Stochastic Analysis of a Single Unit with a Protective Unit Discrete Parametric Markov Chain System Model

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Abstract- The paper deals with cost benefit analysis of a single unit (main unit) system model supported by a helping unit. Main unit has three possible modes – normal (N), partial failure (P) and total failure (F). Whereas the helping unit (protective unit) has two possible modes- normal (N) and total failure (F). The main unit can also work without the protective (helping) unit but with increased failure rate. The system is declared as failed when at least main unit is out of functioning. A single repairman is always available with the system to repair the failed main and protective (helping) unit. The failure and repair times of main unit and protective (helping) unit are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Index Terms- Regenerative point, reliability, MTSF, availability of system, busy period of repairman, net expected profit.

I. Introduction

In the literature of reliability various authors including [1,2,4,8,9,11,12] have analyzed reparable system model with one or more units by assuming different concepts such as three possible modes, slow switch, imperfect switch, waiting time of repairman, two types of repairmen, inspection of a failed unit and pre-emptive repeat priority. They assumed continuous distributions of time to failure and time to repair of a unit. But very few authors Gupta et al [5,6] have obtained the reliability characteristics of redundant system models by considering discrete distributions of failure and repair times. Single unit system models with helping unit having two modes-Normal and Total failure have been studies by few authors including [3,7,10] in the field of reliability theory. The role of helping (protective) unit is to protect the failure of main unit. The main unit may also work without the protective (helping) unit but with increased failure rate. Various measures of system effectiveness are obtained by assuming the failure and repair times as continuous random variables.

The purpose of the present paper is to analyze a single unit (main unit) system model with helping (protective) unit. The main unit has three possible modes- normal (N), partial failure (P) and total failure (F). The system model is analyzed under discrete parametric Markov-Chain i.e. time to failure and repairs are taken as discrete random variables. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availability of the system as well as expected up time of the system upto the epoch (t-1).
iv) Expected busy period of the repairman due to main unit and helping unit upto the epoch (t-1).
v) Net expected profit incurred by the system upto time epoch (t-1) and in steady-state is obtained.

II. MODEL DESCRIPTION AND ASSUMPTIONS

i) The system comprises of a single unit (main unit) is supported by a helping (protective) unit. Initially both the units are operative.
ii) The helping unit is used to protect the failure of main unit. Main unit has three possible modes – normal (N), partial failure (P) and total failure (F), whereas the helping unit has only two possible modes- normal (N) and total failure (F).
iii) The main unit can’t enter into F-mode without passing through the P-mode.
iv) When the helping unit fails, the main unit can also operate with increased failure rate in both N-mode and P-mode.
v) The system stops functioning when main unit is failed and helping unit is good because the operation of helping unit in this situation is worthless.
vi) A single repairman is always available with the system to repair the failed main unit and helping unit. The main unit is repaired only when it enters into F-mode.
vii) The priority in repair is being given to the main unit over the helping unit. Therefore, if during the repair of helping unit the main unit enters into F-mode then main unit is taken up for repair discontinuing the repair of helping unit.
viii) Each repaired unit (main or helping) works as good as new.
ix) Failure and repair times of the units follow independent geometric distributions with different parameters.

III. NOTATIONS AND STATES OF THE SYSTEM

a) Notations :

$p_i q_i^x$ **:** p.m.f. of failure time of main unit from N-mode to P-mode and P-mode to F-mode respectively for i=1, 2 when helping unit is operative $p_i + q_i = 1$.

$p_i' q_i'^x \left(p_i' > p_i\right)$ **:** p.m.f. of failure time of main unit from N-mode to P-mode and P-mode to F-mode respectively for i=1, 2 when helping unit is failed $p_i' + q_i' = 1$.

$p_3 q_3^x$ **:** p.m.f. of failure time of helping unit $p_3 + q_3 = 1$.

$r_i s_i^x$ **:** p.m.f. of repair time of Main and helping unit respectively for i=1, 2 and $r_i + s_i = 1$.

$q_{ij}(g), Q_{ij}(g)$: p.m.f. and C.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : Steady state transition probability from state S_i to S_j.

$p_{ij} = Q_{ij}(\infty)$

$Z_i(t)$: Probability that the system sojourn in state S_i up to epochs 0, 1, 2,…, (t-1).

ψ_i : Mean sojourn time in state S_i.

$*, h$: Symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

b) Symbols for the states of the systems:

MN_0 / MP_0 **:** Main unit is in N-mode/P- mode and operative.

HN_o / HN_g **:** Helping unit is in normal (N) mode and operative/good.

MF_r **:** Main unit is in total failure (F) mode and under repair.

HF_r / HF_w : Helping unit is in total failure (F) mode and under repair/waits for repair.

