Fast and Slow Modes on Dust Ion Acoustic Solitary Waves in A Warm Plasma

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Abstract- Using Korteweg-de Vries (KdV) equation, solitons are investigated in a dusty plasma consisting of positive ion, negatively charged dust grain and Boltzmann distributed electron. Both compressive and rarefactive solitons are found to exist in the dusty plasma for fast and slow modes. Greater Z_d (Z_d is the number of elementary charges residing on the dust grain) is found to yield higher amplitude solitons. Further, it is observed that higher temperature ratio gives higher amplitude in case of slow mode and gives smaller amplitude in case of fast mode.

1. INTRODUCTION

Dusty or Complex plasmas are ordinary plasmas with embedded solid particles. The particles can be made of either dielectric or conducting materials and can have any shape. Dusty plasmas are present in interstellar clouds, circumstellar clouds, planetary rings, comets, cometary tails, asteroid zones, earth's atmosphere and magnetosphere [1- 11]. Dusty plasma can be produced in laboratories by Modified Q-machine, dc discharges, rf discharges etc [1]. Dusty plasma plays an important role in plasma crystals [12], coating and etching of thin films [13] etc. Rao *et al.* [14] have first reported the existence of the dust acoustic waves for low frequency in unmagnetized dust plasma and verified in a laboratory experiment by Barken *et al.* [15]. The linear and nonlinear features of both dust-acoustic wave and dust ion-acoustic wave have also reported in experimental and theoretical observations [16 - 19]. Roychoudhury and Mukharjee [20] have reported that the finite dust temperature restricts the region for the existence of nonlinear solitary waves. Shukla and Silin [21] have reported about the existence of dust ion acoustic waves for higher frequency. Nakamura and Sharma [22] have shown that the presence of negatively charged dust in plasma decreases the velocity of soliton and increases its width for a given hight of the peak. Ghosh [23] has investigated the role of negative ions in dusty plasma with variable dust charge. Tiwari and Mishra [24] have investigated the dust ion acoustic solitons in the plasma consisting positively or negatively charged dust and discussed the conditions in parameter space to obtain compressive and rarefactive dressed solitons. EI-Wakil *et al.* [25] have investigated the dust ion acoustic waves propagation in an inhomogeneous dusty plasma considering positive and negative dust grains. Mowafy *et al.* [26] have investigated the effect of dust charge fluctuation on the propagation of dust ion acoustic waves in inhomogeneous mesospheric dusty plasma. Das and Chatterjee [27] have investigated the formation of large amplitude double layers in a dusty plasma constituting warm dust grains, non-thermal electrons and two temperature isothermal ions using Sagdeev's pseudopotential technique. Baluku *et al.* [28] have investigated the dust ion acoustic solitons in an unmagnetized dusty plasma consisting cold dust particles, adiabatic fluid ions and electrons satisfying kappa distribution. Attia *et al.* [29] have investigated the higher order effects of positive and negative dust charge fluctuation on the propagation of dust ion acoustic waves in a weakly inhomogeneous, weakly coupled, collision less and unmagnetized mesospheric dust plasma consisting of four components dusty plasma. Barman and Talukdar [30] have investigated the propagation of ion-acoustic waves in a warm dusty plasma with electron inertia. Tiwari *et al.* [31] have investigated characteristics of ion acoustic soliton in dusty plasma, including the dynamics of heavily charged massive dust grains using Sagdeev potential method. Alinejad [32] has studied the properties of arbitrary amplitude dust ion acoustic solitary waves in a dusty plasma containing warm adiabatic ions, electrons following flat-trapped velocity distribution and arbitrarily (positively or negatively) charged mobile dust through pseudo-potential method. Recently Maitra [33] has investigated the dust acoustic solitary waves in a magnetized dusty plasma considering the effect of dust size and dust charge variations.
Very recently Malik *et al.* [34] have studied the soliton propagation in a moving electron-positron pair plasma having negatively charged dust grain including finite temperatures of the electrons and positrons. They have obtained two types of modes propagating in opposite directions.

