Isomorphism on Intuitionistic Fuzzy Directed Hypergraphs

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Abstract- Directed hypergraphs are much like standard directed graphs. In intuitionistic fuzzy directed hypergraphs, like directed graphs, standard arcs connect a single tail node to a single head node, hyperarcs connect a set of tail nodes to a set of head nodes. In this paper, the isomorphism between two intuitionistic fuzzy directed hypergraphs is discussed. The condition for two intuitionistic fuzzy directed hypergraphs is isomorphic also discussed and some of its properties are also analyzed.

***Index Terms*-** Intuitionistic fuzzy hypergraph(IFHG), intuitionistic fuzzy directed hypergraph, isomorphism, weak isomorphism, co-weak isomorphism

I. INTRODUCTION

There are several ways of introducing the notion of direction for the edges of a hypergraph. For example, in [1] a directed hypergraph is obtained from a hypergraph H, by partitioning every edge of H into two sets of vertices, namely the tail and the head of the edge. This concept has been extended to intuitionistic fuzzy hypergraphs.

The authors already introduced the concept of intuitionistic fuzzy directed hypergraphs[11]. Generally isomorphism between two graphs is proved to be an equivalence relation. Kalaivani et al., [7] discussed the properties of isomorphism on intuitionistic fuzzy hypergraphs(IFGs) and strong IFGs. Radhamani et al., introduced the concept of isomorphism on fuzzy hypergraphs and their properties[8]. In this paper, an attempt has been made to derive the isomorphism between two intuitionistic fuzzy directed hypergraphs.

II. PRELIMINARIES

In this section, some basic definitions relating to index matrix representation of intuitionistic fuzzy graphs (IMIFG)and intuitionistic fuzzy hypergraphs are given. In [9], Prof. K.T. Atanassov defined cartesian products of intuitionistic fuzzy sets (IFSs) on different universes. Here, the authors have defined six cartesian products of two IFSs over the same universe.

Definition 2.1 [5] Let a set E be fixed. An intuitionistic fuzzy set (IFS) A in E is an object of the form $A=\{(x,\ \mu_A(x),\ \gamma_A(x))/x\in E\}$ where the function $\mu_A: E\rightarrow[0,1]$ and $\gamma_A: E\rightarrow[0,1]$ determine the degree of membership and the degree of nonmembership of the element $X\in E$, respectively and for every $X\in E,\ 0\le\mu_A(x)+\gamma_A(x)\le 1.$

Definition 2.2 Let X be an universal set and let V be an IFS over X in the form $V=\left\{\left\langle v_i,\mu_i\left(v_i\right),\gamma_i\left(v_i\right)\right\rangle \middle| v_i\in V\right\}$, such that $0\le\mu_i(v_i)+\gamma_i(v_i)\le 1$. Six types of Cartesian products of n elements of V over X are defined as

$$v_1\times_1 v_2\times_1 v_3 ...\times_1 v_n=\left\{\left\langle\left\langle v_1,v_2,...,v_n\right\rangle,\prod_{i=1}^{n}\mu_i,\prod_{i=1}^{n}\gamma_i\right\rangle \middle| \left\langle v_1,v_2,...,v_n\right\rangle\in V\right\},$$

$$v_{i_1}\times_2 v_{i_2}\times_2 v_{i_3}...\times_2 v_{i_n}=\left\{\left\langle\left\langle v_1,v_2,...,v_n\right\rangle,\sum_{i=1}^{n}\mu_i-\sum_{i\neq j}\mu_i\mu_j+\sum_{i\neq j\neq k}\mu_i\mu_j\mu_k-...+\left(-1\right)^{n-2}\sum_{i\neq j\neq k\neq n}\mu_i\mu_j\mu_k...\mu_n+\left(-1\right)^{n-1}\prod_{i=1}^{n}\mu_i,\prod_{i=1}^{n}\gamma_i\right\rangle \middle| \left\langle v_1,v_2,...,v_n\right\rangle\in V\right\}$$