With the help of above symbols the possible states of the system are as follows:

$S_0 \equiv (MN_O, HN_O)$, $S_1 \equiv (MN_O, HF_r)$ $S_2 \equiv (MP_O, HN_O)$

$S_3 \equiv (MP_0, HF_r)$, $S_4 \equiv (MF_r, HN_g)$, $S_5 \equiv (MF_r, HF_w)$

The transition diagram of the system model alongwith the transition rates is shown in Fig. 1.

TRANSITION DIAGRAM

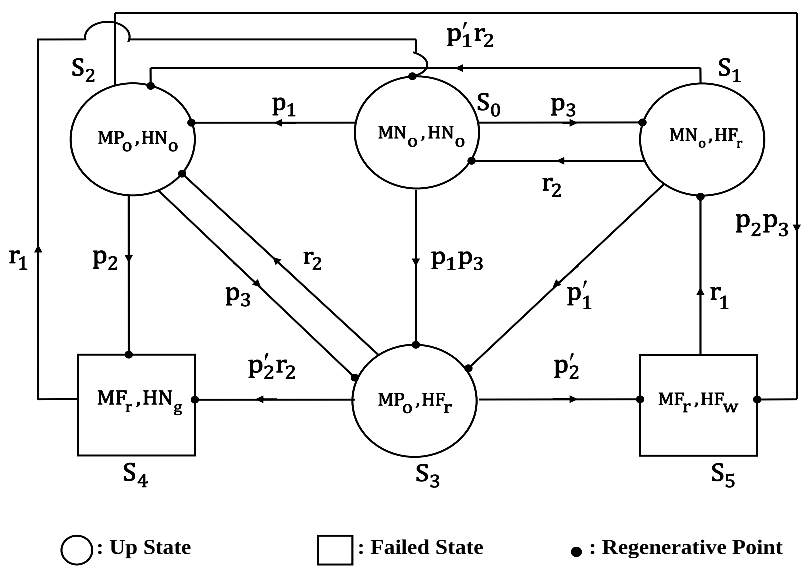

Fig.1

IV. TRANSITION PROBABILITIES

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have

$$Q_{01}(t) = \frac{p_1q_3}{1-q_1q_3}\left[1-(q_1q_3)^{t+1}\right], \qquad Q_{02}(t) = \frac{q_1p_3}{1-q_1q_3}\left[1-(q_1q_3)^{t+1}\right]$$

$$Q_{03}(t) = \frac{p_1p_3}{1-q_1q_3}\left[1-(q_1q_3)^{t+1}\right], \qquad Q_{10}(t) = \frac{q_1'r_2}{1-q_1's_2}\left[1-(q_1's_2)^{t+1}\right]$$

$$Q_{12}(t) = \frac{p_1'r_2}{1-q_1's_2}\left[1-(q_1's_2)^{t+1}\right], \qquad Q_{13}(t) = \frac{p_1's_2}{1-q_1's_2}\left[1-(q_1's_2)^{t+1}\right]$$

$$Q_{23}(t) = \frac{q_2p_3}{1-q_2q_3}\left[1-(q_2q_3)^{t+1}\right], \qquad Q_{24}(t) = \frac{p_2q_3}{1-q_2q_3}\left[1-(q_2q_3)^{t+1}\right]$$

$$Q_{25}(t) = \frac{p_2p_3}{1-q_2q_3}\left[1-(q_2q_3)^{t+1}\right], \qquad Q_{32}(t) = \frac{q_2'r_2}{1-q_2's_2}\left[1-(q_2's_2)^{t+1}\right]$$

$$Q_{34}(t) = \frac{p_2'r_2}{1-q_2's_2}\left[1-(q_2's_2)^{t+1}\right], \qquad Q_{35}(t) = \frac{p_2's_2}{1-q_2's_2}\left[1-(q_2's_2)^{t+1}\right]$$

$$Q_{40}(t) = 1-s_1^{t+1}, \quad Q_{51}(t) = 1-s_1^{t+1} \qquad (1\text{-}14)$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-14) by taking $t \to \infty$, as follows:

$$p_{01} = \frac{p_1q_3}{1-q_1q_3}, \quad p_{02} = \frac{q_1p_3}{1-q_1q_3}, \quad p_{03} = \frac{p_1p_3}{1-q_1q_3}, \quad p_{10} = \frac{q_1'r_2}{1-q_1's_2}$$

$$p_{12} = \frac{p_1'r_2}{1-q_1's_2}, \quad p_{13} = \frac{p_1's_2}{1-q_1's_2}, \quad p_{23} = \frac{q_2p_3}{1-q_2q_3}, \quad p_{24} = \frac{p_2q_3}{1-q_2q_3}$$

$p_{25} = \frac{p_2 p_3}{1-q_2 q_3}$, $p_{32} = \frac{q_2' r_2}{1-q_2' s_2}$, $p_{34} = \frac{p_2' r_2}{1-q_2' s_2}$, $p_{35} = \frac{p_2' s_2}{1-q_2' s_2}$