In this paper, we consider a three component unmagnetized dusty plasma system consisting of positive ion, negatively charged dust grain and Boltzmann distributed electron. Our main objective is to study the effect of temperature on the amplitude and width of dust ion acoustic solitary waves in an unmagnetized plasma for fast and slow modes. The paper is organized as: In Sec 1, introduction, in Sec 2, the basic set of equations governing the plasma model and derivation of the KdV equation, in Sec 3, solitary wave solution, in Sec 4, discussion and finally, conclusions in Sec 5.

2. BASIC EQUATION AND DERIVATION OF KDV EQUATION

We consider a one dimensional, unmagnetized collision less dusty plasma with positive ion, negatively charged dust grain and Boltzmann distributed electron. The dynamics of dust ion acoustic waves is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0 \quad (1)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\alpha}{n_i}\frac{\partial p_i}{\partial x}+Z_d\frac{\partial \phi}{\partial x}=0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)p_i+3p_i\frac{\partial v_i}{\partial x}=0 \quad (3)$$

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d v_d)=0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+v_d\frac{\partial}{\partial x}\right)v_d+\frac{1}{Q'}\left(\frac{\alpha}{n_d}\frac{\partial p_d}{\partial x}-Z_d\frac{\partial \phi}{\partial x}\right)=0 \quad (5)$$

$$\left(\frac{\partial}{\partial t}+v_d\frac{\partial}{\partial x}\right)p_d+3p_d\frac{\partial v_d}{\partial x}=0 \quad (6)$$

$$n_e=e^{\phi} \quad (7)$$

Again for charge imbalances these equations are to be combined by the Poisson equation

$$\frac{\partial^2\phi}{\partial x^2}=n_e-n_i+Z_d n_d \quad (8)$$

where, *i*, e and d stand for positive ion, electron and negatively charged dust grain respectively, $Q'=\frac{m_d}{m_i}$ (= dust grain to ion mass ratio) , $\alpha=\frac{T_i}{T_e}$ (= ion to electron temperature with $T_i=T_d$, T_d is the dust temperature) and Z_d is the number of elementary charges residing on the dust grain.

We have normalized densities n_i, n_e and n_d by the unperturbed densities n_{e0}, pressures p_i, p_d by the characteristic ion pressure $k_b n_{e0} T_i$, time t by the inverse of the characteristic ion plasma frequency i.e., $\omega_{pi}^{-1}=\left(\frac{m_i}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De}=\left(\frac{k_b T_e}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s=\left(\frac{k_b T_e}{m_i}\right)^{\frac{1}{2}}$, and the potential ϕ by $\frac{k_b T_e}{e}$; k_b is the Boltzmann constant.

To derive the KdV equation from the set (1) – (8), we use the stretched variables

$$\xi=\varepsilon^{\frac{1}{2}}(x-Ut),\quad \tau=\varepsilon^{\frac{3}{2}}t \quad (9)$$

with phase velocity U of the wave and using the following expansions of the flow variables in terms of the smallness parameter ε:

$$\begin{aligned}
n_i&=n_{i0}+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\dots,\\
n_d&=n_{d0}+\varepsilon n_{d1}+\varepsilon^2 n_{d2}+\dots,\\
v_i&=\varepsilon v_{i1}+\varepsilon^2 v_{i2}+\dots,\\
v_d&=\varepsilon v_{d1}+\varepsilon^2 v_{d2}+\dots,\\
\phi&=\varepsilon\phi_1+\varepsilon^2\phi_2+\dots\dots
\end{aligned} \quad (10)$$

With the use of of the transformation (9) and the expansion (10) in the normalized set of equations (1) – (8), we get the expression for the phase velocity and the KdV equation as follows-

$$\frac{rZ_{dd}^2}{(Q'U^2-3\alpha)(1-rZ_d)}+\frac{Z_d}{(U^2-3\alpha)(1-rZ_d)}=1,\ r=\frac{n_{d0}}{n_{i0}} \quad (11)$$

This gives $U^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Where $a=(1-rZ_d)$,