$$v_{i_1}\times_3 v_{i_2}\times_3 v_{i_3}...\times_3 v_{i_n}=\left\{\left\langle\left\langle v_1,v_2,...,v_n\right\rangle,\prod_{i=1}^{n}\mu_i,\sum_{i=1}^{n}\gamma_i-\sum_{i\neq j}\gamma_i\gamma_j+\sum_{i\neq j\neq k}\gamma_i\gamma_j\gamma_k-...+\left(-1\right)^{n-2}\sum_{i\neq j\neq k\neq n}\gamma_i\gamma_j\gamma_k...\gamma_n+\left(-1\right)^{n-1}\prod_{i=1}^{n}\gamma_i\right\rangle \middle| \left\langle v_1,v_2,...,v_n\right\rangle\in V\right\}$$

$$v_1\times_4 v_2\times_4 v_3...\times_4 v_n=\left\{\left\langle\left\langle v_1,v_2,...,v_n\right\rangle,\min\left(\mu_1,\mu_2,...,\mu_n\right),\max\left(\gamma_1,\gamma_2,...,\gamma_n\right)\right\rangle \middle| \left\langle v_1,v_2,...,v_n\right\rangle\in V\right\}$$

$$v_1\times_5 v_2\times_5 v_3...\times_5 v_n=\left\{\left\langle\left\langle v_1,v_2,...,v_n\right\rangle,\max\left(\mu_1,\mu_2,...,\mu_n\right),\min\left(\gamma_1,\gamma_2,...,\gamma_n\right)\right\rangle \middle| \left\langle v_1,v_2,...,v_n\right\rangle\in V\right\}$$

$$v_1 \times_6 v_2 \times_6 v_3 \ldots \times_6 v_n = \left\{ \left\langle \langle v_1, v_2, \ldots, v_n \rangle, \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \gamma_i}{n} \right\rangle \middle| \langle v_1, v_2, \ldots, v_n \rangle \in V \right\}$$

It must be noted that $v_i \times_t v_j$ is an IFS, where, $t = 1,2,3,4,5,6$

Definition 2.3 An intuitionistic fuzzy hypergraph *H* is an ordered pair *H*=$\langle V,\ E \rangle$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$, a finite set of vertices

(ii) E = $\{E_1, E_2, \ldots, E_n\}$, a family of intuitionistic fuzzy subsets of *V*

(iii) $(i) E_j = \{(v_i, \mu_j(v_i), \gamma_j(v_i)) : \mu_j(v_i), \gamma_j(v_i) \geq 0$ and $0 \leq \mu_j(v_i) + \gamma_j(v_i) \leq 1\}$, *j* = 1,2,…,*m*

$(ii) E_j \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $\gamma_{ij} : V \times V \to [0,1]$ are such that

$\mu_{ij} \leq \mu_i \square \mu_j$

$\gamma_{ij} \leq \gamma_i \square \gamma_j$

And $0 \leq \mu_{ij} + \gamma_{ij} \leq 1$

Where μ_{ij} and γ_{ij} are the membership and nonmembership values of the edge (v_i, v_j); the values of $\mu_i \square \mu_j$ and $\gamma_i \square \gamma_j$ can be determined by one of the Cartesian products $\times_t$, $t = 1,2,3,4,5,6$ for all i and j given in definition 2.2.

(iv) $E_j \neq \phi$, j = 1,2,…,*m*

(v) $\bigcup_j \operatorname{supp}(E_j) = V$, *j* = 1,2,…,*m*

Here the edges E_j are IFSs. $\mu_j(x_i)$ and $\gamma_j(x_i)$ denote the degree of membership and non-membership of the vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(a_{ij}, \mu_j(v_i), \gamma_j(v_i))$. The sets *(V,E)* are crisp sets.