$p_{40} = 1$, $p_{51} = 1$

We observe that the following relations hold-

$$p_{01}+p_{02}+p_{03}=1,\quad p_{10}+p_{12}+p_{13}=1,\quad p_{23}+p_{24}+p_{25}=1$$

$$p_{32}+p_{34}+p_{35}=1,\quad p_{40}=p_{51}=1 \qquad (15\text{-}19)$$

V. MEAN SOJOURN TIMES

Let T_i be the sojourn time in state S_i (i=0, 1, 2, 3, 4, 5), then mean sojourn time ψ_i in state S_i is given by

$$\psi_i = E(T_i) = \sum_{t=1}^{\infty} P[T_i \geq t]$$

In particular,

$$\psi_0 = \frac{q_1 q_3}{1-q_1 q_3},\quad \psi_1 = \frac{q_1' s_2}{1-q_1' s_2},\quad \psi_2 = \frac{q_2 q_3}{1-q_2 q_3}$$

$$\psi_3 = \frac{q_2' s_2}{1-q_2' s_2},\quad \psi_4 = \psi_5 = \frac{s_1}{r_1} = \psi \text{ (say)} \qquad (20\text{-}24)$$

VI. METHODOLOGY FOR DEVELOPING EQUATIONS

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed states S_4 and S_5 as absorbing state. Now, the expression for $R_i(t)$; i=0, 1, 2, 3; we have the following set of convolution equations.

$$R_0(t) = q_1^t q_3^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u) + \sum_{u=0}^{t-1} q_{02}(u) R_2(t-1-u) + \sum_{u=0}^{t-1} q_{03}(u) R_3(t-1-u)$$

$$= Z_0(t) + q_{01}(t-1) © R_1(t-1) + q_{02}(t-1) © R_2(t-1) + q_{03}(t-1) © R_3(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{12}(t-1) © R_2(t-1) + q_{13}(t-1) © R_3(t-1)$$

$$R_2(t) = Z_2(t) + q_{23}(t-1) © R_3(t-1)$$

$$R_3(t) = Z_3(t) + q_{31}(t-1) © R_1(t-1) \qquad (25\text{-}28)$$

Where,

$Z_1(t) = q_1'^t s_2^t$, $Z_2(t) = q_2'^t q_3^t$, $Z_3(t) = q_2'^t s_2^t$

b) Availability of the system-

Let $A_i^N(t)$ and $A_i^P(t)$ be the respective probabilities that the system is up in normal mode and partial failure mode of main unit at epoch (t-1), when system initially starts from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i^j(t)$; i=0 to 5 and j=N, P.

$$A_0^j(t) = \delta Z_0(t) + q_{01}(t-1) © A_1^j(t-1)$$

$$A_1^j(t)=\delta Z_1(t)+q_{10}(t-1)©A_0^j(t-1)+q_{12}(t-1)©A_2^j(t-1)+q_{13}(t-1)©A_3^j(t-1)$$
$$A_2^j(t)=(1-\delta)Z_2(t)+q_{23}(t-1)©A_3^j(t-1)+q_{24}(t-1)©A_4^j(t-1)+q_{25}(t-1)©A_5^j(t-1)$$
$$A_3^j(t)=(1-\delta)Z_3(t)+q_{32}(t-1)©A_2^j(t-1)+q_{34}(t-1)©A_4^j(t-1)+q_{35}(t-1)©A_5^j(t-1)$$
$$A_4^j(t)=q_{40}(t-1)©A_0^j(t-1)$$
$$A_5^j(t)=q_{51}(t-1)©A_1^j(t-1) \qquad (29\text{-}34)$$

Where,

$\delta=1$ and 0 respectively for j=N and P. The values of $Z_i(t)$; i=0 to 3 are same as given in section 6(a).

c) Busy period of repairman-

Let $B_i^M(t)$ and $B_i^H(t)$ be the respective probabilities that the repairman is busy at epoch (t-1) in the repair of failed main unit and repair of a failed helping unit when system initially starts from state S_i. Using simple probabilistic arguments as in case of reliability and availability analysis, the relations for $B_i^k(t)$; i=0 to 5 and k=M, H can be easily developed as below.

$$B_0^k(t)=q_{01}(t-1)©B_1^k(t-1)+q_{02}(t-1)©B_2^k(t-1)+q_{03}(t-1)©B_3^k(t-1)$$
$$B_1^k(t)=(1-\delta)Z_1(t)+q_{10}(t-1)©B_0^k(t-1)+q_{12}(t-1)©B_2^k(t-1)+q_{13}(t-1)©B_3^k(t-1)$$
$$B_2^k(t)=q_{21}(t-1)©B_1^k(t-1)+q_{23}(t-1)©B_3^k(t-1)+q_{24}(t-1)©B_4^k(t-1)$$
$$B_3^k(t)=(1-\delta)Z_3(t)+q_{32}(t-1)©B_2^k(t-1)+q_{34}(t-1)©B_4^k(t-1)+q_{35}(t-1)©B_5^k(t-1)$$
$$B_4^k(t)=\delta Z_4(t)+q_{40}(t-1)©B_0^k(t-1)$$
$$B_5^k(t)=\delta Z_5(t)+q_{51}(t-1)©B_1^k(t-1) \qquad (35\text{-}40)$$

Where,

$\delta=1$ and 0 respectively for k=M and H. The values of $Z_i(t)$; i=1, 3 are same as given in section 6(a) and $Z_4(t)=Z_5(t)=s_1^t$.

