

Fuzzy rw Super- Continuous Mapping

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Abstract- In this paper we extend the concepts of rw super closed sets and rw super continuous mappings in fuzzy topological spaces and obtain several results concerning the preservation of fuzzy g- super closed sets. Furthermore we characterize fuzzy rw super continuous and fuzzy rw- super closed mappings and obtain some of the basic properties and characterization of these mappings.

Index Terms- Fuzzy super closure fuzzy super interior fuzzy super closed set, fuzzy super open set fuzzy g- super closed sets, fuzzy g- super open sets, fuzzy g- super continuous, fuzzy rw super closed, fuzzy rw -super continuous and fuzzy gc-super irresolute mappings.

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [17] in 1965 and fuzzy topology by Chang [3] in 1968, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Thakur and Malviya [14] introduced the concepts of fuzzy g- closed sets, fuzzy g-continuity and fuzzy gc-irresolute mappings in fuzzy topological spaces.

In this paper we introduce the concepts of fuzzy rw -super closed and fuzzy rw -super continuous mappings using fuzzy g- super closed sets. This definition enables us to obtain conditions under which maps and inverse maps preserve fuzzy g- super closed sets. We also characterize fuzzy $T_{1/2}$ -spaces in terms of fuzzy rw super continuous and fuzzy rw -super closed mappings. Finally some of the basic properties of fuzzy rw super continuous and fuzzy a- super closed mappings are investigated.

II. PRELIMINARIES

Let X be a non empty set and $I = [0,1]$. A fuzzy set on X is a mapping from X into I . The null fuzzy set 0 is the mapping from X into I which assumes only the value 0 and whole fuzzy set 1 is a mapping from X onto I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha \in \Lambda\}$ of fuzzy sets of X is defined by to be the mapping $\sup A_\alpha$ (resp. $\inf A_\alpha$). A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for $y = x$ and $x_\beta(y) = 0$ for $y \neq x, \beta \in [0,1]$ and $y \in X$. A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta qA$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x \in X$ such that $A(x) + B(x) > 1$. For any two fuzzy sets A and B of X , $A \leq B$ if and only if $\overline{(A_qB^c)}$ [5]. A family τ of fuzzy sets of X is called a fuzzy topology [1] on X if $0,1$ belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by $cl(A)$) is the intersection of all the fuzzy closed sets of A and the interior of A (denoted by $int(A)$) is the union of all fuzzy open subsets of A .

Definition 2.1 [6]: Let (X, τ) fuzzy topological space and $A \subseteq X$ then

1. Fuzzy Super closure $scl(A) = \{x \in X: cl(U) \cap A \neq \emptyset\}$
2. Fuzzy Super interior $sint(A) = \{x \in X: cl(U) \leq A \neq \emptyset\}$

Definition 2.2[5,6]: A fuzzy set A of a fuzzy topological space (X, τ) is called:

- (a) Fuzzy super closed if $scl(A) \leq A$.
- (b) Fuzzy super open if $1-A$ is fuzzy super closed $scl(A) = A$

Remark 2.1[5,6]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 2.2[5,6]: The intersection of two fuzzy super closed sets in a fuzzy topological space (X, \mathfrak{S}) may not be fuzzy super closed.
For

Definition 2.2[1,5,6,7]: A fuzzy set A of a fuzzy topological space (X, τ) is called:

- (a) fuzzy semi super open if there exists a super open set O such that $O \leq A \leq \text{cl}(O)$.
- (b) fuzzy semi super closed if its complement $1-A$ is fuzzy semi super open.

Remark 2.3[1,5,7]: Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true .

Definition 2.3[7]: A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy w -super closed if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi super open.

Remark 2.4[5,6]: Every fuzzy super closed set is fuzzy w -super closed but its converse may not be true. For,

Definition 2.4[3,7]: A fuzzy sets A of a fuzzy topological spaces (X, \mathfrak{S}) is called fuzzy regular super open if $A = \text{int}(\text{cl}(A))$.

Definition 2.5[3,7]: A fuzzy sets A of a fuzzy topological spaces (X, \mathfrak{S}) is called fuzzy regular super closed if $A = \text{cl}(\text{int}(A))$.

