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Fractal Dimensional Analysis for Retina

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Abstract- The branching pattern of the retinal blood vessel system resembles fractal, a geometrical pattern whose parts resemble the whole. A new algorithm explains flow analysis of the retina in a recursive manner. The Box Counting method and the Sausage method are used to find out the dimensions of the normal and affected retina in an accurate manner. This fractal approach led to very promising results which improved the analysis of retinal diseases.

Index Terms- Box-Counting method, Fractals, Retina, Sausage method.

I. INTRODUCTION

In this paper we describe the way of finding the fractal dimension for both left and right vision of normal and abnormal retina using images and the model of retina blood vessel using fractals.

A. FRACTALS

A **fractal** is "a rough or fragmented geometric shape that can be subdivided into various parts, each of which is (at least approximately) a reduced-size copy of the whole". A fractal is an object which appears self-similar under varying degrees of magnification, and also an object with its own fractal dimension. Fractals themselves have their own dimension, which is usually a non-integer dimension that greater than their topological dimension D_T and less than their Euclidean dimension D_E. Self-similarity is the major characteristic of the fractal objects [3].Fractal objects and process are therefore said to display 'self-invariant' (self-similar or self-affine) properties. Fractal structures do not have single length scale, while fractal process (time-series) cannot be characterized by a single-time scale. A suspected fractal object is examined by the box counting dimension. Let F be any non-empty bounded subset of R^n and let $N_\delta(F)$ be the smallest number of sets of diameter almost δ which can cover F, the box dimension of F

$$\dim_B F = \lim_{\delta \to 0} -\frac{\log N_\delta(F)}{\log \delta}$$

Sausage method or boundary dilation method is very closely related to the Minkowski dimension. The images were dilated with circles of increasing diameter. As an approximation for a circle we once again used the boxes with pixel sizes of $1\times1,\ 3\times3,\ 5\times5,\ldots,17\times17$ the corresponding approximated radius r in pixels was calculated by

$$r = (A/\pi)^{1/2}$$

where A denotes the area in pixels. The slope k_S of the regression line of the double logarithmic plot of the counted pixels with respect to the radii give

$$D_S = 2 - k_S$$

The estimated fractal capacity dimension is D_S.

B. HUMAN RETINA

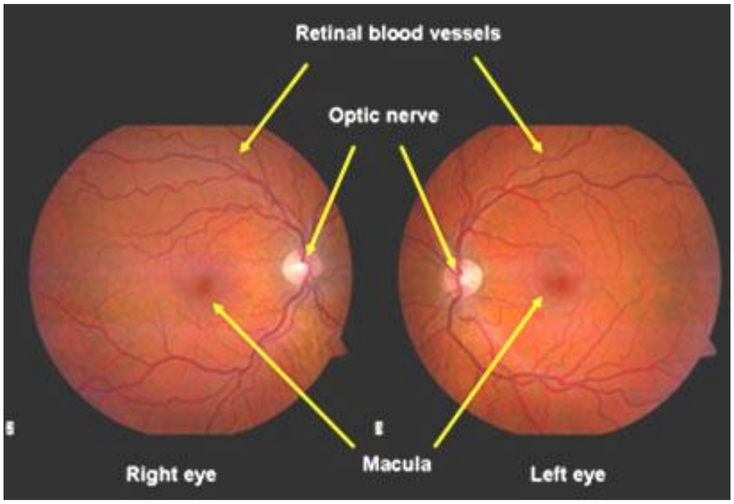

Figure-1 Human retina

Human Retina is an ultra-thin, soft and transparent sheet lining the inner surface at the back of the eye (like a thin light sensitive film), receives light and changes it into electrical signals which are then transmitted to the brain by the optic nerve. The most light-sensitive part of the retina is a tiny spot called the macula and this is where the vision is at its sharpest. Due to the laws of optics, the image formed on the retina is actually upside-down; it is the brain which adjusts it to make it appear the right way up (Figure-1). Retina is the only part in the human body where blood vessels and their circulation observed live giving useful insights into many vascular processes. The pattern of vessels in each person is unique and this property is being increasingly utilized for identification through rapid retina scans at security points. The retinal blood vessel system resembles fractal which satisfies the characteristics of fractal properties [5, 6]. Nowadays retinal diseases are spreading widely which in turn leads to the loss of vision too. This paper predicts the dimension for the normal and abnormal retina using image analysis for both left and right vision.

II. ALGORITHM

The algorithm indicates the traversing of the blood flow in the blood vessel of the human retina in a recursive manner that acts a model for fractal.

Flood-fill (node, target-vessel, replacement-vessel):
1. If the vessel of *node* is not equal to *target-vessel*, return.
2. Set the vessel of *node* to *replacement-vessel*.
3. Perform **Flood-fill** (one step to the west of *node*, *target-vessel*, *replacement-vessel*).

Perform **Flood-fill** (one step to the east of *node*, *target-vessel*, *replacement-vessel*).
Perform **Flood-fill** (one step to the north of *node*, *target-vessel*, *replacement-vessel*).
Perform **Flood-fill** (one step to the south of *node*, *target-vessel*, *replacement-vessel*).
4. Return.

