

Return Stroke VLF Electromagnetic Wave of Oblique Lightning Channel

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Abstract-In this paper, the theories of vertical return stroke, in which Bruce and Golde [1] current is supposed to flow, have been extended to oblique orientations of the channel. The VLF electric field and power density have been calculated for a distance 100 km from the source as function of the channel parameters and of the channel orientation. It is shown that VLF electromagnetic energy radiated from the oblique channel is of Gaussian shape with the maximum appearing at certain frequency depends on orientation of channel.

Index Terms-Current model, current moment, Electromagnetic waves, Frequency spectrum, Lightning discharge, return stroke, VLF electric field.

I. INTRODUCTION

The exact physical processes of electromagnetic wave, generation from lightning discharges, extending from a few hertz in the ELF range to beyond the visible region are not yet fully understood. Using various models for lightning discharge current, many workers [1 – 12] have studied certain characteristics of generating sources and radiated electromagnetic spectrum from them. The effect of orientation of cloud to ground lightning discharges on the radiated energy spectrum has not been thoroughly studied except the some researchers [13 – 15]. In these studies, changes in the field strength and polarization of radiated signals in the extended frequency range have been shown. Visual observations and photographs [16 – 18] have made it obvious that the return strokes are not always vertical. In reality lightning channels are oblique and tortuous. This is a better approximation to the nature. The verticality of return stroke depends on the location of the centre of the charged cloud and the conducting region of the ground. Few researchers [19 - 21] have reported that there are errors found in the magnetic direction caused by non verticality of the return strokes.

The polar diagram of the radiated electromagnetic energy has the usual figure of the eight shapes symmetric along the direction of the return stroke channel, accordingly changes with the changes in the orientation of the return stroke and gives rise to a variability of maximum radiated energy from one return stroke to another return stroke [22]. Thus it is found that the orientation of return stroke controls the magnitude of the field and frequency of the VLF waves, is one of the most important parameters to study the conditions of formation of whistlers. To give a better approximation near to nature, the theory of the vertical electrically conducting channel [22] which simulates return stroke has been extended to oblique orientation of lightning channel. The effect of perfectly conducting ground is taken into account for the calculation of the radiated electric field and power density.

II. RADIATED ELECTRIC FIELD FROM OBLIQUE RETURN STROKE

In order to study the characteristics of radiated electric field, we take the well accepted Bruce and Golde [1] return stroke current model which is given as

$$I_t = I_0 [\exp(-\alpha t) - \exp(-\beta t)] \quad (1)$$

Where, I_0 , α and β are constants which vary from one stroke to another.

Srivastava and Tantry [23] chose $I_0 = 22\text{kA}$, $\alpha = 1.4 \times 10^4 \text{s}^{-1}$ and $\beta = 50 \times 10^4 \text{s}^{-1}$ for the study of VLF characteristics of electromagnetic radiation.

Return stroke velocity model is given by Srivastava [24] as

$$V_t = V_0 [\exp(-at) - \exp(-bt)] \quad (2)$$

Where, $V_0 = 3 \times 10^8 \text{ms}^{-1}$, $a = 6 \times 10^4 \text{s}^{-1}$ and $b = 7 \times 10^5 \text{s}^{-1}$.

An expression very similar to Eq. (2) has been developed by Rai[25].

Few researchers [26 – 28] have reported that the Eq. (2) explains the spectral details of the radiated electromagnetic energy, magnetic field and the increase of electric field with increasing order of the strokes in a multi stroke flash of lightning.

The current moment associated with return stroke channel determines the strength and spectral features of the radiated electromagnetic wave. The current moment associated with the oblique return stroke is defined as

$$M_c = I_t \int_0^t V_t dt = I_t l_t \cos \Phi \tag{3}$$

Where, l_t is the length of the return stroke channel from the ground to charge centre of the thunder cloud (Fig. 1), and Φ is the angle made by the return stroke from the vertical.

