \rho(b) = x, \rho(c) = y$ $\psi(e_1) = e_2^/, \psi(e_2) = e_3^/, \psi(e_3) = e_1^/$. Let $A_1 = \left\{e_1, e_2\right\}$, $A_2 = \left\{e_1, e_2, e_3\right\}$ $B_1 = \left\{e_1^/, e_2^/\right\}$, $B_2 = \left\{e_1^/, e_2^/, e_3^/\right\}$

Let us consider the fuzzy soft sets F_A, F_B in (U, E) and G_A, G_B in (U^*, E^*) as,

$F_A = \{(e_1,\{a/0.9, b/0.4, c/0.6\}), (e_2,\{a/0.6, b/0.9, c/0.8\}), (e_3,\{a/0.7, b/0.5, c/0.8\})\}$
$F_A^C = \{(e_1,\{a/0.1, b/0.6, c/0.4\}), (e_2,\{a/0.4, b/0.1, c/0.2\}), (e_3,\{a/0.3, b/0.5, c/0.2\})\}$
$F_B = \{(e_1,\{a/0.7, b/0.6, c/0.3\}), (e_2,\{a/0.9, b/0.1, c/0.5\}), (e_3,\{a/0.4, b/0.2, c/0.8\})\}$
$F_B^C = \{(e_1,\{a/0.3, b/0.4, c/0.7\}), (e_2,\{a/0.1, b/0.9, c/0.5\}), (e_3,\{a/0.6, b/0.8, c/0.2\})\}$
$G_A = \{(e_1^/,\{x/0.5, y/0.2, z/0.3\}), (e_2^/,\{x/0.6, y/0.4, z/0.1\}), (e_3^/,\{x/0.1, y/0.2, z/0.4\})\}$
$G_B = \{(e_1^/,\{x/0.8, y/0.2, z/0.6\}), (e_2^/,\{x/0.4, y/0.7, z/0.3\}), (e_3^/,\{x/0.9, y/0.5, z/0.1\})\}$

Let $\Im = \{\tilde{\varphi}, \tilde{E}, F_A, F_B\}$ and $\Im^* = \{\tilde{\varphi}^*, \tilde{E}^*, G_A, G_B\}$

where $\tilde{\varphi}^* = \{(e_1^{/}, \{x/0, y/0, z/0\}), (e_2^{/}, \{x/0, y/0, z/0\}), (e_3^{/}, \{x/0, y/0, z/0\})\}$ $\tilde{E}^* = \{(e_1^{/}, \{x/1, y/1, z/1\}), (e_2^{/}, \{x/1, y/1, z/1\})(e_3^{/}, \{x/1, y/1, z/1\})\}$

$\tilde{\varphi} = \{(e_1, \{a/0, b/0, c/0\}), (e_2, \{a/0, b/0, c/0\})\ (e_3, \{a/0, b/0, c/0\})\}.$

$\tilde{E} = \{(e_1, \{a/1, b/1, c/1\}), (e_2, \{a/1, b/1, c/1\}), (e_3, \{a/1, b/1, c/1\})\}.$

Here f is fuzzy soft contra continuous but not fuzzy soft almost contra continuous.

THEOREM 4.11
Every restriction of an fuzzy soft almost contra continuous mapping is fuzzy soft almost contra continuous.

Proof:
Let f be a fuzzy soft almost contra continuous mapping of (U, E) into (U^*, E^*).
Let F_B be any subset of (U, E) .

For any regularly open subset G_B of (U^*, E^*) .

$(f / F_B)^{-1}(G_B) = (F_B) \cap f^{-1}(G_B)$

But f being fuzzy soft almost contra continuous , and hence $f^{-1}(G_B)$ is closed and

Therefore $F_B \cap f^{-1}(G_B)$ is relatively closed subset of F_B .

That implies $(f / F_B)^{-1}(G_B)$ is closed subset of F_B.

Hence $f / (F_B)$ is fuzzy soft almost contra continuous.