Notations

1. Hereafter, $\langle \mu(v_i), \gamma(v_i) \rangle$ or simply $\langle \mu_i, \gamma_i \rangle$ denotes the degrees of membership and nonmembership of the vertex $v_i \in V$, such that $0 \leq \mu_i + \gamma_i \leq 1$
2. $\langle \mu(v_{ij}), \gamma(v_{ij}) \rangle$ or simply $\langle \mu_{ij}, \gamma_{ij} \rangle$ denotes the degrees of membership and nonmembership of the edge $(v_i, v_j) \in V \times V$, such that $0 \leq \mu_{ij} + \gamma_{ij} \leq 1$

Note

If $\mu_{ij} = \gamma_{ij} = 0$, for some i and j, then there is no edge between v_i and v_j, it is indexed by $\langle 0,1 \rangle$. Otherwise there exist an edge between v_i and v_j.

III. INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS

In this section, isomorphism, between two intuitionistic fuzzy directed hypergraphs has been disscussed.

Definition 3.1 An intuitionistic fuzzy directed hypergraph(IFDHG) H is a pair $\langle V, E \rangle$ where V is a non empty set of vertices and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $e \in E$ is defined as a pair $(T(e), h(e))$, where $T(e) \subset V$, with $T(e) \neq \varnothing$, is its tail, and $h(e) \in N - T(e)$ is its head. A vertex s is said to be a source vertex in H if $h(e) \neq s$, for every $e \in E$. A vertex d is said to be a destination vertex in H if $d \neq T(e)$, for every $e \in E$.

Definition 3.2 Let $E = (E^-, E^+)$ be a hyperarc in an IFDHG. Then the vertex sets E^- and E^+ are called the in-set and the out-set of the hyperarc E, respectively. The sets E^- and E^+ need not be disjoint. The hyperarc E is said to be join of the vertices of E^- and the vertices of E^+.

Furthermore, the vertices of E^- are incident to the hyperarc E and the vertices of E^+ are incident from E. The vertices of E^- are adjacent to the vertices of E^+, and the vertices of E^+ are adjacent from the vertices of E^-.

Definition 3.3 The order of an IFDHG $H=\langle V, E\rangle$ is defined to be $O(H)=(O_\mu(H),O_\gamma(H))$ where $O_\mu(H)=\sum_{v_i\in V}\mu_i(v_i)$ $O_\gamma(H)=\sum_{v_i\in V}\gamma_i(v_i)$

Definition 3.4 The size of an IFDHG is defined to be $S(H)=(S_\mu(H),S_\gamma(H))$ where

$S_\mu(H)=\sum_{v_i,v_j\in V}\mu_{ij}(v_i,v_j)$ $S_\gamma(H)=\sum_{v_i,v_j\in V}\gamma_{ij}(v_i,v_j)$

Definition 3.5 The in-degree of v is the number of hyperarcs that contain v in their out-set, and is denoted $d_H^-(v)$. Similarly , the out-degree of v is the number of hyperarcs that contain v in their in-set,and is denoted by $d_H^+(v)$.

Definition 3.6 Consider the two IFDHGs $G=\langle V, E\rangle$ and $G'=\langle V', E'\rangle$.An isomorphism between two IFDHGs G and G' , denoted by $G\cong G'$, is a bijective map $I:V\to V'$ which satisfies

(i) $\mu_i(v_i)=\mu_i'(I(v_i));\gamma_i(v_i)=\gamma_i'(I(v_i))$ for every $v_i\in V$

(ii) $\mu_{ij}(v_i,v_j)=\mu_{ij}'(I(v_i),I(v_j));\gamma_{ij}(v_i,v_j)=\gamma_{ij}'(I(v_i),I(v_j))$ for every $v_i,v_j\in V$

Definition 3.7 A homomorphism between two IFDHGs $G=\langle V, E\rangle$ and $G'=\langle V', E'\rangle$, is defined as $H:V\to V'$ is a map which satisfies