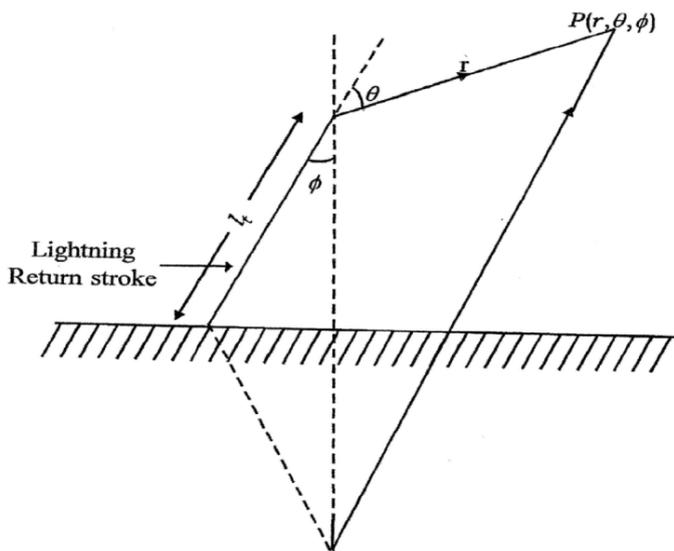


Fig. (1) Geometrical parameter of an oblique return stroke used in Eq. (5)

From simple antenna theory, the component of the radiated electric field at a distance, r from the lightning discharge channel is written as

$$E_d(r, t) = \frac{1}{4\pi\epsilon_0} \frac{1}{c^2 r} \frac{dM_c(t)}{dt} \tag{4}$$

Where, ϵ_0 ($= 8.85 \times 10^{-12} \text{ Fm}^{-1}$) is the permittivity of free space, and c is the velocity of light.

The value of $M_c(t)$ is substituted from Eqs. (1 – 3) into Eq. (4), the direct component of the radiated electric field is obtained as

$$E_d(r, t, \theta, \Phi) = \frac{30I_0V_0 \cos \Phi \sin(\theta+\Phi)}{c a b r} f(t) \tag{5}$$

Where, θ is the observation angle made by the line joining the observation point to the mean direction of the discharge channel and the factor $f(t)$ is similar to that given by Srivastava and Tantry [23]

$$\begin{aligned} f(t) = & (b - a)\{\alpha \exp(-\alpha t) - \beta \exp(-\beta t)\} - \\ & b\{(a + \alpha)\exp[-(a + \alpha)t] - (a + \beta)\exp[-(a + \beta)t]\} + \\ & a\{(b + \alpha)\exp[-(b + \alpha)t] - (b + \beta)\exp[-(b + \beta)t]\} \end{aligned} \tag{6}$$

The image contribution to the electric field of the return stroke due to perfectly conducting ground is given as

$$E_I(r, t, \theta, \Phi) = F E_d(r, t, \theta, \Phi) \tag{7}$$

Where, F is the ground attenuation factor which depends on frequency and distance. As a first approximation, the ground attenuation factor for low frequency waves at large distances of observation remains almost constant and close to unity [29]. Under this approximation, the radiated electric field at the point of observation is expressed as [30]

$$E(r, t, \theta, \Phi) = E_1(r, t, \theta, \Phi) + E_d(r, t, \theta, \Phi) = 2E_d(r, t, \theta, \Phi) \tag{8}$$

Using Eq. (5) into Eq. (8), the radiated electric field is rewritten as

$$E(r, t, \theta, \Phi) = \frac{60I_0V_0 \cos \Phi \sin(\theta + \Phi)}{cabr} f(t) \tag{9}$$

III. FREQUENCY SPECTRUM

Magnitude of the frequency spectrum of direct radiated electric field is written as

$$|E_d(\omega)| = \left| \int_0^\infty E_d(r, t, \theta, \Phi) \exp(-i\omega t) dt \right| \tag{10}$$

In terms of direct radiated electric field, the electromagnetic power density is written as

$$P_d(\omega) = |E_d(\omega)|^2 / \eta \tag{11}$$

Where, ω is the frequency, in rad s⁻¹, at which power is radiated.