EXAMPLE 4.12
Let $(U, E, \Im)$ and $(U^*, E^*, \Im^*)$ be fuzzy soft topological spaces. Let U ={ a, b, c } and $U^* = \{x, y, z\}$ $E = \{e_1, e_2, e_3, e_4\}$. $E^* = \{e_1', e_2', e_3'\}$ and $(U, E), (U^*, E^*)$ classes of fuzzy soft sets.

Let $\rho : \mathrm{U} \rightarrow \mathrm{U}^*$ and $\psi : \mathrm{E} \rightarrow \mathrm{E}^*$ be mappings defined as $\rho(a) = z, \rho(b) = y, \rho(c) = y$ $\psi(e_1) = e_1^{/}, \psi(e_2) = e_1^{/}, \psi(e_3) = e_3^{/}$. Let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_1, e_2, e_3\}$ $B_1 = \{e_1^{/}, e_2^{/}\}$, $B_2 = \{e_1^{/}, e_2^{/}, e_3^{/}\}$

Let us consider the fuzzy soft sets C_1 , C_2 in (U, E) and $F_{B_1}^1, F_{B_2}^2$ in (U^*, E^*) as,

$$C_1 = \{(e_1, \{a/0.7, b/0.7, c/0.7\}), (e_2, \{a/0.7, b/0.7, c/0.7\}), (e_3, \{a/0.4, b/0.4, c/0.4\})\}$$
$$C_1^C = \{(e_1, \{a/0.3, b/0.3, c/0.3\}), (e_2, \{a/0.3, b/0.3, c/0.3\}), (e_3, \{a/0.6, b/0.6, c/0.6\})\}$$

$C_2=\{(e_1,\{a/0.8,b/0.8,c/0.8\}),(e_2,\{a/0.8,b/0.8,c/0.8\}),\{e_3,\{a/0.9,b/0.9,c/0.9\})\}$

$C_2^C=\{(e_1,\{a/0.2,b/0.2,c/0.2\}),(e_2,\{a/0.2,b/0.2,c/0.2\}),\{e_3,\{a/0.1,b/0.1,c/0.1\})\}$

$F_{B_1}^1=\{(e_1^{/},\{x/0.1,y/0.2,z/0.2\}),(e_2^{/},\{x/0.1,y/0.2,z/0.2\}),(e_3^{/},\{x/0.1,y/0.1,z/0.1\})\}$

$F_{B_2}^2=\{(e_1^{/},\{x/0.6,y/0.7,z/0.7\}),(e_2^{/},\{x/0.6,y/0.7,z/0.7\}),(e_3^{/},\{x/0.6,y/0.4,z/0.4\})\}$

Let $\Im=\{\tilde{\varphi},\tilde{E},C_1,C_2\}$ and $\Im^*=\{\tilde{\varphi}^*,\tilde{E}^*,F_{B_1}^1,F_{B_2}^2\}$ $RO(\Im^*)=\{\tilde{\varphi}^*,\tilde{E}^*,F_{B_1}^1\}$

where $\tilde{\varphi}^*=\{(e_1^{/},\{x/0,y/0,z/0\}),(e_2^{/},\{x/0,y/0,z/0\}),(e_3^{/},\{x/0,y/0,z/0\})\}$ $\tilde{E}^*=\{(e_1^{/},\{x/1,y/1,z/1\}),$ $(e_2^{/},\{x/1,y/1,z/1\})(e_3^{/},\{x/1,y/1,z/1\})\}$ $\tilde{\varphi}=\{(e_1,\{a/0,b/0,c/0\})$, $(e_2,\{a/0,b/0,c/0\})$ $(e_3,\{a/0,b/0,c/0\})\}$.
$\tilde{E}=\{(e_1,\{a/1,b/1,c/1\}),\ (e_2,\{a/1,b/1,c/1\}),\ (e_3,\{a/1,b/1,c/1\})\}$.