$b=-\{3\alpha(1+Q')(1-rZ_d)+(Z_dQ'+rZ_d^2)\}$

$c=9\alpha^2(1-rZ_d)+3\alpha(Z_d+rZ_d^2)$

and
$$\frac{\partial\phi_1}{\partial\tau}+p\phi_1\frac{\partial\phi_1}{\partial\xi}+q\frac{\partial^3\phi_1}{\partial\xi^3}=0 \quad (12)$$

where $p=\frac{A}{2B}$ and $q=\frac{1}{2B}$ with

$$A=\frac{3n_{i0}Z_d^2(U^2+\alpha)}{(U^2-3\alpha)^3}-\frac{3n_{d0}Z_d^3(Q'U^2+\alpha)}{(Q'U^2-3\alpha)^3}-1$$

and $B=\frac{n_{d0}Q'Z_d^2U}{(Q'U^2-3\alpha)^2}+\frac{n_{i0}Z_dU}{(U^2-3\alpha)^2}$

From the expressions of A and B, we observe that the nonlinear dust ion-acoustic solitons, in this model of plasma, exist when $U^2\neq 3\alpha,\ \frac{3\alpha}{Q'}$ subject to the condition $Z_d<\frac{1}{r}$.

3. SOLITARY WAVE SOLUTION

To find a stationary solution of the KdV equation (12), we use the transformation $\chi=\eta-V\tau$. Using the boundary conditions $\phi_1=\frac{\partial\phi_1}{\partial\eta}=\frac{\partial^2\phi_1}{\partial\eta^2}=0$ as $|\eta|\to\infty$, the KdV equation (12) can be integrated to give $\phi_1=\frac{3V}{p}\sec h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\chi\right)$, where V is the velocity with which the solitary waves travel to the right.

Thus, the wave amplitude of the soliton is given by $\phi_0=\frac{3V}{p}$ and the corresponding width by $\Delta=2\sqrt{\frac{q}{V}}$.

4. DISCUSSION

In presence of ion temperature in this model of plasma, both fast and slow compressive and rarefactive solitons are found to exist. The ion temperature is responsible for generating the fast and slow modes. For the fast mode, both compressive and rarefactive solitons are exist, but for slow mode only rarefactive solitons of small amplitude are found to exist. It is to be mentioned that the compressive fast ion-acoustic solitons exist only for smaller values of $Z_d(>1)$ [Fig.1 (a)] and its amplitude increase as Z_d increases. Furthermore, they are found to exist only in upper regime of $Z_d(>1)$. On the other hand, rarefactive fast ion-acoustic solitions exist [Fig. 1(a)] only in the upper regime of Z_d for fixed $V=0.05$, $r=0.05$ and $Q'=10$ for different values of $\alpha=0.05(1),0.15(2),0.25(3)$. However, the character of the fast compressive solitons changes to fast rarefactive solitons after certain $Z_d{}^*$ characterizing an uncountable region. The corresponding widths [Fig.1 (b)] of the fast (compressive and rarefactive) ion-acoustic solitons nonlinearly decrease with Z_d with negligible difference for $\alpha=0.05(1),0.15(2),0.25(3)$ for fixed $V=0.05$, $r=0.05$ and $Q'=10$. To the contrary, the amplitudes [Fig. 2(a)] of slow rarefacetive solitons increases parabolically (for higher values of α) with Z_d for fixed $V=0.05$, $r=0.05$ and $Q'=10$ for different values of $\alpha=0.05(1),0.15(2),0.25(3)$. But the corresponding widths [Fig. 2(b)] of the slow rarefactive solitons decreases parabolically (for higher values of α) with Z_d for the same set of values of the parameters. The amplitudes [Fig. 3(a)] of fast compressive soliton decrease linearly but rapidly with α for fixed $V=0.20$, $r=0.05$ and $Q'=10$ for different values of $Z_d=6,7,8$. The corresponding widths [Fig. 3(b)] of the fast compressive solitons are decrease linearly but slowly with α. On the other hand, the amplitudes [Fig. 4(a)] of slow rarefacetive solitons increase (numerically) uniformly with α for fixed $V=0.20$, $r=0.05$ and $Q'=10$ for different values of $Z_d=6,7,8$. But the corresponding widths [Fig. 4(b)] of the slow rarefactive solitons increase uniformly with α for fixed $V=0.20$, $r=0.05$ and $Q'=10$ for different values of $Z_d=6,7,8$. The amplitudes [Fig. 5(a)] of fast compressive solitons decreases sharply in the lower regime of $Q'(\le 4)$ and then decreases very slowly with Q' fixed $V=0.05$, $r=0.05$ and $Z_d=8$ for different values of $\alpha=0.10,0.15,0.20,0.25$. The corresponding widths [Fig. 5(b)] of the fast compressive solitons nonlinearly decrease with Q' fixed $V=0.05$, $r=0.05$ and $Z_d=8$ for different values of $\alpha=0.10,0.15,0.20,0.25$. On the other hand, the amplitudes [Fig. 6(a)] of slow rarefacetive solitons increases (numerically) nonlinearly with Q' for fixed $V=0.05$, $r=0.05$ and $Z_d=8$ for different values of $\alpha=0.10(1),0.15(2),0.20(3),0.25(4)$. But the corresponding widths [Fig. 6(b)] of the slow rarefactive solitons increases sharply in the lower regime of $Q'(\le 4)$ and then decreases very slowly for fixed $V=0.05$, $r=0.05$ and $Z_d=8$ for different values of $\alpha=0.10(1),0.15(2),0.20(3),0.25(4)$.