Remark 2.5: Every fuzzy open (resp. fuzzy regular super closed) set is fuzzy regular super open (resp. fuzzy regular super closed) but the converse may not be true [].The family of all fuzzy regular super open (resp. fuzzy regular super closed) sets of a fuzzy topological (X, \mathfrak{S}) will be denoted by $\text{FRO}(X)$ (resp. $\text{FRC}(X)$).

DEFINITION 2.6[3,5,6,7]: A mapping $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$ is said to be fuzzy almost super continuous if $f^{-1}(G) \in \mathfrak{S}$ for each fuzzy set of $G \in \text{FRO}(Y)$.

Remark 2.6[6]: Every fuzzy super continuous mapping is fuzzy almost super continuous but the converse may not be true [7].

Definition 2.7: A fuzzy sets A of a fuzzy topological spaces (X, \mathfrak{S}) is called fuzzy regular semi super open if there exists a fuzzy regular super open set O such that $O \leq A \leq \text{cl}(O)$ [6]

The family of all fuzzy regular semi super open sets of a fuzzy topological (X, τ) will be denoted by $\text{FRSSO}(X)$.

Remark 2.7: Every fuzzy regular super open set is fuzzy regular semi super open but the converse may not be true .

Definition 2.8: A mapping $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$ is said to be fuzzy almost super irresolute if the inverse image of every fuzzy regular semi super open set of Y is fuzzy semi super open in X . [6]

Remark 2.8: Every fuzzy super irresolute mapping is fuzzy almost super irresolute but the converse may not be true [P_6].

Definition 2.9: A fuzzy set A of a topological spaces (X, \mathfrak{S}) is called fuzzy rg - super closed if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular super open in X .

Remark 2.9: Every fuzzy g - super closed set is fuzzy rg - super closed but its converse may not be true.

Definition 2.10: A mapping $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$ is said to be fuzzy rg -super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy rg - super closed set in X .

Remark 2.10: Every fuzzy g -super continuous mapping is fuzzy rg -super continuous but the converse may not be true .

III. FUZZY RW SUPER-CLOSED SETS

In the present section we introduce the concepts of fuzzy rw super-closed sets in fuzzy topology and obtained some of its basic properties.

Definition 3.1: A fuzzy set A of a topological spaces (X, \mathfrak{S}) is called fuzzy rw super-closed if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular semi super open in X .

Remark 3.1: Every fuzzy w - super closed set is fuzzy rw - super closed but its converse may not be true for,

Example 3.1: Let $X = \{a, b\}$ and the fuzzy sets A and U be defined as follows:

$$A(a)=0.7, A(b)=0.8, U(a)=0.7, U(b)=0.6$$

Let $\tau = \{0, U, 1\}$ be a fuzzy topology on X . Then A is fuzzy rw -super closed but not fuzzy w -super closed.

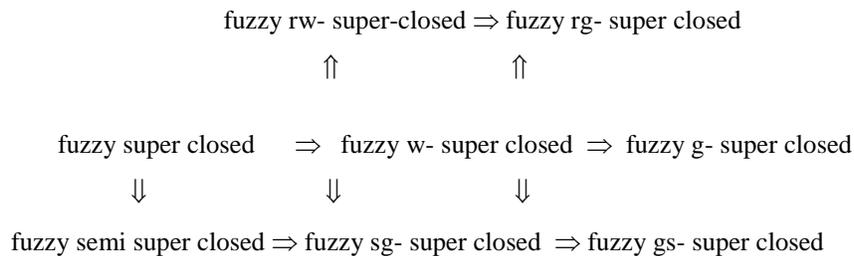
Remark 3.2: Every fuzzy rw -super-closed set is fuzzy rg -super closed but not conversely. For,

Example 3.2: Let $X = \{a, b, c, d\}$ and the fuzzy sets $O, U, V, W,$ and A are defined as follows:

$$O(a) = 1, O(b) = 0, O(c) = 0, O(d) = 0, U(a) = 0, U(b) = 1, U(c) = 0, U(d) = 0$$

$$V(a)=1, V(b) = 1, V(c) = 0, V(d) = 0, W(a) = 0, W(b) = 0, W(c) = 1, W(d) = 1$$

$A(a) = 0, A(b) = 0, A(c) = 1, A(d) = 0$, Let $\tau = \{0, O, U, V, W, 1\}$ be the fuzzy topology on X . then A is rw super-closed but not rg -super closed. Thus we have the following diagram of implications:



Theorem 3.1: Let (X, \mathfrak{T}) be a fuzzy topological spaces and A is fuzzy subset of X . Then A is fuzzy rw super-closed if and only if $\overline{\text{cl}}(A_q F) \Rightarrow \overline{\text{cl}}(\text{cl}(A)_q F)$ for every fuzzy regular semi super closed set F of X .

