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ON GENERALIZED PSEUDO-PROJECTIVE ϕ-RECURRENTN(k)-CONTACT METRIC MANIFOLD

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Abstract: The object of the present paper is to study generalized pseudo-projective ϕ-recurrent $N(k)$-contact metric manifold.

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1. Introduction

In 1988, S. Tanno [11]introduced the notion of k-nullity distribution of a contact metric manifold as a distribution such that the characteristic vector fieldξ of the contact metric manifold belongs to the distribution. The Contact metric manifold with ξ belonging to the k-nullity distribution is called $N(k)$-contact metric manifold and such a manifold is also studied by various authors. Generalizing this notion, in 1995 Blair, Koufogiorgos and Papantoniou [3] introduced the notion of a contact metric manifold with ξ belonging to the (k, μ)-nullity distribution, where k and μ are real constants. In particular if μ=0, then the notion of (k, μ)-nullity distribution reduces to the notion of k-nullity distribution. Later in 2008, De, Gazi, [6] studied ϕ-recurrent $N(k)$-contact metric manifold.

In this paper we study Generalized pseudo-projective ϕ-recurrent $N(k)$-contact metric manifold. Here we show that Generalized pseudo-projective ϕ-recurrent $N(k)$-contact metric manifold is an η-Einstein manifold, and we find a relation between the associated 1-forms$A$and B. We also prove that the characteristic vector field ξ and vector field ρ associated to the 1-forms A and $B$are co-directional. Finally we prove that a generalized pseudo-projective ϕ-recurrent $N(k)$-contact metric manifold is of constant curvature.

2. Contact Metric Manifold

A (2n+1)-dimensional manifold M^{2n+1} is said to admit an almost Contact structure if itadmits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying

(2.1)$(a)\ \phi^2(X) = -X + \eta(X)\xi, (b)\ \eta(\xi) = 1, (c)\ \eta \circ \phi = 0, (d)\ \phi\xi = 0.$

An almost contact metric structure is said to be normal if the induced almost complexstructure J on the product manifold$M^{2n+1} \times \boldsymbol{R}$defined by

$$J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt})$$

is integrable, where X is tangent to M, $t$is the coordinate of $\boldsymbol{R}$ and f is a smooth functionon $M \times \boldsymbol{R}$. Let g be a compatible Riemannian metric with almost contact structure (ϕ, ξ, η),
that is

(2.2)$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$.

Then M becomes an almost contact metric manifold equipped with an almost contact structure(ϕ, ξ, η, g). From (2.1) it can be easily seen that

(2.3)$(a)\ g(X, \phi Y) = -g(\phi X, Y), (b)\ g(X, \xi) = \eta(X)$

for all vector fields X, Y . An almost contact metric structure becomes a contact metric structure if
(2.4) $g(X, \phi Y) = d\eta(X, Y)$,

for all vector fields X, Y. The 1-form η is then a contact form and ξ is its characteristic vector field. We define a (1,1) tensor field h by $h = \frac{1}{2}£_{\xi}\phi$, where £ denotes the Lie-differentiation. Then h is symmetric and satisfies $h\phi = -\phi h$.We have $Tr.h = Tr.\phi h = 0$ and $h\xi = 0$. Also,
(2.5)$\nabla_X \xi = -\phi X - \phi h X$,

holds in a Contact metric manifold. A normal contact metric manifold is a Sasakian manifold.An almost contact metric manifold is Sasakian if and only if

(2.6) $(\nabla_X \phi)(Y) = g(X, Y)\xi - \eta(Y)X, \qquad X, Y \in TM$

where ∇ is the Levi-Civita connection of the Riemannian metric g. A contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ for which ξ is a killing vector is said to be a K-contact manifold. It is well known that the tangent sphere bundle of a flat Riemannian manifold admits a contact metric structure satisfying $R(X, Y)\xi = 0$ ([2]). On the other hand, on a Sasakian manifold, the following holds:

(2.7)$R(X, Y)\xi = \eta(Y)X - \eta(X)Y$.