5. CONCLUSION

In this investigation, we have studied the effect of ion temperature and dust temperature in a three-component dusty plasma using reductive perturbation method. Two modes namely slow and fast are observed corresponding to different phase velocities. Both compressive and rarefactive solitons are found to exist in the dusty plasma for fast and slow modes. Greater Z_d is found to yield higher amplitude solitons. Further, it is observed that higher temperature ratio gives higher amplitude in case of slow mode and gives smaller amplitude in case of fast mode.

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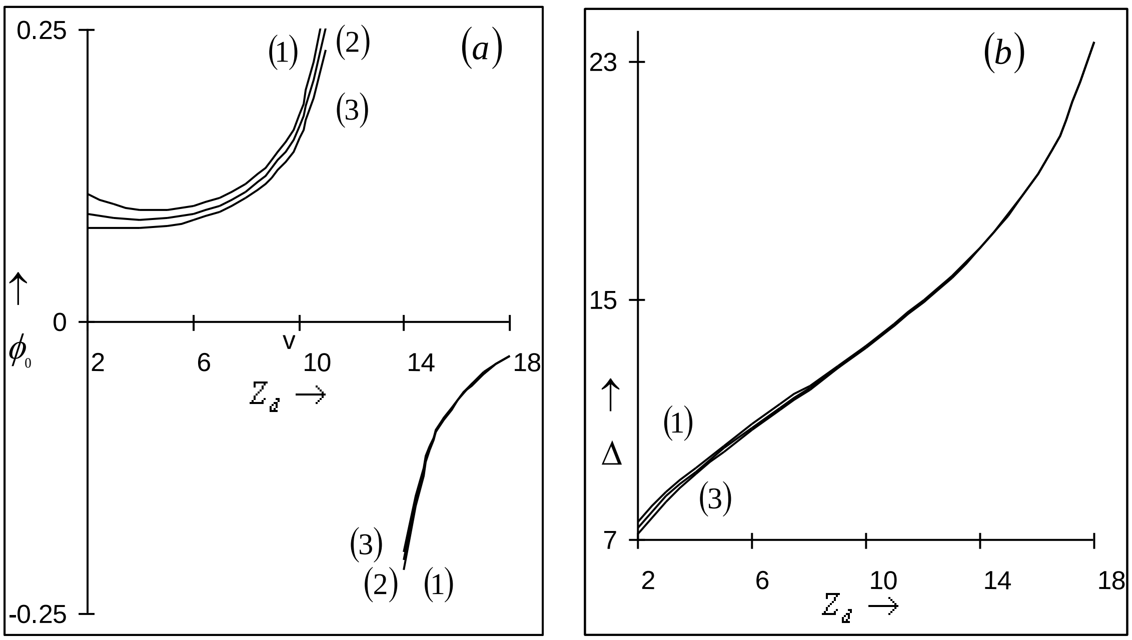

Figure1. Amplitudes (a) and widths (b) of fast compressive and rarefactive dust ion-acoustic solitons versus Z_d for fixed $V = 0.05$, $r = 0.05$ and $Q' = 10$ for different values of $\alpha = 0.05(1), 0.15(2), 0.25(3)$.