VII. ANALYSIS OF RELIABILITY AND MTSF

Taking geometric transforms of (25-28) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h)=\frac{N_1(h)}{D_1(h)} \qquad (41)$$

Where,

$$N_1(h)=Z_0^*\left[1-h^2q_{23}^*q_{32}^*\right]\left(Z_0^*+hq_{01}^*Z_1^*\right)+\left[hq_{01}^*\left(hq_{12}^*+h^2q_{13}^*q_{32}^*\right)+hq_{02}^*+h^2q_{03}^*q_{32}^*\right]Z_2^*$$
$$+\left[hq_{01}^*\left(h^2q_{12}^*q_{23}^*+hq_{13}^*\right)+h^2q_{02}^*q_{23}^*+hq_{03}^*\right]Z_3^*$$
$$D_1(h)=1-h^2q_{23}^*q_{32}^*-h^2q_{10}^*q_{01}^*\left(1-h^2q_{23}^*q_{32}^*\right)-h^2q_{12}^*q_{23}^*\left(hq_{31}^*+h^2q_{34}^*q_{41}^*\right)$$

Collecting the coefficient of h^t from expression (41), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}\sum_{t=1}^{\infty}h^tR(t)=\frac{N_1(1)}{D_1(1)}-1 \qquad (42)$$

Where,

$$N_1(1)=(1-p_{34}p_{43})(\psi_0+p_{01}\psi_1)+\left[(p_{12}+p_{13}p_{32})+p_{02}+p_{03}p_{32}\right]\psi_2+\left[p_{01}(p_{13}+p_{12}p_{23})+p_{02}p_{23}+p_{03}\right]\psi_3$$
$$D_1(1)=(1-p_{23}p_{32})(1-p_{01}p_{10})$$

VIII. AVAILABILITY ANALYSIS

On taking geometric transforms of (29-34) and simplifying the resulting equations for j=N and P we get

$$A_0^{N*}(h) = N_2(h)/D_2(h) \quad \text{and} \quad A_0^{P*}(h) = N_3(h)/D_2(h) \qquad (43\text{-}44)$$

Where,

$$N_2(h) = Z_0^*\left[1-h^2q_{23}^*q_{32}^* - h^2q_{51}^*q_{12}^*\left(h^2q_{23}^*q_{35}^* + hq_{25}^*\right) - h^2q_{51}^*q_{13}^*\left(hq_{35}^* + h^2q_{32}^*q_{25}^*\right)\right]$$
$$+Z_1^*\left[hq_{01}^*\left(1-h^2q_{23}^*q_{32}^*\right) + h^2q_{02}^*q_{51}^*\left(h^2q_{23}^*q_{35}^* + hq_{25}^*\right) + h^2q_{03}^*q_{51}^*\left(hq_{35}^* + h^2q_{32}^*q_{25}^*\right)\right]$$
$$N_3(h) = Z_2^*\left[hq_{01}^*\left(hq_{12}^* + h^2q_{13}^*q_{32}^*\right) + hq_{02}^*\left(1-h^3q_{13}^*q_{35}^*q_{51}^*\right) + hq_{03}^*\left(hq_{32}^* + h^3q_{35}^*q_{51}^*q_{12}^*\right)\right]$$
$$+Z_3^*\left[hq_{01}^*\left(hq_{13}^* + h^2q_{12}^*q_{23}^*\right) + hq_{02}^*\left(hq_{23}^* + h^3q_{25}^*q_{51}^*q_{13}^*\right) + hq_{03}^*\left(1-h^3q_{12}^*q_{25}^*q_{51}^*\right)\right]$$