The above algorithm has been programmed and run by using C++ [1].

III. DISTANCE MEASURE

Distance measure is the distance from the centre of mass to the perimeter point (x_i, y_i). So the radial distance is defined as

$$d(i) = \sqrt{(x(i)-\bar{x})^2 + (y(i)-\bar{y})^2}$$

i=0, 1,2,…… .N-1

Here $d(i)$ is a vector obtained by the distance measure of the boundary pixels. A normalized vector $r(i)$ is obtained by dividing $d(i)$ by the maximum value of $d(i)$ [2].

The quantitative parameters such as Area, Perimeter, Formfactor and Invaslog can found out using Sausage method. Perimeter stands for the sum of all individual perimeters of all individual objects in the image and was calculated correspondingly.

$$Formfactor = 4\pi\, Area / perimeter^2$$

$$Invas\log = -\log(Formfactor)$$

The value of Invaslog has been specially proven to be a strong quantitative measure for the invasiveness.

IV. RESULTS BASED ON REINAL IMAGES

The dimension D of the retinal images have been estimated using Box-counting Method (D_B) and Sausage Method (D_s).The amount of invasiveness is found out by Form Factor and Invaslog. The above factors have been calculated for the normal as well as abnormal image for both left and right retina[4]. The table 1 and table 2 depict the dimension for normal retinal image and table 3 and table 4 for abnormal retinal image from the images given below. (Figure-2,3 and 4)

Figure-2 Normal Image of Right and left retina

Figure-3 Abnormal Image of Right retina

Figure-4 Abnormal Image of left retina

Table - 1. Box-Counting Method

Scaling	Right eye image						Left eye image					
	Area	Perimeter	Total area	Form factor	Invaslog	Dimension D_b	Area	Perimeter	Total area	Form factor	Invaslog	Dimension D_b
2	68156	1080	69236	0.734	0.134	1.9750	68395	1190	69585	0.607	0.217	1.9739
3	30053	939	30992	0.428	0.369		30092	1071	31163	0.330	0.481	
4	16778	749	17527	0.376	0.425		16799	838	17637	0.301	0.521	
5	10662	603	11265	0.368	0.434		10670	679	11349	0.291	0.536	
6	7381	481	7862	0.401	0.397		7373	534	7907	0.325	0.488	
7	5386	419.1429	5805.1431	0.385	0.415		5382	456	5838	0.325	0.488	
8	4105	369	4474	0.379	0.421		4106	400	4506	0.322	0.492	
9	3218	323.6666	3541.6665	0.386	0.413		3214	348.3333	3562.3333	0.333	0.478	
10	2593	290	2883	0.387	0.412		2592	315	2907	0.328	0.484	

Table - 2. Sausage method

Scaling	Right eye image				Left eye image			
	Area	Radius	K_s	$D_s = 2 - K_s$	Area	Radius	K_s	$D_s = 2 - K_s$
3	2923	30.503	0.3956	1.6044	738	15.327	0.5118	1.4882
5	1009	17.921			245	8.831		
7	490	12.489			118	6.129		
9	283	9.491			68	4.652		
11	185	7.674			42	3.656		
13	127	6.358			28	2.985		
15	92	5.412			19	2.459		
17	68	4.652			14	2.111		

Table - 3. Box-Counting Method

Scaling	Right eye image						Left eye image					
	Area	Perimeter	Total area	Form factor	Invaslog	Dimension D_b	Area	Perimeter	Total area	Form factor	Invaslog	Dimension D_b
2	46793	1850	48643	0.172	0.764		38184	760	38944	0.831	0.080	
3	20628	1290	21920	0.156	0.807		16812	651	17463	0.499	0.302	
4	11523	922	12445	0.170	0.770		9381	508	9889	0.457	0.340	
5	7331	723	8054	0.176	0.754		5967	398	6365	0.473	0.325	
6	5054	545	5599	0.214	0.670	1.9611	4127	319	4446	0.510	0.292	1.9689
7	3687	456	4143	0.223	0.652		3000	281	3281	0.477	0.321	
8	2804	398	3202	0.223	0.652		2283	249	2532	0.463	0.334	
9	2202	328	2530	0.257	0.590		1794	223	2017	0.453	0.344	
10	1770	316	2086	0.223	0.652		1439	197	1636	0.466	0.332	

Table - 4. Sausage method

Scaling	Right eye image				Left eye image			
	Area	Radius	K_s	$D_s = 2 - K_s$	Area	Radius	K_s	$D_s = 2 - K_s$
3	20628	80.321			16812	73.153		
5	7331	48.307			5967	43.582		
7	3687	34.258			3000	30.902		
9	2202	26.475	0.2464	1.7536	1794	23.897	0.2616	1.7384
11	1459	21.550			1177	19.356		
13	1027	18.081			835	16.303		
15	763	15.584			617	14.014		
17	586	13.658			468	12.205		

V. CONCLUSION

The Box-counting method and Sausage method are analyzed for the normal image as well as the pathological images. The comparison between the two methods for the normal image as well as the pathological image shows us the Sausage method gives the affordable results. As the dimension increases the invasiveness also increases which can be found from the above tables.

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