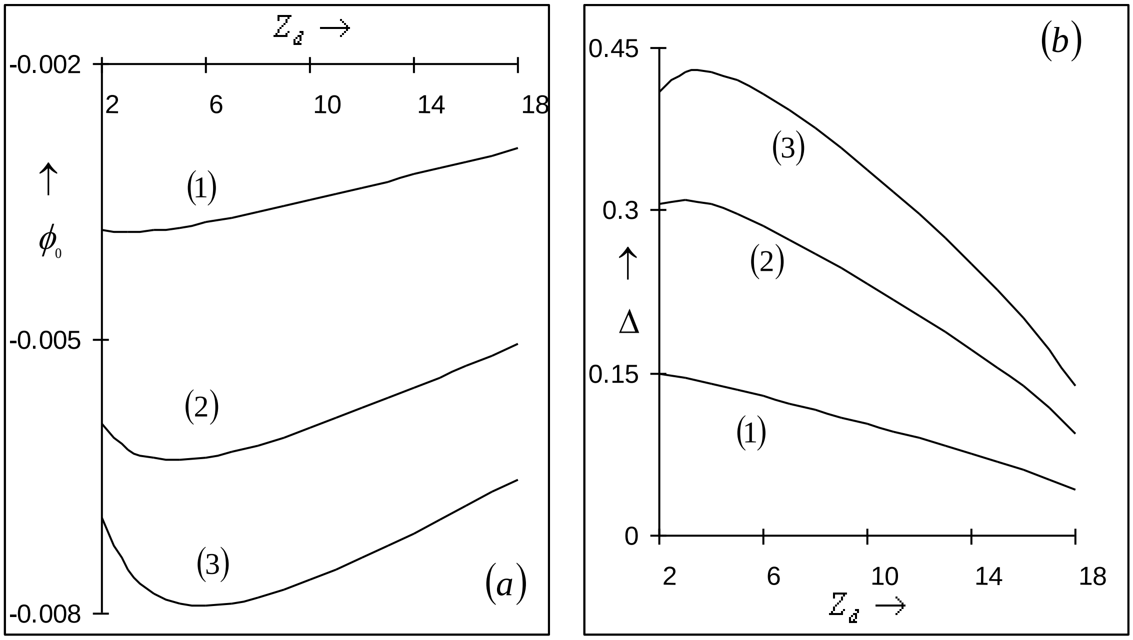

Figure 2. Amplitudes (a) and widths (b) of slow rarefactive dust ion-acoustic solitons versus Z_d for fixed $V = 0.05$, $r = 0.05$ and $Q' = 10$ for different values of $\alpha = 0.05(1), 0.15(2), 0.25(3)$.