and

$$D_2(h) = 1-h^2q_{23}^*q_{32}^* - h^2q_{51}^*q_{12}^*\left(h^2q_{23}^*q_{35}^* + hq_{25}^*\right) - h^2q_{51}^*q_{13}^*\left(hq_{35}^* + h^2q_{32}^*q_{25}^*\right)$$
$$-q_{10}^*\left[hq_{01}^*\left(1-h^2q_{23}^*q_{32}^*\right) + h^2q_{02}^*q_{51}^*\left(h^2q_{23}^*q_{35}^* + hq_{25}^*\right) + h^2q_{03}^*q_{51}^*\left(hq_{35}^* + h^2q_{32}^*q_{25}^*\right)\right]$$
$$-hq_{40}^*\left[hq_{01}^*\left(hq_{12}^*\left(h^2q_{23}^*q_{34}^* + hq_{24}^*\right) + hq_{13}^*\left(h^2q_{32}^*q_{24}^* + hq_{34}^*\right)\right) + hq_{02}^*\left(\left(h^2q_{23}^*q_{34}^*\right.\right.\right.$$
$$\left.\left.\left.+hq_{24}^*\right) + h^2q_{51}^*q_{13}^*\left(h^2q_{25}^*q_{34}^* - h^2q_{24}^*q_{35}^*\right)\right) + hq_{03}^*\left(\left(hq_{34}^* + h^2q_{32}^*q_{24}^*\right) + h^2q_{51}^*q_{12}^*\left(h^2q_{24}^*q_{35}^* - h^2q_{25}^*q_{34}^*\right)\right)\right]$$

The steady state availabilities of the system due to operation of the system in N-mode and P-mode of main unit are given by-

$$A_0^N = \lim_{t\to\infty} A_0^N(t) = \lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

and

$$A_0^P = \lim_{t\to\infty} A_0^P(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

Now since $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule we get

$$A_0^N = -N_2(1)/D_2'(1) \quad \text{and} \quad A_0^P = -N_3(1)/D_2'(1) \qquad (45\text{-}46)$$

Where,

$$N_2(1) = \psi_0\left[1-p_{23}p_{32} - p_{12}\left(p_{23}p_{35} + p_{25}\right) - p_{13}\left(p_{35} + p_{32}p_{25}\right)\right]1$$
$$+\psi_1\left[1-p_{23}p_{32} - p_{02}\left(p_{23}p_{34} + p_{24}\right) - p_{03}\left(p_{34} + p_{32}p_{24}\right)\right]$$
$$N_3(1) = \psi_2\left[1-p_{01}p_{10} - p_{34}\left(p_{03} + p_{01}p_{13}\right) - p_{35}\left(p_{13} + p_{10}p_{03}\right)\right]$$
$$+\psi_3\left[1-p_{01}p_{10} - p_{24}\left(p_{02} + p_{01}p_{12}\right) - p_{25}\left(p_{12} + p_{10}p_{02}\right)\right]$$

and

$$D_2'(1) = -\left[\psi_0\left(1-p_{23}p_{32} - p_{12}\left(p_{23}p_{35} + p_{25}\right) - p_{13}\left(p_{35} + p_{32}p_{25}\right)\right) + \psi_1\left(1-p_{23}p_{32} - p_{02}\left(p_{23}p_{34}\right.\right.\right.$$
$$\left.\left.+p_{23}p_{34}\right) - p_{03}\left(p_{34} + p_{32}p_{24}\right)\right) + \psi_2\left(1-p_{01}p_{10} - p_{34}\left(p_{03} + p_{01}p_{13}\right) - p_{35}\left(p_{13} + p_{10}p_{03}\right)\right)$$
$$+\psi_3\left(1-p_{01}p_{10} - p_{24}\left(p_{02} + p_{01}p_{12}\right) - p_{25}\left(p_{12} + p_{10}p_{02}\right)\right) + \psi\left(\left(p_{12} + p_{10}p_{02}\right)\left(p_{24} + p_{23}p_{34}\right)\right.$$
$$\left.\left.+\left(p_{13} + p_{10}p_{03}\right)\left(p_{34} + p_{32}p_{24}\right) + \left(p_{01}p_{12} + p_{02}\right)\left(p_{25} + p_{23}p_{35}\right) + \left(p_{01}p_{13} + p_{03}\right)\left(p_{35} + p_{32}p_{25}\right)\right)\right]$$

Now the expected uptime of the system in N- mode and P-mode of main unit up to epoch (t-1) are respectively given by

$$\mu_{up}^N(t) = \sum_{x=0}^{t-1} A_0^N(x) \quad \text{and} \quad \mu_{up}^P(t) = \sum_{x=0}^{t-1} A_0^P(x)$$

so that

$$\mu_{up}^{N*}(h) = A_0^{N*}(h)/(1-h) \quad \text{and} \quad \mu_{up}^{P*}(h) = A_0^{P*}(h)/(1-h) \qquad (47\text{-}48)$$

IX. BUSY PERIOD ANALYSIS

On taking geometric transform of (35-40) and simplifying the resulting equations for k=M and H we get

$$B_0^{M*}(h)=\frac{N_4(h)}{D_2(h)} \quad \text{and} \quad B_0^{H*}(h)=\frac{N_5(h)}{D_2(h)} \qquad (49\text{-}50)$$