(i) $\mu_i(v_i)\le\mu_i'(H(v_i));\gamma_i(v_i)\ge\gamma_i'(H(v_i))$ for every $v_i\in V$

(ii) $\mu_{ij}(v_i,v_j)\le\mu_{ij}'(H(v_i),H(v_j));\gamma_{ij}(v_i,v_j)\ge\gamma_{ij}'(H(v_i),H(v_j))$ for every $v_i,v_j\in V$

Definition 3.8 A weak isomorphism between two IFDHGs $G=\langle V, E\rangle$ and $G'=\langle V', E'\rangle$ is defined as $I:V\to V'$ is a bijective homomorphism that satisfies

$$\mu_i(v_i)=\mu_i'(I(v_i));\gamma_i(v_i)=\gamma_i'(I(v_i)) \text{ for every } v_i\in V$$

Definition 3.9 A co-weak isomorphism between two IFDHGs $G=\langle V, E\rangle$ and $G'=\langle V', E'\rangle$ is defined as $I:V\to V'$ is a bijective homomorphism that satisfies

$$\mu_{ij}(v_i,v_j)=\mu_{ij}'(I(v_i),I(v_j)) \textit{ and } \gamma_{ij}(v_i,v_j)=\gamma_{ij}'(I(v_i),I(v_j)) \textit{ for every } v_i,v_j\in V$$

IV. SOME PROPERTIES OF ISOMORPHISM ON INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS

Theorem 4.1 For any two isomorphic IFDHG their order and size are same.

Proof . If $I:G\to G'$ is an isomorphism between the IFDHGs G and G' with the underlying sets V and V' respectively, then $\mu_i(v_i)=\mu_i'(I(v_i));\gamma_i(v_i)=\gamma_i'(I(v_i))$ for every $v_i\in V$ and $\mu_{ij}(v_i,v_j)=\mu_{ij}'(I(v_i),I(v_j));\gamma_{ij}(v_i,v_j)=\gamma_{ij}'(I(v_i),I(v_j))$ for every $v_i,v_j\in V$

We know that

$$O_\mu(G)=\sum_{v_i\in V}\mu_i(v_i)=\sum_{v_i\in V}\mu_i'(I(v_i))=O_\mu(G')$$

$$O_\gamma(G)=\sum_{v_i\in V}\gamma_i(v_i)=\sum_{v_i\in V}\gamma_i'(I(v_i))=O_\gamma(G')$$

$$S_\mu(H)=\sum_{v_i,v_j\in V}\mu_{ij}(v_i,v_j)=\sum_{v_i,v_j\in V}\mu_{ij}'(h(v_i),h(v_j))=S_\mu'(H)$$

$$S_\gamma(H)=\sum_{v_i,v_j\in V}\gamma_{ij}(v_i,v_j)=\sum_{v_i,v_j\in V}\gamma_{ij}'(h(v_i),h(v_j))=S_\gamma'(H)$$

Corollary 4.2 Converse of the above theorem need not be true.

Remark 4.3 If the IFDHGs are weak isomorphic then their order are same. But the IFDHGs of same order need not be weak isomorphic. The following example illustrates this.

Example 4.4 Let $G=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ and $G'=\{v_1',v_2',v_3',v_4',v_5',v_6'\}$ be two IFDHGs as given in Figure 1 and Figure 2 respectively.