As a generalization of both $R(X, Y)\xi = 0$ and the Sasakian case; D.Blair, T.Koufogiorgos and B.J.Papantoniou [3] considered the (k, μ)-nullity condition on a Contact metric manifold andgave several reasons for studying it. The (k, μ)- nullity distribution $N(k, \mu)$ of a Contactmetric manifold M is defined by

$$N(k, \mu) : p \rightarrow N_p(k, \mu)$$
$$= \{W \in T_pM : R(X, Y)W = (kI + \mu h)(g(Y, W)X - g(X, W)Y)\},$$

for all$X, Y \in TM$, where $(k, \mu) \in R^2$. A contact metric manifold M^{2n+1} with $\xi \in N(k, \mu)$ is called a (k, μ)-manifold. In particular on a (k, μ)-manifold, we have

(2.8)$R(X, Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - \eta(X)hY]$.

On a (k, μ)-manifold $k \leq 1$. If $k = 1$, the structure is Sasakian ($h = 0$ and μ is indeterminant)andif k < 1, the (k, μ)-nullity condition determines the curvature of M^{2n+1}completely [3].Infact, for a (k, μ)-manifold, the condition of being a Sasakian manifold, ak-contact manifold, $k = 1$ and $h = 0$ are all equivalent. The k-nullity distribution$N(k)$ of a Riemannian manifold$M$ is defined by

$$N(k): p \rightarrow Np(k) = \{Z \in TpM : R(X, Y)Z = g(Y, Z)X - g(X, Z)Y\},$$

k being a constant. If the characteristic vector $\xi \in N(k)$,then we call a contact metric manifold an $N(k)$-contact metric manifold. If $k = 1$, then $N(k)$-contact metric manifold is

Sasakian and if $k = 0$, then $N(k)$-contact metric manifold is locally isometric to the product $E^{n+1}\times S^n(4)$ for n > 1 and flat for $n = 1$. If $k < 1$, the scalar curvature is $r = 2n(2n-2+k)$. If μ = 0, then a (k,μ)-contact metric manifold reduces to a $N(k)$-contact metric manifold.

In $N(k)$-contact metric manifold the following relations hold:

(2.9) $$h^2 = (k-1)\phi^2, \qquad k \le 1,$$

(2.10) $$(\nabla_X\phi)(Y) = g(X+hX,Y)\xi - \eta(Y)(X+hX),$$

(2.11) $$R(\xi,X)Y = k[g(X,Y)\xi - \eta(Y)X],$$

(2.12) $$S(X,\xi) = 2nk\eta(X),$$

(2.13) $$S(X,Y) = 2(n-1)g(X,Y) + 2(n-1)g(hX,Y) + [2(1-n) + 2nk]\eta(X)\eta(Y), \qquad n \ge 1,$$

(2.14) $$r = 2n(2n-2+k),$$

(2.15) $$S(\phi X,\phi Y) = S(X,Y) - 2nk\eta(X)\eta(Y) - 4(n-1)g(hX,Y),$$

(2.16) $$(\nabla_X\eta)(Y) = g(X+hX,\phi Y),$$

(2.17) $$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y],$$

(2.18) $$\eta(R(X,Y)Z) = k(g(Y,Z)\eta(X) - g(X,Z)\eta(Y)].$$

Definition 2.1. ([13]) A $N(k)$-contact metric manifold is said to be locally pseudo-projective ϕ-symmetric if

(2.19) $$\phi^2((\nabla_W\bar{P})(X,Y)Z) = 0,$$

for all vector fields X,Y,Z,W orthogonal to ξ.

Definition 2.2. ([13]) A $N(k)$-contact metric manifold is said to be pseudo-projective ϕ-recurrent if there exists a non-zero 1-form A such that

(2.20) $$\phi^2((\nabla_W\bar{P})(X,Y)Z) = A(W)R(X,Y)Z$$

for arbitrary vector fields X,Y,Z and W, where $\bar{P}$ is a pseudo-projective curvature tensorgiven by .

(2.21) $$\bar{P}(X,Y)Z = aR(X,Y)Z - b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{(2n+1)}\left[\frac{a}{2n} + b\right][g(Y,Z)X - g(X,Z)Y]$$

where R is the curvature tensor, and r is the scalar curvature. If the 1-form A vanishes, then the manifold reduces to locally pseudo projective ϕ-symmetricmanifold.

Definition 2.3. A $N(k)$-contact metric manifold is said to be generalized pseudo-projective ϕ-recurrent if its curvature tensor $\bar{P}$ satisfies the condition

(2.22) $$\phi^2\big((\nabla_W\bar{P})(X,Y)Z\big) = A(W)P(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

where A and B are two 1-forms, B is non-zero and these are defined by

$$A(W) = g(W,\rho_1), \qquad B(W) = g(W,\rho_2),$$

and ρ_1, ρ_2 are vector fields associated with 1-forms A and B, respectively.