Where,

$$N_4(h)=Z_4^*\Big[hq_{01}^*\Big(hq_{12}^*\big(h^2q_{23}^*q_{34}^*+hq_{24}^*\big)+hq_{13}^*\big(h^2q_{32}^*q_{24}^*+hq_{34}^*\big)\Big)+hq_{02}^*\Big(\big(h^2q_{23}^*q_{34}^*+hq_{34}^*\big)$$
$$+h^2q_{51}^*q_{13}^*\big(h^2q_{25}^*q_{34}^*-h^2q_{24}^*q_{35}^*\big)\Big)+hq_{03}^*\Big(\big(hq_{34}^*+h^2q_{32}^*q_{24}^*\big)+h^2q_{51}^*q_{12}^*\big(h^2q_{24}^*q_{35}^*$$
$$-h^2q_{25}^*q_{34}^*\big)\Big)\Big]+Z_5^*\Big[hq_{01}^*\Big(hq_{12}^*\big(h^2q_{23}^*q_{35}^*+hq_{25}^*\big)+hq_{13}^*\big(hq_{35}^*+h^2q_{32}^*q_{25}^*\big)\Big)+hq_{02}^*$$
$$\big(h^2q_{23}^*q_{35}^*+hq_{25}^*\big)+hq_{03}^*\big(hq_{35}^*+h^2q_{32}^*q_{25}^*\big)\Big]$$

$$N_5(h)=Z_1^*\Big[hq_{01}^*\big(1-h^2q_{23}^*q_{32}^*\big)+h^2q_{02}^*q_{51}^*\big(h^2q_{23}^*q_{35}^*+hq_{25}^*\big)+h^2q_{03}^*q_{51}^*\big(hq_{35}^*+h^2q_{32}^*q_{25}^*\big)\Big]$$
$$+Z_3^*\Big[hq_{01}^*\big(hq_{13}^*+h^2q_{12}^*q_{23}^*\big)+hq_{02}^*\big(hq_{23}^*+h^3q_{25}^*q_{51}^*q_{13}^*\big)+hq_{03}^*\big(1-h^3q_{12}^*q_{25}^*q_{51}^*\big)\Big]$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the repair of failed main unit and failed helping unit are given by-

$$B_0^M=\lim_{t\to\infty}B_o^M(t)=\lim_{h\to1}(1-h)\frac{N_4(h)}{D_2(h)}$$

$$B_0^H=\lim_{t\to\infty}B_o^H(t)=\lim_{h\to1}(1-h)\frac{N_5(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^M=-\frac{N_4(1)}{D_2'(1)} \quad \text{and} \quad B_0^H=-\frac{N_5(1)}{D_2'(1)} \qquad (51\text{-}52)$$

Where,

$$N_4(1)=\psi\big[(p_{12}+p_{10}p_{02})(p_{24}+p_{23}p_{34})+(p_{13}+p_{10}p_{03})(p_{34}+p_{32}p_{24})$$
$$+(p_{01}p_{12}+p_{02})(p_{25}+p_{23}p_{35})+(p_{01}p_{13}+p_{03})(p_{35}+p_{32}p_{25})\big]$$

$$N_5(1)=\psi_1\big[1-p_{23}p_{32}-p_{02}(p_{23}p_{34}+p_{24})-p_{03}(p_{34}+p_{32}p_{24})\big]$$
$$+\psi_3\big[1-p_{01}p_{10}-p_{24}(p_{02}+p_{01}p_{12})-p_{25}(p_{12}+p_{10}p_{02})\big]$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in repair of failed main unit and failed helping unit up to epoch (t-1) are respectively given by-

$$\mu_b^M(t)=\sum_{x=0}^{t-1}B_0^M(x), \qquad \mu_b^H(t)=\sum_{x=0}^{t-1}B_0^H(x)$$

So that,

$$\mu_b^{M*}(h)=\frac{B_0^{M*}(h)}{(1-h)}, \qquad \mu_b^{H*}(h)=\frac{B_0^{H*}(h)}{(1-h)} \qquad (53\text{-}54)$$

X. PROFIT FUNCTION ANALYSIS

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

K_0 =revenue per-unit time by the system when main unit is operative in normal mode (N).

K_1 =revenue per-unit time by the system when main unit is operative in partial mode (P).

K_2 =cost per-unit time when repairman is busy in the repairing failed main unit.

K_3 =cost per-unit time when repairman is busy in the repairing failed helping unit.