Proof: Necessity: Let F be a fuzzy regular semi super closed subsets of X and $\overline{\text{cl}}(A_q F)$. Then $A \leq 1-F$ and $1-F$ is fuzzy regular semi super open in X . Therefore $\text{cl}(A) \leq 1-F$ because A is fuzzy rw -super closed. Hence $\overline{\text{cl}}(\text{cl}(A)_q F)$.

Sufficiency: Let $U \in \text{FRSSO}(X)$ such that $A \leq U$ then $\overline{\text{cl}}(A_q(1-U))$ and $1-U$ is fuzzy regular semi super closed in X . Hence by hypothesis $\overline{\text{cl}}(\text{cl}(A)_q(1-U))$. Therefore $\text{cl}(A) \leq U$, Hence A is fuzzy rw super closed in X .

Theorem 3.2: Let A be a fuzzy rw -super closed set in a fuzzy topological space (X, τ) and x_β be a fuzzy point of X such that $x_\beta q(\text{cl}(\text{int}(A)))$ then $\text{cl}(\text{int}(x_\beta))_q A$.

Proof: If $\overline{\text{cl}}(\text{cl}(\text{int}(x_\beta))_q A)$ then $A \leq 1-\text{cl}(\text{int}(x_\beta))$ and so $\text{cl}(A) \leq 1-\text{cl}(\text{int}(x_\beta)) \leq 1-x_\beta$ because A is fuzzy rw super closed in X . Hence $\overline{\text{cl}}(x_\beta q \text{cl}(\text{int}(A)))$, a contradiction.

Theorem 3.3: If A and B are fuzzy rw -super closed sets in a fuzzy topological space (X, τ) then $A \cup B$ is fuzzy rg -super closed.

Proof: Let $U \in \text{FRSO}(X)$ such that $A \cup B \leq U$. Then $A \leq U$ and $B \leq U$, so $\text{cl}(A) \leq U$ and $\text{cl}(B) \leq U$. Therefore $\text{cl}(A) \cup \text{cl}(B) \leq \text{cl}(A \cup B) \leq U$. Hence $A \cup B$ is fuzzy rw -super closed.

Remark 3.3: The intersection of any two fuzzy rw -super closed sets in a fuzzy topological space (X, τ) may not be fuzzy rw -super closed for,

Example 3.3: Let $X = \{a, b, c, d\}$ and the fuzzy sets A and B are defined as follows;

$$A(a)=1, \quad A(b)=1, \quad A(c)=0, \quad A(d)=0,$$

$$B(a)=1, \quad B(b)=0, \quad B(c)=1, \quad B(d)=1,$$

Let $\mathfrak{T} = \{0, A, B, A \cap B, 1\}$ be the fuzzy topology on X . Then A and B are fuzzy rw -super closed but their intersection $A \cap B$ is not fuzzy rw -super closed.

Theorem 3.4: Let $A \leq B \leq \text{cl}(A)$ and A is fuzzy rw -super closed set in a fuzzy topological space (X, τ) then B is fuzzy rw -super closed.

Proof: Let $U \in \text{FRSO}(X)$ such that $B \leq U$. Then $A \leq U$ and since A is fuzzy rw- super closed. Then $\text{cl}(A) \leq U$. Now $B \leq \text{cl}(A) \Rightarrow \text{cl}(B) \leq \text{cl}(A) \leq U$. Consequently B is fuzzy rw -super closed.

Definition 3.2: A fuzzy set A of a fuzzy topological space (X, \mathfrak{T}) is called fuzzy rw -super open) if and only if $1 - A$ is fuzzy rw -super closed.

Remark 3.4: Every fuzzy w- super open set is fuzzy rw- super open .But converse may not be true. For the fuzzy set B defined by $B(a)=0.5$ and $B(b)=0.7$ in the fuzzy topological (X, \mathfrak{T}) of example 6.1.1 is fuzzy rg- super open but not fuzzy regular super open.

Theorem 3.5: A fuzzy set A of a fuzzy rw-super open if and only if $F \leq \text{int}(A)$ whenever $F \leq A$ and F is fuzzy regular semi super open.