3. Generalized Pseudo-Projective ϕ-Recurrent $N(k)$-Contact Metric Manifold

Let us consider a Generalized pseudo-projective ϕ-recurrent $N(k)$ −contact metric manifold. Then by virtue of 2.1(a) and (2.22) we have

$$\begin{aligned} & -((\nabla_W \bar{P})(X,Y)Z) + \eta((\nabla_W \bar{P})(X,Y)Z)\xi \\ (3.1) \quad & = A(W)\bar{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y], \end{aligned}$$

from which it follows that

$$\begin{aligned} & -g((\nabla_W \bar{P})(X,Y)Z,U) + \eta((\nabla_W \bar{P})(X,Y)Z)\eta(U) \\ (3.2) \quad & = A(W)g(\bar{P}(X,Y)Z,U) + B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned}$$

Let {ei}, $i = 1, 2, \dots, 2n+1$ be an orthonormal basis of the tangent space at any point of themanifold. Then putting $X = U = \{e_i\}$ in (3.2) and taking summation over $i, 1 \le i \le 2n+1$,we get

$$(3.3)(\nabla_W S)(Y,Z) = A(W)\left[S(Y,Z) - \frac{r}{2n+1}g(Y,Z)\right] + 2nB(W)g(Y,Z).$$

Replacing Z by ξ in (3.3) and using (2.12), we have

$$(3.4)(\nabla_W S)(Y,\xi) = A(W)\eta(Y)\left[2nk - \frac{r}{2n+1}\right] + 2nB(W)\eta(Y).$$

Now we have,

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi).$$

Using (3.4) and (2.16) in the above relation, it follows that

$$(3.5)(\nabla_W S)(Y,\xi) = -2nkg(\phi W + \phi hW, \quad Y) + S(Y, \quad \phi W + \phi hW).$$

In view of (3.4) and (3.5), we have

$$\begin{aligned} (3.6) \quad S(Y,\phi W + \phi hW) = \; & 2nkg(\phi W + \phi hW, Y) \\ & + A(W)\eta(Y)\left[2nk - \frac{r}{2n+1}\right] + 2nB(W)\eta(Y). \end{aligned}$$

Replacing Y by ϕY in (3.6), and after a brief simplification, we get

$$\begin{aligned} S(Y,W) = \; & 2[(n+k-1) + n(k-1)(nk+n-1)]g(Y,W) \\ & + 2[(n-1)(k-1) - n(k-1)(nk+n-1)]\eta(Y)\eta(W) \end{aligned}$$

or,

$$(3.7) \qquad S(Y,W) = ag(Y,W) + b\eta(Y)\eta(W),$$

Where $a = 2[(n+k-1)] + n(k-1)(nk+n-1)$,
$b = 2[(n-1)(k-1) - n(k-1)(nk+n-1)]$ are constants.
Therefore we state the following:

Theorem 3.1. A Generalized pseudo-projective ϕ-recurrent $N(k)$-contact metric manifold isan η-Einstein manifold with constant coefficients.

Now putting $Y = Z = ei$ in (3.2) and taking summation over i, $i = 1, 2, \dots, 2n+1$, we get

$$- (a - b)(\nabla_W S)(X,U) - b\nabla_W r g(X,U) + \frac{dr(w)}{2n+1}\left(\frac{a}{2n} + b\right)[2ng(X,U)]$$
$$+ (a - b)(\nabla_W S)(X,\xi)\eta(U) + b\nabla_W r\eta(X)\eta(U) - \frac{dr(W)}{2n+1}\left(\frac{a}{2n} + b\right)[2n\eta(X)\eta(U)]$$

(3.8)
$$= A(W)[(a - b)S(X,U) + brg(X,U)] - \frac{r}{2n+1}\left(\frac{a}{2n} + b\right)2ng(X,U)] + 2nB(W)g(X,U).$$

Putting $U = \xi$ in (3.8) we have,

(3.9) $A(W)\left[2nk(a - b)\eta(X) + br\,\eta(X) - \frac{r}{2n+1}\left(\frac{a}{2n} + b\right)2n\,\eta(X)\right] + 2nB(W)\eta(X) = 0.$