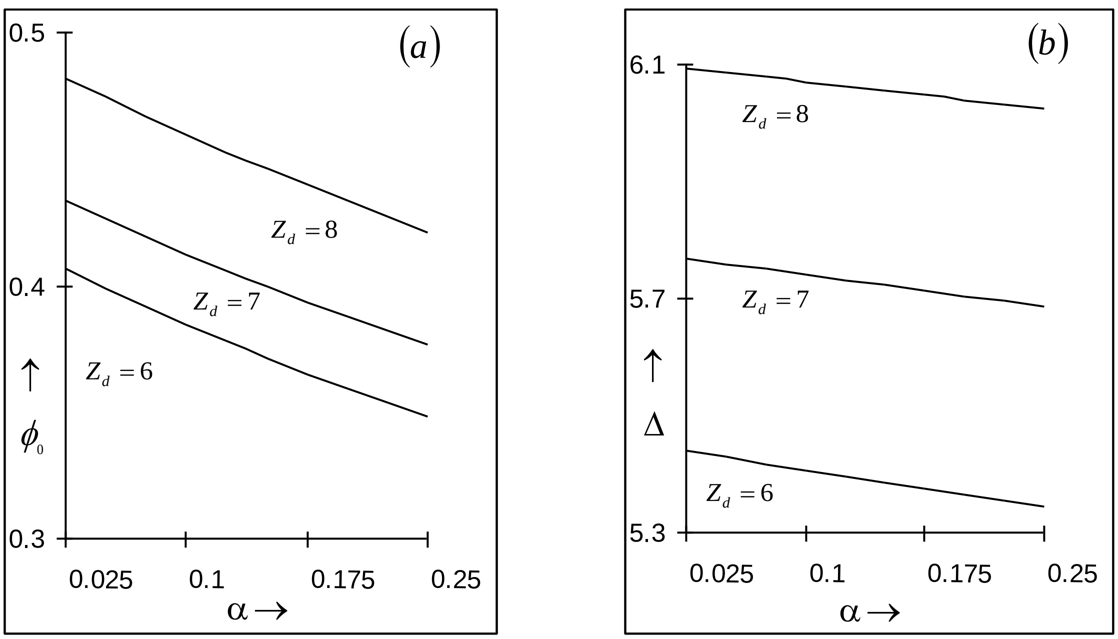

Figure 3. Amplitudes (a) and widths (b) of fast compressive dust ion-acoustic solitons versus α for fixed $V = 0.20$, $r = 0.05$ and $Q' = 10$ for different values of $Z_d = 6, 7, 8$.

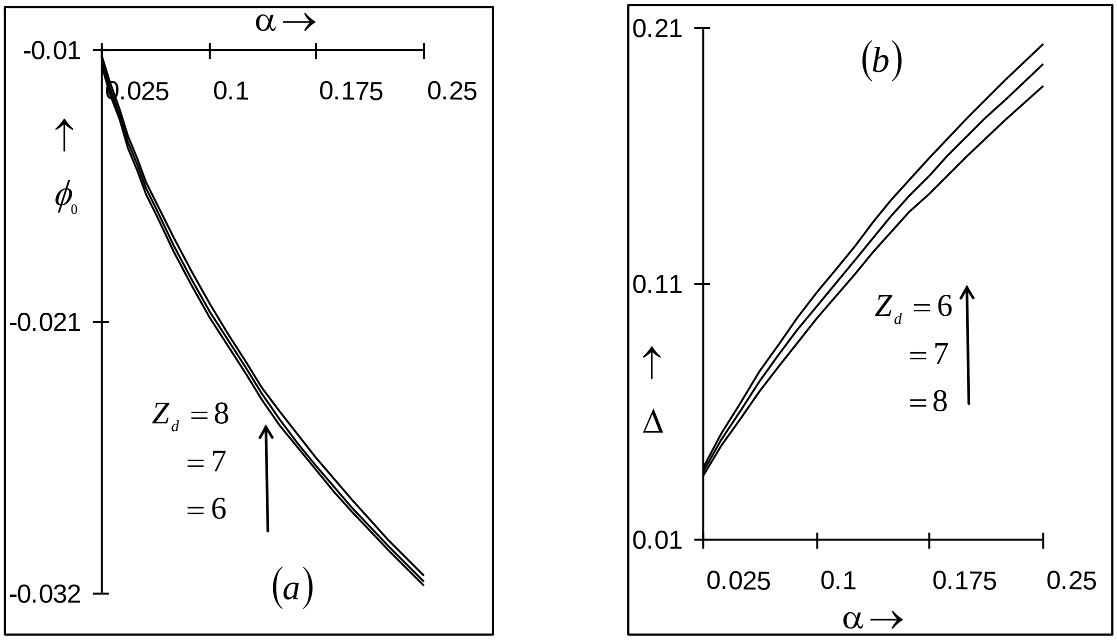

Figure 4. Amplitudes (a) and widths (b) of slow rarefactive dust ion-acoustic solitons versus α for fixed $V = 0.20$, $r = 0.05$ and $Q' = 10$ for different values of $Z_d = 6, 7, 8$.