Proof: Obvious.

Theorem 3.6: Let A be a fuzzy rw -super open set in a fuzzy topological spaces (X, \mathfrak{T}) and $\text{int}(A) \leq B \leq A$ then B is fuzzy rw- super open.

Proof : Obvious .

Theorem 3.7: Let (X, \mathfrak{T}) be a fuzzy topological space and $\text{FC}(X)$ be the family of all fuzzy super closed sets of X .Then $\text{FRSSO}(X) \subseteq \text{FSC}(X)$ if and only if every fuzzy subset of X is fuzzy rw- super closed.

Proof: Necessity: Suppose that $\text{FRSO}(X) \subseteq \text{FC}(X)$ and that $A \leq U \in \text{FRSSO}(X)$ then $\text{cl}(A) \leq \text{cl}(U) = U$ and A is fuzzy rw -super closed.

Sufficiency: Suppose that every fuzzy subset of X is fuzzy rw- super closed .If $U \in \text{FRSSO}(X)$ then since $U \leq U$ and U is fuzzy rw -super closed, $\text{cl}(U) \leq U$ and $U \in \text{FC}(X)$. Thus $\text{FRSO}(X) \subseteq \text{FC}(X)$.

Theorem 3.8: Let A be a fuzzy w- super closed set in a fuzzy topological space (X, \mathfrak{T}) and $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is a fuzzy almost super irresolute and fuzzy super closed mappings then $f(A)$ is fuzzy rw - super closed in Y .

Proof: If $f(A) \leq G$ where $G \in \text{FRSSO}(Y)$. Then $A \leq f^{-1}(G) \in \text{FSSO}(X)$ and hence $\text{cl}(A) \leq f^{-1}(G)$, because A is a fuzzy w- super closed in X . Since f is fuzzy super closed, $f(\text{cl}(A))$ is a fuzzy super closed set in Y . It follows that $\text{cl}(f(A)) \leq \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \leq G$. Thus $\text{cl}(f(A)) \leq G$ and $f(A)$ is a fuzzy rw –super closed set in Y .

Definition 3.2: A mapping $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is said to be fuzzy regular semi super irresolute if the inverse image of each fuzzy regular semi super open in X .

Theorem 3.9: Let A be the fuzzy rw -super closed set in a fuzzy topological space (X, \mathfrak{T}) and $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is a fuzzy regular semi super irresolute and fuzzy super closed mapping then $f(A)$ is fuzzy rw super closed sets in Y .

Proof: If $f(A) \leq G$ where $G \in \text{FRSO}(Y)$ then $A \leq f^{-1}(G) \in \text{FRSO}(X)$ and hence $\text{cl}(A) \leq f^{-1}(G)$ because A is fuzzy rw -super-closed in X . Since f is fuzzy closed $f(\text{cl}(A))$ is a fuzzy closed set in Y . It follows that $\text{cl}(f(A)) \leq \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \leq G$ thus $\text{cl}(f(A)) \leq G$ and $f(A)$ is fuzzy rw -super closed sets in Y .

Definition 3.3: A collection $\{G_\alpha: \alpha \in \Lambda\}$ of fuzzy rw- super open sets in a fuzzy topological space (X, \mathfrak{T}) is called a fuzzy rw- super open cover of a fuzzy set A of X if $A \leq \bigcup \{G_\alpha: \alpha \in \Lambda\}$.

Definition 3.4: A fuzzy set of a topological space (X, \mathfrak{T}) is said to be fuzzy rw -super compact if every fuzzy rw –super open cover of X has a finite sub cover.

Definition 3.5: A fuzzy topological space (X, \mathfrak{T}) is said to be fuzzy rw- super compact relative to X , if for every collection $\{G_\alpha: \alpha \in \Lambda\}$ of fuzzy rw -super open subsets of X such that $A \leq \bigcup \{G_{\alpha_j}: \alpha_j \in \Lambda_0\}$.

Definition 3.6: A crisp subset of A of a fuzzy topological space (X, \mathfrak{T}) is said to be fuzzy rw –super compact if A is fuzzy rw- super compact as a fuzzy subspace of X .

Theorem 3.10: Fuzzy rw-super closed crisp subsets of a fuzzy rw-super compact space are fuzzy rw-super compact relative to X.