The Restriction of function f is $f/(C_2):(C_2)\to(U^*,E^*)$ defined by the inverse image under $f/(C_2)$ of any $F_{B_1}^1\in RO(\Im^*)$ is a fuzzy soft set $F_S\in\Im$, that is $(f/(C_2)^{-1}(F_{B_1}^1)\in\Im$

Whenever $F_{B_1}^1\in RO(\Im^*)$. Now $(f/(C_2))^{-1}(F_{B_1}^1)=C_2^C$.

Therefore restriction of f is fuzzy soft contra continuous.

V. Conclusion

Fuzzy soft sets are very popular subject for researchers. In this paper the notions of fuzzy soft contra continuous mapping and fuzzy soft almost contra continuous mapping have been introduced and some related theorems have been established. We have put forward some examples to illustrate our notions.

References

[1] Ahmad B. and Kharal A, " ", Advances in Fuzzy Systems,Volume 2009,Article ID 586507,pp.1-6.

[2] Banashree Bora, " ",International Journal for Basic Sciences and Social Sciences (IJBSS) ISSN:2319-2968,Vol.1,Issue 2, August-2012 pp.50-64.

[3] Dontchev .J,"Contra Continuous Functions and Strongly S-closed Spaces",International Journal of math.math.Sci 19 (1996),303-310.

[4] Erdal Ekici,"Almost Contra- Precontinuous Functions",Bulletin of the Malaysian Mathematical Sciences Society (Second Series) 27 (2004) pp.53-65.

[5] Joseph J.E and Kwack M.H (1980)"On S-closed Spaces",Proc.Amer.math.Soc.No.80,pp.341-348.

[6] Maji P.K. and Roy A.R., " ",Computers and Mathematics with Applications 45, 2003,pp. 555-562.

[7] Maji P.K., Biswas R. and Roy A.R., " ", Journal of Fuzzy Mathematics, Vol 9,no.3.pp. 2001,589-602,

[8] Molodstov, D.A., " - ", Computers and Mathematics with Applications, Vol. 37,pp. 19-31,1999.

[9] NeogT.j., Sut D.K., " ", International Journal of latest Trends in computing, Vol 2,No 3,September 2011,pp.439-450.

[10] Neog T.J.,Sut D.K. and Hazarika G.C., " ", International journal of Latest trends in Mathematics,Vol.2, No.1,March 2012,pp.54-67.

[11] Ponselvakumari A.,and Selvi R.," Fuzzy Soft Almost Continuous Mapping",Global Journal of Pure and Applied Mathematics ,Volume 10,Number 3 (2014),pp.337-347.

[12] Ponselvakumari A.,and Selvi R.," Fuzzy Soft Pre Continuous Mapping", International journal of Mathematical Science and Applications(paper Accepted)

[13] Ponselvakumari A.,and Selvi R.," Fuzzy Soft Semi Continuous Mapping", International journal of Organisations and Scientific Research, Volume 10,Issue 6 Ver.II(Nov-Dec.2014),PP01-09.

[14] Roy S. and SamantaT.k., " ", Annals of Fuzzy Mathematics and Informatics ,2011.

[15] Singal M.K., and Asha Rani Singal., " ",Yokohama Math.3(1968), 63-73.

[16] Tanay B. and Kandemir M. Burc, " ", Computer and Mathematics with Applications 61 (2011)pp.2952-2957.

[17] TugbahanSimsekler.,SaziyeYuksel., " " , Annals of Fuzzy Mathematics and Informatics ,Vol 5,No.1,(January 2013),pp.87-96.

Authors

First Author – A.Ponselvakumari. M.Sc. Mphil University V. O. Chidambaranar College of Engineering, Anna University:Tuticorin Campus e:mail : Ponselvakumari.p@gmail.com
Second Author – Dr.R.Selvi, M.Sc, M.Phil. Ph.D, Sri ParasakthiCollege for women,.Courtallam., MS University. Tirunelveli e-mail :r.selvimuthu@gmail.com