By including the effect of perfectly conducting ground the radiated power density is written as

$$P_d(\omega) = |E_d(\omega)|^2 / \eta = 4 |E_d(\omega)|^2 / \eta = 4P_d(\omega) \tag{12}$$

Substituting for $E_d(r, t, \theta, \Phi)$ from Eq. (5) into Eq. (9), we obtain

$$|E_d(\omega)| = \frac{30I_0V_0 \cos \Phi \sin(\theta + \Phi)}{cabr} \left[\left\{ \left(\frac{\beta^2(b-a)}{\beta^2 + \omega^2} + \frac{b(a+\alpha)^2}{(a+\alpha)^2 + \omega^2} + \frac{a(b+\beta)^2}{(b+\beta)^2 + \omega^2} \right) - \left(\frac{\alpha^2(b-a)}{\alpha^2 + \omega^2} + \frac{b(a+\beta)^2}{(a+\beta)^2 + \omega^2} + \frac{a(b+\alpha)^2}{(b+\alpha)^2 + \omega^2} \right) \right\}^2 + \omega^2 \left\{ \left(\frac{\beta(b-a)}{\alpha^2 + \omega^2} + \frac{b(a+\beta)}{(a+\beta)^2 + \omega^2} + \frac{a(b+\alpha)}{(b+\beta)^2 + \omega^2} \right) - \left(\frac{\beta(b-a)}{\beta^2 + \omega^2} + \frac{b(a+\alpha)}{(a+\alpha)^2 + \omega^2} + \frac{a(b+\beta)}{(b+\beta)^2 + \omega^2} \right) \right\}^2 \right]^{1/2} \tag{13}$$

Taking the effect of perfectly conducting ground, the expression for frequency spectrum of the radiated electric field is written as

$$|E(\omega)| = 2|E_d(\omega)| \tag{14}$$

IV. RESULTS AND DISCUSSIONS

So far the theory developed for this study shows that the effect of orientation of lightning channel are introduced into the expressions of VLF electric field and into the radiated power density through the factors $\cos \Phi \sin(\theta + \Phi)$ and $\cos^2 \Phi \sin^2(\theta + \Phi)$, respectively.

Calculation of the radiated electric field from the current source has been done by using Eq. (9). The variation of computed VLF electric field arriving at a distance of 100 km and making different observation angles with the channel for different orientation of return stroke is shown in Fig.(2). It is observed that maximum radiated electric field is at observation angle $\theta = 90^\circ$ and at orientation of the

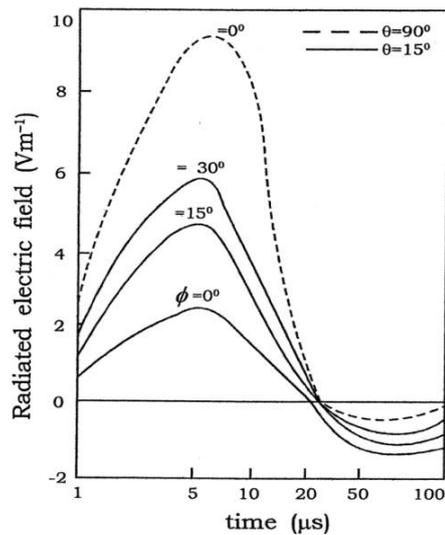


Fig.(2) Variation of the radiated electric field from oblique return stroke with time at distance, 100 km for different orientation of channel.

channel $\phi = 0^\circ$ which is consistent with the dipolar characteristics of the source. The electric field is seen to have maximum value at $5 \mu\text{s}$ for every orientation of oblique channel and for every observation angle. After $5 \mu\text{s}$, VLF electric field decreases with increasing time and finally exhibits an excursion to negative values. It is clear from the figure that at same angle of observation, $\theta = 15^\circ$ and at observation distance, $r = 100 \text{ km}$ the peak value of the VLF electric field decreases with decreasing orientation of channel. Zero crossing of the radiated electric field depends on orientation of return stroke.