Then, the net expected profit incurred up to epoch (t-1) given by

$$P(t)=K_0\mu_{up}^{N}(t)+K_1\mu_{up}^{P}(t)-K_2\mu_{b}^{M}(t)-K_3\mu_{b}^{H}(t) \qquad (55)$$

The expected profit per unit time in steady state is given by-

$$P=\lim_{t\to\infty}\frac{P(t)}{t}=\lim_{h\to1}(1-h)^2P^*(h)$$

$$=K_0\lim_{h\to1}(1-h)^2\frac{A_0^{N*}(h)}{(1-h)}+K_1\lim_{h\to1}(1-h)^2\frac{A_0^{P*}(h)}{(1-h)}-K_2\lim_{h\to1}(1-h)^2\frac{B_0^{M*}(h)}{(1-h)}$$

$$-K_3\lim_{h\to1}(1-h)^2\frac{B_0^{H*}(h)}{(1-h)}$$

$$=K_0A_0^N+K_1A_0^P-K_2B_0^M-K_3B_0^H \qquad (56)$$

XI. GRAPHICAL REPRESENTATION

The curves for MTSF and profit function have been drawn for different values of parameters. **Fig. 2** depicts the variations in MTSF with respect to repair rate (r_2) of failed helping unit for three different values of failure rate (p_1) of main unit from its normal mode to partial failure mode when helping unit is good and two different values of failure rate (p_3) of helping unit whereas the values of other parameters are kept fixed as p'_1=0.6, p'_2=0.8, p_2=0.2 and r_1=0.6. From the curves, we observe that expected life of the system increases too much slowly with almost linear trend as the value of r increases and it decreases with the increase values of p_1 and p_3.

Similarly **Fig. 3** reveals the variations in profit function (P) with respect to r_2 for varying values of p_1 and p_3, when the values of other parameters are kept fixed as p'_1=0.6, p'_2=0.8, p_2=0.2, r_1=0.6, K_0=20, K_1=15, K_2=40 and K_3=50. From the figure it is clearly observed from the smooth curves that the system is profitable only if repair parameter r_2 is greater than 0.059, 0.104 and 0.15 for p_1=0.05, 0.10 and 0.15 respectively for fixed values of p_3=0.03 from dotted curves, we conclude that the system is profitable only if r_1 is greater than 0.074, 0.120 and 0.165 for p_1=0.05, 0.10 and 0.15 respectively for fixed values of p_3=0.06.

REFERENCES

[1] Goel LR and Sharma SC, "Stochastic analysis of two unit standby system with two switching devices", Microelectron. Reliab., 26(5), 859-861 (1986).

[2] Goel LR, Tyagi PK and Gupta R, "Analysis of a stsndby system with dependent repair time and slow switching device", Microelectron. Reliab., 34(2), 383-386(1994).

[3] Goel LR and Mumtaz SZ, "Analysis of a single unit system with helping unit", Microelectron Reliab., Vol.-25, No. 3, pp. 297-301 (1993).

[4] Gopalan MN and Naidu RS, "Stochastic behavior of a two-unit reparable system subject to inspection", Microelectron. Reliab., 22(4), 717-722(1982).

[5] Gupta R and Varshney G, "A two identical unit parallel system with Geometric failure and repair time distributions", J. of Comb. Info. & System Sciences, Vol. 32, No. 1-4, pp 127–136(2007).

[6] Gupta R and Varshney G, "A two non–identical unit parallel system with Geometric failure and repair time distributions", IAPQR Trans. Vol. 31, No. 2, pp 127–139 (2006).

[7] Gupta R, Chaudhary P and Kumar K, "Cost-Benefit Analysis of a Single-Unit System Model with Helping Unit" Pure and Applied Mathematika Sciences, Vol. LXVII; No. 1-2,March 2008.

[8] Gupta SM, Jaiswal NK and Goel LR, "Analysis of two-unit standby redundant system under partial failure and pre-emptive repair priority", System Sci. 13, 675-681 (1982).

[9] Gupta SM, Jaiswal NK and Goel LR, "Stochastic behavior of a standby redundant system with three modes", Microelectron Reliab. 23(2), 329-331(1983).

[10] Kumar P and Gupta R, "Reliability analysis of a single unit M/G/1 system model with helping unit", J. of Comb. Info. & System Sciences, Vol. 32, No. 1-4, pp. 209-219 (2007).

[11] Murari K and Goel V, "Reliability of a system with two types of repair facilities", Microelectron. Reliab., 23, 1015-1025(1983).

[12] Nakagawa T and Osaki S, "Stochastic behavior of a two unit standby redundant system with imperfect switch over," IEEE Trans. Reliab. R-24, 193-196 (1975).