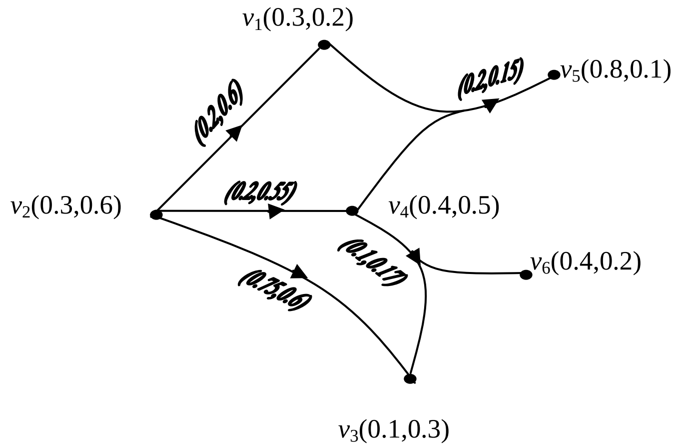

Figure 1: G

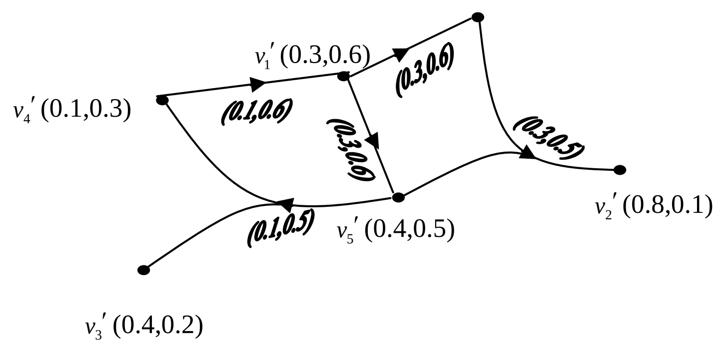

Figure 2: G'

Remark 4.5 If the IFDHGs are co-weak isomorphic, then their sizes are same. But the IFDHGs of same size need not be co-weak isomorphic.

Theorem 4.6 If G and G' are isomorphic IFDHGs then the degrees of their vertices are preserved.
Proof. Let $I:V \rightarrow V'$ be an isomorphism of G and G'. By definition 3.6, we have
$\mu_{ij}\left(v_i,v_j\right)=\mu_{ij}'\left(I\left(v_i\right),I\left(v_j\right)\right);\gamma_{ij}\left(v_i,v_j\right)=\gamma_{ij}'\left(I\left(v_i\right),I\left(v_j\right)\right)$ for every $v_i,v_j \in V$

$$d_\mu^-\left(v_i\right)=\left|E_\mu^+\right|=d_\mu^{-\prime}I\left(v_i\right) \qquad d_\gamma^-\left(v_i\right)=\left|E_\gamma^+\right|=d_\gamma^{-\prime}I\left(v_i\right)$$

$$d_\mu^+\left(v_i\right)=\left|E_\mu^-\right|=d_\mu^{+\prime}I\left(v_i\right) \qquad d_\gamma^+\left(v_i\right)=\left|E_\gamma^-\right|=d_\gamma^{+\prime}I\left(v_i\right)$$

Example 4.7 Consider the two IFDHGs G and G' which preserve the degree of vertices but G and G' are not isomorphic.