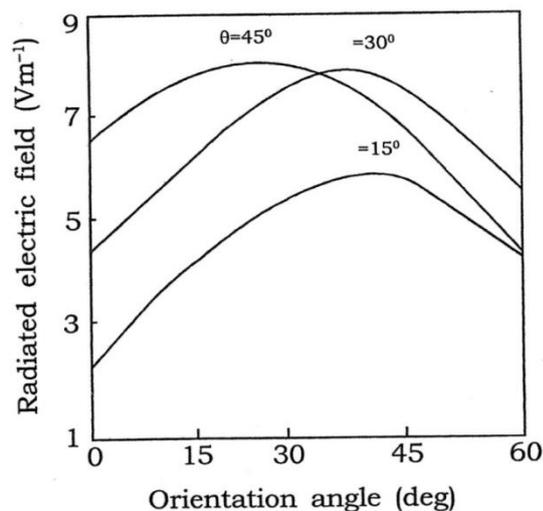


Fig.(3) Variation of radiated electric field from return stroke with orientation at observation distance, 100 km and at $t = 5 \mu\text{s}$.

The variation of the radiated electric field with orientation of oblique lightning channel for observation angles, $\theta = 15^\circ, 30^\circ$ and 45° at time $5 \mu\text{s}$ and at observation distance, $r = 100 \text{ km}$ is depicted in Fig. (3). The radiated electric field increases

with increasing orientation and is seen to attain a maximum value and there after decreases. At orientation, $\phi = 31.5^\circ$ value of the radiated electric field is the same for observation angles, $\theta = 30^\circ$ and 45° ; after orientation 31.5° the electric field decreases more rapidly.

Orientation of the return stroke is considered from the vertical line joining the base of the channel. Therefore, the front of the draining process continuously moves away from the observation point. Thus, the polar diagram of radiated electromagnetic energy accordingly changes and gives rise to variability of the level of the radiated electric field [22].

In order to study the variation of the radiated electric field with frequency, computation is done by using expression (14) for different orientations of oblique return stroke. In this calculation the observation distance is kept at 100 km and angle of observation is assumed to be $\theta = 90^\circ$. The variations of the radiated electric field with frequency are shown in Fig. (4). The radiated VLF electric field variation with frequency has a Gaussian shape with peak at certain frequency. The orientation of return stroke governs the shape of the Gaussian curves.

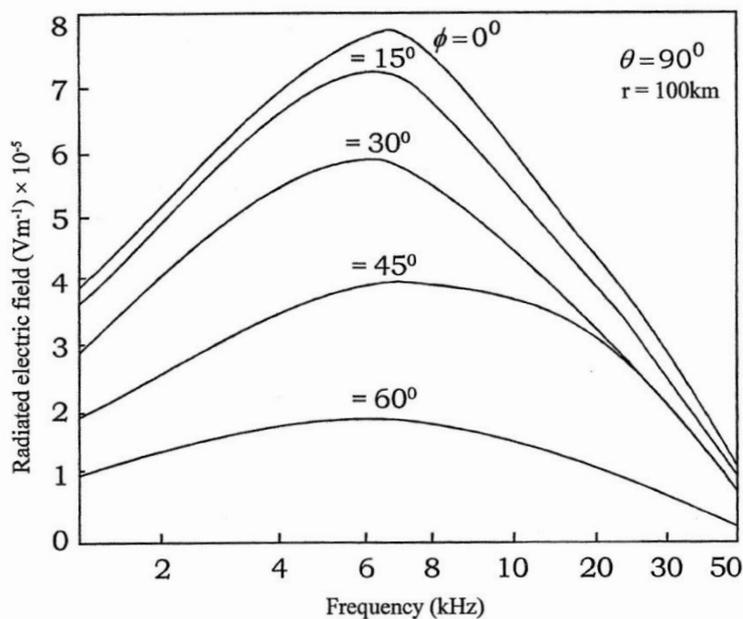


Fig. (4) Variation of radiated electric field of oblique return stroke with frequency at $r = 100$ km.

