Computation of Fuzzy Transportation Problem with Dual Simplex method

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Abstract- This paper deals with the transportation problem that has fuzzy cost coefficient. In this paper the aim of work is to introduce dual simplex method to solve transportation problem with fuzzy objective functions. The fuzzy objective functions have fuzzy demand and supply coefficients, which are represented as fuzzy numbers. For this we are solving fuzzy transportation problem by dual simplex method.

Index Terms- Dual simplex method, Fuzzy Transportation Problem, Operation research, Trapezoidal fuzzy numbers.

I. Introduction

The transportation problem is a special type of linear programming problem which arises in many practical applications. The transportation model has wide practical applications, not only in transportation systems, but also in other systems such as production planning [3]. The concept of fuzzy set theory, first introduced by Zadeh [19] is used for solving different types of linear programming problems [1, 2, 3, 14, 6, 9, 15, 16, 18, 19, 21]. After that, especially after 1990s various models and algorithms under both crisp environment and uncertain environment are presented. For ex. Bit Et [2]. Al, Jimnez and Verdegay [7,8], Li et. Al. [9], Srinivasan and Thompsom [18], L. Liu and B. Liu [12].

Liu and Kao [13] developed a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply quantities, and the demand quantities are fuzzy numbers. Jimenez and Verdegay [8] proposed a GA to deal with the fuzzy solid transportation problem in which the fuzziness affects only in the constraint set. They concluded that GA showed a good performance in finding parametric solutions in comparison with nonparametric solutions obtained with other nonlinear solution methods. Lin and Tsai [11] investigated solving the transportation problem with fuzzy demands and fuzzy supplies using a two-stage GA. This study suggests methods of linear programming approach to solving the transportation problem with fuzzy demands and fuzzy supplies. The numerical solved by dual simplex method. In this method the coefficients of objective function are in the form of fuzzy numbers and changing problem in linear programming problem then solved by dual simplex method.

At first Danting G.A. and P. Wolfe [5] (1955) generalised simplex method for minimizing a linear form under inequality restraints. The simplex method starts with a dictionary which is feasible but does not satisfy the optimality condition on the Z equation. It performs successive pivot operations preserving feasibility to find a dictionary which is both feasible and optimal. The dual simplex method starts with a dictionary which satisfies the optimality condition on the Z equation, but is not feasible. It then performs successive pivot operations, which preserve optimality, to find a dictionary which is both feasible and optimal.

II. Methodology

Dual simplex method is helpful in finding the solution of an L.P. problem for a number of different right hand side vectors b_i. It is also used when new constraints are added to an L.P.P. for which the optimal solution has already been obtained. Firstly, we recall some preliminaries with respect to the possibility measure. Then convert the problem in LPP. In such situations we have an infeasible basic primal solution whose associated dual solution is feasible. This method consists of the following steps:

Step 1: Convert the problem into maximization problem if it is initially in the minimization form

Step 2: Convert ≥ type constraints, if any, into ≤ type by multiplying both sides of such constraints by -1.

Step 3: Convert the inequality constraints into equalities by the addition of slack variables and obtain the initial basic solution. Express the above information in the form of a table called the dual simplex table.

Step 4: Compute $C_j – E_j$ for every column.

1. If all $C_j – E_j$ are negative or zero and all bj are non-negative, the solution found above is the optimum basic feasible solution.
2. If all $C_j – E_j$ are negative or zero and at least one bi is negative, then proceed to step 5.
3. If all $C_j – E_j$ are positive, the method fails.

Step 5: Select the row that contains the most negative bi. This row is called the key row or the pivot row. The corresponding basic variable leaves the current solution.

Step 6: Look at elementary of the key row

1. If all elements are non-negative, the problem does not have a feasible solution.
2. If at least one element is negative, find the ratios of the corresponding elements of $C_j – E_j$ row to these elements. Ignore the ratio associated with positive or zero element of the key row. Choose the smallest of these ratios. The corresponding column is the key column and the associated variable is the entering variable.

ISSN 2250-3153

Step 7: Make this key element unity. Carry out the row operation as in the regular simplex method and repeat iterations until either an optimal feasible solution is obtained in a finite number of steps or there is an indication of non-existence of a feasible solution.

Numerical Example

We use an example to illustrate the effectiveness of the proposed dual simplex method approach to solve the fuzzy transportation problem with fuzzy demands and fuzzy supplies. Consider a fuzzy transportation problem with three supplies (m=3) and three demands (n=3). Table 1 shows the fuzzy supplies $\widetilde{a_{ij}}$ and fuzzy demands $\widetilde{b_{ij}}$, i=1, 2, 3 and j=1, 2, 3.

TABLE 1: FUZZY COEFFICIENTS IN THE EXAMPLE

Fuzzy supplies		Fuzzy demands	
$\widetilde{a_1}$	(4,5,7,8)	$\widetilde{b_1}$	(6,7,8,9)
$\widetilde{a_2}$	(5,6,7,8)	$\widetilde{b_2}$	(4,5,6,7)
$\widetilde{a_3}$	(2,3,4,5)	$\widetilde{b_3}$	(1,2,4,5)

The problem has the following form:

Min Z = $4x_{11}+3x_{12}+2x_{13}+7x_{21}+7x_{22}+4x_{23}+9x_{31}+25x_{32}+12x_{33}$

Such that

$x_{11}+x_{12}+x_{13} \cong \widetilde{a_1}$

$x_{21}+x_{22}+x_{23} \cong \widetilde{a_2}$

$x_{31}+x_{32}+x_{33} \cong \widetilde{a_3}$

$x_{11}+x_{21}+x_{31} \cong \widetilde{b_1}$

$x_{12}+x_{22}+x_{32} \cong \widetilde{b_2}$

$x_{13}+x_{23}+x_{33} \cong \widetilde{b_3}$

$x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33} \geq 0.$

The estimated values of fuzzy coefficients are as follows

$\widetilde{a_1} = 6$, $\widetilde{a_2} = 7$, $\widetilde{a_3} = 4$, $\widetilde{b_1} = 6$, $\widetilde{b_2} = 6$, $\widetilde{b_3} = 3$.

Next we use dual simplex method to obtain the optimal solution and the optimal solutions are

$x_{11} = 0$ $x_{12} = 6$ $x_{21} = 3$ $x_{23} = 4$ $x_{31} = 4$

And the transportation cost is Z = 91.

III. Conclusion

This study investigates a dual simplex method approach to solve the transportation problem with fuzzy demands and fuzzy supplies. When the dual simplex method is applied to solve fuzzy transportation problem, the computation needs not to define membership function of the fuzzy numbers, and neither to use the extension principle nor interval arithmetic and α cuts. Instead of the proposed method uses usual evolution. The empirical results show that the proposed method approach outperforms the other fuzzy method for solving the transportation problem with fuzzy demands and fuzzy supplies.

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