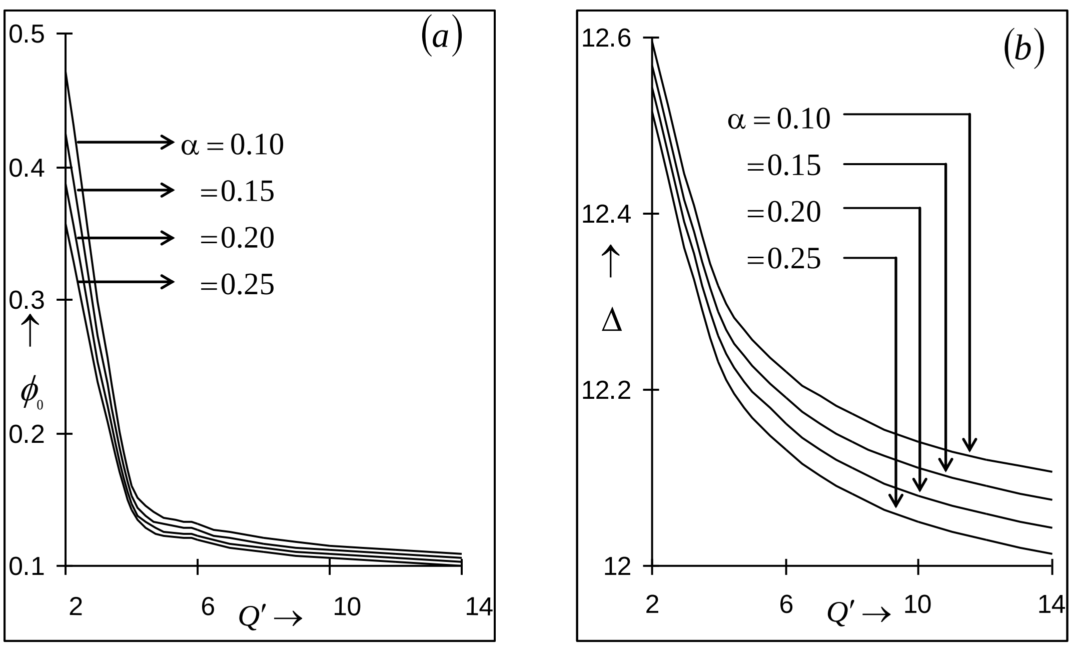

Figure 5. Amplitudes (a) and widths (b) of fast compressive dust ion-acoustic solitons versus Q' for fixed $V = 0.05$, $r = 0.05$ and $Z_d = 8$ for different values of $\alpha = 0.10, 0.15, 0.20, 0.25$.

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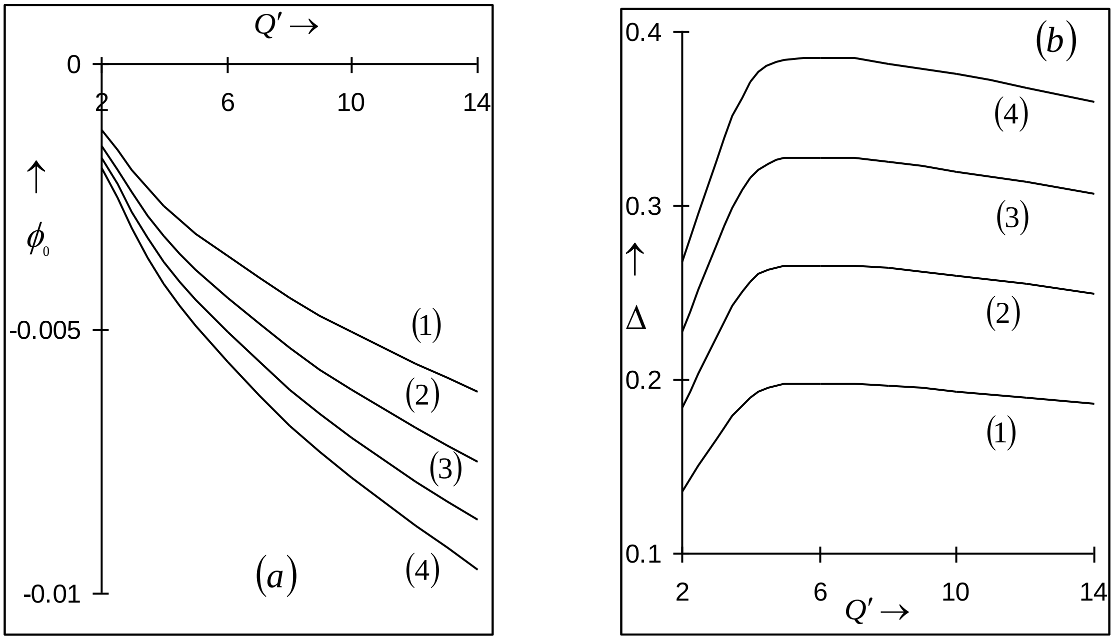

Figure 6. Amplitudes (a) and widths (b) of slow rarefactive dust ion-acoustic solitons versus Q' **for fixed** $V = 0.05$**,** $r = 0.05$ **and** $Z_d = 8$ **for different values of** $\alpha = 0.10(1), 0.15(2), 0.20(3), 0.25(4)$**.**