The radiated electric field increases with frequency and has its maximum value at 6 kHz for every orientation and there after decreases with frequency. The peak value of the field decreases with increasing orientation.

Fig. (5) shows the variations of the peak value of the radiated electric field from lightning return stroke with orientation at observation distance, $r = 100$ km and at frequency 6 kHz. The radiated electric field at frequency 6 kHz decreases from its maximum value with increasing orientation of lightning channel. This variation is might be due to the dependency of length of return stroke on the orientation of channel and change in the observation angle from θ to $\theta + \phi$.

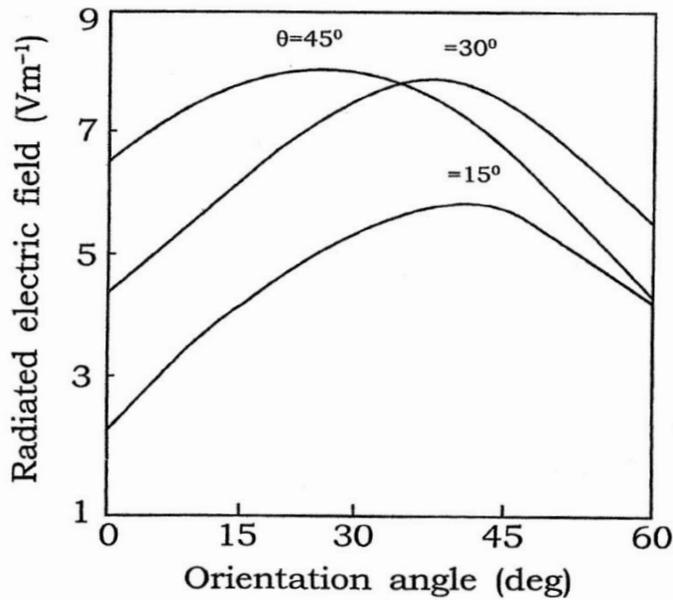


Fig. (5) Variation of radiated electric field from oblique return stroke with orientation at observation distance, $r = 100 \text{ km}$ and at frequency, $f = 6 \text{ kHz}$.

Variations of the radiated power density with frequency for different orientations are shown in Fig. (6). In this computation the observation distance is kept at 100 km and observation angle is taken as $\theta = 90^\circ$. The power density variation with frequency for different orientations of lightning channel has Gaussian shape.

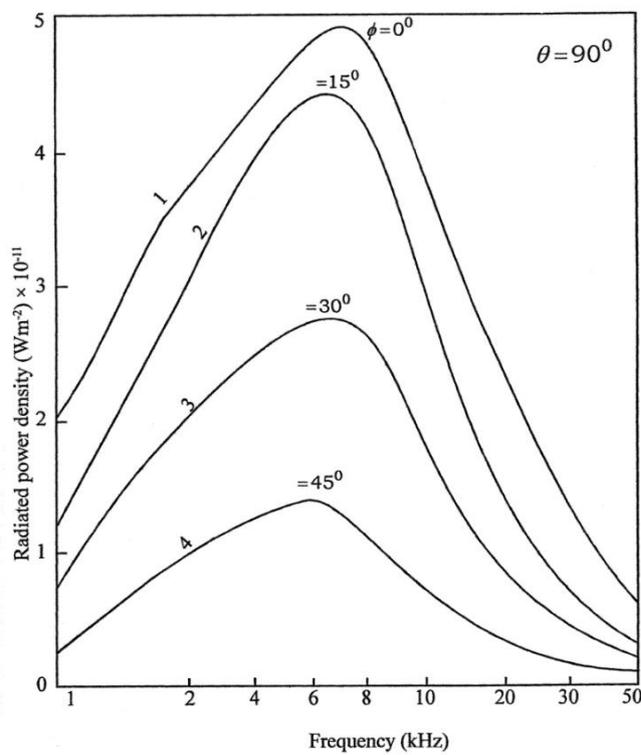


Fig. (6) Frequency spectrum of radiated power density of oblique return stroke at observation distance, $r = 100 \text{ km}$.