Putting $X = \xi$ in (3.9) we have,

(3.10)
$$B(W) = \left[kb - ka - \frac{br}{2n} - \frac{r}{2n(2n+1)}(a + 2nb)\right]A(W).$$

This leads to the following theorem:

Theorem 3.2.*In a generalized pseudo-projective ϕ-recurrent N(k)-contact metric manifold(M^{2n+1}, g), the 1-forms A and B are related as in (3.10)*

Now from (3.1) we have

(3.11)
$$(\nabla_W \bar{P})(X,Y)Z = \eta((\nabla_W \bar{P})(X,Y)Z)\xi - A(W)\bar{P}(X,Y)Z - B(W)[g(Y,Z)X - g(X,Z)Y].$$

This implies,

$$a(\nabla_W R)(X,Y)Z = a\eta((\nabla_W R)(X,Y)Z)\xi - aA(W)R(X,Y)Z$$
$$+ b[(\nabla_W S)(Y,Z)\eta(X) - (\nabla_W S)(X,Z)\eta(Y)]\xi$$
$$- b[(\nabla_W S)(Y,Z)X - (\nabla_W S)(X,Z)Y]$$
$$- bA(W)[S(Y,Z)X - S(X,Z)Y]$$
$$+ \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(W)[g(Y,Z)X - g(X,Z)Y]$$

(3.12)
$$- B(W)[g(Y,Z)X - g(X,Z)Y].$$

From (3.12) and the Bianchi identity we get

$$aA(W)\eta(R(X,Y)Z) + aA(X)\eta(R(Y,W)Z) + aA(Y)\eta(R(W,X)Z)$$
$$= bA(W)[S(X,Z)\eta(Y) - S(Y,Z)\eta(X)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(W)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)]$$
$$+bA(X)[S(Y,Z)\eta(W) - S(W,Z)\eta(Y)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(X)[g(Y,Z)\eta(W) - g(W,Z)\eta(Y)]$$
$$+bA(Y)[S(W,Z)\eta(X) - S(X,Z)\eta(W)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(Y)[g(W,Z)\eta(X) - g(X,Z)\eta(W)]$$
$$+B(W)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] + B(X)[g(W,Z)\ \eta(Y) - g(Y,Z)\eta(W)]$$

(3.13)
$$+ B(Y)[g(X,Z)\eta(W) - g(W,Z)\eta(X)].$$

By virtue of (2.8), we obtain from (3.13) that

$$aA(W)[k[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] + aA(X)[k[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)]$$
$$+ aA(Y)[k[g(X,Z)\eta(W) - [g(W,Z)\eta(X)]$$
$$= bA(W)[S(X,Z)\eta(Y) - S(Y,Z)\eta(X)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(W)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)]$$
$$+bA(X)[S(Y,Z)\eta(W) - S(W,Z)\eta(Y)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(X)[g(Y,Z)\eta(W) - g(W,Z)\eta(Y)]$$

$$+bA(Y)[S(W,Z)\eta(X) - S(X,Z)\eta(W)] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right]A(Y)[g(W,Z)\eta(X) - g(X,Z)\eta(W)]$$
$$+B(W)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)] + B(X)[g(Y,Z)\ \eta(W) - g(W,Z)\eta(Y)]$$
$$+ B(Y)[g(W,Z)\eta(X) - g(X,Z)\eta(W)]. \quad (3.14)$$

Putting $Y = Z = e_i$ in (3.10)and taking summation over $i,\ 1 \le i \le 2n+1$, we get

$$(a)\ A(W)\eta(X) = A(X)\eta(W)$$
$$(b)\ B(W)\eta(X) = B(X)\eta(W), \quad (3.15)$$

for all vector fields X, W.

Replacing X by ξ in (3.11) we get

$$(a) A(W) = \eta(W)\eta(\rho_1)$$
$$(b) B(W) = \eta(W)\eta(\rho_2), \quad (3.16)$$

for any vector field W, where $A(\xi) = g(\xi, \rho_1) = \eta(\rho_1)$ and $B(\xi) = g(\xi, \rho_2) = \eta(\rho_2)$, $\rho_1$and $\rho_2$being the vector fields associated to the 1-forms A and B.