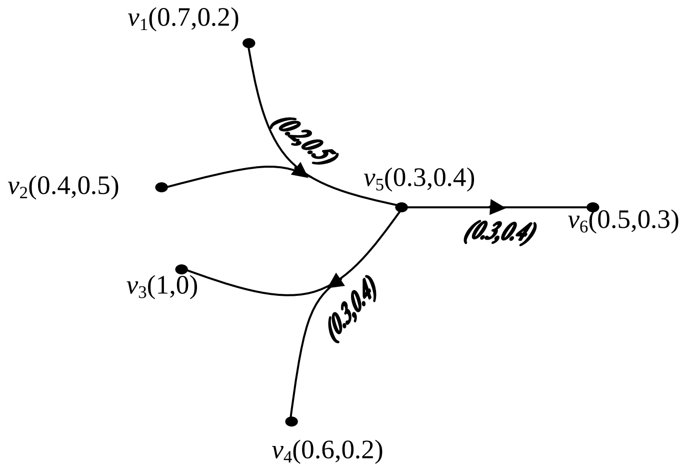

Figure 3: G

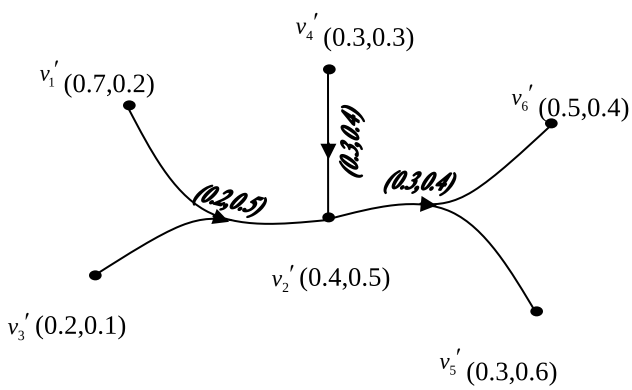

Figure 4: G'

V. ISOMORPHISM ON INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS USING INDEX MATRIX

In this section, the index matrix representation of two isomorphisc intuitionistic fuzzy directed hypergraphs is discussed. To check isomorphism between two intuitionistic fuzzy directed hypergraphs, it is necessary to check whether (i) they have the same number of vertices, (ii) they have the same number of hyperarcs and (iii) they have the same number of vertices with the same degrees.

Consider the two IFDHGs $G_1 = \{v_1, v_2, v_3, v_4, v_5\}$ and $G_2 = \{u_1, u_2, u_3, u_4, u_5\}$ given in Figure 5 and Figure 6 as follows.

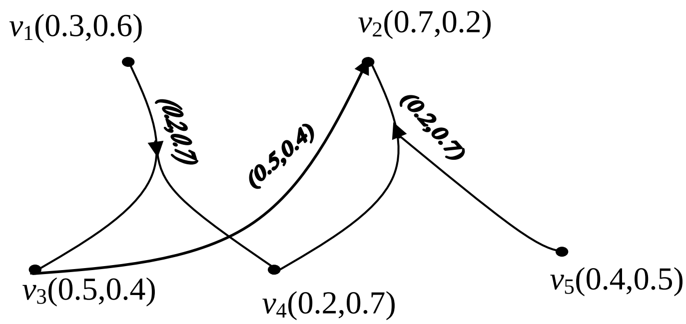

Figure 5: G_1

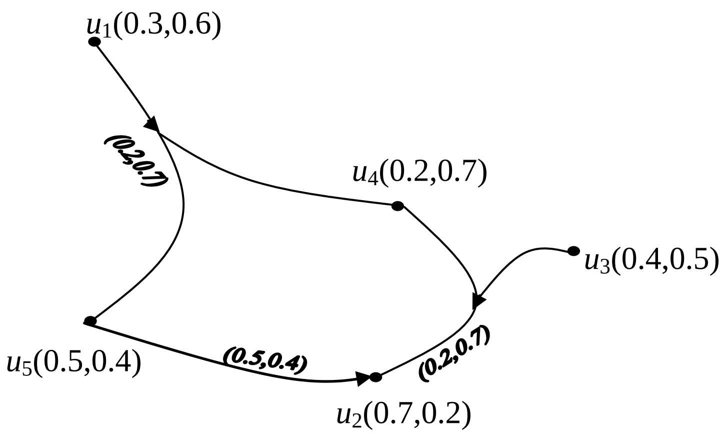

Figure 6: G_2

The Index matrix of G_1 is $G_1 = [V_1, \mu_{ij}, \gamma_{ij}]$ where $V_1 = \{v_1, v_2, v_3, v_4, v_5\}$ and

$\langle \mu_{ij}, \gamma_{ij} \rangle \equiv$

	v_1	v_2	v_3	v_4	v_5
v_1	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0.2, 0.7 \rangle$
v_2	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_3	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_4	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_5	$\langle 0,1 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$