Proof: Let A be a fuzzy rw super-closed crisp set off a fuzzy rw- super compact space (X, \mathfrak{T}) . Then $1 - A$ is fuzzy rw-super open in X. Let $G = \{G_\alpha: \alpha \in \Lambda\}$. Be a cover of A fuzzy rw- super open sets in X. Then the family $\{G, 1-A\}$ is a fuzzy rg- super open in X is fuzzy rw-super compact. it had sub cover $\{G_{\alpha_1} G_{\alpha_2} G_{\alpha_3} \dots G_{\alpha_n}\}$. If the sub cover contain $1-A$, we discard it thus we have obtained a finite fuzzy rg-open sub cover of A is fuzzy rw-super-compact relative to X.

IV. FUZZY RW-SUPER CONTINUOUS MAPPINGS

In the present section we introduce the concept of fuzzy regular super w-Super continuous mappings in fuzzy topology and obtained some of its basic properties.

Definition 4.1: A fuzzy set A of topological spaces (X, \mathfrak{T}) is called fuzzy regular w- super continuous mapping (written as fuzzy rw-super continuous mapping) if the inverse image of every fuzzy closed set of Y is fuzzy rw-super closed in X.

Theorem 4.2: A mapping $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is fuzzy rw-super continuous if and only if the inverse image of every fuzzy super open set of Y is fuzzy rw-super open in X.

Proof: It is obvious because $f^{-1}(1-A) = 1-f^{-1}(A)$ for every fuzzy set A of Y.

Remark 4.2: Every fuzzy w-super continuous mapping is rw-super continuous but its converse may not be true for,

Example 4.1: Let $X = \{a, b\}$ and $Y = \{x, y\}$ the fuzzy sets U and V be defined as follows;

$$U(a)=0.7, \quad U(b)=0.6$$

$$V(x)=0.2, \quad V(y)=0.2$$

Let $\mathfrak{T} = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be a fuzzy topology on X and Y respectively. Then the mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy rw-super continuous mapping but not fuzzy w-super continuous mapping.

Remark 4.2: Every fuzzy rw-super continuous mapping is rg-super continuous but its converse may not be true for,

Example 4.2: Let $X = \{a, b\}$ and $Y = \{x, y\}$ the fuzzy sets U and V be defined as follows;

$$U(a)=0.7, \quad U(b)=0.6$$

$$V(x)=0.2, \quad V(y)=0.2$$

Let $\mathfrak{T} = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be a fuzzy topology on X and Y respectively. Then the mapping $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy rw-super continuous mapping but not fuzzy w-super continuous mapping.

Theorem 4.2: If $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is fuzzy w-super continuous then for each fuzzy point x_β of X and each fuzzy open set B, $f(x_\beta) \in B$ then there exists a fuzzy rw-super open set A such that $x_\beta \in A$ and $f(A) \leq B$.

Proof: Let x_β be fuzzy point of X and B be a fuzzy open set such that $f(x_\beta) \in B$ put $B = f^{-1}(A)$, then by the hypothesis A is a fuzzy rw-super open set of X such that $x_\beta \in A$ and $f(A) = f(f^{-1}(B)) \leq B$.

Theorem 4.3: If $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is fuzzy rw super continuous then for each fuzzy point x_β of X and each fuzzy open set B of Y such that, $f(x_\beta) \in B$ then there exists a fuzzy rw-super open set A such that $x_\beta \in A$ and $f(A) \leq B$.

Proof: Let x_β be fuzzy point of X and B be a fuzzy super open set such that $f(x_\beta) \in B$ put $B = f^{-1}(A)$, then by the hypothesis A is a fuzzy rw-super open set of X such that $x_\beta \in A$ and $f(A) = f(f^{-1}(B)) \leq B$.

Definition 4.2: Let (X, \mathfrak{T}) be a fuzzy topological. The rw-super closure of the fuzzy set A of X denoted by rw-super $cl(A)$ is defined as follows $rw-super cl(A) = \inf\{B: B \geq A, B \text{ is fuzzy rw-super closed set of } X\}$.

Remark 4.3: It is clear that $A \leq rw\text{-super } cl(A) \leq cl(A)$ for any fuzzy set A of X .

Theorem 4.4: If $f: (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy $rw\text{-super}$ continuous then $f(rw\text{-super } cl(A)) \leq cl(f(A))$ for every fuzzy set A of X .