Considering the half power density points, the band width of the radiated spectrum has been calculated. It is found that a band of frequencies containing different ranges is emitted out from lightning strokes depends on the orientation of the channel. The band widths of the radiated VLF waves as obtained from the Fig. (6) are given as

1. (1.7 - 15.0) kHz for $\phi = 0^{\circ}$
2. (1.8 - 12.5) kHz for $\phi = 15^{\circ}$
3. (2.0 - 09.6) kHz for $\phi = 30^{\circ}$, and
4. (2.1 - 08.8) kHz for $\phi = 45^{\circ}$.

The lower cut off frequency increases with increasing orientation of lightning channel while upper cut off frequency decreases with increasing of orientation. Thus, it can be argued that the lightning return stroke, where input energy is supplied by thunder cloud, shows the behavior of amplification. Prasad and Singh [22] have argued that the lower and upper frequency cut offs observed in whistler sonograms can be used to derive approximate values of constants, I_0 , α and β .

The maximum value of the power density is found at 6 kHz for each value of orientation, and it is found to decrease with increasing orientation.

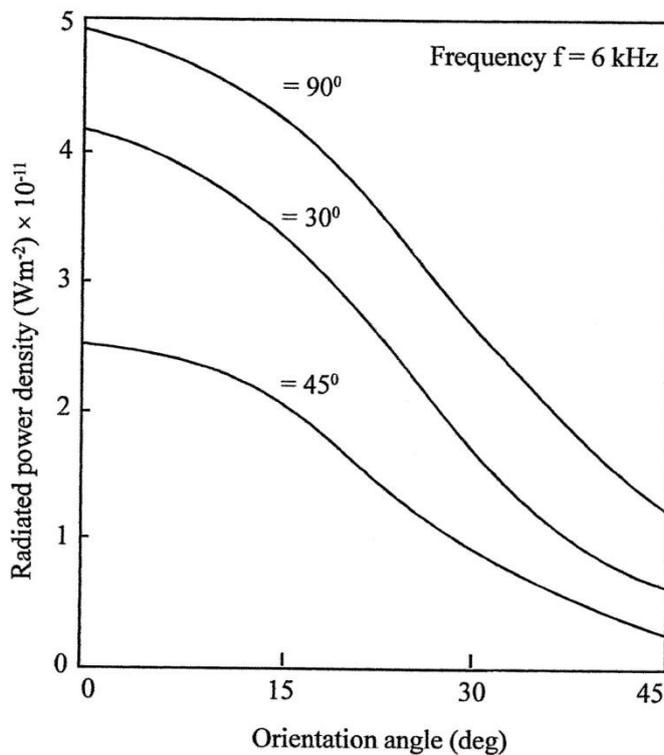


Fig. (7) Variation of radiated power density associated with oblique return stroke with orientation at observation distance, $r = 100$ km.

The variation of the radiated power density at frequency, 6 kHz with orientation of the oblique return stroke is shown in Fig. (7). In this computation, observation distance is kept at 100 km and observation angles are taken as $\theta = 90^{\circ}$, 45° and 30° . It is clear from this figure that the changes in the radiated power density associated with return stroke depend on the orientation of lightning channel.

V. CONCLUSIONS

In this study an attempt is made to calculate the radiated VLF electric field and power density from an oblique lightning return stroke at distance 100 km from the source. The aim of this study is to show that the orientation of oblique lightning channel which simulates return stroke plays an important role in controlling the shape and waveforms of the radiated VLF electric field and power density.

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