The Index matrix of G_2 is $G_2 = [V_2, \mu_{ij}, \gamma_{ij}]$ where $V_2 = \{u_1, u_2, u_3, u_4, u_5\}$ and

$\langle \mu_{ij}, \gamma_{ij} \rangle \equiv$

	u_1	u_2	u_3	u_4	u_5
u_1	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0,1 \rangle$
u_2	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
u_3	$\langle 0,1 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
u_4	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
u_5	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$

Therefore, the degrees of vertices are calculated and displayed as follows:

$$
\begin{aligned}
&d_H^-(u_1)=0,\ d_H^+(u_1)=1; \quad d_H(u_1)=(0,1)\\
&d_H^-(u_2)=2,\ d_H^+(u_2)=0; \quad d_H(u_2)=(2,0)\\
&d_H^-(u_3)=0,\ d_H^+(u_3)=1; \quad d_H(u_3)=(0,1)\\
&d_H^-(u_4)=1,\ d_H^+(u_4)=1; \quad d_H(u_4)=(1,1)\\
&d_H^-(u_5)=1,\ d_H^+(u_5)=1; \quad d_H(u_5)=(1,1)\\
&d_H^-(v_1)=0,\ d_H^+(v_1)=1; \quad d_H(v_1)=(0,1)\\
&d_H^-(v_2)=2,\ d_H^+(v_2)=0; \quad d_H(v_2)=(2,0)\\
&d_H^-(v_3)=1,\ d_H^+(v_3)=1; \quad d_H(v_3)=(1,1)\\
&d_H^-(v_4)=1,\ d_H^+(v_4)=1; \quad d_H(v_4)=(1,1)\\
&d_H^-(v_5)=0,\ d_H^+(v_5)=1; \quad d_H(v_5)=(0,1)
\end{aligned}
$$

$d_H(u_1)=d_H(u_3)=d_H(v_1)=d_H(v_5)$, we must have either
(i) $f(u_1)=v_1$ and $f(u_3)=v_5$ or (ii) $f(u_1)=v_5$ and $f(u_3)=v_1$. Perhaps either will work.
$d_H(u_2)=d_H(v_2)$. So $f(u_2)=v_2$
Finally, since $d_H(u_4)=d_H(u_5)=d_H(v_3)=d_H(v_4)$, we must have either
(i) $f(u_4)=v_3$ and $f(u_5)=v_4$ or (ii) $f(u_4)=v_4$ and $f(u_5)=v_3$
The relabeling is done by using (i) in each of the above cases to get the map $1\to1;\ 3\to5;\ 2\to2;\ 4\to3;\ 5\to4$ permute the rows and coloumns of the index matrix of G_1 using this map to see if we get the index matrix of G_2. Otherwise, change the labels of the graph G_2 to produce the graph G_2^*
according to the above permutation and recalculate the index matrix. Therefore, the new index matrix of G_2^* (after labeling of G_2) becomes

$\langle \mu_{ij},\gamma_{ij}\rangle \equiv$

	u_1	u_2	u_3	u_4	u_5
u_1	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.7\rangle$	$\langle 0,1\rangle$
u_2	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
u_3	$\langle 0,1\rangle$	$\langle 0.2,0.7\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
u_4	$\langle 0,1\rangle$	$\langle 0.2,0.7\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
u_5	$\langle 0,1\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$

Which is same as G_1. Hence, $G_1 \cong G_2$

VI. CONCLUSION

A graph isomorphism search is an important problem of graph theory. It provides a bijective correspondence which preserve adjacent relation between vertex sets of two graphs. In this paper, an attempt has been made to define an isomorphism between two intuitionistic fuzzy hypergraphs. For, six types of cartesian products of two IFSs over the same universe are defined. Also intuitionistic fuzzy directed hypergraph is defined using cartesian products, in addition to the index matrix representation of isomorphism between two IFDHGs.

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