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Behavior of MTSF with respect to p_1, r_1 and p_3

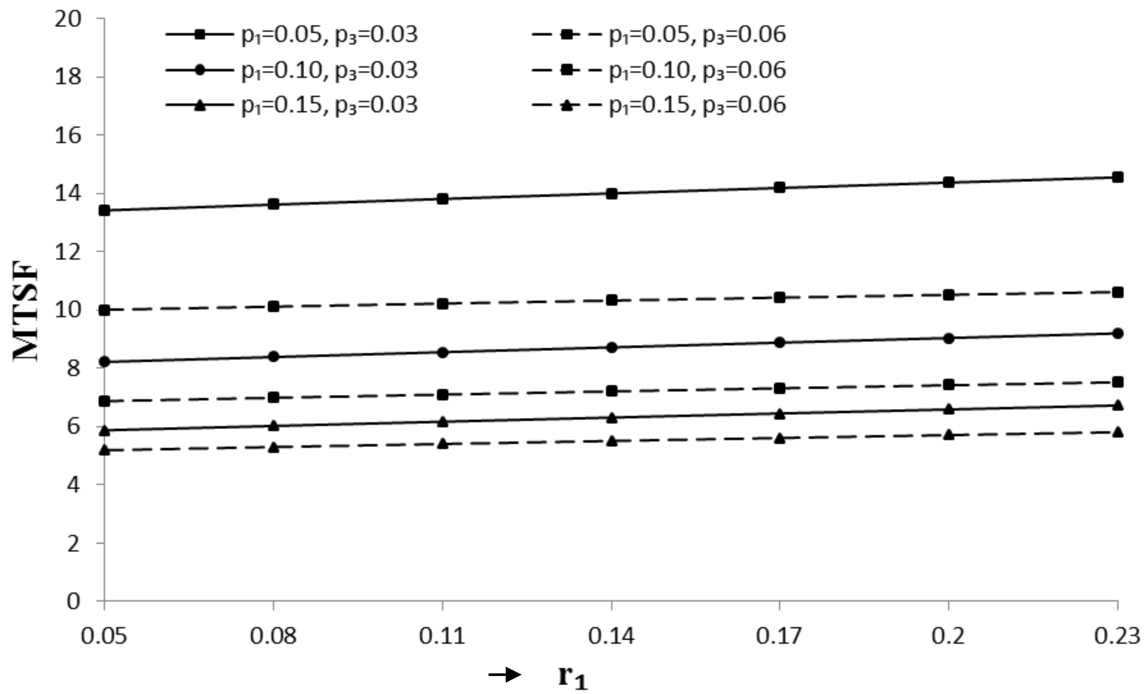

Fig. 2

Behavior of Profit (P) with respect to p_1, r_1 and p_3

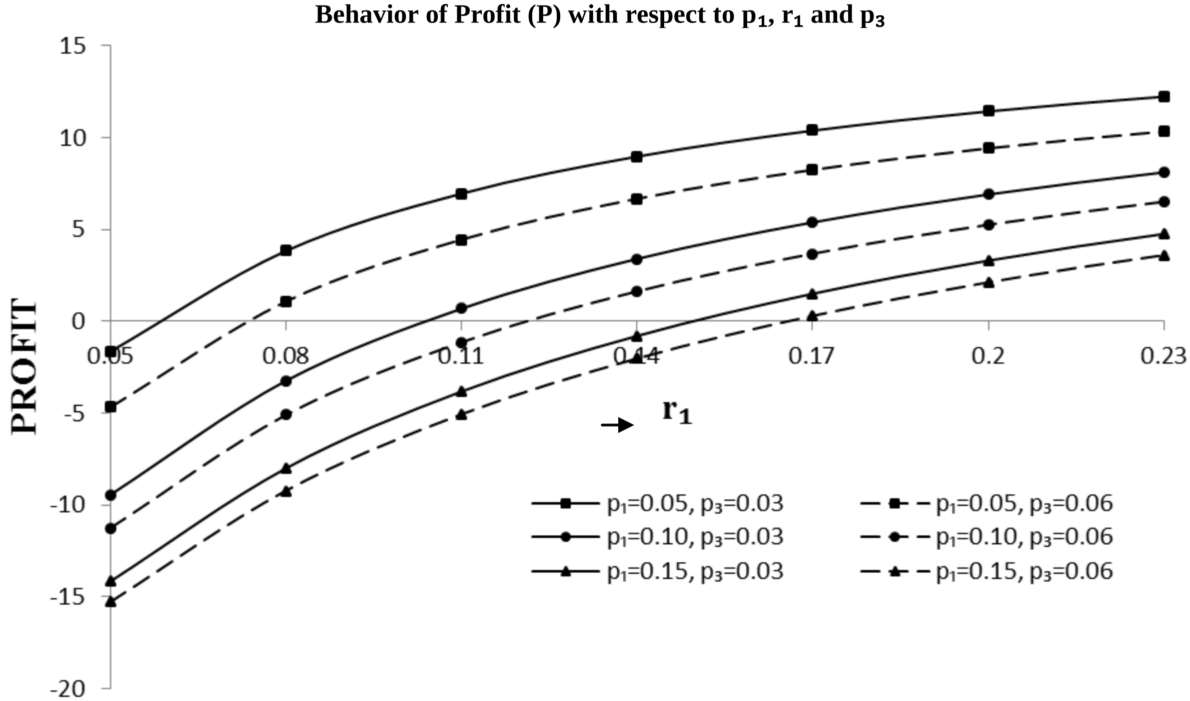

Fig. 3