From (3.15) and (3.16), we can state the following theorem:

***Theorem 3.3**. In a generalized pseudo-projective ϕ-recurrent N(k)-contact metric manifold,the characteristic field ξ and the vector fields ρ_1 and $\rho_2$associated to the 1-forms A and Brespectively are co-directional and the 1-forms A and B are given by (3.16).*

4. 3-dimensional Generalized Pseudo-Projective ϕ-Recurrent $N(k)$ −Contact Metric Manifold

In a 3-dimensional $N(k)$ −contact metric Manifold (M^3, g), we have

$$R(X,Y)Z = \left(\frac{r}{2} - 2k\right)[g(Y,Z)X - g(X,Z)Y] + \left(3k - \frac{r}{2}\right)[g(Y,Z)\eta(X)\xi$$
$$- g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \quad (4.1)$$

$$S(X,Y) = \left(\frac{r}{2} - k\right) g(X,Y) + \left(3k - \frac{r}{2}\right) \eta(X)\eta(Y). \quad (4.2)$$

Using (4.1) and (4.2) in (2.21), we get

$$\bar{P}(X,Y)Z = \left[(a+b)\left(\frac{r}{2} - 2k\right)\right][g(Y,Z)X - g(X,Z)Y] + (a+b)\left(3k - \frac{r}{2}\right)[\eta(Y)\eta(Z)X$$
$$-\eta(X)\eta(Z)Y] + \left(3k - \frac{r}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi]. \quad (4.3)$$

Differentiating the equation (4.3) covariantly, we get

$$(\nabla_W \bar{P})(X,Y)Z = \left[\frac{adr(W)}{2} + \frac{bdr(W)}{2} - \frac{dr(W)}{2n+1}\left(\frac{a}{2n} + b\right)\right][g(Y,Z)X - g(X,Z)Y\ ]$$
$$- \frac{adr(W)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi]$$
$$-\left[\frac{adr\,(W)}{2} + \frac{bdr\,(W)}{2}\right][\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]$$
$$+a\left[3k - \frac{r}{2}\right][g(Y,Z)(\nabla_W\eta)(X)\xi + g(Y,Z)\eta(X)(\nabla_W\xi)$$
$$- g(X,Z)(\nabla_W\eta)(Y)\xi - g(X,Z)\eta(Y)(\nabla_W\xi)]$$

$$+ (a + b)\left[3k - \frac{r}{2}\right][(\nabla_W\eta)(Y)\eta(Z)X + (\nabla_W\eta)(Z)\eta(Y)X$$
$$(4.4) \qquad - (\nabla_W\eta)(X)\eta(Z)Y - (\nabla_W\eta)(Z)\eta(X)Y].$$

Noting that we may assume that all vector fields X, Y, Z, W are orthogonal to ξ and using (2.1), we get

$$(\nabla_W\bar{P})(X,Y)Z = dr(W)\left[\frac{10a}{21} + \frac{5b}{14}\right][g(Y,Z)X) - g(X,Z)Y)]$$
$$(4.5) \qquad +a\left[3k - \frac{r}{2}\right][g(Y,Z)(\nabla_W\eta)(X) - g(X,Z)(\nabla_W\eta)(Y)]\xi.$$

Applying $\phi^2$to the both sides of (4.5) and using (2.1), we get

$$(4.6) \qquad \phi^2(\nabla_W\bar{P})(X,Y)Z = dr(W)\left[\frac{10a}{21} + \frac{5b}{14}\right][g(Y,Z)X - g(X,Z)Y].$$

Using(2.22), the equation (4.6) reduces to,

$$A(W)\bar{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y]$$
$$(4.7) \qquad = dr(W)\left[\frac{10a}{21} + \frac{5b}{14}\right][g(Y,Z)X - g(X,Z)Y].$$

Putting $W = \{ei\}$,where $\{ei\}$, $i = 1,2,3$, is an orthonormal basis of the tangent space at anypoint of the manifold and taking summation over i, $1 \leq i \leq 3$, we obtain

$$(4.8) \ \bar{P}(X,Y)Z = \lambda[g(Y,Z)X - g(X,Z)Y],$$

where $\lambda = \left[\frac{dr(e_i)}{A(e_i)}\left(\frac{10a}{21} + \frac{5b}{14}\right) - \frac{B(e_i)}{A(e_i)}\right]$is a scalar, since A and B are non-zero 1-forms. Thenby Schur's theorem λ will be a constant on the manifold. Therefore, (M^3, g) is of constantcurvature λ. Thus we get the following theorem:

Theorem 4.4. *A 3-dimensional Generalized pseudo-projective ϕ-recurrent N(k)-contact metricmanifold is of constant curvature.*

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