Proof: Let A be a fuzzy set of X . Then $cl(f(A))$ is a fuzzy super closed set of Y . Since f is fuzzy $rw\text{-super}$ continuous $f^{-1}(cl(f(A)))$ is fuzzy $rw\text{-super}$ closed in X . Clearly $A \leq f^{-1}(cl(f(A)))$ therefore, $rw\text{-super } cl(A) \leq rw\text{-super } cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$, hence $f(rw\text{-super } cl(A)) \leq cl(f(A))$.

Remark 4.4: The converse of the theorem 6.2.2 is not true .

Definition 4.3: A fuzzy topological space (X, \mathfrak{S}) is said to be fuzzy $rw\text{-super-}T_{1/2}$ if every fuzzy $rw\text{-super}$ closed set in X is fuzzy super closed in X .

Theorem 4.5: Let f be a mapping from a fuzzy $rw\text{-super-}T_{1/2}$ space (X, \mathfrak{S}) to a fuzzy topological space (Y, σ) then the following condition are equivalent:

- (a) f is fuzzy super continuous.
- (b) f is fuzzy $w\text{-super}$ continuous.
- (c) f is fuzzy $rw\text{-super}$ continuous.

Proof: Obvious.

Theorem 4.6: If $f: (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy $rw\text{-super}$ continuous and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is fuzzy super continuous .Then $g \circ f: (X, \mathfrak{S}) \rightarrow (Z, \eta)$ is fuzzy $rw\text{-super}$ continuous.

Proof: If A is fuzzy closed in Z then $f^{-1}(A)$ is fuzzy closed in Y because g is fuzzy super continuous. Therefore $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy $rw\text{-super}$ closed in X . Hence $g \circ f$ is fuzzy $rw\text{-super}$ continuous.

Theorem 4.7: If $f: (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are two fuzzy $rw\text{-super}$ continuous mapping and (Y, σ) is fuzzy $rw\text{-super-}T_{1/2}$ super continuous. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is fuzzy $rw\text{-super}$ continuous.

Proof: Obvious.

Theorem 4.8: A fuzzy $rw\text{-super}$ continuous image of a fuzzy $rw\text{-super}$ compact space is fuzzy compact.

Proof: Let $f: (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a fuzzy $rw\text{-super}$ continuous mapping from a fuzzy $rw\text{-super}$ compact space (X, τ) on to a fuzzy topological space (Y, σ) . Let $\{A_i; i \in \Lambda\}$ be a fuzzy super open cover of Y , then $\{f^{-1}(A_i); i \in \Lambda\}$ is a fuzzy $rw\text{-super}$ open cover of X . Since X is fuzzy $rw\text{-super}$ compact it has a finite sub cover, say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3) \dots f^{-1}(A_n)\}$ since f is onto $\{A_1, A_2, \dots, A_n\}$ is a fuzzy super open cover of Y so (Y, σ) is fuzzy compact.

Definition 4.4: A fuzzy topological space (X, \mathfrak{S}) is fuzzy $rw\text{-super}$ connected if there is no proper fuzzy set of X which is both fuzzy $rw\text{-super}$ open and fuzzy $rw\text{-super}$ closed.

Remark 4.5: Every fuzzy $rw\text{-super}$ connected space is fuzzy super connected but he converse may not be true. For;

Example 4.4: Let $X = \{a, b\}$ and A be defined as follows, $A(a) = 0.5, A(b) = 0.7$. Let $\mathfrak{S} = \{0, A, 1\}$ be topology on X , then (X, \mathfrak{S}) is fuzzy super connected but not fuzzy $rw\text{-super}$ connected.

Theorem 4.10: If (X, \mathfrak{S}) fuzzy $rw\text{-super-}T_{1/2}$ connected if and only if it is fuzzy $rw\text{-super}$ connected.

Proof: Obvious.

Theorem 4.10: If $f: (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ fuzzy $rw\text{-super}$ continuous surjection and X is fuzzy $rw\text{-super}$ connected then Y is fuzzy super connected.

Proof: Suppose Y is not fuzzy super connected .Then there exists a proper fuzzy set A of Y which is both fuzzy super open and fuzzy super closed, therefore $f^{-1}(A)$ is proper fuzzy set of X , which is both fuzzy $rw\text{-super}$ open and fuzzy $rw\text{-super}$ closed, because f is fuzzy $rw\text{-super}$ continuous surjection. Hence, X is not fuzzy $rw\text{-super}$ connected, which is a contradiction

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